# Year End Review Answer Key

1 a) $\dfrac{v(6)-v(2)}{6-2} = \dfrac{-1-3}{4} = -1 \text{ m/s}^2$

b) $\dfrac{v(10)-v(2)}{10-2} = \dfrac{-8-3}{8} = \dfrac{-11}{8} \text{ m/s}^2$ or $\dfrac{1}{10-2}\displaystyle\int_2^{10} a(t) = \dfrac{1}{8}\left[v(10)-v(2)\right]$

$= \dfrac{1}{8}\left[-8-3\right]$

$= -11/8 \text{ m/s}^2$

c) $\dfrac{1}{6}\displaystyle\int_0^6 v(t)\,dt =$ average velocity in m/s over the time interval $t=0s$ to $t=6s$

d) $\displaystyle\int_0^{10} v(t)\,dt =$ Displacement of the particle in metres over the interval $t=0s$ to $t=10s$

e) $5 + 2(3) + 4(-1) + 4(-8) = 5+6-4-32$
$= -25\text{m}$

f) speed is decreasing because $v(2) > 0$ and $a(2) < 0$.

2. $x(t) = \cos(3t) - \sin(4t)$
$v(t) = -3\sin 3t - 4\cos 4t$
$a(t) = -9\cos 3t + 16\sin 4t$
$a(0) = -9(\cos 0) + 16(\sin 0) = -9$

3. $x(t) = 3t^2 + 1$
ave vel $= \dfrac{s(6)-s(1)}{6-1} = \dfrac{(3(6)^2+1)-(3(1)^2+1)}{5} = \dfrac{109-4}{5} = 21$

4. $x(t) = t^3 - 9t^2 + 15t + 3$
$v(t) = 3t^2 - 18t + 15$

$3t^2 - 18t + 15 = 0$
$3(t^2 - 6t + 5) = 0$
$3(t-1)(t-5) = 0$
$t=1 \quad t=5$

$v(t)$: $+$ ($\frac{1}{2}$) | 1 | $-$ (2) | 5 | $+$ (6)

$t = 5$

$t = 0 \quad x(0) = 3$
$t = 5 \quad x(5) = -22$
$t = 10 \quad x(10) = 253$

5. a) ave accel $= \dfrac{V(.75) - V(.25)}{.75 - .25} = \dfrac{12.8\ m/h - 10.3\ m/h}{.5h} = 5\ m/h^2$

b) $\displaystyle\int_0^2 V(t)\,dt = 0.5\left[10.3 + 12.8 + 20.1 + 14.3\right]$

$= 28.75$ miles

c) yes vehicle has only traveled 28.75 miles in first 2 hours ∴ only has 35 miles to go, and
$28.75 + 35 < 75$ miles

6. a) Area $= \displaystyle\int_0^4 \sqrt{x}\,dx + \int_4^6 (6-x)\,dx$

$= \dfrac{2}{3}x^{3/2}\Big|_0^4 + \left(6x - \dfrac{x^2}{2}\right)\Big|_4^6$

$= \dfrac{2}{3}(4)^{3/2} + \left[\left(6(6) - \dfrac{6^2}{2}\right) - \left(6(4) - \dfrac{4^2}{2}\right)\right]$

$= \dfrac{16}{3} + \left[18 - 16\right]$

$= \dfrac{16}{3} + 2 = \dfrac{16}{3} + \dfrac{6}{3} = \dfrac{22}{3}$

b) $x = y^2$ $\qquad x = 6 - y$

$V = \pi \displaystyle\int_0^2 (6-y+1)^2 - (y^2+1)^2\,dy$

$= \pi \displaystyle\int_0^2 \left[(7-y)^2 - (y^2+1)^2\right]dy$

$= 185.145$

c) $V = \pi \displaystyle\int_0^4 (\sqrt{x})^2\,dx = \pi \int_0^4 x\,dx = \pi\left[\dfrac{x^2}{2}\Big|_0^4\right]$

$= \pi\left[\dfrac{4^2}{2} - 0\right]$

$= 8\pi$

6d) $V = \int_0 (6-y-y^2)(2y)\,dy = \frac{32}{3}$

7. a) $A = \int_0^2 (6-4\ln(3-x))\,dx$

$A = 6.817$

b) $V = \pi \int_0^2 (8-4\ln(3-x))^2 - (2)^2\,dx = 168.180$

c) $V = \int_0^2 (6-4\ln(3-x))^2\,dx = 26.267$

8.

$A = 1.032831888$

a) $A = \int_A^2 (f(x)-g(x))\,dx = 1.007$

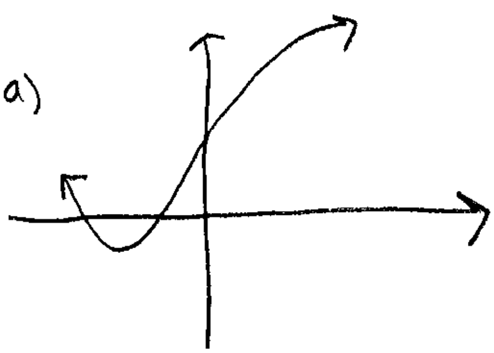
b) $V = \frac{\pi}{2}\int_0^A \left(\frac{g(x)-f(x)}{2}\right)^2 dx = \frac{\pi}{8}\int_0^A (g(x)-f(x))^2 = 0.459$

c) $m = f(x) - g(x)$

$m' = f'(x) - g'(x)$

$m'(1) = f'(1) - g'(1) = 4.000$

9. a)



b) $m = (3-1)\cos(0) = 2$

$y - 1 = 2(x-0)$

$y = 2x + 1$

$y = 2(0.2) + 1$

$y = 1.4$

c) $\int \frac{dy}{y-3} = \int -\cos x \, dx$

$\ln|y-3| = -\sin x + C$

$|y-3| = e^{-\sin x} \cdot e^{C}$ let $\pm e^{C} = A$

$y-3 = Ae^{-\sin x}$

$1-3 = Ae^{-\sin 0}$

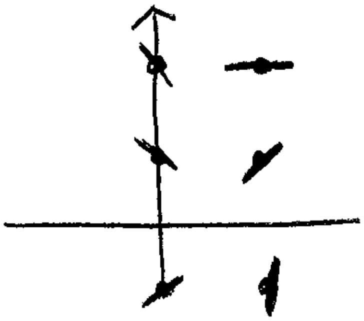
$-2 = A$

$y-3 = -2e^{-\sin x}$

$y = -2e^{-\sin x} + 3$

10. $\frac{dy}{dx} = 2x - y$

a)



b) $\frac{d^2y}{dx^2} = 2 - \frac{dy}{dx}$

$= 2 - (2x - y)$

$\frac{d^2y}{dx^2} = 2 - 2x + y > 0$ in quad II

as x coordinates are $<0$
and y coordinates are $>0$.

$\therefore$ concave up

11. a) $\frac{dy}{dx} = xy^3$

$m = 1(2)^3 = 8$

$y - 2 = 8(x-1)$

$y = 8x - 8 + 2$

$y = 8x - 6$

b) $y = 8(1.1) - 6$

$y = 2.8$

c) $\int y^{-3} dy = \int x \, dx$

$\frac{-y^{-2}}{2} = \frac{x^2}{2} + C$

$\frac{-1}{2y^2} = \frac{x^2}{2} + C$

$\frac{-1}{2(2)^2} = \frac{(1)^2}{2} + C$

$-\frac{1}{8} = \frac{1}{2} + C$

$-\frac{5}{8} = C$

$\frac{-1}{2y^2} = \frac{x^2}{2} - \frac{5}{8}$

$\frac{-4}{y^2} = 4x^2 - 5$

$\frac{-4}{4x^2-5} = y^2$

$\pm\sqrt{\frac{-4}{4x^2-5}} = y = \sqrt{\frac{-4}{4x^2-5}}$

12. $\int e^{2y}\,dy = \int 3x^2\,dx$

$\frac{e^{2y}}{2} = x^3 + C$

$-\frac{1}{2} = 0^3 + C$

$-\frac{1}{2} = C$

$\frac{e^{2y}}{2} = x^3 - \frac{1}{2}$

$e^{2y} = 2x^3 - 1$

$2y = \ln(2x^3 - 1)$

$y = \frac{1}{2}\ln(2x^3 - 1)$

13. $g(2) = 1$

$g'(2) = \frac{1}{f'(g^{-1}(2))} = \frac{1}{f'(1)} = \frac{1}{4}$

$f'(x) = 3x^2 + 1$

$f'(1) = 3(1)^2 + 1 = 4$

14. $g'(-4) = \frac{1}{f'(g^{-1}(4))} = \frac{1}{f'(5)} = \frac{1}{-\frac{1}{2}} = -2$

15 a) $f(x) = \csc(3x^4 - 7x)$

$f'(x) = -\csc(3x^4 - 7x)\cot(3x^4 - 7x) \cdot (12x^3 - 7)$

$= -(12x^3 - 7)\csc(3x^4 - 7x)\cot(3x^4 - 7x)$

b) $f(x) = x^4 \sec(5x^3)$

$f'(x) = x^4 \cdot \sec(5x^3)\tan(5x^3) \cdot 15x^2 + \sec(5x^3) \cdot 4x^3$

$= 15x^6 \sec(5x^3)\tan(5x^3) + 4x^3 \sec(5x^3)$

$= x^3 \sec(5x^3)\left[15x^3 \tan(5x^3) + 4\right]$

c) $f(x) = \cot(\ln(3x+2))$

$f'(x) = -\csc^2(\ln(3x+2)) \cdot \frac{1}{3x+2} \cdot 3$

$= \frac{-3\csc^2(\ln(3x+2))}{3x+2}$

d) $f(x) = \sin^{-1}(4x^2 - 9x)$

$$f'(x) = \frac{1}{\sqrt{1-(4x^2-9x)^2}} \cdot (8x-9) = \frac{8x-9}{\sqrt{1-(4x^2-9x)^2}}$$

e) $f(x) = \cos^{-1}(3e^{2x})$

$$f'(x) = \frac{-1}{\sqrt{1-(3e^{2x})^2}} \cdot 3e^{2x} \cdot 2 = \frac{-6e^{2x}}{\sqrt{1-9e^{4x}}}$$

15f) * see last page

g) $f(x) = 2x^3 \cot^{-1}(3x-7)$

$$f'(x) = 2x^3 \cdot \frac{-1}{1+(3x-7)^2} \cdot 3 + \cot^{-1}(3x-7) \cdot 6x^2$$

$$= \frac{-6x^3}{1+(3x-7)^2} + 6x^2 \cot^{-1}(3x-7)$$

$$= 6x^2 \left[ \frac{-x}{1+(3x-7)^2} + \cot^{-1}(3x-7) \right]$$

h) $f(x) = \tan^{-1}(5x^7 - 3x)$

$$f'(x) = \frac{1}{1+(5x^7-3x)^2} \cdot (35x^6 - 3) = \frac{35x^6-3}{1+(5x^7-3x)^2}$$

16a) $\lim_{x \to 0} \frac{\sin x}{x} = \frac{0}{0}$

$\lim_{x \to 0} \frac{\cos x}{1} = \cos 0 = 1$

b) $\lim_{t \to 1} \frac{5t^4 - 4t^2 - 1}{10 - t - 9t^3} = \frac{0}{0}$

$\lim_{t \to 1} \frac{20t^3 - 8t}{-1 - 27t^2} = \frac{20-8}{-1-27} = \frac{12}{-28} = -\frac{3}{7}$

c) $\lim_{w \to -4} \frac{\sin(\pi w)}{w^2 - 16} = \frac{0}{0}$

$\lim_{w \to -4} \frac{\pi \cos \pi w}{2w} = \frac{\pi(\cos(-4\pi))}{2(-4)} = \frac{\pi}{-8} = -\frac{\pi}{8}$

d) $\lim_{t \to \infty} \frac{\ln(3t)}{t^2} = \frac{\infty}{\infty}$

$\lim_{t \to \infty} \frac{\frac{3}{3t}}{2t}$

$= \lim_{t \to \infty} \frac{1}{2t^2} = \frac{1}{\infty} = 0$

17. a) $\int (3x+5)\cos\left(\frac{x}{4}\right)dx$

let $u = 3x+5$ $\quad v = 4\sin\frac{1}{4}x$

$du = 3dx$ $\quad dv = \cos\left(\frac{x}{4}\right)dx$

$= (3x+5)\cdot 4\sin\frac{x}{4} - \int 4\sin\frac{x}{4}\cdot 3\,dx$

$= 4(3x+5)\sin\frac{x}{4} - 12\int \sin\frac{x}{4}\,dx$

$= 4(3x+5)\sin\frac{x}{4} - 12\left[-4\cos\frac{x}{4}\right] + C$

$= 4(3x+5)\sin\frac{x}{4} + 48\cos\frac{x}{4} + C$

b) $\int w^2 \sin(10w)\,dw$

let $u = w^2$ $\quad v = -\frac{1}{10}\cos(10w)$

$du = 2w\,dw$ $\quad dv = \sin(10w)\,dw$

$= -\frac{w^2}{10}\cos(10w) - \int -\frac{2w}{10}\cos(10w)\,dw$

$= -\frac{w^2}{10}\cos(10w) + \frac{1}{5}\int w\cos(10w)\,dw$ $\quad$ let $u = w$ $\quad v = \frac{1}{10}\sin(10w)$

$du = dw$ $\quad dv = \cos(10w)\,dw$

$= -\frac{w^2}{10}\cos(10w) + \frac{1}{5}\left[\frac{w}{10}\sin(10w) - \int \frac{1}{10}\sin(10w)\,dw\right]$

$= -\frac{w^2}{10}\cos(10w) + \frac{w}{50}\sin(10w) - \frac{1}{50}\left(-\frac{1}{10}\cos(10w)\right) + C$

$= -\frac{w^2}{10}\cos(10w) + \frac{w}{50}\sin(10w) + \frac{1}{500}\cos(10w) + C$

c) $\displaystyle\int (3x+x^2)\sin 2x\,dx$

*

let $u = 3x+x^2$ $\quad v = -\frac{1}{2}\cos 2x$

$du = (3+2x)\,dx$ $\quad dv = \sin 2x\,dx$

$$= -\frac{1}{2}(3x+y^2)\cos 2x - \int -\frac{1}{2}\cos 2x\,(3+2x)\,dx$$

$$= -\frac{1}{2}(3x+x^2)\cos 2x + \frac{1}{2}\int (3+2x)\cos 2x\,dx$$

let $u = 3+2x$ $\quad v = \frac{1}{2}\sin 2$

$du = 2\,dx$ $\quad dv = \cos 2x\,dx$

$$= -\frac{1}{2}(3x+x^2)\cos 2x + \frac{1}{2}\left[(3+2x)(\tfrac{1}{2}\sin 2x) - \int \frac{1}{2}\sin 2x \cdot 2\,dx\right]$$

$$= -\frac{1}{2}(3x+y^2)\cos 2x + \frac{1}{4}(3+2x)\sin 2x - \frac{1}{2}\int \sin 2x\,dx$$

$$= -\frac{1}{2}(3x+x^2)\cos 2x + \frac{1}{4}(3+2x)\sin 2x + \frac{1}{4}\cos 2x + C$$

d) $\displaystyle\int e^x\cos x\,dx$

let $u = e^x$ $\quad v = \sin x$

$du = e^x\,dx$ $\quad dv = \cos x\,dx$

$$= e^x\sin x - \int e^x\sin x\,dx$$

let $u = e^x$ $\quad v = -\cos x$

$du = e^x\,dx$ $\quad dv = \sin x\,dx$

$$e^x\cos x\,dx = e^x\sin x - \left[-e^x\cos x - \int -e^x\cos x\,dx\right]$$

$$\int e^x\cos x\,dx = e^x\sin x + e^x\cos x - \int e^x\cos x\,dx$$

$$2\int e^x\cos x\,dx = e^x\sin x + e^x\cos x$$

$$\int e^x\cos x\,dx = \frac{1}{2}\left(e^x\sin x + e^x\cos x\right) + C$$

e) $\displaystyle\int \frac{3x+11}{x^2-x-6}\,dx$

$$\frac{3x+11}{(x-3)(x+2)} = \frac{A}{(x-3)} + \frac{B}{(x+2)}$$

$$3x+11 = A(x+2) + B(x-3)$$

Let $x=-2$

$$-6+11 = B(-2-3)$$

$$5 = B(-5)$$

$$\boxed{-1=B}$$

Let $x=3$

$$3(3)+11 = A(3+2)$$

$$20 = 5A$$

$$\boxed{4=A}$$

$$\int \frac{3x+11}{x^2-x-6}\,dx = \int\left(\frac{4}{x-3} + \frac{-1}{x+2}\right)dx$$

$$= 4\ln|x-3| - \ln|x+2| + C$$

f) $\displaystyle\int \frac{8}{x(3x+4)(x+1)}\,dx$

$$\frac{8}{x(3x+4)(x+1)} = \frac{A}{x} + \frac{B}{3x+4} + \frac{C}{x+1}$$

$$8 = A(3x+4)(x+1) + B(x)(x+1) + C(x)(3x+4)$$

Let $x=-1$

$$8 = C(-1)(1)$$

$$\boxed{-8=C}$$

Let $x=0$

$$8 = A(4)(1)$$

$$\boxed{2=A}$$

Let $x=-4/3$

$$8 = B\left(-\tfrac{4}{3}\right)\left(-\tfrac{4}{3}+1\right)$$

$$8 = B\left(-\tfrac{4}{3}\right)\left(-\tfrac{1}{3}\right)$$

$$8 = B(4/9)$$

$$\frac{72}{4} = B$$

$$18 = B$$

Let $u = 3x+4$
$du = 3\,dx$

$$\int \frac{8}{3x^3+7x^2+4x}\,dx = \int\left(\frac{2}{x} + \frac{18}{3x+4} + \frac{-8}{x+1}\right)dx$$

$$= 2\ln|x| + \tfrac{1}{3}\cdot 18\ln|3x+4| - 8\ln|x+1| + C$$

$$= 2\ln|x| + 6\ln|3x+4| - 8\ln|x+1| + C$$

g) $$\int \frac{x^4-5x^3+6x^2-18}{x^3-3x^2}\,dx$$

$$\begin{array}{r}
x - 2 \\
x^3-3x^2 \overline{\big)\, x^4-5x^3+6x^2-0x-18} \\
-x^4+3x^3 \qquad\qquad\quad \\
\hline
-2x^3+6x^2 \qquad\quad \\
+2x^3 - 6x^2 \qquad\quad \\
\hline
0 - 0x - 18
\end{array}$$

$$= \int \left[(x-2) - \frac{18}{x^3-3x^2}\right] dx$$

$$= \frac{x^2}{2} - 2x - \int \frac{18}{x^2(x-3)}\,dx$$

$$\frac{18}{x^2(x-3)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-3}$$

$$18 = A(x)(x-3) + B(x-3) + C(x^2)$$

Let $x=0$

$$18 = B(0-3)$$

$$\boxed{-6 = B}$$

Let $x=3$

$$18 = C(3)^2$$

$$\boxed{2 = C}$$

Let $x=1$

$$18 = A(1)(1-3) + B(1-3) + C$$

$$18 = -2A - 2(-6) + 2$$

$$18 = -2A + 14$$

$$4 = -2A$$

$$\boxed{-2 = A}$$

$$= \frac{x^2}{2} - 2x - \int \left(\frac{-2}{x} + \frac{-6}{x^2} + \frac{2}{x-3}\right) dx$$

$$= \frac{x^2}{2} - 2x + 2\ln|x| + \frac{6}{x} + 2\ln|x-3| + C$$

$$\int \frac{\quad}{x^3-9x^2}\,dx$$

$$\frac{4x-11}{x^2(x-9)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-9}$$

$$4x-11 = A(x)(x-9) + B(x-9) + C(x^2)$$

let $x=0$

$-11 = B(-9)$

$\boxed{11/9 = B}$

let $x=9$

$36-11 = 81C$

$\boxed{\frac{25}{81} = C}$

let $x=3$

$4(3)-11 = A(3)(-6) + \frac{11}{9}(-6) + \frac{25}{81}(9)$

$1 = -18A - \frac{22}{3} + \frac{25}{9}$

$9 = -162A - 66 + 25$

$50 = -162A$

$-\frac{25}{81} = A$

$$\int \frac{4x-11}{x^3-9x^2}\,dx = \int \left( \frac{-\frac{25}{81}}{x} + \frac{\frac{11}{9}}{x^2} + \frac{\frac{25}{81}}{x-9} \right) dx$$

$$= -\frac{25}{81}\ln|x| - \frac{11}{9x} + \frac{25}{81}\ln|x-9| + C$$

i) $\int \sin^3 x \cos^2 x \, dx$

$\int \sin^2 x \cos^2 x \sin x \, dx$

$\int (1-\cos^2 x)\cos^2 x \sin x \, dx$ let $u = \cos x$

$du = -\sin x \, dx$

$-du = \sin x \, dx$

$$= -\int (1-u^2)u^2\,du = -\int (u^2-u^4)\,du$$

$$= -\left(\frac{u^3}{3} - \frac{u^5}{5}\right) + C$$

$$= -\frac{\cos^3 x}{3} + \frac{\cos^5 x}{5} + C$$

j) $\int \tan^2 x \sec^4 x \, dx$

$\int \tan^2 x \sec^2 x \sec^2 x \, dx$

$\int \tan^2 x (1+\tan^2 x) \sec^2 x \, dx$ Let $u = \tan x$
$du = \sec^2 x \, dx$

$\int u^2 (1+u^2) \, du = \int (u^2 + u^4) \, du$

$= \frac{u^3}{3} + \frac{u^5}{5} + C$

$= \frac{\tan^3 x}{3} + \frac{\tan^5 x}{5} + C$

k) $\int 3\cos^2 5x \, dx$

$= 3 \int \cos^2 5x \, dx$ Let $u = 5x$
$du = 5\,dx$
$\frac{1}{5} du = dx$

$= \frac{3}{5} \int \cos^2 u \, du$

$= \frac{3}{5} \left[ \int \frac{1}{2} (1 + \cos 2u) \, du \right]$

$= \frac{3}{10} \left[ u + \frac{\sin 2u}{2} \right] + C$

$= \frac{3}{10} u + \frac{3}{20} \sin 2u + C$

$= \frac{3}{10}(5x) + \frac{3}{20} \sin 2(5x) + C$

$= \frac{3}{2}x + \frac{3}{20} \sin 10x + C$

OR

$= \frac{3}{2}x + \frac{3}{10} \sin(5x)\cos(5x) + C$

l) $\int \tan^5 x \sec^3 x \, dx$

$\int \tan^4 x \sec^2 x \tan x \sec x \, dx$

$\int (\sec^2 x - 1)^2 \sec^2 x \tan x \sec x \, dx$ let $u = \sec x$

$du = \sec x \tan x$

$\int (u^2 - 1)^2 u^2 \, du$

$\int (u^4 - 2u^2 + 1) u^2 \, du = \int (u^6 - 2u^4 + u^2) \, du$

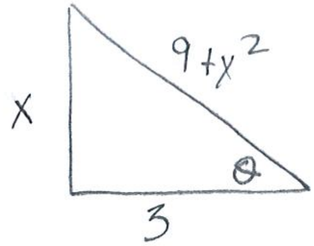
$= \frac{u^7}{7} - \frac{2u^5}{5} + \frac{u^3}{3} + C$

$= \frac{\sec^7 x}{7} - \frac{2\sec^5 x}{5} + \frac{\sec^3 x}{3} + C$

m) $\displaystyle\int \frac{1}{\sqrt{9+x^2}}\,dx$

$x = 3\tan\theta$

$dx = 3\sec^2\theta\, d\theta$




$$\int \frac{3\sec^2\theta\, d\theta}{3\sec\theta}$$

$$= \int \sec\theta\, d\theta$$

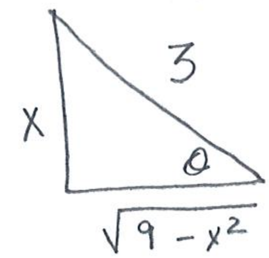
$$= \ln|\sec\theta + \tan\theta| + C$$

$$= \ln\left|\frac{9+x^2}{3} + \frac{x}{3}\right| + C$$

n) $\displaystyle\int x^3\sqrt{9-x^2}\,dx$

$x = 3\sin\theta$

$dx = 3\cos\theta\, d\theta$




$$\int (3\sin\theta)^3 \cdot 3\cos\theta \cdot 3\cos\theta\, d\theta$$

$$\int 27\sin^3\theta \cdot 9\cos^2\theta\, d\theta$$

$$243\int \cos^2\theta\,(1-\cos^2\theta)\sin\theta\, d\theta$$

Let $u = \cos\theta$

$du = -\sin\theta\, d\theta$

$-du = \sin\theta\, d\theta$

$$= -243\int u^2(1-u^2)\,du$$

$$= -243\int (u^2-u^4)\,du$$

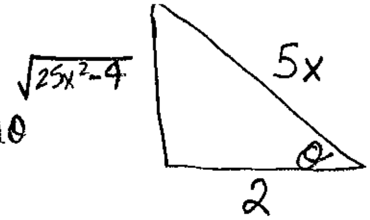
$$= -243\left[\frac{u^3}{3} - \frac{u^5}{5}\right] du$$

$$= -243\left[\frac{\cos^3\theta}{3} - \frac{\cos^5\theta}{5}\right] = -243\left[\frac{1}{3}\left(\frac{\sqrt{9-x^2}}{3}\right)^3 - \frac{1}{5}\left[\left(\frac{\sqrt{9-x^2}}{3}\right)\right]^5\right]$$

$+C$

o) $\displaystyle\int \frac{\sqrt{25x^2-4}}{x}\,dx$

$5x = 2\sec\theta$
$x = \frac{2}{5}\sec\theta$
$dx = \frac{2}{5}\sec\theta\tan\theta\,d\theta$

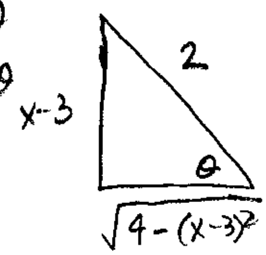



$$\int \frac{2\tan\theta \cdot \frac{2}{5}\sec\theta\tan\theta\,d\theta}{\frac{2}{5}\sec\theta}$$

$$2\int \tan^2\theta\,d\theta = 2\int(\sec^2\theta - 1)\,d\theta$$
$$= 2\left[\tan\theta - \theta\right] + C$$
$$= 2\left[\frac{\sqrt{25x^2-4}}{2} - \operatorname{arcsec}\frac{5}{2x}\right] + C$$

p) $\displaystyle\int \sqrt{6x-x^2-5}\,dx = \int \sqrt{4-(x-3)^2}\,dx$

$x-3 = 2\sin\theta$
$dx = 2\cos\theta\,d\theta$




$-5 - (x^2-6x)$
$9 - 5 - (x^2-6x+9)$
$4-(x-3)^2$

$$= \int 2\cos\theta \cdot 2\cos\theta\,d\theta$$
$$= 4\int \cos^2\theta\,d\theta$$
$$= 4\int \tfrac{1}{2}(1+\cos 2\theta)\,d\theta$$
$$= 2\left[\theta + \tfrac{1}{2}\sin 2\theta\right] + C$$
$$= 2\theta + 2\sin\theta\cos\theta + C$$
$$2\sin^{-1}\left(\frac{x-3}{2}\right) + 2\left(\frac{x-3}{2}\right)\left(\frac{\sqrt{4-(x-3)^2}}{2}\right) + C$$
$$= 2\sin^{-1}\left(\frac{x-3}{2}\right) + (x-3)\left(\frac{\sqrt{4-(x-3)^2}}{2}\right) + C$$

15 f) $f(x) = \sec^{-1}(6x^2)$

$$f'(x) = \frac{1}{|6x^2|\sqrt{(6x^2)^2 - 1}} \cdot 12x = \frac{2}{x\sqrt{36x^4 - 1}}$$