Year End Review Integral Calculus 30

1.

The table below gives values for the velocity and acceleration of a particle moving along the x-axis for selected values of time t. Both velocity and acceleration are differentiable functions of time t. The velocity is decreasing for all values of t, $0 \le t \le 10$. Use the data in the table to answer the questions that follow.

Time, t	0	2	6	10
Velocity, $v(t)$	5	3	-1	-8
Acceleration, $a(t)$	0	-1	-3	-5

******Assume time is in seconds, velocity in m/s and acceleration is in m/s^2.**

a) Find an approximation for the acceleration at $t = 4$.

b) Find the average acceleration from $t = 2$ to $t = 10$.

c) Given $\frac{1}{6}\int_0^6 v(t)dt$. Using correct units, explain the meaning of this integral in the context of the problem.

d) Using correct units, explain the meaning of $\int_0^{10} v(t)dt$.

e) If the initial position of the particle is $s(0) = 5$. Use a right hand Riemann Sum with the intervals provided in the table to find the position of the particle at $t = 10$.

f) Is the speed of the particle increasing or decreasing at $t = 2$. Justify your answer.

2. The position of a particle is given by $x(t) = \cos(3t) - \sin(4t)$. Find the acceleration at $t = 0$.

3. If the position of an ant traveling along the x-axis at time t is $x(t) = 3t^2 + 1$, what is the ant's average velocity from $t = 1$ to $t = 6$?

4. The position of a particle traveling along a straight line is given by $x(t) = t^3 - 9t^2 + 15t + 3$. On the interval $t = 0$ to $t = 10$, when is the particle furthest to the left?

5.

t (hours)	v (miles per hour)
0	0
0.25	10.3
0.5	13.1
0.75	12.8
1	16.2
1.25	20.1
1.5	20.2
1.75	14.3
2	9.6

The table represents data collected in an experiment on a new type of electric engine for a small neighborhood vehicle (*i.e.*, one that is licensed for travel on roads with speed limits of 35 mph or less).

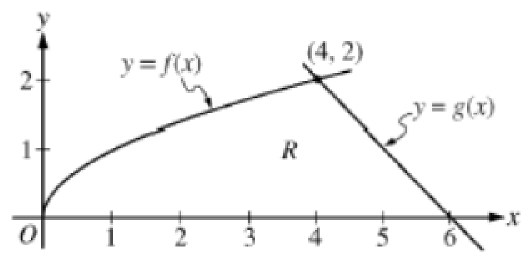
The readings represent velocity, in miles per hour, taken in 15-minute intervals on a 2 hour trip.

a) What is the average acceleration over [0.25, 0.75]?

b) Use a midpoint Riemann sum with four sub intervals to approximate $\int_0^2 v(t)dt$.

c) At the end of two hours the vehicle is 35 miles from a source for recharging the battery. Assuming the car can travel 75 miles on a single charge, can the vehicle get back to the source without being towed or pushed? Justify your answer.

6. The functions $f(x) = \sqrt{x}$ and $g(x) = 6 - x$ are shown in the diagram. Let region R be the region bound by the curves and the x-axis.



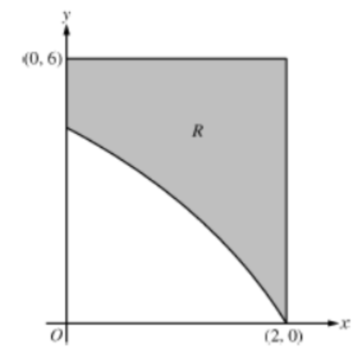
a) Find the area of region R.

b) Find the volume when region R is revolved around the line $x = -1$.

c) Let region R be the region bounded by $f(x)$, the x-axis and the vertical line $x = 4$. Find the volume of the solid if region R is revolved about the x-axis.

d) The region R is the base of a solid. Cross sections taken perpendicular to the y-axis are a rectangles whose height is $2y$. Find the volume of the solid.

7. In the figure proivded, R is the shade region in ther first quadrant bounded by the graph of $y = 4\ln(3 - x)$, the horizontal line $y = 6$, and the vertical line $x = 2$.

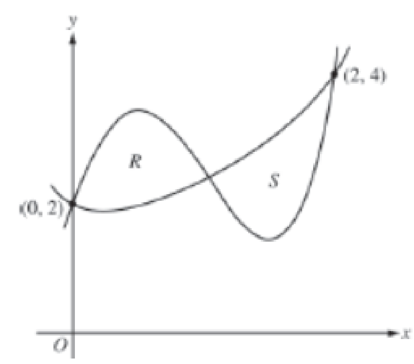


a) Find the area of region R.

b) Find the volume of the solid generated when region R is revolved about the line $y = 8$.

c) The region R is the base of a solid. If each cross section perpendicular to the x-axis is a square, find the volume of the solid.

8. Let f and g be the functions defined by $f(x) = 1 + x + e^{x^2-2x}$ and $g(x) = x^4 - 6.5x^2 + 6x + 2$. Let R and S be the two regions enclose by the graphs of f and g shown in the figure.



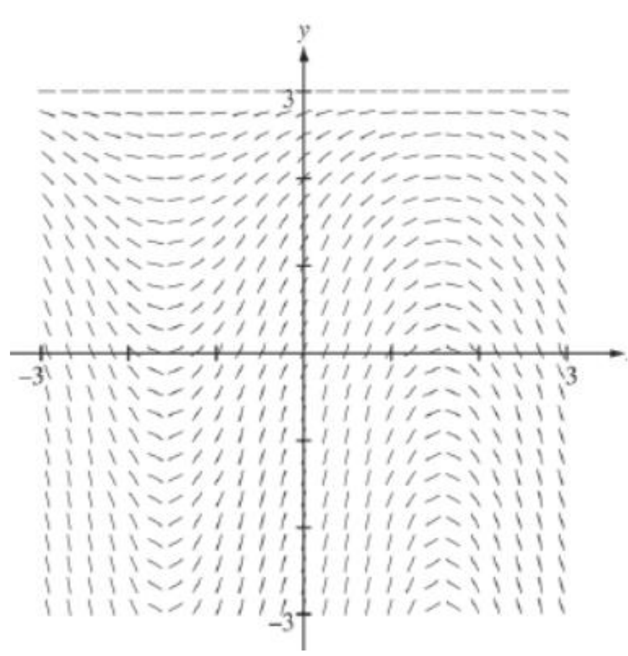
a) Find the area of region S

b) Region R is the base of a solid whose cross sections perpendicular to the x-axis are semi-circles. Find the volume of the solid.

c) Let m be the vertical distance between graphs f and g in region S. Find the rate at which m changes with respect to x when $x = 1$.

9. Consider the differential equation $\frac{dy}{dx} = (3 - y)cosx$. Let $y = f(x)$ be a partricular solution to the differential equation with the initial condition $f(0) = 1$.

a) A portion of the slope of the differential equation is given below. Sketch the solution curve through $(0,1)$.

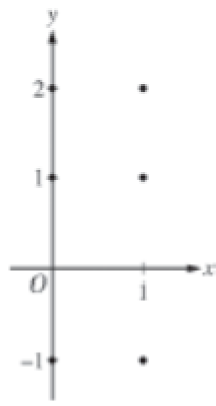


b) Write the equation of the tangent line to the solution curve in part a) at the point $(0,1)$. Use this equation to apporximate $f(0.2)$.

c) Find $y = f(x)$, the particular solution to the differential equation with the initial condition $f(0) = 1$.

10. Consider the differential equation $\frac{dy}{dx} = 2x - y$.

a) On the axes provided, sketch the slope field for the given differential equation at the six points indicated.



b) Find $\frac{d^2y}{dx^2}$ in terms of x and y. Determine the concavity of all solution curves for the given differential equation in Quadrant II. Given a reason for your answer.

11. Let $y = f(x)$ be a particular solution to the differential equation $\frac{dy}{dx} = xy^3$ with $f(1) = 2$.

a) Write an equation for the line tangent to the graph of $y = f(x)$ at $x = 1$.

b) Use the tangent line approximation to approximate $f(1.1)$.

c) Find the particular solution $y = f(x)$ with the initial condition $f(1) = 2$.

12. Find the particular solution $y = f(x)$ to the differential equation $\frac{dy}{dx} = \frac{3x^2}{e^{2y}}$, at the initial condition $y(1) = 0$.

13. Let $f(x) = x^3 + x$. If g is the inverse of f, then find $g'(2)$. **You may have to use some deductive reasoning to find $g(2)$.**

14. If g is the inverse of f, and $g(-4) = 5$ and $f'(5) = -\frac{1}{2}$, then find $g'(-4)$

15. Find the derivative of the following:

a) $f(x) = \csc(3x^4 - 7x)$

b) $f(x) = x^4\sec(5x^3)$

c) $f(x) = \cot(\ln(3x + 2))$

d) $f(x) = sin^{-1}(4x^2 - 9x)$

e) $f(x) = cos^{-1}(3e^{2x})$

f) $f(x) = sec^{-1}(6x^2)$

g) $f(x) = 2x^3cot^{-1}(3x - 7)$

h) $f(x) = tan^{-1}(5x^7 - 3x)$

16. Evaluate the following limits:

a) $\lim_{x \to 0} \frac{sinx}{x}$

b) $\lim_{t \to 1} \frac{5t^4 - 4t^2 - 1}{10 - t - 9t^3}$

c) $\lim_{w \to -4} \frac{sin(\pi w)}{w^2 - 16}$

d) $\lim_{t \to \infty} \frac{\ln(3t)}{t^2}$

17. Evaluate the following integrals:

a) $\int (3x + 5)cos\left(\frac{x}{4}\right)dx$

b) $\int w^2 sin(10w)dw$

c) $\int (3x + x^2)sin(2x)dx$

d) $\int e^x cosxdx$

e) $\int \frac{3x+11}{x^2 - x - 6}dx$

f) $\int \frac{8}{3x^3 + 7x^2 + 4x}dx$

g) $\int \frac{x^4 - 5x^3 + 6x^2 - 18}{x^3 - 3x^2}dx$

h) $\int \frac{4x - 11}{x^3 - 9x^2}dx$

i) $\int sin^3xcos^2xdx$

j) $\int tan^2x\, sec^4xdx$

k) $\int 3cos^2 5xdx$

l) $\int tan^5x\, sec^3xdx$

m) $\int \frac{1}{\sqrt{9 + x^2}}dx$

n) $\int x^3\sqrt{9 - x^2}\, dx$

o) $\int \frac{\sqrt{25x^2 - 4}}{x}dx$

p) $\int \sqrt{6x - x^2 - 5}dx$

18. The rate at which oil flows through an underground pipeline is provided in the chart below. The rate of flow is recorded in $\frac{gallons}{min}$ and the time is measured in $minutes$.

Rate	12	9	8	17	14	13	10	9	7
Time	0	5	10	15	20	25	30	35	40

a) Use a midpoint Riemann sum with 4 equal subintervals to approximate $\int_0^{40} R(t)dt$.

b) Use a left end Riemann sum with 8 equal subintervals to approximate $\int_0^{40} R(t)dt$.

c) Use a trapezoidal approximation to approximate $\int_0^{40} R(t)dt$.

d) Using correct units, explain the meaning of $\int_0^{40} R(t)dt$.

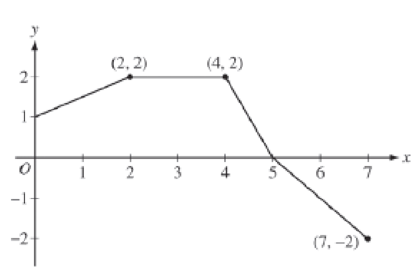
19. Given the graph of $f(x)$ below, find the following:

a) $\int_0^4 f(x)dx$

b) $\int_5^2 f(x)dx$

c) $\int_0^7 f(x)dx$

d) $\int_0^2 4f(x)dx - \int_4^5 \frac{1}{2}f(x)dx$



20. Find the derivative of the following integrals.

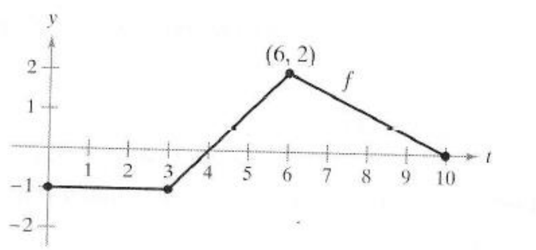
a) $\int_2^x (3t^3)dt$

b) $\int_{2x}^4 (2m^2 - m)dm$

c) $\int_3^{x^2} (4b)^3 db$

d) $\int_{3x^2}^7 (\sin(4t))dt$

21. Given that $g(x) = \int_0^x f(t)dt$, find the following:

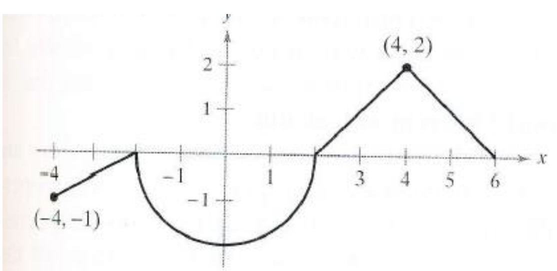


a) $g(4)$ b) $g(6)$ c) $g'(6)$ d) $g''(8)$

e) At what x coordinates does $g(x)$ have inflection points. Justify.

f) Find the equation of the tangent line to function g at the coordinate $x = 6$.

22. Given the graph of $f'(x)$ below over the closed interval [-4,6] and that $f(-2) = 4$ find the following:



a) $f(-4)$

b) $f(4)$

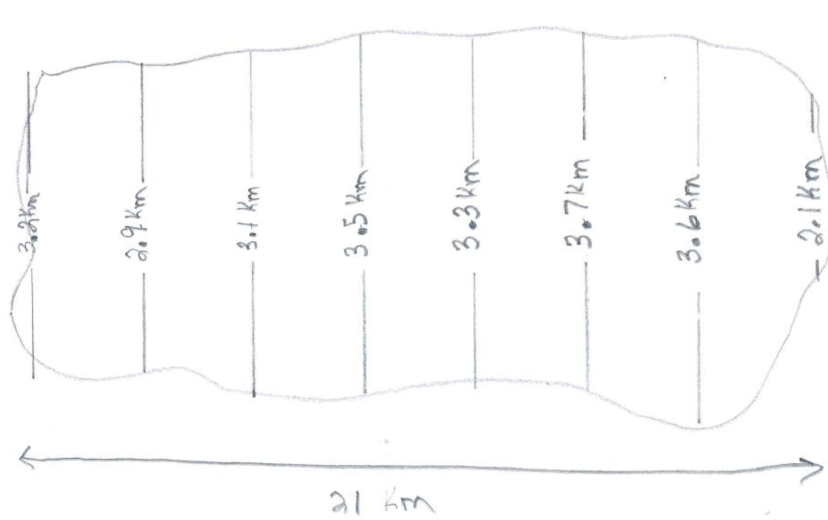
c) $f(6)$

d) Find the absolute minimum over the interval [-4,6]

e) Find all inflection points of $f(x)$ and justify.

f) Find the interval(s) where $f(x)$ is both decreasing and concave down. Justify.

23. During recent northern forest fires, photos from airplanes were used to approximate the area of the burning forest fires. A sketch of one photo is provided below. Approximate distances across the burning forest where calculated every 3 horizontal kilometres. Using trapezoidal approximation estimate the total area of the burning fire.



24. Suppose the decay equation for a radioactive element is known to be $y = y_o e^{-0.24t}$, where t is minutes. How long will it take this element to decay to 70% of its original amount?

25. Suppose a population of fruit flies increases according to the law of exponential growth. If there were 100 fruit flies present after day 2 and 300 fruit flies present after day 4, approximately how many fruit flies were there initially?

26. The process of raw sugar has an "inversion" step that changes the sugar's molecular structure. Once the process begins, the amount of sugar decays at a rate of $\frac{dy}{dt} = -ky.$ If 1000 kg of raw sugar reduces to 800 kg of raw sugar during the first 10 hours, how much sugar will remain after another 14 hours?

Answers

1. a) -1 m/s^2 b) $-\frac{11}{8}m/s^2$ c) average velocity in m/s over the time interval t=0s to t=6s.
d) displacement of the particle in mteres over the interval t=0s to t=10s. e) -25m
f) speed is decreasing because v(2)>0 and a(2)<0. 2. -9 3. 21 4. t=5 5.a) 5 miles/h^2
b) 28.75 miles c) Yes vehicle has only traveled 28.75 miles and is only 35 miles from charging station and 28.75 + 35 < 75. 6.a) $\frac{22}{3}$ b) 185.145 c) 8π d) $\frac{32}{3}$ 7. a) 6.817 b) 168.180 c) 26.267
8. a) 1.007 b) 0.459 c) 4.000 9. a) See teacher answer key. b) $y = 2x + 1, f(0.2) = 1.4$
c) $y = -2e^{-sinx} + 3$ 10. a) see teachers answer key b) $\frac{d^2y}{dx^2} = 2 - 2x + y$, concave up because in quadrant 2, $x < 0$ and $y > 0$, therefore $f''(x) > 0$ in quadrant 2 11. a) $y = 8x - 6$
b) $f(1.1) = 2.8$ c) $y = \sqrt{\frac{-4}{4x^2-5}}$ 12. $y = \frac{1}{2}ln(2x^3 - 1)$ 13. $\frac{1}{4}$ 14. -2
15.a) $-(12x^3 - 7)csc(3x^4 - 7x)cot(3x^4 - 7x)$ b) $x^3sec(5x^3)(15x^3tan(5x^3) + 4)$
c) $-\frac{3csc^2(ln(3x+2))}{3x+2}$ d) $\frac{8x-9}{\sqrt{1-(4x^2-9x)^2}}$ e) $-\frac{6e^{2x}}{\sqrt{1-9e^{4x}}}$ f) $\frac{2}{x\sqrt{36x^4-1}}$
g) $6x^2\left(-\frac{x}{1+(3x-7)^2} + cot^{-1}(3x-7)\right)$ h) $\frac{35x^6-3}{1+(5x^7-3x)^2}$ 16. a) 1 b) $-\frac{3}{7}$ c) $-\frac{\pi}{8}$ d) 0
17.a) $4(3x+5)sin\frac{x}{4} + 48cos\frac{x}{4} + C$ b) $-\frac{w^2}{10}cos10w + \frac{w}{50}sin10w + \frac{1}{500}cos10w + C$
c) $-\frac{1}{2}(3x + x^2)cos2x + \frac{1}{4}(3 + 2x)sin2x + \frac{1}{4}cos2x + C$ d) $\frac{1}{2}(e^x sinx + e^x cosx) + C$
e) $4ln|x-3| - ln|x+2| + C$ f) $2ln|x| + 6ln|3x+4| - 8ln|x+1| + C$
g) $\frac{x^2}{2} - 2x + 2ln|x| - \frac{6}{x} - 2ln|x-3| + C$ h) $-\frac{25}{81}ln|x| - \frac{11}{9x} + \frac{25}{81}ln|x-9| + C$
i) $-\frac{cos^3(x)}{3} + \frac{cos^5(x)}{5} + C$ j) $\frac{tan^3(x)}{3} + \frac{tan^5(x)}{5} + C$ k) $\frac{3}{2}x + \frac{3}{10}sin(5x)cos(5x) + C$
l) $\frac{sec^7(x)}{7} - \frac{2sec^5(x)}{5} + \frac{sec^3(x)}{3} + C$ m) $ln\left|\frac{9+x^2}{3} + \frac{x}{3}\right| + C$
n) $-243\left(\frac{1}{3}\left(\frac{\sqrt{9-x^2}}{3}\right)^3 - \frac{1}{5}\left(\frac{\sqrt{9-x^2}}{3}\right)^5\right) + C$ o) $2\left(\frac{\sqrt{25x^2-4}}{2} - sec^{-1}\left(\frac{5}{2x}\right)\right) + C$
p) $2sin^{-1}\left(\frac{x-3}{2}\right) + (x-3)\left(\frac{\sqrt{4-(x-3)^2}}{2}\right) + C$ 18. a) 480 gallons b) 460 gallons c) 447.5 gallons
d) the number of gallons to flow through the pipe from t = 0 to t = 40 min. 19. a) 7 b) -5 c) 6
d) $\frac{23}{2}$ 20. a) $3x^3$b) $-2(8x^2 - 2x)$ c) $128x^7$ d) $-6x(sin12x^2)$ 21. a) $\frac{-7}{2}$ b) $\frac{-3}{2}$ c) 2 d) -1/2 e) 6 because g' changes from inc to dec. f) $y = 2x - \frac{27}{2}$ 22. a) 5 b) $6 - 2\pi$ c) $8 - 2\pi$ d) $4 - 2\pi$ e) $x = -2,4$ because f' changes from inc to dec. x = 0 because f' changes from dec. to inc.
f) $(-2,0)$ because f'<0 and f' is decreasing 23. 68.25 km^2 24. 1.486 min 25. Approx. 34 years
26. 585.350 kg