Lesson 6.1 **Slope of a Line** **Exercises (pages 339–343)**

A

4. Count squares to determine the rise and run.

a) The rise is 2.
The run is 11.

Slope = $\frac{\text{rise}}{\text{run}}$

Slope = $\frac{2}{11}$

b) The rise is 4.
The run is 14.

Slope = $\frac{\text{rise}}{\text{run}}$

Slope = $\frac{4}{14}$ Simplify.

Slope = $\frac{2}{7}$

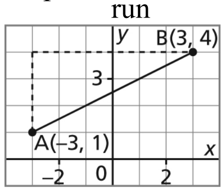
5. a) The segment goes down to the right, so its slope is negative.

b) The segment goes up to the right, so its slope is positive.

c) The segment is vertical, so its slope is not defined.

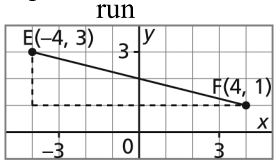
d) The segment is horizontal, so its slope is 0.

6. a) Slope = $\frac{\text{rise}}{\text{run}}$



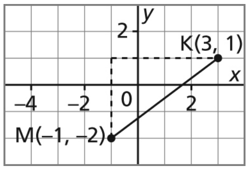
Slope = $\frac{3}{6}$, or $\frac{1}{2}$

b) Slope = $\frac{\text{rise}}{\text{run}}$



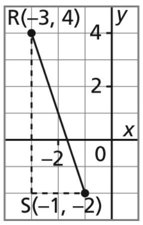
Slope = $\frac{-2}{8}$, or $-\frac{1}{4}$

c) Slope $= \dfrac{\text{rise}}{\text{run}}$



Slope $= \dfrac{3}{4}$

d) Slope $= \dfrac{\text{rise}}{\text{run}}$



Slope $= \dfrac{-6}{2}$, or –3

7. a) Since x increases by 1, the run is 1.
Since y increases by 3, the rise is 3.

Slope $= \dfrac{\text{rise}}{\text{run}}$

Slope $= \dfrac{3}{1}$, or 3

b) Since x increases by 2, the run is 2.
Since y decreases by 7, the rise is –7.

Slope $= \dfrac{\text{rise}}{\text{run}}$

Slope $= \dfrac{-7}{2}$, or $-\dfrac{7}{2}$

c) Since x decreases by 4, the run is –4.
Since y decreases by 2, the rise is –2.

Slope $= \dfrac{\text{rise}}{\text{run}}$

Slope $= \dfrac{-2}{-4}$, or $\dfrac{1}{2}$

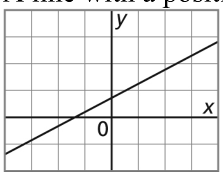
d) Since x decreases by 2, the run is –2.
Since y increases by 1, the rise is 1.

$$\text{Slope} = \frac{\text{rise}}{\text{run}}$$

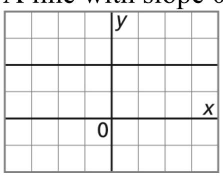
$$\text{Slope} = \frac{1}{-2}, \text{ or } -\frac{1}{2}$$

8. Sketches may vary. The lines may be in different positions on the grid but they should have the same orientations as those shown.

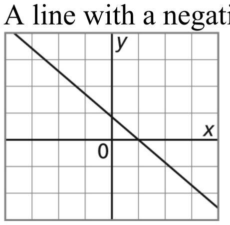
a) A line with a positive slope goes up to the right.



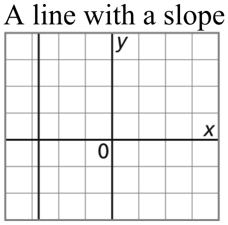
b) A line with slope 0 is horizontal.



c) A line with a negative slope goes down to the right.



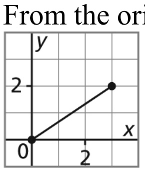
d) A line with a slope that is not defined is vertical.



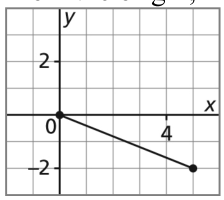
9. Diagrams may vary. The line segments may have different lengths and may have different endpoints at the origin but they should have the same orientations as those shown.

a) Mark a point at the origin. For a slope of $\frac{2}{3}$, the rise is 2 and the run is 3.

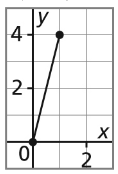
From the origin, move 2 squares up and 3 squares right. Mark a point. Join the points.



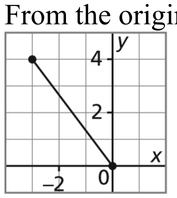
b) Mark a point at the origin. Write the slope $-\frac{2}{5}$ as $\frac{-2}{5}$; then the rise is –2 and the run is 5. From the origin, move 2 squares down and 5 squares right. Mark a point. Join the points.



c) Mark a point at the origin. For a slope of 4, write 4 as $\frac{4}{1}$; then the rise is 4 and the run is 1. From the origin, move 4 squares up and 1 square right. Mark a point. Join the points.

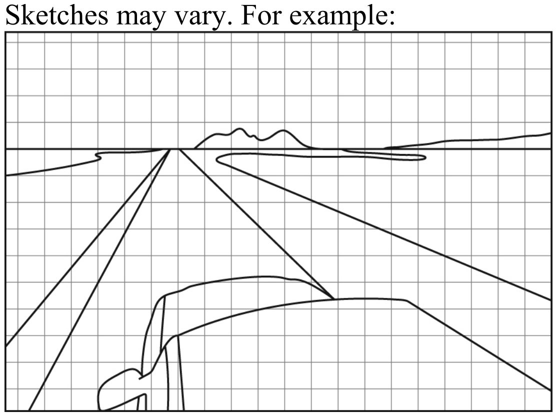


d) Mark a point at the origin. Write the slope $-\frac{4}{3}$ as $\frac{4}{-3}$; then the rise is 4 and the run is –3. From the origin, move 4 squares up and 3 squares left. Mark a point. Join the points.



10. a) Answers may vary; for example, I can use the slopes of the lines to draw them at the correct angle when I copy them. The horizon, and the edges of the crops, road, and the vehicle are straight lines. So, I determine the rise and run of these lines, then draw these lines on a grid to represent the edges of each object in the copy of the picture.

b) Sketches may vary. For example:



B

11. Chosen points may vary.

a) I chose the points with coordinates (4, 1) and (–4, –3).

$$\text{Slope} = \frac{\text{rise}}{\text{run}}$$

$$\text{Slope} = \frac{\text{change in } y\text{-coordinates}}{\text{change in } x\text{-coordinates}}$$

$$\text{Slope} = \frac{-3-1}{-4-4}$$

$$\text{Slope} = \frac{-4}{-8}, \text{ or } \frac{1}{2}$$

b) I chose the points with coordinates (2, 0) and (–2, –2).

$$\text{Slope} = \frac{\text{rise}}{\text{run}}$$

$$\text{Slope} = \frac{\text{change in } y\text{-coordinates}}{\text{change in } x\text{-coordinates}}$$

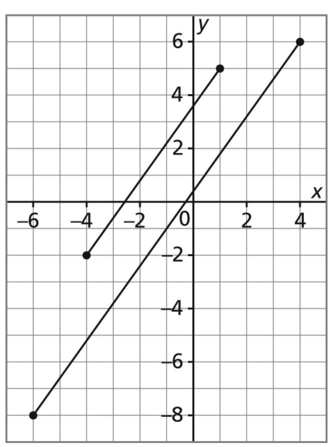
$$\text{Slope} = \frac{-2-0}{-2-2}$$

$$\text{Slope} = \frac{-2}{-4}, \text{ or } \frac{1}{2}$$

c) The slopes in parts a and b are equal because the slopes of all segments of the same line are equal.

12. Diagrams may vary. The line segments may be different lengths and may have different endpoints but they should have the same orientation as those shown.

a)



b) The line segments have the same slope; that is, both of them go up to the right with a rise of 7 and a run of 5. The line segments have different lengths and they are in different places on the grid.

13. a) Slope = $\dfrac{\text{change in } y\text{-coordinates}}{\text{change in } x\text{-coordinates}}$

i) Slope of PQ = $\dfrac{6-2}{3-1}$

Slope of PQ = $\dfrac{4}{2}$, or 2

ii) Slope of ST = $\dfrac{5-1}{8-0}$

Slope of ST = $\dfrac{4}{8}$, or $\dfrac{1}{2}$

iii) Slope of VR = $\dfrac{-8-4}{3-(-1)}$

Slope of VR = $\dfrac{-12}{4}$, or –3

iv) Slope of UW = $\dfrac{-5-(-7)}{-6-(-12)}$

Slope of UW = $\dfrac{2}{6}$, or $\dfrac{1}{3}$

b) i) The slope of PQ is 2. It tells me that the line goes up to the right with a rise of 2 and a run of 1.

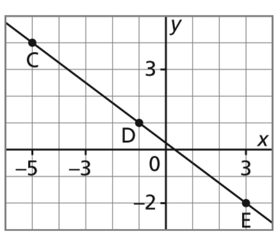
ii) The slope of ST is $\dfrac{1}{2}$. It tells me that the line goes up to the right with a rise of 1 and a run of 2. This line is less steep than the line in part a.

iii) The slope of VR is –3. This tells me that the line goes down to the right with a rise of –3 and a run of 1. I can also think of this line as having a rise of 3 and a run of –1.

iv) The slope of UW is $\dfrac{1}{3}$. This tells me that the line goes up to the right with a rise of 1 and a run of 3. It is less steep than both the lines PQ and ST.

14. Diagrams may vary.

a)



b) $\text{Slope} = \dfrac{\text{change in } y\text{-coordinates}}{\text{change in } x\text{-coordinates}}$

i) $\text{Slope of CD} = \dfrac{1-4}{-1-(-5)}$

$\text{Slope of CD} = \dfrac{-3}{4}$, or $-\dfrac{3}{4}$

ii) $\text{Slope of DE} = \dfrac{-2-1}{3-(-1)}$

$\text{Slope of DE} = \dfrac{-3}{4}$, or $-\dfrac{3}{4}$

iii) $\text{Slope of CE} = \dfrac{-2-4}{3-(-5)}$

$\text{Slope of CE} = \dfrac{-6}{8}$, or $-\dfrac{3}{4}$

The slopes of the segments are equal because all segments on the same line have the same slope.

15. a) $\text{Slope} = \dfrac{\text{rise}}{\text{run}}$ Substitute the values for the rise and run.

$\text{Slope} = \dfrac{6}{90}$

$\text{Slope} = \dfrac{1}{15}$

The treadmill has a slope of $\dfrac{1}{15}$.

b) $\text{Slope} = \dfrac{\text{rise}}{\text{run}}$

Substitute the values for the slope and the run. Let the rise be represented by r.

$0.15 = \dfrac{r}{90}$ Solve for the rise. Multiply each side by 90.

$90(0.15) = \dfrac{r}{90}(90)$

$r = 90(0.15)$

$r = 13.5$

At the maximum slope, the rise is 13.5 in.

16. a) Slope = $\frac{\text{rise}}{\text{run}}$ Substitute the values for the rise and run. Write 4 ft. as 48 in.

Slope = $\frac{-1}{48}$, or $-\frac{1}{48}$

The trench has a slope of $-\frac{1}{48}$, or the trench has a slope of $\frac{1}{48}$ downward.

b) The length of the trench is the run.

Slope = $\frac{\text{rise}}{\text{run}}$

Substitute the values for the slope and the rise. Write $6\frac{1}{2}$ as 6.5. Let the run be represented by r.

$\frac{1}{48} = \frac{6.5}{r}$ Solve for r. Multiply each side by $48r$.

$$\frac{1}{48}(48r) = \frac{6.5}{r}(48r)$$

$$r = 312$$

A length of 312 in. is $\frac{312}{12}$ ft., or 26 ft.

The trench is 26 ft. measured horizontally.

c) The length of the drop is the rise.

Slope = $\frac{\text{rise}}{\text{run}}$

Substitute the values for the slope and the run. Let the rise be represented by r.

$$\frac{1}{48} = \frac{r}{18}$$

Solve for r. Multiply each side by a common denominator of 48 and 18; that is, 144.

$$\frac{1}{48}(144) = \frac{r}{18}(144)$$

$3 = 8r$ Divide each side by 8.

$$r = \frac{3}{8}$$

A depth of $\frac{3}{8}$ ft. is $\frac{3}{8}$(12 in.), or $4\frac{1}{2}$ in.

The trench drops $4\frac{1}{2}$ in. over a horizontal distance of 18 ft.

17. a) A line with a slope of –2 goes down to the right, so the line could be line ii or line iv.

From the graph, the slope of line ii is: $\frac{-1}{2}$, or $-\frac{1}{2}$.

From the graph, the slope of line iv is: $-\frac{2}{1}$, or –2

So, the line with a slope of –2 is line iv.

b) A line with a slope of $\frac{1}{2}$ goes up to the right, so the line could be line i or line iii.

From the graph, the slope of line i is: $\frac{2}{1}$, or 2

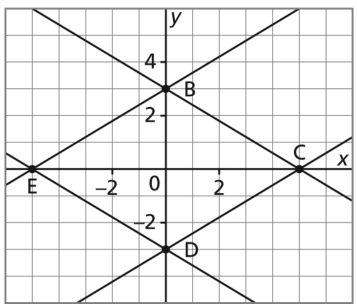
From the graph, the slope of line iii is: $\frac{1}{2}$

So, the line with a slope of $\frac{1}{2}$ is line iii.

c) From part a, line ii has a slope of $-\frac{1}{2}$.

d) From part b, line i has a slope of 2.

18. a)



From the graph:

i) Slope of BC = $\frac{-3}{5}$, or $-\frac{3}{5}$

ii) Slope of DC = $\frac{3}{5}$

iii) Slope of DE = $\frac{3}{-5}$, or $-\frac{3}{5}$

iv) Slope of BE = $\frac{3}{5}$

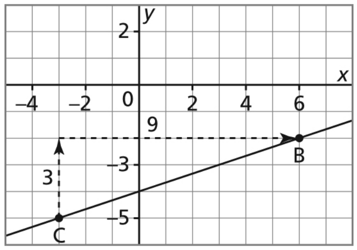
b) The slopes of BC and ED are equal. The slopes of BE and CD are equal.
The two different slopes are opposites.

19. Slope = $\frac{\text{rise}}{\text{run}}$

a) The slope of a horizontal line is 0 because its rise is 0, and the quotient of 0 and any number is zero.

b) The slope of a vertical line is undefined because its run is 0, and the quotient of any number and 0 is undefined; that is, I cannot divide by 0.

20. a)



From the graph:

Slope of BC = $\dfrac{\text{rise}}{\text{run}}$

Slope of BC = $\dfrac{3}{9}$, or $\dfrac{1}{3}$

The correct answer is $\dfrac{1}{3}$.

b) The student who got the answer 3 may have written the expression for the slope as $\dfrac{\text{run}}{\text{rise}}$, and then written:

Slope of BC = $\dfrac{9}{3}$, or 3

The student who got the answer –3 may have written the expression for the slope as $\dfrac{\text{run}}{\text{rise}}$, and then wrote the wrong sign for either the rise or the run:

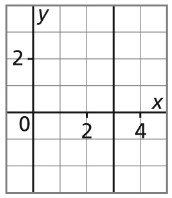
Slope of BC = $\dfrac{-9}{3}$, or –3

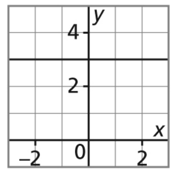
The student who got the answer $-\dfrac{1}{3}$ may have subtracted one set of coordinates in the wrong order and written:

$$\begin{aligned}\text{Slope of BC} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{-5 - (-2)}{6 - (-3)} \\ &= \frac{-3}{9}, \text{ or } -\frac{1}{3}\end{aligned}$$

21. Positions of lines on the grid may vary.

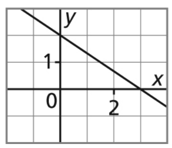
a) i)





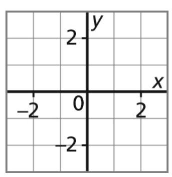
b) i) I could draw more lines than I can count, but each one would be vertical with a slope not defined, or horizontal with a slope equal to 0.

a) ii)



b) ii) I could draw more lines than I can count, but each one would be oblique with a slope equal to any real number except 0.

a) iii)



b) iii) A line that coincides with the x-axis or with the y-axis has more intercepts than I can count. The slope of the x-axis is 0 and the slope of the y-axis is not defined.

22. Slope $= \dfrac{\text{rise}}{\text{run}}$

The slope must be less than $\dfrac{1}{12}$.

So, $\dfrac{\text{rise}}{\text{run}} < \dfrac{1}{12}$

Let r represent the minimum horizontal distance.
Substitute: rise = 70 and run = r

$\dfrac{70}{r} < \dfrac{1}{12}$ Multiply each side of the inequality by $12r$.

$$12r\left(\frac{70}{r}\right) < 12r\left(\frac{1}{12}\right)$$
$$12(70) < r$$
$$r > 840$$

The minimum horizontal distance must be 840 cm, or 8.4 m.
To justify the answer, determine the slope for a greater horizontal distance, such as 900 cm.

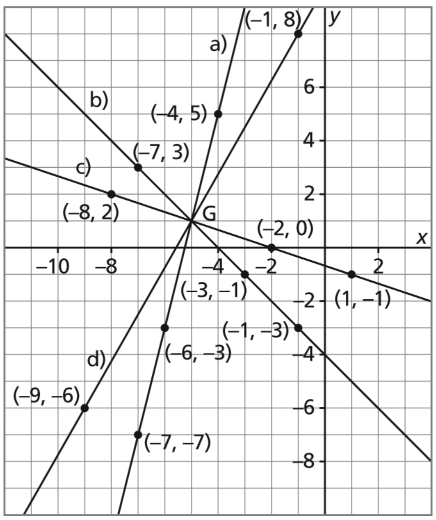
Slope = $\dfrac{\text{rise}}{\text{run}}$

Slope = $\dfrac{70}{900}$, or $0.0\overline{7}$

Compare this value with $\dfrac{1}{12} = 0.08\overline{3}$

Since $0.0\overline{7} < 0.08\overline{3}$, a ramp with a run of 900 cm will be less steep than a ramp with a run of 840 cm.

23.



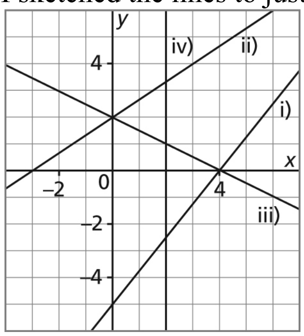
a) A line with slope 4 has a rise of 4 and a run of 1; so, from point G I moved 4 squares up and 1 square right to point (–4, 5). A line with slope 4 also has a rise of –4 and a run of –1: so from point G I moved 4 squares down and 1 square left to reach point (–6, –3), then I moved 4 squares down and 1 square left again to reach point (–7, –7).

b) A line with slope –1 has a rise of 2 and a run of –2; so from point G I moved 2 squares up and 2 squares left to reach point (–7, 3). A line with slope –1 also has a run of –2 and a rise of 2, so from point G I moved 2 squares down and 2 squares right to reach (–3, –1), then I moved 2 squares down and 2 squares right again to reach (–1, –3).

c) A line with slope $-\dfrac{1}{3}$ has a rise of 1 and a run of –3; so from point G I moved 1 square up and 3 squares left to reach point (–8, 2). A line with slope $-\dfrac{1}{3}$ also has a rise of –1 and a run of 3; so from point G I moved 1 square down and 3 squares right to reach point (–2, 0), then I moved 1 square down and 3 squares right again to reach point (1, –1).

d) A line with slope $\dfrac{7}{4}$ has a rise of 7 and a run of 4; so from point G I moved 7 squares up and 4 squares right to reach point (–1, 8). A line with slope $\dfrac{7}{4}$ also has a rise of –7 and a run of –4; so from point G I moved 7 squares down and 4 squares left to reach point (–9, –6). My grid wasn't big enough to plot another point, so I added –4 to –9 to get –13; and I added –7 to –6 to get –13; so my third point has coordinates (–13, –13).

24. I sketched the lines to justify my answers.



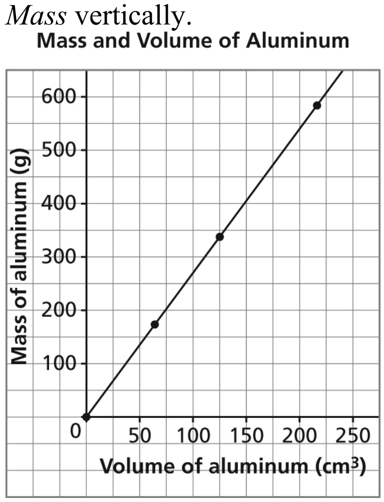
a), b) i) The slope of the line is positive because it goes up to the right.

ii) The slope of the line is positive because it goes up to the right.

iii) The slope of the line is negative because it goes down to the right.

iv) The slope of the line is not defined because it is vertical.

25. a) Since the mass of aluminum depends on its volume, I plotted *Volume* horizontally and *Mass* vertically.



b) I checked that the points lie on the same straight line. I calculated the slope of the line through different pairs of points.
For the points (64, 172.8) and (125, 337.5):

$$\text{Slope} = \frac{337.5 - 172.8}{125 - 64}$$

$$\text{Slope} = \frac{164.7}{61}$$

Slope = 2.7
For the points (125, 337.5) and (216, 583.2):

$$\text{Slope} = \frac{583.2 - 337.5}{216 - 125}$$

$$\text{Slope} = \frac{245.7}{91}$$

Slope = 2.7
Since the slopes are equal, the points lie on the same line with slope 2.7.

c) The slope is 2.7 g/cm^3; that is, it shows that the mass of 1 cm^3 of aluminum is 2.7 g; this is the density of aluminum.

d) 1 cm^3 of aluminum has a mass of 2.7 g.

i) 50 cm^3 has a mass of 50(2.7 g) = 135 g

ii) 275 cm^3 has a mass of 275(2.7 g) = 742.5 g

e) The volume of 1 g is $\frac{1}{2.7}$ cm^3.

i) The volume of 100 g is: $100\left(\frac{1}{2.7}\right)$ cm^3 = 37.0370… cm^3

The volume of 100 g is approximately 37 cm^3.

ii) The volume of 450 g is: $450\left(\frac{1}{2.7}\right)$ cm^3 = $166.\overline{6}$ cm^3

The volume of 450 g is approximately 167 cm^3.

26. a) The number of text messages is a whole number, so there are no permissible values between those represented by the plotted points. So, the points may not be joined.

b) From the graph, the cost for 10 messages is \$1.50.

So, the cost for 1 message is: $\frac{\$1.50}{10}$ = \$0.15, or 15¢

c) The cost for 33 messages is: 33(\$0.15) = \$4.95

d) The number of messages that can be sent for \$7.20 is: $\frac{\$7.20}{\$0.15\,/\,\text{message}}$ = 48 messages

e) I assumed that all messages cost the same.

27. a) I subtracted the months: 5 – 2 = 3
I subtracted the amounts saved: \$280 – \$145 = \$135

To calculate the amount saved each month, I divided: $\frac{\$135}{3}$ = \$45

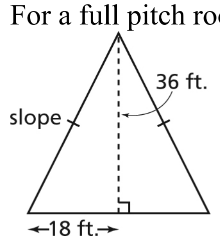
If I graphed the Account balance against the Months saved, the slope of the graph would be the amount saved per month.

b) Charin saves \$45 a month.
So, in 10 months, Charin will have saved: 10(\$45) = \$450

c) Charin saves \$45 a month. So, in 2 months, he saves \$90. I subtract this amount from the account balance after 2 months to determine how much money was in the account at the start: \$145 – \$90 = \$55

d) I assumed that the savings account did not earn any interest, or, if it did, then it was not paid into the account in the first 5 months.

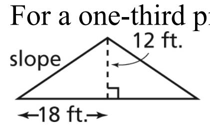
28. a) For a full pitch roof, the height and span are equal.



To calculate the slope, I divided: $\frac{36 \text{ ft.}}{18 \text{ ft.}} = 2$

So, the slope is 2.

b) For a one-third pitch roof, the height is one-third the span.



To calculate the slope, I divided: $\frac{12 \text{ ft.}}{18 \text{ ft.}} = \frac{2}{3}$

So, the slope is $\frac{2}{3}$.

C

29. I calculated the slope of the glide path.
The rise is the change in altitude: 5500 m – 7000 m = –1500 m, or –1.5 km
The run is 18 km.

Slope = $\frac{-1.5 \text{ km}}{18 \text{ km}}$, or $-0.08\overline{3}$

Determine the run, r, when the rise is –2600 m, or –2.6 km.

$\text{Slope} = \frac{\text{rise}}{\text{run}}$ Substitute for slope, rise, and run.

$-0.08\overline{3} = \frac{-2.6 \text{ km}}{r}$ Multiply each side by r.

$-0.08\overline{3}\,r = -2.6 \text{ km}$ Divide each side by $-0.08\overline{3}$.

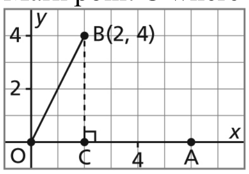
$r = \frac{-2.6 \text{ km}}{-0.08\overline{3}}$

$r = 31.2 \text{ km}$

When the rise is –2.6 km, the run is 31.2 km. This means that when the plane was at an altitude of 2600 m, it travelled a horizontal distance of 31.2 km before it landed. Since Winnipeg was 63 km away, the plane could not have reached Winnipeg.

30. Positions of point A may vary.

a), b) Draw segment OB. Mark a point A anywhere on the positive x-axis, such as at (6, 0). Mark point C where the vertical line through B intersects the x-axis.



From the diagram,

Slope of OB is: $\frac{4}{2} = 2$

In right ΔOBC

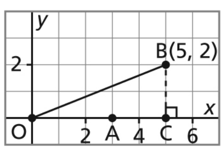
$\tan \angle COB = \frac{\text{opposite}}{\text{adjacent}}$

$\tan \angle COB = \frac{4}{2}$

$\tan \angle COB = 2$

Since both C and A are on the same line, $\tan \angle COB = \tan \angle AOB = 2$

c) Draw segment OB. Mark a point A anywhere on the positive x-axis, such as at (3, 0). Mark point C where the vertical line through B intersects the x-axis.



From the diagram,

Slope of OB is: $\frac{2}{5}$

In right ΔOBC

$\tan \angle COB = \frac{\text{opposite}}{\text{adjacent}}$

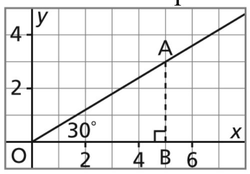
$\tan \angle COB = \frac{2}{5}$

Since both C and A are on the same line, $\tan \angle COB = \tan \angle AOB = \frac{2}{5}$

d) The slope of a line segment is equal to the tangent of the angle formed by the segment and the positive x-axis. Both the slope and the tangent are equal to the quotient of the same two numbers.

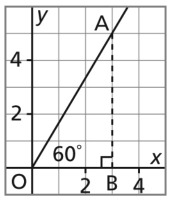
31. (Solution if Exercise 30 has been previously completed.)

a) Label the oblique arm of the 30° angle as OA.



From exercise 30, the slope of OA is tan 30° = 0.5773…

b) Label the oblique arm of the 60° angle as OB.

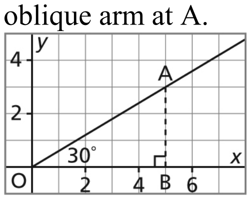


From exercise 30, the slope of OB is tan 60° = 1.7320…

c) For an angle with one arm horizontal and one arm oblique:
Since 1.7320… is not double 0.5773…; when an angle doubles, the slope of the oblique arm does not double.

31. (Solution if Exercise 30 has not been previously completed.)

a) Draw the 30° angle. From B(5, 0), draw the perpendicular to the *x*-axis to intersect the oblique arm at A.



Determine the rise AB.
In right ΔAOB:

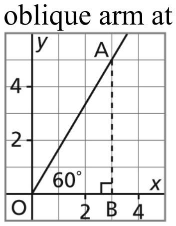
$\tan \angle AOB = \dfrac{AB}{OB}$ Substitute: ∠AOB = 30° and OB = 5

$\tan 30° = \dfrac{AB}{5}$ Multiply each side by 5.

$AB = 5 \tan 30°$

So, the slope of OA is: $\dfrac{5 \tan 30°}{5} = \tan 30°$

b) Draw the 60° angle. From B(3, 0), draw the perpendicular to the *x*-axis to intersect the oblique arm at A.



Determine the rise AB.
In right ΔAOB:

$\tan \angle AOB = \frac{AB}{OB}$ Substitute: $\angle AOB = 60°$ and $OB = 3$

$\tan 60° = \frac{AB}{3}$ Multiply each side by 3.

$AB = 3 \tan 60°$

So, the slope of OA is: $\frac{3 \tan 60°}{3} = \tan 60°$

c) $\tan 30° = 0.5773\ldots$ and $\tan 60° = 1.7320\ldots$
For an angle with one arm horizontal and one arm oblique:
Since 1.7320… is not double 0.5773…; when an angle doubles, the slope of the oblique arm does not double.

Lesson 6.2 **Slopes of Parallel and Perpendicular Lines** **Exercises (pages 349–351)**

A

3. Parallel lines have the same slope.

a) A parallel line has slope $\frac{4}{5}$.

b) A parallel line has slope $-\frac{4}{3}$.

c) A parallel line has slope 3.

d) A parallel line has slope 0.

4. Perpendicular lines have slopes that are negative reciprocals. When the slope of a line is a fraction, to determine the slope of a perpendicular line, invert the fraction and change its sign. When the slope of a line is an integer, write it in fraction form with denominator 1; then invert and change its sign.

a) A perpendicular line has slope $-\frac{6}{7}$.

b) A perpendicular line has slope $\frac{8}{5}$.

c) Write the slope 9 as $\frac{9}{1}$. A perpendicular line has slope $-\frac{1}{9}$.

d) Write the slope –5 as $-\frac{5}{1}$. A perpendicular line has slope $\frac{1}{5}$.

5. Two lines are parallel when their slopes are equal. Two lines are perpendicular when the product of their slopes is –1.

a) The slopes are equal, so the lines are parallel.

b) The slopes are not equal and their product is not –1, so the lines are neither parallel nor perpendicular.

c) The slopes are not equal and their product is not –1, so the lines are neither parallel nor perpendicular.

d) The product of the slopes is –1, so the lines are perpendicular.

6. Two lines are parallel when their slopes are equal.
Perpendicular lines have slopes that are negative reciprocals. When the slope of a line is a fraction, to determine the slope of a perpendicular line, invert the fraction and change its sign.

When the slope of a line is an integer, write it in fraction form with denominator 1; then invert and change its sign.

a) For a line with slope $-\frac{4}{9}$:

i) A parallel line has slope $-\frac{4}{9}$.

ii) A perpendicular line has slope $\frac{9}{4}$.

b) For a line with slope 5:

i) A parallel line has slope 5.

ii) Write the slope as $\frac{5}{1}$. A perpendicular line has slope $-\frac{1}{5}$.

c) For a line with slope $\frac{7}{3}$:

i) A parallel line has slope $\frac{7}{3}$.

ii) A perpendicular line has slope $-\frac{3}{7}$.

d) For a line with slope –4:

i) A parallel line has slope –4.

ii) Write the slope as $\frac{-4}{1}$. A perpendicular line has slope $\frac{1}{4}$.

B

7. Calculate the slope of the club and the slope of the line through the tips of the golfer's shoes.
For the slope of the club:
From the diagram, the rise is approximately –1 and the run is approximately 6.

Slope of the club is approximately: $-\frac{1}{6}$

For the slope of the line through the tips of the golfer's shoes:
From the diagram, the rise is approximately –1 and the run is approximately 6.

Slope of the line through tips of shoes is approximately: $-\frac{1}{6}$

Since the slopes are the same, the club is parallel to the line through the golfer's shoes, and the golfer is set up correctly.

8. Lines are parallel if they have the same slope. Perpendicular lines have slopes that are negative reciprocals. So, calculate the slope of each line segment then check to see if they are equal or are negative reciprocals. Use the formula: Slope = $\frac{\text{change in } y}{\text{change in } x}$

a) i) A(–5, –2), B(1, 5); and C(–1, –4), D(4, 1)

ii) Slope of AB = $\frac{5-(-2)}{1-(-5)}$

Slope of AB = $\frac{7}{6}$

Slope of CD = $\frac{1-(-4)}{4-(-1)}$

Slope of CD = $\frac{5}{5}$, or 1

The slopes of AB and CD are not equal, nor are they negative reciprocals, so the lines are neither parallel nor perpendicular.

b) i) E(–3, 4), F(3, 2); and G(2, 5), H(0, –1)

ii) Slope of EF = $\frac{2-4}{3-(-3)}$

Slope of EF = $\frac{-2}{6}$, or $-\frac{1}{3}$

Slope of GH = $\frac{5-(-1)}{2-0}$

Slope of GH = $\frac{6}{2}$, or 3

Since $3\left(-\frac{1}{3}\right) = -1$; the slopes of EF and GH are negative reciprocals, so the lines are perpendicular.

c) i) J(–2, 3), K(1, –3); and M(3, 1), N(–4, –2)

ii) Slope of JK = $\frac{-3-3}{1-(-2)}$

Slope of JK = $\frac{-6}{3}$, or –2

Slope of MN = $\frac{-2-1}{-4-3}$

Slope of MN = $\frac{-3}{-7}$, or $\frac{3}{7}$

The slopes of JK and MN are not equal, nor are they negative reciprocals, so the lines are neither parallel nor perpendicular.

d) i) P(0, 5), Q(6, 2); and R(–4, –1), S(0, –3)

ii) Slope of PQ = $\frac{2-5}{6-0}$

Slope of PQ = $\frac{-3}{6}$, or $-\frac{1}{2}$

Slope of RS = $\frac{-3-(-1)}{0-(-4)}$

Slope of RS = $\frac{-2}{4}$, or $-\frac{1}{2}$

The slopes of AB and CD are equal, so the lines are parallel.

9. Lines are parallel if they have the same slope. Perpendicular lines have slopes that are negative reciprocals. So, calculate the slope of each line segment then check to see if the slopes are equal or are negative reciprocals. Use the formula: Slope = $\frac{\text{change in } y}{\text{change in } x}$

a) Slope of ST = $\frac{5-(-1)}{-1-(-4)}$

Slope of ST = $\frac{6}{3}$, or 2

Slope of UV = $\frac{-1-1}{5-1}$

Slope of UV = $\frac{-2}{4}$, or $-\frac{1}{2}$

The slopes of ST and UV are negative reciprocals, so the line segments are perpendicular.

b) Slope of BC = $\frac{3-(-2)}{-3-(-6)}$

Slope of BC = $\frac{5}{3}$

Slope of DE = $\frac{5-0}{5-2}$

Slope of DE = $\frac{5}{3}$

The slopes of BC and DE are equal, so the line segments are parallel.

c) Slope of NP = $\frac{-4-2}{-3-(-6)}$

Slope of NP = $\frac{-6}{3}$, or –2

Slope of QR = $\dfrac{4-(-3)}{3-1}$

Slope of QR = $\dfrac{7}{2}$

The slopes of NP and QR are not equal, nor are they negative reciprocals, so the line segments are neither parallel nor perpendicular.

d) Slope of GH = $\dfrac{1-5}{4-(-2)}$

Slope of GH = $\dfrac{-4}{6}$, or $-\dfrac{2}{3}$

Slope of JK = $\dfrac{0-(-4)}{7-1}$

Slope of JK = $\dfrac{4}{6}$, or $\dfrac{2}{3}$

The slopes of GH and JK are not equal, nor are they negative reciprocals, so the line segments are neither parallel nor perpendicular.

10. a) For DE, the coordinates of the points at the intercepts are: (4, 0) and (0, –6)

Slope of DE = $\dfrac{-6-0}{0-4}$

Slope of DE = $\dfrac{-6}{-4}$, or $\dfrac{3}{2}$

For FG, the coordinates of the points at the intercepts are: (–6, 0) and (0, 4)

Slope of FG = $\dfrac{4-0}{0-(-6)}$

Slope of FG = $\dfrac{4}{6}$, or $\dfrac{2}{3}$

Both lines have positive slopes, which are reciprocals.

b) For HJ, the coordinates of the points at the intercepts are: (–2, 0) and (0, 3)

Slope of HJ = $\dfrac{3-0}{0-(-2)}$

Slope of HJ = $\dfrac{3}{2}$

For KM, the coordinates of the points at the intercepts are: (–9, 0) and (0, 6)

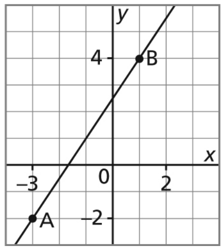
Slope of KM = $\dfrac{6-0}{0-(-9)}$

Slope of KM = $\dfrac{6}{9}$, or $\dfrac{2}{3}$

Both lines have positive slopes, which are reciprocals.

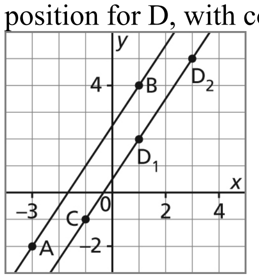
11. a) Slope of AB = $\dfrac{4-(-2)}{1-(-3)}$

Slope of AB = $\dfrac{6}{4}$, or $\dfrac{3}{2}$



b) Line CD is parallel to line AB, so line CD has slope $\dfrac{3}{2}$.

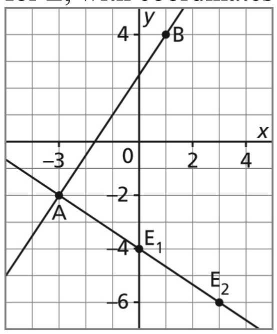
c) Since line CD has slope $\dfrac{3}{2}$, its rise is 3 and its run is 2. From point C, move 3 squares up and 2 squares right to get to one possible position for D, with coordinates (1, 2). From this point, move 3 squares up and 2 squares right again to get to a second possible position for D, with coordinates (3, 5).



My answers may be different because a classmate may choose the coordinates of any other point on the line CD that I drew.

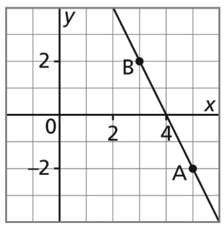
d) Line AE is perpendicular to line AB; so the slope of AE is the negative reciprocal of $\dfrac{3}{2}$, which is $-\dfrac{2}{3}$.

e) Since line AE has slope $-\dfrac{2}{3}$, its rise is –2 and its run is 3. From point A, move 2 squares down and 3 squares right to get to one possible for E, with coordinates (0, –4). From this point, move 2 squares down and 3 squares right again to get to a second possible position for E, with coordinates (3, –6).



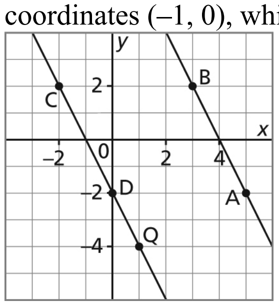
12. a) Slope of AB = $\dfrac{2-(-2)}{3-5}$

Slope of AB = $\dfrac{4}{-2}$, or –2



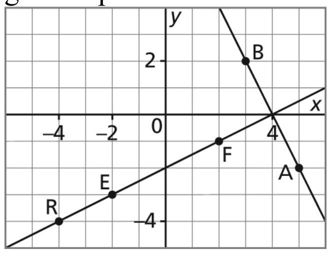
b) Line CD is parallel to line AB, so line CD has slope –2.

c) Since line CD has slope –2, its rise is 2 and its run is –1. From point Q, move 2 squares up and 1 square left to get to a point with coordinates (0, –2), which is the *y*-intercept. From this point, move 2 squares up and 1 square left again to get to a point with coordinates (–1, 0), which is the *x*-intercept.



d) Line EF is perpendicular to line AB; so the slope of EF is the negative reciprocal of –2, which is $\dfrac{1}{2}$.

e) Since line EF has slope $\dfrac{1}{2}$, its rise is 1 and its run is 2. From point R, move 1 square up and 2 squares right to get to a point with coordinates (–2, –3). From this point, move 1 square up and 2 squares right again to get to a point with coordinates (0, –2), which is the *y*-intercept. From this point, move 1 square up and 2 squares right again to get to a point with coordinates (2, –1). From this point, move 1 square up and 2 squares right again to get to a point with coordinates (4, 0), which is the x-intercept.



13. a) Quadrilateral HJKM is a parallelogram if its opposite sides are parallel.
I counted squares to determine the rise and the run of each side.

Slope of HJ = $\frac{2}{7}$

Slope of MK = $\frac{2}{7}$

Since the slopes of HJ and MK are equal, those sides are parallel.

Slope of HM = $\frac{-6}{2}$, or -3

Slope of JK = $\frac{-6}{2}$, or -3

Since the slopes of HM and JK are equal, those sides are parallel.
Since both pairs of opposite sides are parallel, quadrilateral HJKM is a parallelogram.

b) Quadrilateral HJKM is a rectangle if its angles are right angles; that is, if adjacent sides are perpendicular.

The slopes of adjacent sides are $\frac{2}{7}$ and -3; since these are not negative reciprocals, the sides are not perpendicular, and the parallelogram is not a rectangle.

14. From the diagram, I can see that sides GF and DE are not parallel.
I check to see if sides GD and FE are parallel.
I counted squares to determine the rise and the run of each side.

Slope of GD = $\frac{3}{3}$, or 1

Slope of FE = $\frac{4}{4}$, or 1

Since the slopes of GD and FE are equal, those sides are parallel and quadrilateral DEFG is a trapezoid.

15. When I know the coordinates of Q and R, I can calculate the slope of QR. This slope is equal to the slope of the opposite side of the rectangle, TS. The other pair of opposite sides, QT and RS, also have equal slopes. This slope is the negative reciprocal of the slope of QR because the sides are perpendicular to QR. But I do not know the lengths of sides RS and QT, so I cannot determine the coordinates of S and T.

16. I calculate the slopes of the 3 sides of ΔABC, then check to see which two of the slopes are negative reciprocals.
A(–3, 1), B(6, –2), and C(3, 4)

Slope = $\dfrac{\text{change in } y}{\text{change in } x}$

Slope of AB = $\dfrac{-2-1}{6-(-3)}$

Slope of AB = $-\dfrac{3}{9}$

Slope of AB = $-\dfrac{1}{3}$

Slope of BC = $\dfrac{4-(-2)}{3-6}$

Slope of BC = $\dfrac{6}{-3}$, or -2

Slope of AC = $\dfrac{4-1}{3-(-3)}$

Slope of AC = $\dfrac{3}{6}$, or $\dfrac{1}{2}$

Since -2 and $\dfrac{1}{2}$ are negative reciprocals, then BC is perpendicular to AC, and ΔABC is a right triangle, with $\angle C = 90°$.

17. I calculate the slopes of the 3 sides of ΔDEF, then check to see if any two of the slopes are negative reciprocals.
D(–3, –2), E(1, 4), and F(4, 2)

Slope = $\dfrac{\text{change in } y}{\text{change in } x}$

Slope of DE = $\dfrac{4-(-2)}{1-(-3)}$

Slope of DE = $\dfrac{6}{4}$, or $\dfrac{3}{2}$

Slope of DF = $\dfrac{2-(-2)}{4-(-3)}$

Slope of DF = $\dfrac{4}{7}$

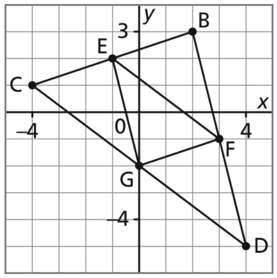
Slope of EF = $\dfrac{2-4}{4-1}$

Slope of EF = $-\dfrac{2}{3}$

Since $\dfrac{3}{2}$ and $-\dfrac{2}{3}$ are negative reciprocals, then DE is perpendicular to EF, and ΔDEF is a right triangle, with $\angle E = 90°$.

18. Triangles may vary. I chose vertices so that the midpoints of the lines lie where grid lines intersect. Triangle BCD has vertices B(2, 3), C(–4, 1), and D(4, –5).

a)



$$\text{Slope} = \frac{\text{change in } y}{\text{change in } x}$$

$$\text{Slope of BC} = \frac{3-1}{2-(-4)}$$

$$\text{Slope of BC} = \frac{2}{6}, \text{ or } \frac{1}{3}$$

$$\text{Slope of CD} = \frac{-5-1}{4-(-4)}$$

$$\text{Slope of CD} = \frac{-6}{8}, \text{ or } -\frac{3}{4}$$

$$\text{Slope of BD} = \frac{-5-3}{4-2}$$

$$\text{Slope of BD} = \frac{-8}{2}, \text{ or } -4$$

b) The midpoints of the sides are: E(–1, 2), F(3, –1), and G(0, –2); they form ΔEFG.

$$\text{Slope} = \frac{\text{change in } y}{\text{change in } x}$$

$$\text{Slope of EF} = \frac{-1-2}{3-(-1)}$$

$$\text{Slope of EF} = -\frac{3}{4}$$

$$\text{Slope of FG} = \frac{-2-(-1)}{0-3}$$

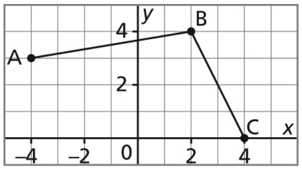
$$\text{Slope of FG} = \frac{-1}{-3}, \text{ or } \frac{1}{3}$$

$$\text{Slope of EG} = \frac{-2-2}{0-(-1)}$$

$$\text{Slope of EG} = \frac{-4}{1}, \text{ or } -4$$

c) The slopes of the sides of ΔBCD are equal to the slopes of the sides of ΔEFG; that is, BC is parallel to FG; BD is parallel to EG, and CD is parallel to EF. In each case, the segment that joins the midpoint of two sides of a triangle is parallel to the third side of the triangle.

19.



a) If ABCD was a rectangle, then AB would be perpendicular to BC. From the diagram, they do not look perpendicular, but I shall determine the slopes of these sides to verify.

$$\text{Slope} = \frac{\text{change in } y}{\text{change in } x}$$

$$\text{Slope of AB} = \frac{4 - 3}{2 - (-4)}$$

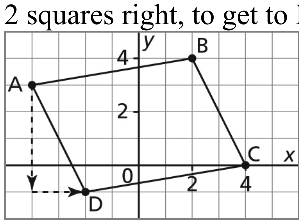
$$\text{Slope of AB} = \frac{1}{6}$$

$$\text{Slope of BC} = \frac{0 - 4}{4 - 2}$$

$$\text{Slope of BC} = \frac{-4}{2}, \text{ or } -2$$

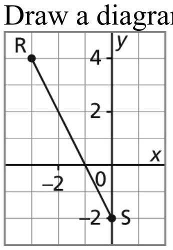
Since $\frac{1}{6}$ and –2 are not negative reciprocals, then AB is not perpendicular to BC and ABCD is not a rectangle.

b) Side AD is parallel to side BC, so they have the same rise and the same run.
BC has a rise of –4 and a run of 2. So, from point A, move 4 squares down, then 2 squares right, to get to D(–2, –1).



c) I could have used the fact that AB is parallel to DC, and moved 1 square down from C, then 6 squares left; I would get to the same point D.
I could also have added coordinates:
Point C has coordinates (4, 0). I could have added –1 to the y-coordinate, to represent a rise of –1, then added –6 to the x-coordinate to represent a rise of –6. Then the coordinates of D are: (4 – 6, 0 – 1), or (–2, –1)

20. Draw a diagram.



For a right triangle, ∠R, ∠S, or ∠T is a right angle.

Suppose ∠R is the right angle.
Then, RT is perpendicular to RS at R.
The slope of RT is the negative reciprocal of the slope of RS.
From the diagram, the rise of RS is –6 and the run of RS is 3.

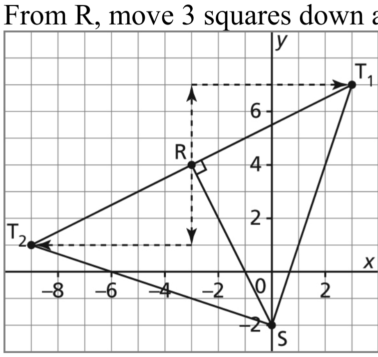
So the slope of RS is $\frac{-6}{3}$.

The negative reciprocal of $\frac{-6}{3}$ can be written as $\frac{3}{6}$.

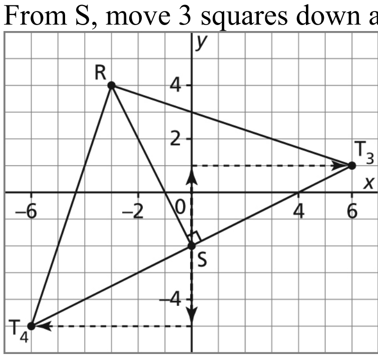
So, the rise of RT is 3 and the run of RT is 6.
From R, move 3 squares up and 6 squares right to reach point $T_1(3, 7)$.

The negative reciprocal of $\frac{-6}{3}$ can also be written as $\frac{-3}{-6}$

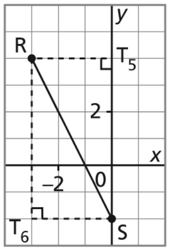
So, the rise of RT is –3 and the run of RT is –6.
From R, move 3 squares down and 6 squares left to reach $T_2(-9, 1)$.



Suppose ∠S is the right angle.
Repeat a similar process as for ∠R.
Then, ST is perpendicular to RS at S.
The slope of ST is the negative reciprocal of the slope of RS.
So, the rise of ST is 3 and the run of ST is 6.
From S, move 3 squares up and 6 squares right to reach point $T_3(6, 1)$.
So, the rise of ST is –3 and the run of ST is –6.
From S, move 3 squares down and 6 squares left to reach $T_4(-6, -5)$.



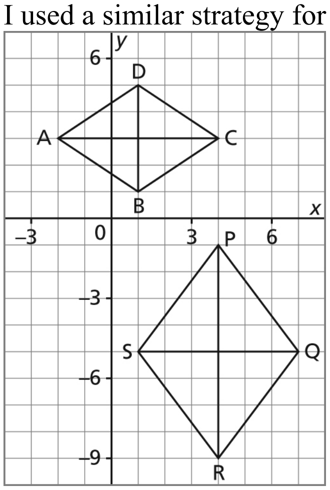
Suppose ∠T is the right angle. From the Angle in a Semicircle Property, T is a point on the circumference of a circle with diameter RS.



Possible coordinates for T are: (3, 7), (–9, 1), (6, 1), (–6, –5), (4, 0), and (–3, –2)

C

21. Rhombuses may vary.
For rhombus ABCD, to make sure all the sides were the same length and to make sure I didn't draw a square, I started at A and moved 2 squares down and 3 squares right to get to B. From point B, I moved 3 squares right and 2 squares up to reach C. From C I moved 2 squares up and 3 squares left to reach D. I then joined AD.
I used a similar strategy for rhombus PQRS.



Wherever I drew the rhombuses, the diagonals were always vertical and horizontal, so the diagonals intersect at right angles. To verify this, I calculated the slopes of the diagonals.
For rhombus ABCD, the diagonals are AC and BD.

Slope = $\dfrac{\text{change in } y}{\text{change in } x}$

Slope of AC = $\dfrac{3-3}{4-(-2)}$

Slope of AC = $\dfrac{0}{6}$, or 0; so AC is horizontal.

Slope of BD = $\dfrac{5-1}{1-1}$

Slope of BD = $\dfrac{4}{0}$, which is undefined; so BD is vertical.

Since AC is horizontal and BD is vertical, then the diagonals of the rhombus are perpendicular.
For rhombus PQRS, the diagonals are PR and SQ.

Slope = $\dfrac{\text{change in } y}{\text{change in } x}$

Slope of SQ = $\dfrac{-5-(-5)}{7-1}$

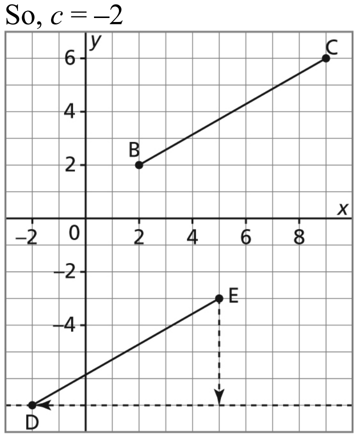
Slope of SQ = $\dfrac{0}{6}$, or 0; so SQ is horizontal.

Slope of PR = $\dfrac{-9-(-1)}{4-4}$

Slope of PR = $\dfrac{-8}{0}$, which is undefined; so PR is vertical.

Since SQ is horizontal and PR is vertical, then the diagonals of the rhombus are perpendicular.

22. Method 1: Use grid paper.
Plot points B(2, 2), C(9, 6), and E(5, –3). Draw a broken line through $y = -7$ because D lies on that line.
Since BC and DE are parallel, their slopes are equal.
From the graph, the rise from C to B is –4 and the run is –7.
So, from E, move 4 squares down and 7 squares left to reach D at (–2, –7).
So, $c = -2$



Method 2: Use algebra.
Determine the slope of each segment.

Slope = $\dfrac{\text{change in } y}{\text{change in } x}$

Slope of BC = $\dfrac{6-2}{9-2}$

Slope of BC = $\dfrac{4}{7}$

Slope of DE = $\dfrac{-3-(-7)}{5-c}$, or $\dfrac{4}{5-c}$

Since BC and DE are parallel, equate the two slopes:

$\dfrac{4}{7} = \dfrac{4}{5-c}$ Multiply each side by 7.

$$7\left(\frac{4}{7}\right) = 7\left(\frac{4}{5-c}\right)$$

$$4 = \frac{28}{5-c}$$ Multiply each side by $(5 - c)$.

$$4(5-c) = (5-c)\left(\frac{28}{5-c}\right)$$

$20 - 4c = 28$ Collect like terms.

$-4c = 28 - 20$

$-4c = 8$ Divide each side by –4.

$$c = \frac{8}{-4}$$

$c = -2$

23. Determine the slope of each segment.

$$\text{Slope} = \frac{\text{change in } y}{\text{change in } x}$$

$$\text{Slope of AB} = \frac{10-5}{7-3}$$

$$\text{Slope of AB} = \frac{5}{4}$$

$$\text{Slope of CD} = \frac{a-2}{1-0}$$

$$\text{Slope of CD} = \frac{a-2}{1}, \text{ or } a-2$$

a) Since AB and CD are parallel, equate the two slopes:

$$\frac{5}{4} = a - 2$$ Multiply each side by 4.

$$4\left(\frac{5}{4}\right) = 4(a-2)$$

$5 = 4a - 8$ Collect like terms.

$13 = 4a$ Divide each side by 4.

$$\frac{13}{4} = a$$

$a = 3.25$

b) Since AB and CD are perpendicular, then their slopes are negative reciprocals. That is, the product of the slopes is equal to –1.

$$\left(\frac{5}{4}\right)(a-2) = -1$$ Multiply each side by 4.

$$4\left(\frac{5}{4}\right)(a-2) = 4(-1)$$

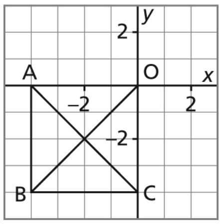
$5(a - 2) = -4$

$5a - 10 = -4$

$$5a = 6$$
$$a = \frac{6}{5}$$
$$a = 1.2$$

24. Positions of squares may vary.

a) The coordinates of the vertices are: O(0, 0), A(–4, 0), B(–4, –4), and C(0, –4)



The diagonals are AC and OB.

$$\text{Slope} = \frac{\text{change in } y}{\text{change in } x}$$

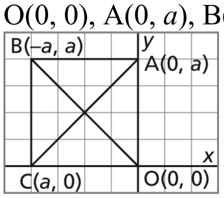
$$\text{Slope of AC} = \frac{-4-0}{0-(-4)}$$

$$\text{Slope of AC} = \frac{-4}{4}\text{, or } -1$$

$$\text{Slope of OB} = \frac{-4-0}{-4-0}\text{, or } 1$$

Since the slopes of OB and AC are negative reciprocals, then OB and AC are perpendicular.

b) Since each side of the square has length a units, the coordinates of the vertices are: O(0, 0), A(0, a), B($-a$, a), and C($-a$, $-a$)



The diagonals are AC and OB.

$$\text{Slope} = \frac{\text{change in } y}{\text{change in } x}$$

$$\text{Slope of AC} = \frac{0-a}{-a-0}$$

$$\text{Slope of AC} = \frac{-a}{-a}\text{, or } 1$$

$$\text{Slope of OB} = \frac{a-0}{-a-0}$$

$$\text{Slope of OB} = \frac{a}{-a}\text{, or } -1$$

Since the slopes of OB and AC are negative reciprocals, then OB and AC are perpendicular.

Checkpoint 1 **Assess Your Understanding (page 353)**

6.1

1. Slope = $\dfrac{\text{change in } y}{\text{change in } x}$

Point A has coordinates (–6, 5). Point B has coordinates (3, –1).

Slope of AB = $\dfrac{-1-5}{3-(-6)}$

Slope of AB = $\dfrac{-6}{9}$

Slope of AB = $-\dfrac{2}{3}$

Point C has coordinates (–5, –1). Point D has coordinates (3, 1).

Slope of CD = $\dfrac{1-(-1)}{3-(-5)}$

Slope of CD = $\dfrac{2}{8}$

Slope of CD = $\dfrac{1}{4}$

2. Slope = $\dfrac{\text{change in } y}{\text{change in } x}$

a) Point Q has coordinates (–2, 5). Point R has coordinates (2, –10).

Slope of QR = $\dfrac{-10-5}{2-(-2)}$

Slope of QR = $\dfrac{-15}{4}$

b) The point at the x-intercept has coordinates A(3, 0). The point at the y-intercept has coordinates B(0, –5).

Slope of AB = $\dfrac{-5-0}{0-3}$

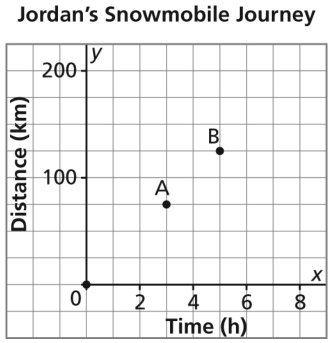
Slope of AB = $\dfrac{-5}{-3}$

Slope of AB = $\dfrac{5}{3}$

The slope of the line through the intercepts is $\dfrac{5}{3}$.

3. The slope of a line is equal to the slope of any segment of the line, so we can use any two points that form that segment to determine the slope of the line.

4. a) Label two points on the line: A(3, 75) and B(5, 125)
Label the axes x and y.



$$\text{Slope} = \frac{\text{change in } y}{\text{change in } x}$$

$$\text{Slope of AB} = \frac{125\text{ km} - 75\text{ km}}{5\text{ h} - 3\text{ h}}$$

$$\text{Slope of AB} = \frac{50\text{ km}}{2\text{ h}}$$

Slope of AB = 25 km/h
The slope is Jordan's average speed.

b) Since Jordan travels at an average speed of 25 km/h, in $1\frac{1}{4}$ h, he will travel:

25 km/h (1.25 h) = 31.25 km

In $1\frac{1}{4}$ h, Jordan will travel approximately 31 km.

c) Jordan travels at an average speed of 25 km/h. To travel 65 km, it will take him:

$$\frac{65\text{ km}}{25\text{ km/h}} = 2.6\text{ h}$$

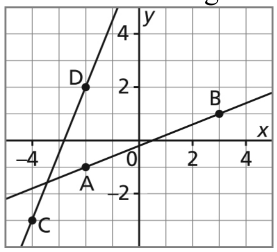
There are 60 min in 1 h, so 0.6 h is 0.6(60 min) = 36 min
It will take Jordan 2 h 36 min to travel 65 km.

6.2

5. The positions of the lines on the grids and their labels may vary.

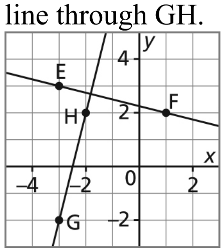
a) The lines are neither parallel nor perpendicular because the slopes are neither equal nor are they negative reciprocals.

A line with slope $\frac{2}{5}$ has a rise of 2 and a run of 5. To draw line AB with slope $\frac{2}{5}$, I began at point A, moved 2 squares up and 5 squares right to reach point B. I drew a line through AB. A line with slope $\frac{5}{2}$ has a rise of 5 and a run of 2. To draw line CD with slope $\frac{5}{2}$, I began at point C, moved 5 squares up and 2 squares right to reach point D. I drew a line through CD.

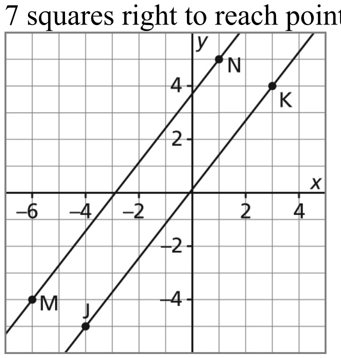


b) The lines are perpendicular because their slopes are negative reciprocals.

A line with slope $-\frac{1}{4}$ has a rise of –1 and a run of 4. To draw line EF with slope $-\frac{1}{4}$, I began at point E, moved 1 square down and 4 squares right to reach point F. I drew a line through EF. A line with slope 4 has a rise of 4 and a run of 1. To draw line GH with slope 4, I began at point G, moved 4 squares up and 1 squares right to reach point H. I drew a line through GH.



c) The lines are parallel because their slopes are equal; that is, the fraction $\frac{18}{14}$ simplifies to $\frac{9}{7}$. A line with slope $\frac{9}{7}$ has a rise of 9 and a run of 7. To draw line JK with slope $\frac{9}{7}$, I began at point J, moved 9 squares up and 7 squares right to reach point K. I drew a line through JK. To draw line MN with slope $\frac{9}{7}$, I began at point M, moved 9 squares up and 7 squares right to reach point N. I drew a line through MN.



6. Coordinates may vary.
Draw a line through the points D(–6, –1) and E(2, 5).

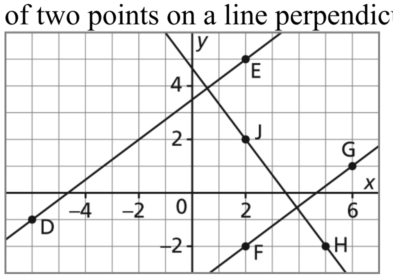
Slope = $\dfrac{\text{change in } y}{\text{change in } x}$

Slope of DE $= \dfrac{5-(-1)}{2-(-6)}$

Slope of DE $= \dfrac{6}{8}$, or $\dfrac{3}{4}$

a) A line parallel to DE has slope $\dfrac{3}{4}$. To draw a parallel line FG, choose any point such as F(2, –2). From F, move 3 squares up and 4 squares right to reach point G with coordinates (6, 1). So, the coordinates of two points on a line parallel to DE are (2, –2) and (6, 1).

b) A line perpendicular to DE has a slope that is the negative reciprocal of $\dfrac{3}{4}$; that is, $-\dfrac{4}{3}$. To draw a perpendicular line HJ, choose any point such as H(5, –2). From H, move 4 squares up and 3 squares left to reach point J with coordinates (2, 2). So, the coordinates of two points on a line perpendicular to DE are (5, –2) and (2, 2).



7. The triangle is a right triangle if two sides are perpendicular; that is, they form a right angle.

Slope $= \dfrac{\text{change in } y}{\text{change in } x}$

Slope of AB $= \dfrac{-6-5}{-5-(-1)}$

Slope of AB $= \dfrac{-11}{-4}$, or $\dfrac{11}{4}$

Slope of BC $= \dfrac{1-(-6)}{3-(-5)}$

Slope of BC $= \dfrac{7}{8}$

Slope of AC $= \dfrac{1-5}{3-(-1)}$

Slope of AC $= \dfrac{-4}{4}$, or -1

Since no two slopes are negative reciprocals, no two sides are perpendicular, and the triangle is not a right triangle.

8. Plot M and P on a grid.

Slope $= \dfrac{\text{change in } y}{\text{change in } x}$

Slope of MP = $\dfrac{-3-6}{3-(-3)}$

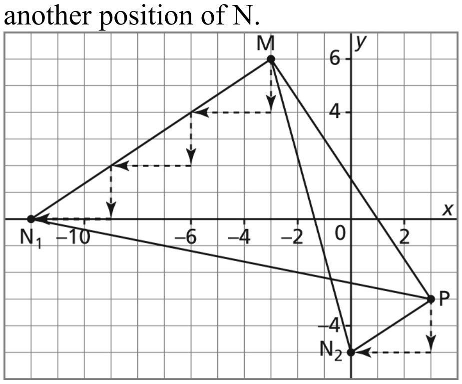
Slope of MP = $\dfrac{-9}{6}$, or $-\dfrac{3}{2}$

So, if the right angle is at M, the slope of MN is the negative reciprocal of $-\dfrac{3}{2}$, which is $\dfrac{2}{3}$.

From point M, I move 2 squares down and 3 squares left. I continue to do this until I reach the x-axis at (–12, 0). This is one position of N.

If the right angle is at P, the slope of PN is $\dfrac{2}{3}$.

From point P, I move 2 squares down and 3 squares left. I reach the y-axis at (0, –5). This is another position of N.



Lesson 6.3 **Math Lab: Investigating Graphs of Linear Functions** **Assess Your Understanding (page 356)**

1. Each line has an equation of the form $y = mx + b$, where m is the slope and b is the y-intercept.

a) Since the slope of each line is $\frac{1}{2}$, then the equation of each line has the form $y = \frac{1}{2}x + b$

From the top of the screen to the bottom, the equations are:

$y = \frac{1}{2}x + 4$, $y = \frac{1}{2}x + 2$, $y = \frac{1}{2}x - 1$, $y = \frac{1}{2}x - 2$, $y = \frac{1}{2}x - 3$

b) Since the slope of each line is $-\frac{1}{3}$, then the equation of each line has the form

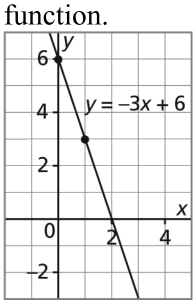
$y = -\frac{1}{3}x + b$

From the top of the screen to the bottom, the equations are:

$y = -\frac{1}{3}x + 4$, $y = -\frac{1}{3}x + 3$, $y = -\frac{1}{3}x + 1$, $y = -\frac{1}{3}x - 2$, $y = -\frac{1}{3}x - 3$

2. The graph of a function with an equation of the form $y = mx + b$ has a slope of m and a y-intercept of b. To graph the function, I would mark a point at the y-intercept, then use the slope to identify the rise and run. From the y-intercept, I would move vertically a number of squares equal to the rise, then horizontally a number of squares equal to the run, then mark another point. When I draw a line through the points, I have drawn a graph of the function.

3. The graph of the linear function with equation $y = -3x + 6$ has a y-intercept of 6 and a slope of –3. From 6 on the y-axis, I move 3 squares down, then 1 square right and mark a point. I draw a line through this point and the point at the y-intercept. This line is the graph of the function.



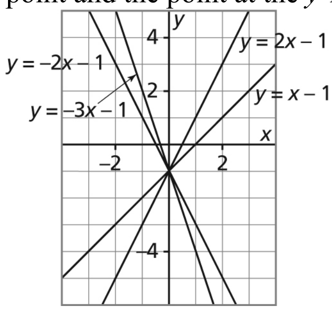
4. **a)** Predictions may vary. For example:
When m is positive, the graphs go up to the right. As m increases, the graphs get steeper.
When m is negative, the graphs go down to the right. As m decreases, the graphs get steeper.

b) **i)** The graph of $y = x - 1$ has slope 1 and y-intercept –1. From –1 on the y-axis, I move 1 square up, then 1 square right and mark a point. I draw a line through this point and the point at the y-intercept.

ii) The graph of $y = 2x - 1$ has slope 2 and y-intercept –1. From –1 on the y-axis, I move 2 squares up, then 1 square right and mark a point. I draw a line through this point and the point at the y-intercept.

iii) The graph of $y = -3x - 1$ has slope –3 and y-intercept –1. From –1 on the y-axis, I move 3 squares down, then 1 square right and mark a point. I draw a line through this point and the point at the y-intercept.

iv) The graph of $y = -2x - 1$ has slope –2 and y-intercept –1. From –1 on the y-axis, I move 2 squares down, then 1 square right and mark a point. I draw a line through this point and the point at the y-intercept.



The graphs show that my predictions in part a were correct.

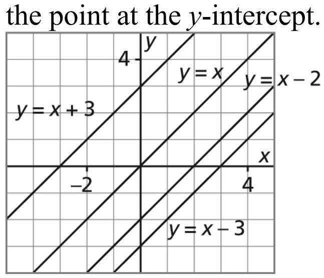
5. **a)** Predictions may vary. For example:
The graph moves up if b is increasing and the graph moves down if b is decreasing.

b) **i)** The graph of $y = x - 3$ has slope 1 and y-intercept –3. From –3 on the y-axis, I move 1 square up, then 1 square right and mark a point. I draw a line through this point and the point at the y-intercept.

ii) The graph of $y = x - 2$ has slope 1 and y-intercept –2. From –2 on the y-axis, I move 1 square up, then 1 square right and mark a point. I draw a line through this point and the point at the y-intercept.

iii) The graph of $y = x$ has slope 1 and y-intercept 0. From the origin, I move 1 square up, then 1 square right and mark a point. I draw a line through this point and the point at the y-intercept.

iv) The graph of $y = x + 3$ has slope 1 and y-intercept 3. From 3 on the y-axis, I move 1 square up, then 1 square right and mark a point. I draw a line through this point and the point at the y-intercept.

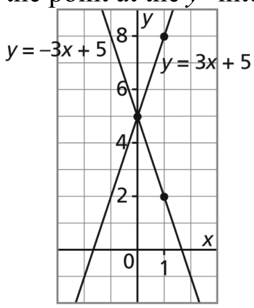


The graphs show that my prediction in part a was correct.

6. I used the same strategy as in question 3.

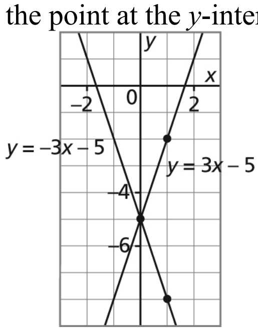
a) The graph of $y = 3x + 5$ has slope 3 and y-intercept 5. From 5 on the y-axis, I move 3 squares up, then 1 square right and mark a point. I draw a line through this point and the point at the y-intercept.

b) The graph of $y = –3x + 5$ has slope –3 and y-intercept 5. From 5 on the y-axis, I move 3 squares down, then 1 square right and mark a point. I draw a line through this point and the point at the y- intercept.

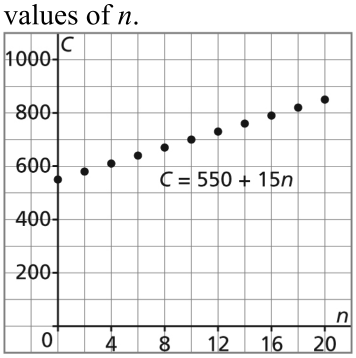


c) The graph of $y = 3x – 5$ has slope 3 and y-intercept –5. From –5 on the y-axis, I move 3 squares up, then 1 square right and mark a point. I draw a line through this point and the point at the y-intercept.

d) The graph of $y = –3x – 5$ has slope –3 and y-intercept –5. From –5 on the y-axis, I move 3 squares down, then 1 square right and mark a point. I draw a line through this point and the point at the y-intercept.



7. a) The graph of $C = 550 + 15n$ has a C-intercept of 550 and a slope of 15. Since n is a whole number, the graph is a set of points. From 550 on the C-axis, I move 15 up and 1 right. Since it is difficult to move 15 up when the scale is 1 square represents \$100, I move 150 up and 10 right, then mark a point. From this point, I move 150 up and 10 right, then mark another point. I use a straightedge to mark more points along this line at different values of n.



b) *m* represents the slope or rate of change; that is, \$15 per person. *b* represents the initial cost of \$550 to rent the hall. After this has been paid, each additional person costs \$15.

Lesson 6.4 **Slope-Intercept Form of the Equation for a Linear Function** **Exercises (pages 362–364)**

A

4. Compare each given equation with the equation of the form $y = mx + b$, where m is the slope and b is the y-intercept.

a) The graph of $y = 4x - 7$ has slope 4 and y-intercept –7.

b) When a coefficient of a variable is not indicated, the coefficient is 1. The graph of $y = x + 12$ has slope 1 and y-intercept 12.

c) The graph of $y = -\frac{4}{9}x + 7$ has slope $-\frac{4}{9}$ and y-intercept 7.

d) The graph of $y = 11x - \frac{3}{8}$ has slope 11 and y-intercept $-\frac{3}{8}$.

e) The equation $y = \frac{1}{5}x$ can be written as $y = \frac{1}{5}x + 0$. The slope is $\frac{1}{5}$ and the y-intercept is 0; the graph passes through the origin.

f) The equation $y = 3$ can be written as $y = 0x + 3$. The slope is 0 and the y-intercept is 3.

5. Substitute the given values for the slope m and the y-intercept b into the equation $y = mx + b$.

a) With slope 7 and y-intercept 16, the equation is: $y = 7x + 16$

b) With slope $-\frac{3}{8}$ and y-intercept 5, the equation is: $y = -\frac{3}{8}x + 5$

c) The point H(0, –3) represents a y-intercept of –3; the slope is $\frac{7}{16}$; so the equation is:

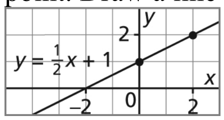
$$y = \frac{7}{16}x - 3$$

d) With slope $-\frac{6}{5}$ and y-intercept –8, the equation is: $y = -\frac{6}{5}x - 8$

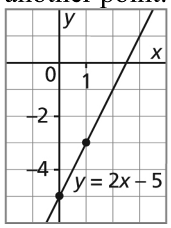
e) The origin O(0, 0) represents a y-intercept of 0; the slope is $-\frac{5}{12}$; so the equation is:

$$y = -\frac{5}{12}x$$

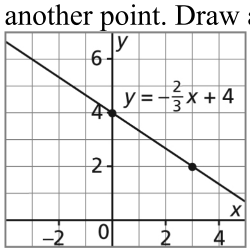
6. a) Mark a point at 1 on the y-axis, then move 1 square up and 2 squares right. Mark another point. Draw a line through the points.



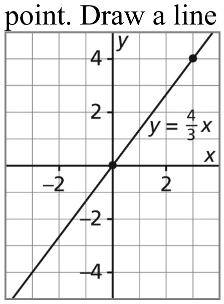
b) Mark a point at –5 on the y-axis, then move 2 squares up and 1 square right. Mark another point. Draw a line through the points.



c) Mark a point at 4 on the y-axis, then move 2 squares down and 3 squares right. Mark another point. Draw a line through the points.

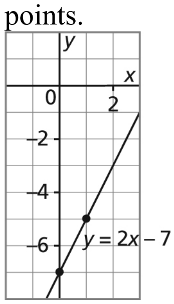


d) Mark a point at the origin, then move 4 squares up and 3 squares right. Mark another point. Draw a line through the points.

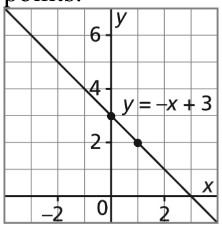


B

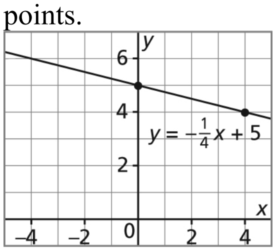
7. a) The slope of the line is 2 and its y-intercept is –7. Mark a point at –7 on the y-axis, then move 2 squares up and 1 square right. Mark another point. Draw a line through the points.



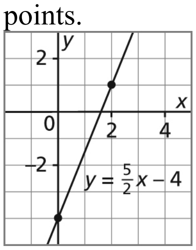
b) The slope of the line is –1 and its y-intercept is 3. Mark a point at 3 on the y-axis, then move 1 square down and 1 square right. Mark another point. Draw a line through the points.



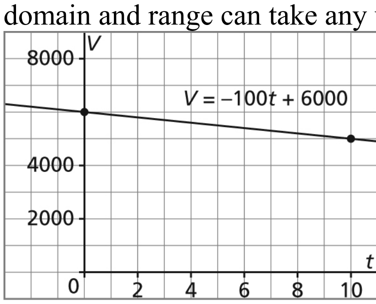
c) The slope of the line is $-\frac{1}{4}$ and its y-intercept is 5. Mark a point at 5 on the y-axis, then move 1 square down and 4 squares right. Mark another point. Draw a line through the points.



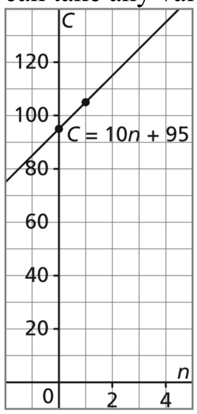
d) The slope of the line is $\frac{5}{2}$ and its y-intercept is –4. Mark a point at –4 on the y-axis, then move 5 squares up and 2 squares right. Mark another point. Draw a line through the points.



e) The slope of the line is –100 and its V-intercept is 6000. Mark a point at 6000 on the V-axis, then move 1000 units down and 10 units right. Mark another point. I assume the domain and range can take any values so I draw a line through the points.



f) The slope of the line is 10 and its C-intercept is 95. Mark a point at 95 on the C-axis, then move 10 units up and 1 unit right. Mark another point. I assume the domain and range can take any values so I draw a line through the points.



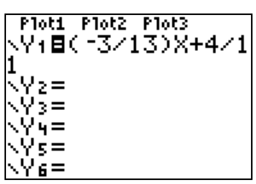
8. a) An equation of the line has the form $C = mt + b$.
Substitute $m = 50$ because the slope or rate of change is \$50/h.
Substitute $b = 80$ because the initial fee is \$80.
An equation is: $C = 50t + 80$

b) Use the general form of the equation, $C = mt + b$.
Substitute $m = 40$ because the new slope or rate of change is \$40/h.
Substitute $b = 100$ because the new initial fee is \$100.
An equation is: $C = 40t + 100$

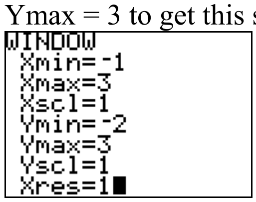
9. An equation of the line has the form $F = md + b$.
Substitute $m = 0.02$ because the slope or rate of change is 2% of the amount d dollars.
Substitute $b = 3.50$ because the withdrawal fee is \$3.50.
An equation is: $F = 0.02d + 3.50$

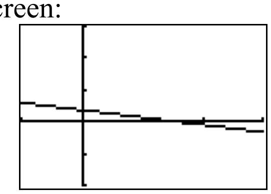
10. Screens may vary.
I used a graphing calculator.

a) I pressed $\boxed{\text{Y=}}$, then I input the expression $\text{Y1} = \left(\frac{-3}{13}\right)\text{X}+\frac{4}{11}$.

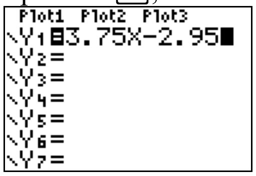


I pressed $\boxed{\text{GRAPH}}$. I changed the $\boxed{\text{WINDOW}}$ to Xmin = –1, Xmax = 3, Ymin = –2, and Ymax = 3 to get this screen:

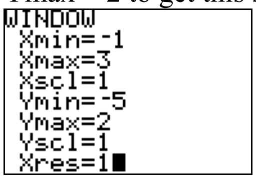


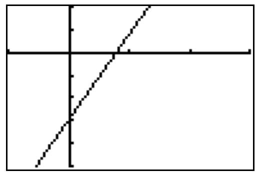


b) I pressed [Y=], then I input the expression Y1 = 3.75X–2.95.

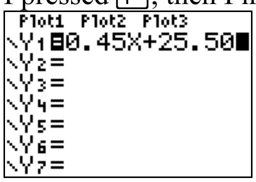


I pressed [GRAPH]. I changed the [WINDOW] to Xmin = –1, Xmax = 3, Ymin = –5, and Ymax = 2 to get this screen:

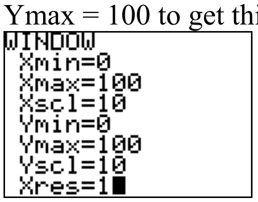


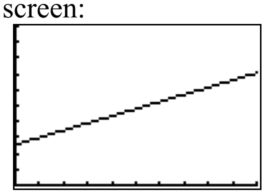


c) I pressed [Y=], then I input the expression Y1 = 0.45X+25.50.

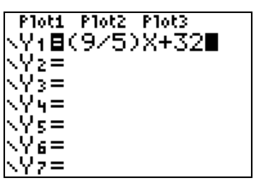


I pressed GRAPH. I changed the [WINDOW] to Xmin = 0, Xmax = 100, Ymin = 0, and Ymax = 100 to get this screen:

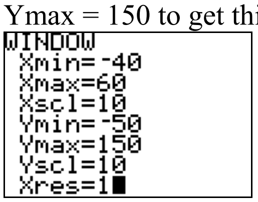


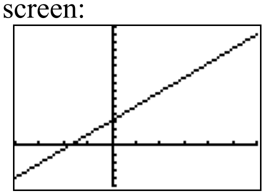


d) I pressed [Y=], then I input the expression Y1 = $\left(\frac{9}{5}\right)$X+32.



I pressed [GRAPH]. I changed the [WINDOW] to Xmin = –40, Xmax = 60, Ymin = –50, and Ymax = 150 to get this screen:





11. a) The student may have confused the values of the slope and the *y*-intercept.

b) From the graph:
Two points on the line have coordinates: (1, 1) and (0, –3)

$$\text{Slope} = \frac{\text{change in } y\text{-coordinates}}{\text{change in } x\text{-coordinates}}$$

Slope of the line is: $\dfrac{1-(-3)}{1-0} = 4$

The y-intercept is –3.
Substitute $m = 4$ and $b = -3$ in the equation $y = mx + b$.
The correct equation is: $y = 4x - 3$

12. a) i) From the graph:
Two points on the line have coordinates: (–2, 3) and (2, 1)

Slope = $\dfrac{\text{change in } y\text{-coordinates}}{\text{change in } x\text{-coordinates}}$

Slope of the line is: $\dfrac{1-3}{2-(-2)} = \dfrac{-2}{4}$, or $-\dfrac{1}{2}$

The y-intercept is 2.

ii) Substitute $m = -\dfrac{1}{2}$ and $b = 2$ in the equation $y = mx + b$.

The equation is: $y = -\dfrac{1}{2}x + 2$

To verify the equation, substitute the coordinates of another point on the line, such as (4, 0).

Substitute $x = 4$ and $y = 0$ in $y = -\dfrac{1}{2}x + 2$.

L.S. $= y$	R.S. $= -\dfrac{1}{2}x + 2$
L.S. $= 0$	R.S. $= -\dfrac{1}{2}(4) + 2$
	R.S. $= -2 + 2$
	R.S. $= 0$
	$=$ L.S.

Since the left side is equal to the right side, the equation is correct.

iii) Substitute $x = 10$ in $y = -\dfrac{1}{2}x + 2$.

$y = -\dfrac{1}{2}(10) + 2$

$y = -5 + 2$

$y = -3$

When $x = 10$, $y = -3$

b) i) From the graph:
Two points on the line have coordinates: (0, –6) and (1, –2)

Slope = $\dfrac{\text{change in } y\text{-coordinates}}{\text{change in } x\text{-coordinates}}$

Slope of the line is: $\dfrac{-2-(-6)}{1-0} = \dfrac{4}{1}$, or 4

The y-intercept is –6.

ii) Substitute $m = 4$ and $b = –6$ in the equation $y = mx + b$.
The equation is: $y = 4x – 6$
To verify the equation, substitute the coordinates of another point on the line, such as (2, 2).
Substitute $x = 2$ and $y = 2$ in $y = 4x – 6$.

L.S. = y	R.S. = $4x – 6$
L.S. = 2	R.S. = $4(2) – 6$
	R.S. = $8 – 6$
	R.S. = 2
	= L.S.

Since the left side is equal to the right side, the equation is correct.

iii) Substitute $x = 10$ in $y = 4x – 6$.
$y = 4(10) – 6$
$y = 40 – 6$
$y = 34$
When $x = 10$, $y = 34$

c) **i)** From the graph:
Two points on the line have coordinates: (–4, –2) and (0, 1)

$$\text{Slope} = \frac{\text{change in } y\text{-coordinates}}{\text{change in } x\text{-coordinates}}$$

Slope of the line is: $\dfrac{1-(-2)}{0-(-4)} = \dfrac{3}{4}$

The y-intercept is 1.

ii) Substitute $m = \dfrac{3}{4}$ and $b = 1$ in the equation $y = mx + b$.

The equation is: $y = \dfrac{3}{4}x + 1$

To verify the equation, substitute the coordinates of another point on the line, such as (4, 4).

Substitute $x = 4$ and $y = 4$ in $y = \dfrac{3}{4}x + 1$.

L.S. = y	R.S. = $\dfrac{3}{4}(4) + 1$
L.S. = 4	R.S. = $3 + 1$
	R.S. = 4
	= L.S.

Since the left side is equal to the right side, the equation is correct.

iii) Substitute $x = 10$ in $y = \dfrac{3}{4}x + 1$.

$y = \dfrac{3}{4}(10) + 1$

$y = \dfrac{30}{4} + 1$

$y = \frac{30}{4} + \frac{4}{4}$

$y = \frac{34}{4}$, or $\frac{17}{2}$

When $x = 10$, $y = 8.5$

d) i) From the graph:
Two points on the line have coordinates: (0, –2) and (3, –3)

Slope = $\frac{\text{change in } y\text{-coordinates}}{\text{change in } x\text{-coordinates}}$

Slope of the line is: $\frac{-3-(-2)}{3-0} = \frac{-1}{3}$, or $-\frac{1}{3}$

The y-intercept is –2.

ii) Substitute $m = -\frac{1}{3}$ and $b = -2$ in the equation $y = mx + b$.

The equation is: $y = -\frac{1}{3}x - 2$

To verify the equation, substitute the coordinates of another point on the line, such as (6, –4).

Substitute $x = 6$ and $y = -4$ in $y = -\frac{1}{3}x - 2$.

L.S. = y	R.S. = $-\frac{1}{3}(6) - 2$
L.S. = –4	R.S. = –2 – 2
	R.S. = –4
	= L.S.

Since the left side is equal to the right side, the equation is correct.

iii) Substitute $x = 10$ in $y = -\frac{1}{3}x - 2$.

$y = -\frac{1}{3}(10) - 2$

$y = -\frac{10}{3} - \frac{6}{3}$

$y = -\frac{16}{3}$

When $x = 10$, $y = -\frac{16}{3}$

13. a) From the graph:
Two points on the line have coordinates: (0, 900) and (10, 100)

Slope = $\frac{\text{change in } h\text{-coordinates}}{\text{change in } t\text{-coordinates}}$

Slope of the line is: $\dfrac{100-900}{10-0} = \dfrac{-800}{10}$, or –80

The slope is the quotient of height in metres and time in minutes, so the slope is –80 m/min; this is the rate at which the plane is descending; every minute, it descends 80 m.
The *h*-intercept is 900; this is the height in metres of the plane before it began its descent.

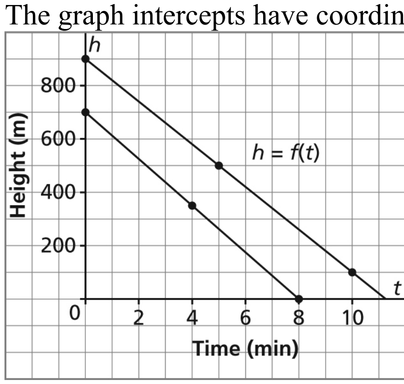
b) Rewrite the equation $y = mx + b$ as $h = mt + b$, to use the dependent and independent variables on the graph.
Substitute $m = -80$ and $b = 900$ in the equation $h = mt + b$.
The equation is: $h = -80t + 900$
To verify the equation, substitute the coordinates of another point on the line, such as (5, 500).
Substitute $t = 5$ and $h = 500$ in $h = -80t + 900$.

L.S. = h	R.S. = $-80t + 900$
L.S. = 500	R.S. = $-80(5) + 900$
	R.S. = $-400 + 900$
	= 500
	= L.S.

Since the left side is equal to the right side, the equation is correct.

c) Substitute $t = 5.5$ in $h = -80t + 900$.
$h = -80(5.5) + 900$
$h = -440 + 900$
$h = 460$
When $t = 5.5$ min, $h = 460$ m; that is, 5.5 min after beginning its descent, the plane is 460 m high.

d) **i)** The graph intercepts have coordinates: (0, 700) and (8, 0)



The graph is steeper.

Slope = $\dfrac{\text{change in } h\text{-coordinates}}{\text{change in } t\text{-coordinates}}$

Slope of the line is: $\dfrac{0-700}{8-0} = \dfrac{-700}{8}$, or –87.5

The slope is –87.5 m/min; the plane is descending at a faster rate; every minute, it descends 87.5 m.

ii) Substitute $m = -87.5$ and $b = 700$ in the equation $h = mt + b$.
The equation is: $h = -87.5t + 700$

To verify the equation, substitute the coordinates of another point on the line, such as (4, 350).
Substitute $t = 4$ and $h = 350$ in $h = -87.5t + 700$.

L.S. $= h$	R.S. $= -87.5t + 700$
L.S. $= 350$	R.S. $= -87.5(4) + 700$
	R.S. $= -350 + 700$
	$= 350$
	$=$ L.S.

Since the left side is equal to the right side, the equation is correct.

14. a) From the given data:
Rewrite the equation $y = mx + b$ as $C = mn + b$, to use the dependent and independent variables given.
The C-intercept is 20, which is the initial cost in dollars.
The slope is 0.80, which is the cost per song downloaded.
Substitute $m = 0.80$ and $b = 20$ in the equation $C = mn + b$.
The equation is: $C = 0.80n + 20$

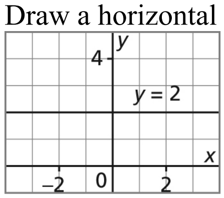
b) Substitute $n = 109$ in the equation: $C = 0.80n + 20$
$C = 0.80(109) + 20$
$C = 87.2 + 20$
$C = 107.2$
The total cost to download 109 songs is \$107.20.

c) Substitute $C = 120$ in the equation: $C = 0.80n + 20$

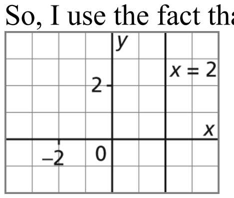
$120 = 0.80n + 20$	Solve for n.
$100 = 0.80n$	Divide each side by 0.80.
$125 = n$	

Michelle downloaded 125 songs.

15. a) A horizontal line has a slope of 0, so the equation
$y = mx + b$ becomes
$y = 0x + b$, or
$y = b$
So, for the line $y = 2$, the slope is 0 and the y-intercept is 2.
Draw a horizontal line through 2 on the y-axis.



b) I cannot use $y = mx + b$ to graph $x = 2$ because it has an infinite slope.
So, I use the fact that $x = 2$ is a vertical line through 2 on the x-axis.



16. a) From the given data:
Rewrite the equation $y = mx + b$ as $E = mt + b$, to use the dependent and independent variables given.
The E-intercept is 34, which is Alun's nightly earnings in dollars before tips are added.
The slope is 0.05, which is the percent of tips that Alun gets, written as a decimal.
Substitute $m = 0.05$ and $b = 34$ in the equation $E = mt + b$.
The equation is: $E = 0.05t + 34$

b) Substitute $t = 400$ in the equation: $E = 0.05t + 34$
$E = 0.05(400) + 34$
$E = 20 + 34$
$E = 54$
Alun earns \$54 when the tips are \$400.

c) Substitute $E = 64$ in the equation: $E = 0.05t + 34$
$64 = 0.05t + 34$ Solve for t.
$30 = 0.05t$ Divide each side by 0.05.
$600 = t$
When Alun earned \$64, the tips were \$600.

17. a) From the graph:
Two points on the line have coordinates: (0, 1) and (1, 5)
$$\text{Slope} = \frac{\text{change in } y\text{-coordinates}}{\text{change in } x\text{-coordinates}}$$
Slope of the line is: $\frac{5-1}{1-0} = 4$
The y-intercept is 1.
For the equation of the line, substitute $m = 4$ and $b = 1$ in $y = mx + b$.
So, the equation is: $y = 4x + 1$

b) From the graph:
Two points on the line have coordinates: (0, –1) and (3, 1)
$$\text{Slope} = \frac{\text{change in } y\text{-coordinates}}{\text{change in } x\text{-coordinates}}$$
Slope of the line is: $\frac{1-(-1)}{3-0} = \frac{2}{3}$
The y-intercept is –1.
For the equation of the line, substitute $m = \frac{2}{3}$ and $b = -1$ in $y = mx + b$.
So, the equation is: $y = \frac{2}{3}x - 1$

c) From the graph:
Two points on the line have coordinates: (–3, –2) and (0, –7)
$$\text{Slope} = \frac{\text{change in } y\text{-coordinates}}{\text{change in } x\text{-coordinates}}$$

Slope of the line is: $\dfrac{-7-(-2)}{0-(-3)} = \dfrac{-5}{3}$, or $-\dfrac{5}{3}$

The y-intercept is –7.

For the equation of the line, substitute $m = -\dfrac{5}{3}$ and $b = -7$ in $y = mx + b$.

So, the equation is: $y = -\dfrac{5}{3}x - 7$

18. From Graph A:
Two points on the line have coordinates: (1, 2) and (0, –1)

Slope = $\dfrac{\text{change in } y\text{-coordinates}}{\text{change in } x\text{-coordinates}}$

Slope of the line is: $\dfrac{-1-2}{0-1} = \dfrac{-3}{-1}$, or 3

The y-intercept is –1.
For the equation of the line, substitute $m = 3$ and $b = -1$ in $y = mx + b$.
So, the equation is $y = 3x - 1$, which is the equation in part b.

From Graph B:
Two points on the line have coordinates: (3, 0) and (0, –1)

Slope = $\dfrac{\text{change in } y\text{-coordinates}}{\text{change in } x\text{-coordinates}}$

Slope of the line is: $\dfrac{-1-0}{0-3} = \dfrac{-1}{-3}$, or $\dfrac{1}{3}$

The y-intercept is –1.

For the equation of the line, substitute $m = \dfrac{1}{3}$ and $b = -1$ in $y = mx + b$.

So, the equation is $y = \dfrac{1}{3}x - 1$, which is the equation in part d.

From Graph C:
Two points on the line have coordinates: (2, 3) and (1, 1)

Slope = $\dfrac{\text{change in } y\text{-coordinates}}{\text{change in } x\text{-coordinates}}$

Slope of the line is: $\dfrac{1-3}{1-2} = \dfrac{-2}{-1}$, or 2

The y-intercept is –1.
For the equation of the line, substitute $m = 2$ and $b = -1$ in $y = mx + b$.
So, the equation is $y = 2x - 1$, which is the equation in part a.

From Graph D:
Two points on the line have coordinates: (–2, 1) and (1, –2)

Slope = $\dfrac{\text{change in } y\text{-coordinates}}{\text{change in } x\text{-coordinates}}$

Slope of the line is: $\frac{1-(-2)}{-2-1} = \frac{3}{-3}$, or –1

The y-intercept is –1.
For the equation of the line, substitute $m = -1$ and $b = -1$ in $y = mx + b$.
So, the equation is $y = -x - 1$, which is the equation in part c.

19. From Graph A:
Two points on the line have coordinates: (4, 3) and (2, 1)

Slope = $\frac{\text{change in } y\text{-coordinates}}{\text{change in } x\text{-coordinates}}$

Slope of the line is: $\frac{1-3}{2-4} = \frac{-2}{-2}$, or 1

The y-intercept is –1.
For the equation of the line, substitute $m = 1$ and $b = -1$ in $y = mx + b$.
So, the equation is $y = x - 1$, which is the equation in part d.

From Graph B:
Two points on the line have coordinates: (2, 5) and (–2, 1)

Slope = $\frac{\text{change in } y\text{-coordinates}}{\text{change in } x\text{-coordinates}}$

Slope of the line is: $\frac{1-5}{-2-2} = \frac{-4}{-4}$, or 1

The y-intercept is 3.
For the equation of the line, substitute $m = 1$ and $b = 3$ in $y = mx + b$.
So, the equation is $y = x + 3$, which is the equation in part c.

From Graph C:
Two points on the line have coordinates: (–3, –1) and (1, –5)

Slope = $\frac{\text{change in } y\text{-coordinates}}{\text{change in } x\text{-coordinates}}$

Slope of the line is: $\frac{-5-(-1)}{1-(-3)} = \frac{-4}{4}$, or –1

The y-intercept is –4.
For the equation of the line, substitute $m = -1$ and $b = -4$ in $y = mx + b$.
So, the equation is $y = -x - 4$, which is the equation in part a.

From Graph D:
Two points on the line have coordinates: (–2, 3) and (2, –1)

Slope = $\frac{\text{change in } y\text{-coordinates}}{\text{change in } x\text{-coordinates}}$

Slope of the line is: $\frac{-1-3}{2-(-2)} = \frac{-4}{4}$, or –1

The y-intercept is 1.
For the equation of the line, substitute $m = -1$ and $b = 1$ in $y = mx + b$.
So, the equation is $y = -x + 1$, which is the equation in part b.

20. From Graph A:
Two points on the line have coordinates: (–3, 3) and (3, 1)

Slope = $\dfrac{\text{change in } y\text{-coordinates}}{\text{change in } x\text{-coordinates}}$

Slope of the line is: $\dfrac{1-3}{3-(-3)} = \dfrac{-2}{6}$, or $-\dfrac{1}{3}$

The y-intercept is 2.
These match the slope and y-intercept in part d.

From Graph B:
Two points on the line have coordinates: (1, 5) and (–1, –1)

Slope = $\dfrac{\text{change in } y\text{-coordinates}}{\text{change in } x\text{-coordinates}}$

Slope of the line is: $\dfrac{-1-5}{-1-1} = \dfrac{-6}{-2}$, or 3

The y-intercept is 2.
These match the slope and y-intercept in part a.

From Graph C:
Two points on the line have coordinates: (3, –1) and (–3, –3)

Slope = $\dfrac{\text{change in } y\text{-coordinates}}{\text{change in } x\text{-coordinates}}$

Slope of the line is: $\dfrac{-3-(-1)}{-3-3} = \dfrac{-2}{-6}$, or $\dfrac{1}{3}$

The y-intercept is –2.
These match the slope and y-intercept in part b.

From Graph D:
Two points on the line have coordinates: (–1, 1) and (0, –2)

Slope = $\dfrac{\text{change in } y\text{-coordinates}}{\text{change in } x\text{-coordinates}}$

Slope of the line is: $\dfrac{-2-1}{0-(-1)} = -3$

The y-intercept is –2.
These match the slope and y-intercept in part c.

21. Parallel lines have the same slope.

Look for equations with the same value of m.
The graphs of these equations have the same slope of –5, so they represent parallel lines:
$y = -5x - 7$ and $y = -5x + 13$
The graphs of these equations have the same slope of 5, so they represent parallel lines:
$y = 5x + 15$ and $y = 5x + 24$

The graphs of these equations have the same slope of $\dfrac{1}{5}$, so they represent parallel lines:

$y = \frac{1}{5}x + 9$ and $y = \frac{1}{5}x + 21$

The graphs of these equations have the same slope of $-\frac{1}{5}$, so they represent parallel lines:

$y = -\frac{1}{5}x + 15$ and $y = -\frac{1}{5}x$

Perpendicular lines have slopes that are negative reciprocals.

Look for equations whose values of m have the product of –1.
The graphs of these equations have slopes with the product of –1, so they represent perpendicular lines:

$y = -5x - 7$ and $y = \frac{1}{5}x + 9$; $y = -5x - 7$ and $y = \frac{1}{5}x + 21$;

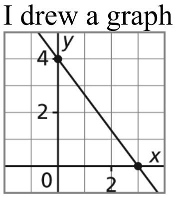
$y = -5x + 13$ and $y = \frac{1}{5}x + 9$; $y = -5x + 13$ and $y = \frac{1}{5}x + 21$;

$y = 5x + 15$ and $y = -\frac{1}{5}x + 15$; $y = 5x + 15$ and $y = -\frac{1}{5}x$;

$y = 5x + 24$ and $y = -\frac{1}{5}x + 15$; $y = 5x + 24$ and $y = -\frac{1}{5}x$

C

22. I drew a graph with y-intercept 4 and x-intercept 3.



$$\text{Slope} = \frac{\text{change in } y\text{-coordinates}}{\text{change in } x\text{-coordinates}}$$

Slope of the line is: $\frac{0-4}{3-0} = \frac{-4}{3}$, or $-\frac{4}{3}$

The y-intercept is 4.

For the equation of the line, substitute $m = -\frac{4}{3}$ and $b = 4$ in $y = mx + b$.

So, the equation is: $y = -\frac{4}{3}x + 4$

23. Since F(4, –6) lies on the line with equation $y = \frac{5}{3}x + c$, the coordinates of F must satisfy the equation.

Substitute $x = 4$ and $y = -6$ in $y = \frac{5}{3}x + c$.

$-6 = \frac{5}{3}(4) + c$ Solve for c.

$$-6 = \frac{20}{3} + c$$

$$-\frac{18}{3} - \frac{20}{3} = c$$

$$c = -\frac{38}{3}$$

24. Since E(–3, 5) lies on the line with equation $y = mx - \frac{7}{8}$, the coordinates of E must satisfy the equation.

Substitute $x = -3$ and $y = 5$ in $y = mx - \frac{7}{8}$.

$$5 = m(-3) - \frac{7}{8}$$ Solve for m.

$$5 = -3m - \frac{7}{8}$$

$$\frac{40}{8} + \frac{7}{8} = -3m$$

$$\frac{47}{8} = -3m$$ Divide each side by –3.

$$m = \frac{47}{-24}, \text{ or } -\frac{47}{24}$$

Lesson 6.5 **Slope-Point Form of the Equation for a Linear Function** **Exercises (pages 372–374)**

A

4. Compare each given equation with slope-point form: $y - y_1 = m(x - x_1)$, where m is the slope and the coordinates of a point on the line are (x_1, y_1).

a) $y - 5 = -4(x - 1)$
The slope m is –4.
The coordinates of a point on the line are (1, 5).

b) $y + 7 = 3(x - 8)$; write this equation as:
$y - (-7) = 3(x - 8)$
The slope m is 3.
The coordinates of a point on the line are (8, –7).

c) $y + 11 = (x + 15)$; write this equation as:
$y - (-11) = 1(x - (-15))$
The slope m is 1.
The coordinates of a point on the line are (–15, –11).

d) $y = 5(x - 2)$; write this equation as:
$y - 0 = 5(x - 2)$
The slope m is 5.
The coordinates of a point on the line are (2, 0).

e) $y + 6 = \frac{4}{7}(x + 3)$; write this equation as:

$y - (-6) = \frac{4}{7}(x - (-3))$

The slope m is $\frac{4}{7}$.

The coordinates of a point are on the line (–3, –6).

f) $y - 21 = -\frac{8}{5}(x + 16)$; write this equation as:

$y - 21 = -\frac{8}{5}(x - (-16))$

The slope m is $-\frac{8}{5}$.

The coordinates of a point are on the line (–16, 21).

5. Substitute the given values of slope and coordinates into the equation in slope-point form: $y - y_1 = m(x - x_1)$, where m is the slope and the coordinates of a point on the line are (x_1, y_1).

a) The graph has slope –5 and passes through P(–4, 2).
Substitute: $y_1 = 2$, $m = -5$, and $x_1 = -4$
$y - 2 = -5(x - (-4))$
An equation is: $y - 2 = -5(x + 4)$

b) The graph has slope 7 and passes through Q(6, –8).
Substitute: $y_1 = -8$, $m = 7$, and $x_1 = 6$
$y - (-8) = 7(x - 6)$
An equation is: $y + 8 = 7(x - 6)$

c) The graph has slope $-\frac{3}{4}$ and passes through R(7, –5).

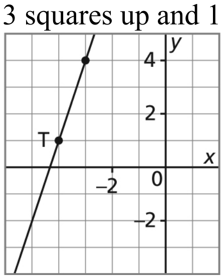
Substitute: $y_1 = -5$, $m = -\frac{3}{4}$, and $x_1 = 7$

$y - (-5) = -\frac{3}{4}(x - 7)$

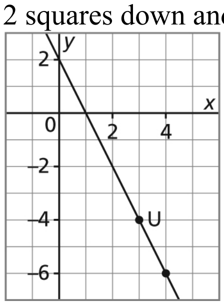
An equation is: $y + 5 = -\frac{3}{4}(x - 7)$

d) The graph has slope 0 and passes through S(3, –8).
A line with slope 0 is horizontal and it intersects the y-axis.
The y-coordinate of P is –8.
So, an equation is: $y = -8$

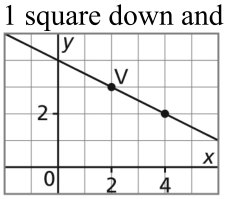
6. **a)** Mark a point at T(–4, 1). The slope is 3, so the rise is 3 and the run is 1. From T, move 3 squares up and 1 square right. Mark another point. Draw a line through the points.



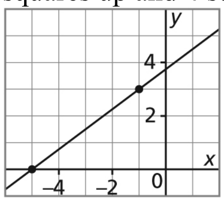
b) Mark a point at U(3, –4). The slope is –2, so the rise is –2 and the run is 1. From U, move 2 squares down and 1 square right. Mark another point. Draw a line through the points.



c) Mark a point at V(2, 3). The slope is $-\frac{1}{2}$, so the rise is –1 and the run is 2. From V, move 1 square down and 2 squares right. Mark another point. Draw a line through the points.



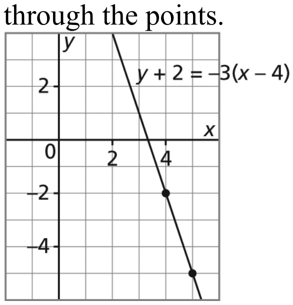
d) The point at an x-intercept of –5 has coordinates (–5, 0). Mark a point at (–5, 0). The slope is $\frac{3}{4}$, so the rise is 3 and the run is 4. From the point at the intercept, move 3 squares up and 4 squares right. Mark another point. Draw a line through the points.



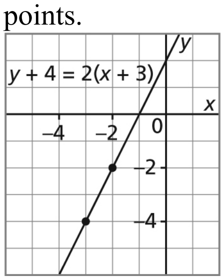
B

7. Compare each given equation with the equation $y - y_1 = m(x - x_1)$, to identify the slope m and the coordinates of a point on the line (x_1, y_1).

a) $y + 2 = -3(x - 4)$; write this equation as:
$y - (-2) = -3(x - 4)$
The slope m is –3.
The coordinates of a point on the line are (4, –2).
The graph has slope –3 and passes through the point (4, –2).
Mark a point at (4, –2). The slope is –3, so the rise is –3 and the run is 1. From the marked point, move 3 squares down and 1 square right. Mark another point. Draw a line through the points.

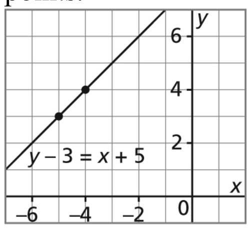


b) $y + 4 = 2(x + 3)$; write this equation as:
$y - (-4) = 2(x - (-3))$
The slope m is 2.
The coordinates of a point on the line are (–3, –4).
The graph has slope 2 and passes through the point (–3, –4).
Mark a point at (–3, –4). The slope is 2, so the rise is 2 and the run is 1. From the marked point, move 2 squares up and 1 square right. Mark another point. Draw a line through the points.

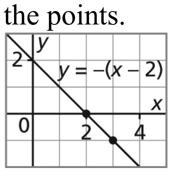


c) $y - 3 = (x + 5)$; write this equation as:
$y - 3 = 1(x - (-5))$

The slope m is 1.
The coordinates of a point on the line are (–5, 3).
The graph has slope 1 and passes through the point (–5, 3).
Mark a point at (–5, 3). The slope is 1, so the rise is 1 and the run is 1. From the marked point, move 1 square up and 1 square right. Mark another point. Draw a line through the points.



d) $y = -(x - 2)$; write this equation as:
$y - 0 = -1(x - 2)$
The slope m is –1.
The coordinates of a point on the line are (2, 0).
The graph has slope –1 and passes through the point (2, 0).
Mark a point at (2, 0). The slope is –1, so the rise is –1 and the run is 1. From the marked point, move 1 square down and 1 square right. Mark another point. Draw a line through the points.



8. **a)** To determine the equation of a line, I can use the slope-point of the equation:
$y - y_1 = m(x - x_1)$, and substitute the given values.
I substitute: $y_1 = 5$, $m = -4$, and $x_1 = -3$
$y - y_1 = m(x - x_1)$ becomes
$y - 5 = -4(x - (-3))$
$y - 5 = -4(x + 3)$
This is the same equation as that given in the question.

b) I apply the distributive property to the equation in part a.
$y - 5 = -4(x + 3)$
$y - 5 = -4x - 12$ Collect like terms.
$y = -4x - 12 + 5$
$y = -4x - 7$
This is the same equation as that given in the question.

9. **a)** **i)** From the graph:
A point on the line has coordinates P(–2, 4). Another point on the line has coordinates (1, 0). Use the coordinates of these points to calculate the slope of the line.

$$\text{Slope} = \frac{\text{change in } y\text{-coordinates}}{\text{change in } x\text{-coordinates}}$$

$$\text{Slope} = \frac{0 - 4}{1 - (-2)}$$

Slope = $\frac{-4}{3}$, or $-\frac{4}{3}$

Substitute the slope and the coordinates of point P in the equation: $y - y_1 = m(x - x_1)$

Substitute: $y_1 = 4$, $m = -\frac{4}{3}$, and $x_1 = -2$

$y - 4 = -\frac{4}{3}(x - (-2))$

An equation is: $y - 4 = -\frac{4}{3}(x + 2)$

b) i) $y - 4 = -\frac{4}{3}(x + 2)$ Solve for y.

$y - 4 = -\frac{4}{3}x - \frac{8}{3}$

$y = -\frac{4}{3}x - \frac{8}{3} + 4$

$y = -\frac{4}{3}x - \frac{8}{3} + \frac{12}{3}$

$y = -\frac{4}{3}x + \frac{4}{3}$

From the equation, the y-intercept is $\frac{4}{3}$.

For the x-intercept, substitute $y = 0$ in the equation above.

$y = -\frac{4}{3}x + \frac{4}{3}$

$0 = -\frac{4}{3}x + \frac{4}{3}$ Solve for x.

$\frac{4}{3}x = \frac{4}{3}$ Divide each side by $\frac{4}{3}$.

$x = 1$

The x-intercept is 1.

a) ii) From the graph:

A point on the line has coordinates P(3, 3). Another point on the line has coordinates (–2, 1). Use the coordinates of these points to calculate the slope of the line.

Slope = $\frac{\text{change in } y\text{-coordinates}}{\text{change in } x\text{-coordinates}}$

Slope = $\frac{1 - 3}{-2 - 3}$

Slope = $\frac{-2}{-5}$, or $\frac{2}{5}$

Substitute the slope and the coordinates of point P in the equation: $y - y_1 = m(x - x_1)$

Substitute: $y_1 = 3$, $m = \frac{2}{5}$, and $x_1 = 3$

$y - 3 = \frac{2}{5}(x - 3)$

An equation is: $y - 3 = \frac{2}{5}(x - 3)$

b) ii) $y - 3 = \frac{2}{5}(x - 3)$ Solve for y.

$y - 3 = \frac{2}{5}x - \frac{6}{5}$

$y = \frac{2}{5}x - \frac{6}{5} + 3$

$y = \frac{2}{5}x - \frac{6}{5} + \frac{15}{5}$

$y = \frac{2}{5}x + \frac{9}{5}$

From the equation, the y-intercept is $\frac{9}{5}$.

For the x-intercept, substitute $y = 0$ in the equation above.

$y = \frac{2}{5}x + \frac{9}{5}$

$0 = \frac{2}{5}x + \frac{9}{5}$ Solve for x.

$\frac{2}{5}x = -\frac{9}{5}$ Divide each side by $\frac{2}{5}$.

$x = \left(-\frac{9}{5}\right)\left(\frac{5}{2}\right)$

$x = -\frac{9}{2}$

The x-intercept is $-\frac{9}{2}$.

a) iii) From the graph:
A point on the line has coordinates P(–4, –2). Another point on the line has coordinates (–1, –1). Use the coordinates of these points to calculate the slope of the line.

$\text{Slope} = \frac{\text{change in } y\text{-coordinates}}{\text{change in } x\text{-coordinates}}$

$\text{Slope} = \frac{-1 - (-2)}{-1 - (-4)}$

$\text{Slope} = \frac{1}{3}$

Substitute the slope and the coordinates of point P in the equation: $y - y_1 = m(x - x_1)$

Substitute: $y_1 = -2$, $m = \frac{1}{3}$, and $x_1 = -4$

$y-(-2)=\frac{1}{3}(x-(-4))$

An equation is: $y+2=\frac{1}{3}(x+4)$

b) iii) $y+2=\frac{1}{3}(x+4)$ Solve for y.

$y+2=\frac{1}{3}x+\frac{4}{3}$

$y=\frac{1}{3}x+\frac{4}{3}-2$

$y=\frac{1}{3}x+\frac{4}{3}-\frac{6}{3}$

$y=\frac{1}{3}x-\frac{2}{3}$

From the equation, the y-intercept is $-\frac{2}{3}$.

For the x-intercept, substitute $y = 0$ in the equation above.

$y=\frac{1}{3}x-\frac{2}{3}$

$0=\frac{1}{3}x-\frac{2}{3}$ Solve for x.

$\frac{1}{3}x=\frac{2}{3}$ Multiply each side by 3.

$x=3\left(\frac{2}{3}\right)$

$x = 2$

The x-intercept is 2.

a) iv) From the graph:

A point on the line has coordinates P(1, –2). Another point on the line has coordinates (–1, 3). Use the coordinates of these points to calculate the slope of the line.

$\text{Slope}=\frac{\text{change in } y\text{-coordinates}}{\text{change in } x\text{-coordinates}}$

$\text{Slope}=\frac{3-(-2)}{-1-1}$

$\text{Slope}=\frac{5}{-2}$, or $-\frac{5}{2}$

Substitute the slope and the coordinates of point P in the equation: $y-y_1=m(x-x_1)$

Substitute: $y_1=-2$, $m=-\frac{5}{2}$, and $x_1=1$

$y-(-2) = -\frac{5}{2}(x-1)$

An equation is: $y+2 = -\frac{5}{2}(x-1)$

b) iv) $y+2 = -\frac{5}{2}(x-1)$ Solve for y.

$y+2 = -\frac{5}{2}x + \frac{5}{2}$

$y = -\frac{5}{2}x + \frac{5}{2} - 2$

$y = -\frac{5}{2}x + \frac{5}{2} - \frac{4}{2}$

$y = -\frac{5}{2}x + \frac{1}{2}$

From the equation, the y-intercept is $\frac{1}{2}$.

For the x-intercept, substitute $y = 0$ in the equation above.

$y = -\frac{5}{2}x + \frac{1}{2}$

$0 = -\frac{5}{2}x + \frac{1}{2}$ Solve for x.

$\frac{5}{2}x = \frac{1}{2}$ Multiply each side by $\frac{2}{5}$.

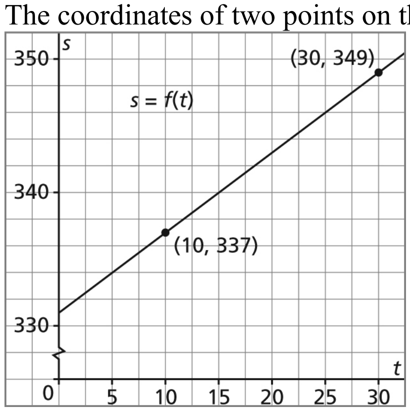
$x = \left(\frac{1}{2}\right)\left(\frac{2}{5}\right)$

$x = \frac{1}{5}$

The x-intercept is $\frac{1}{5}$.

10. Different variables may be used.

a) Sketch a graph of the speed of sound as a function of the air temperature.
The coordinates of two points on the graph are: (10, 337) and (30, 349)



Use the form of a linear equation that involves the coordinates of two points on the line:

$$\frac{s - s_1}{t - t_1} = \frac{s_2 - s_1}{t_2 - t_1}$$

Substitute: $s_1 = 337$, $t_1 = 10$, $s_2 = 349$, $t_2 = 30$

$$\frac{s - 337}{t - 10} = \frac{349 - 337}{30 - 10}$$

$$\frac{s - 337}{t - 10} = \frac{12}{20}$$

$$\frac{s - 337}{t - 10} = 0.6$$ Multiply each side by $(t - 10)$.

$$(t - 10)\left(\frac{s - 337}{t - 10}\right) = 0.6(t - 10)$$

$s - 337 = 0.6(t - 10)$

The linear equation above represents the function.

b) Substitute $t = 0$ in the equation: $s - 337 = 0.6(t - 10)$

$s - 337 = 0.6(0 - 10)$ Solve for s.

$s - 337 = -6$

$s = 337 - 6$

$s = 331$

When the air temperature is 0°C, the speed of sound is 331 m/s.

11. Use the form of a linear equation that involves the coordinates of two points on the line:

$$\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$$

a) The two points are B(–2, –5) and C(1, 1).

Substitute: $y_1 = -5$, $x_1 = -2$, $y_2 = 1$, and $x_2 = 1$

$$\frac{y - (-5)}{x - (-2)} = \frac{1 - (-5)}{1 - (-2)}$$

$$\frac{y + 5}{x + 2} = \frac{6}{3}$$

$$\frac{y + 5}{x + 2} = 2$$ Multiply each side by $(x + 2)$.

$$(x + 2)\left(\frac{y + 5}{x + 2}\right) = 2(x + 2)$$

$$y + 5 = 2(x + 2)$$

The equation above is in slope-point form.

The slope is 2. Another form of the equation is: $y - y_1 = m(x - x_1)$

Substitute: $y_1 = 1$, $m = 2$, and $x_1 = 1$

$y - 1 = 2(x - 1)$

Apply the distributive property to one of the equations above.

$y + 5 = 2(x + 2)$
$y + 5 = 2x + 4$
$y = 2x + 4 - 5$
$y = 2x - 1$
The equation above is in slope-intercept form.

b) The two points are Q(–4, 7) and R(5, –2).
Substitute: $y_1 = 7$, $x_1 = -4$, $y_2 = -2$, and $x_2 = 5$

$$\frac{y - 7}{x - (-4)} = \frac{-2 - 7}{5 - (-4)}$$

$$\frac{y - 7}{x + 4} = \frac{-9}{9}$$

$$\frac{y - 7}{x + 4} = -1 \qquad \text{Multiply each side by } (x + 4).$$

$$(x + 4)\left(\frac{y - 7}{x + 4}\right) = -1(x + 4)$$

$$y - 7 = -(x + 4)$$

The equation above is in slope-point form.

The slope is –1. Another form of the equation is: $y - y_1 = m(x - x_1)$
Substitute: $y_1 = -2$, $m = -1$, and $x_1 = 5$
$y + 2 = -(x - 5)$

Apply the distributive property to one of the equations above.
$y - 7 = -(x + 4)$
$y - 7 = -x - 4$
$y = -x - 4 + 7$
$y = -x + 3$
The equation above is in slope-intercept form.

c) The two points are U(–3, –7) and V(2, 8).
Substitute: $y_1 = -7$, $x_1 = -3$, $y_2 = 8$, and $x_2 = 2$

$$\frac{y - (-7)}{x - (-3)} = \frac{8 - (-7)}{2 - (-3)}$$

$$\frac{y - (-7)}{x - (-3)} = \frac{15}{5}$$

$$\frac{y + 7}{x + 3} = 3 \qquad \text{Multiply each side by } (x + 3).$$

$$(x + 3)\left(\frac{y + 7}{x + 3}\right) = 3(x + 3)$$

$$y + 7 = 3(x + 3)$$

The equation above is in slope-point form.

The slope is 3. Another form of the equation is: $y - y_1 = m(x - x_1)$
Substitute: $y_1 = 8$, $m = 3$, and $x_1 = 2$
$y - 8 = 3(x - 2)$

Apply the distributive property to one of the equations above.

$y + 7 = 3(x + 3)$
$y + 7 = 3x + 9$
$y = 3x + 9 - 7$
$y = 3x + 2$

The equation above is in slope-intercept form.

d) The two points are H(–7, –1) and J(–5, –5).
Substitute: $y_1 = -1$, $x_1 = -7$, $y_2 = -5$, and $x_2 = -5$

$$\frac{y-(-1)}{x-(-7)} = \frac{-5-(-1)}{-5-(-7)}$$

$$\frac{y-(-1)}{x-(-7)} = \frac{-4}{2}$$

$$\frac{y+1}{x+7} = -2 \qquad \text{Multiply each side by } (x + 7).$$

$$(x+7)\left(\frac{y+1}{x+7}\right) = -2(x+7)$$

$$y + 1 = -2(x + 7)$$

The equation above is in slope-point form.
Apply the distributive property to the equation above.

$y + 1 = -2(x + 7)$
$y + 1 = -2x - 14$
$y = -2x - 14 - 1$
$y = -2x - 15$

The equation above is in slope-intercept form.

12. From Graph A:
Its slope is 1 and its y-intercept is 1.
The only given equation with slope 1 is the equation in part b: $y - 3 = (x - 2)$
Apply the distributive property to determine this equation in slope-intercept form.

$y - 3 = (x - 2)$
$y - 3 = x - 2$
$y = x - 2 + 3$
$y = x + 1$

The above equation represents a line with slope 1 and y-intercept 1.
So, the equation in part b represents Graph A.

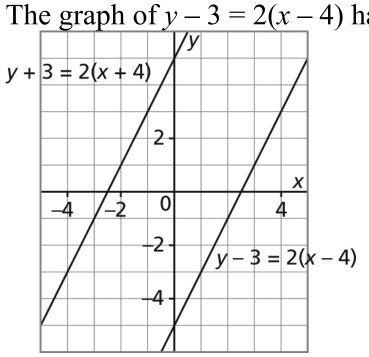
From Graph B:
Its slope is 2 and its y-intercept is 5.
There are two given equations with slope 2.
They are the equation in part a: $y + 3 = 2(x - 1)$ and the equation in part c: $y - 3 = 2(x + 1)$
Apply the distributive property to determine each equation in slope-intercept form.
For the equation in part a:

$y + 3 = 2(x - 1)$
$y + 3 = 2x - 2$
$y = 2x - 2 - 3$
$y = 2x - 5$

The above equation represents a line with slope 2 and y-intercept –5.
Graph C has slope 2 and y-intercept –5.
So, the equation in part a represents Graph C.
For the equation in part c:
$y - 3 = 2(x + 1)$
$y - 3 = 2x + 2$
$y = 2x + 2 + 3$
$y = 2x + 5$
The above equation represents a line with slope 2 and y-intercept 5.
Graph B has slope 2 and y-intercept 5
So, the equation in part c represents Graph B.

From Graph D:
Its slope is –1 and its y-intercept is –5.
The only given equation with slope –1 is the equation in part d: $y + 3 = -(x + 2)$
Apply the distributive property to determine this equation in slope-intercept form
$y + 3 = -(x + 2)$
$y + 3 = -x - 2$
$y = -x - 2 - 3$
$y = -x - 1$
The above equation represents a line with slope –1 and y-intercept –5.
So, the equation in part d represents Graph D.

13. The graph of $y + y_1 = m(x + x_1)$ has slope m and passes through the point with coordinates $(-x_1, -y_1)$.
The graph of $y - y_1 = m(x - x_1)$ has slope m and passes through the point with coordinates (x_1, y_1).
So, the lines are parallel.
For example:
The graph of $y + 3 = 2(x + 4)$ has slope 2 and passes through the point with coordinates (–4, –3).
The graph of $y - 3 = 2(x - 4)$ has slope 2 and passes through the point with coordinates (4, 3).



From the graph, it looks as though the y-intercepts are opposite numbers and the x-intercepts are opposite numbers. Determine the intercepts of each graph to check.
For $y + 3 = 2(x + 4)$:
Substitute $x = 0$ to determine the y-intercept.
$y + 3 = 2(0 + 4)$
$y + 3 = 8$
$y = 5$
Substitute $y = 0$ to determine the x-intercept.
$0 + 3 = 2(x + 4)$

$3 = 2x + 8$
$-5 = 2x$
$x = -2.5$
For $y - 3 = 2(x - 4)$:
Substitute $x = 0$ to determine the y-intercept.
$y - 3 = 2(0 - 4)$
$y - 3 = -8$
$y = -5$
Substitute $y = 0$ to determine the x-intercept.
$0 - 3 = 2(x - 4)$
$-3 = 2x - 8$
$5 = 2x$
$x = 2.5$
So, the graphs have opposite y-intercepts, opposite x-intercepts, and equal slopes.

14. a) The graph has slope 2 and y-intercept 4.
Three of the given equations represent graphs with slope 2.
Write each equation in slope-intercept form.
$y + 1 = 2(x - 2)$ becomes
$y + 1 = 2x - 4$
$y = 2x - 4 - 1$
$y = 2x - 5$
The above equation represents a graph with y-intercept 5, so it is not the correct equation.
$y + 2 = 2(x - 1)$ becomes
$y + 2 = 2x - 2$
$y = 2x - 2 - 2$
$y = 2x - 4$
The above equation represents a graph with y-intercept –4, so it is not the correct equation.
$y - 2 = 2(x + 1)$ becomes
$y - 2 = 2x + 2$
$y = 2x + 2 + 2$
$y = 2x + 4$
The above equation represents a graph with y-intercept 4, so it is the correct equation.

b) The graph has slope $\frac{1}{3}$ and y-intercept between 1 and 2.

Three of the given equations represent graphs with slope $\frac{1}{3}$.

Write each equation in slope-intercept form.

$y - 1 = \frac{1}{3}(x - 2)$ becomes

$y - 1 = \frac{1}{3}x - \frac{2}{3}$

$y = \frac{1}{3}x - \frac{2}{3} + 1$

$y = \frac{1}{3}x + \frac{1}{3}$

The above equation represents a graph with y-intercept $\frac{1}{3}$, so it is not the correct equation.

$y + 2 = \frac{1}{3}(x + 1)$ becomes

$$y + 2 = \frac{1}{3}x + \frac{1}{3}$$

$$y = \frac{1}{3}x + \frac{1}{3} - 2$$

$$y = \frac{1}{3}x - \frac{5}{3}$$

The above equation represents a graph with y-intercept $-\frac{5}{3}$, so it is not the correct equation.

$y - 2 = \frac{1}{3}(x - 1)$ becomes

$$y - 2 = \frac{1}{3}x - \frac{1}{3}$$

$$y = \frac{1}{3}x - \frac{1}{3} + 2$$

$$y = \frac{1}{3}x + \frac{5}{3}$$

The above equation represents a graph with y-intercept $\frac{5}{3}$; since this number is between 1 and 2, this is the correct equation.

c) The graph has slope $-\frac{2}{3}$ and y-intercept between 2 and 3.

Two of the given equations represent graphs with slope $-\frac{2}{3}$.

Write each equation in slope-intercept form.

$y - 1 = -\frac{2}{3}(x - 2)$ becomes

$$y - 1 = -\frac{2}{3}x + \frac{4}{3}$$

$$y = -\frac{2}{3}x + \frac{4}{3} + 1$$

$$y = -\frac{2}{3}x + \frac{7}{3}$$

The above equation represents a graph with y-intercept $\frac{7}{3}$, since this number is between 2 and 3, this might be the correct equation.

$y - 2 = -\frac{2}{3}(x - 1)$ becomes

$$y - 2 = -\frac{2}{3}x + \frac{2}{3}$$
$$y = -\frac{2}{3}x + \frac{2}{3} + 2$$
$$y = -\frac{2}{3}x + \frac{8}{3}$$

The above equation represents a graph with y-intercept $\frac{8}{3}$; since this number is between 2 and 3, this might be the correct equation.

The graph passes through (2, 1). Substitute $x = 2$ in both equations to check the y values.

$$y = -\frac{2}{3}x + \frac{7}{3}$$
$$y = -\frac{2}{3} \times 2 + \frac{7}{3}$$
$$y = -\frac{4}{3} + \frac{7}{3}$$
$$y = \frac{3}{3}\text{, or } 1$$

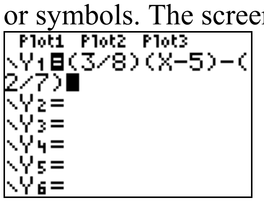
$$y = -\frac{2}{3}x + \frac{8}{3}$$
$$y = -\frac{2}{3} \times 2 + \frac{8}{3}$$
$$y = -\frac{4}{3} + \frac{8}{3}$$
$$y = \frac{4}{3}$$

So, equation $y = -\frac{2}{3}x + \frac{7}{3}$ is the correct equation.

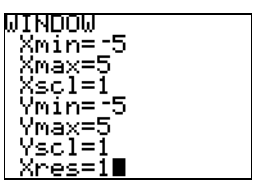
15. I used a graphing calculator. I wrote each equation in the form $y = f(x)$.

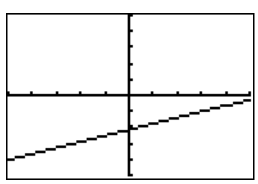
a) $y + \frac{2}{7} = \frac{3}{8}(x - 5)$ becomes $y = \frac{3}{8}(x - 5) - \frac{2}{7}$

On a TI-83 calculator, press Y=, then input the equation with no spaces between the terms or symbols. The screen should look like this:



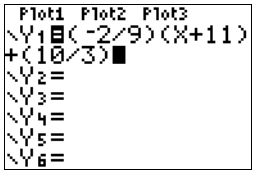
Press GRAPH. To get the screen below, press WINDOW and set the window as follows:



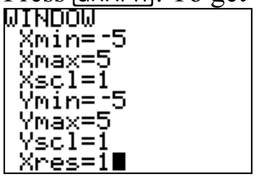


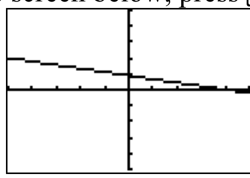
b) $y - \frac{10}{3} = -\frac{2}{9}(x + 11)$ becomes $y = -\frac{2}{9}(x + 11) + \frac{10}{3}$

On a TI-83 calculator, press Y=, then input the equation with no spaces between the terms or symbols. The screen should look like this:



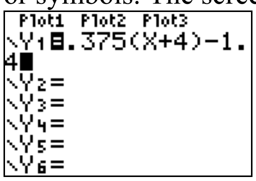
Press GRAPH. To get the screen below, press WINDOW and set the window as follows:



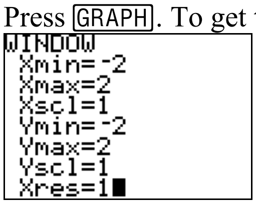


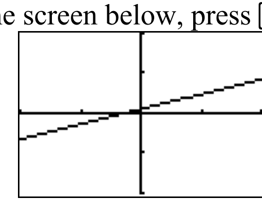
c) $y + 1.4 = 0.375(x + 4)$ becomes $y = 0.375(x + 4) - 1.4$

On a TI-83 calculator, press Y=, then input the equation with no spaces between the terms or symbols. The screen should look like this:



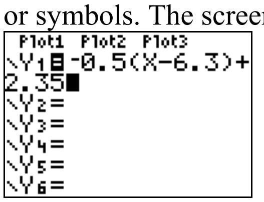
Press GRAPH. To get the screen below, press WINDOW and set the window as follows:



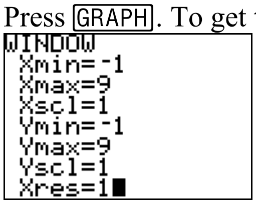


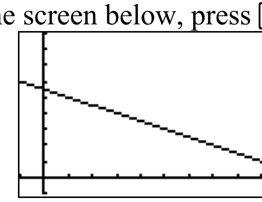
d) $y - 2.35 = -0.5(x - 6.3)$ becomes $y = -0.5(x - 6.3) + 2.35$

On a TI-83 calculator, press Y=, then input the equation with no spaces between the terms or symbols. The screen should look like this:

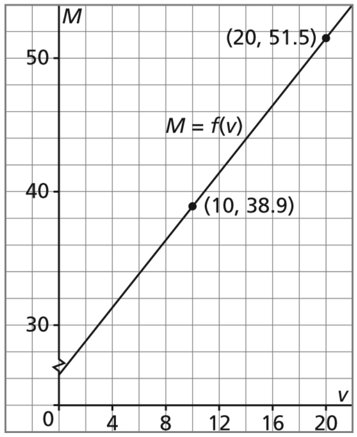


Press GRAPH. To get the screen below, press WINDOW and set the window as follows:





16. a) Sketch a graph of the mass, M grams, of the cylinder and liquid as a function of the volume, v millilitres, of liquid.
The coordinates of two points on the graph are: (10, 38.9) and (20, 51.5)



$$\text{Slope} = \frac{\text{change in } M\text{-coordinates}}{\text{change in } v\text{-coordinates}}$$

$$\text{Slope} = \frac{51.5 - 38.9}{20 - 10}$$

$$\text{Slope} = \frac{12.6}{10}$$

Slope = 1.26

The slope is the quotient of a mass in grams and a volume in millilitres, so the slope is 1.26 g/mL.
The slope is the mass in grams of 1 mL of liquid.

b) Use the slope-point form of the equation: $y - y_1 = m(x - x_1)$
Replace y with M, and replace x with v.
$M - M_1 = m(v - v_1)$
Substitute: $M_1 = 51.5$, $m = 1.26$, and $v_1 = 20$
$M - 51.5 = 1.26(v - 20)$
The linear equation above represents the function.

c) Substitute $v = 30$ in the equation: $M - 51.5 = 1.26(v - 20)$
$M - 51.5 = 1.26(30 - 20)$
$M - 51.5 = 1.26(10)$
$M - 51.5 = 12.6$
$M = 12.6 + 51.5$
$M = 64.1$
When the volume of liquid is 30 mL, the mass of the cylinder and liquid is 64.1 g.

d) When the volume of the liquid is 0, the mass of the cylinder and liquid, M, will represent the mass of just the cylinder.
Substitute $v = 0$ in the equation: $M - 51.5 = 1.26(v - 20)$
$M - 51.5 = 1.26(0 - 20)$
$M - 51.5 = 1.26(-20)$
$M - 51.5 = -25.2$
$M = -25.2 + 51.5$
$M = 26.3$
The mass of the empty graduated cylinder is 26.3 g.

17. a) The mass of potash is a linear function of the time in years since 2005.
The year 2005 is 0 years since 2005.
The year 2007 is 2 years since 2005.
So, two ordered pairs that satisfy this function are: (0, 8.2) and (2, 9.4)
Use these ordered pairs to determine the slope of a graph of the function.
Let t represent the time in years since 2005.
Let p represent the mass of potash in millions of tonnes.

$$\text{Slope} = \frac{\text{change in } p\text{-coordinates}}{\text{change in } t\text{-coordinates}}$$

$$\text{Slope} = \frac{9.4 - 8.2}{2 - 0}$$

$$\text{Slope} = \frac{1.2}{2}$$

Slope = 0.6
Use the slope-point form of the equation: $y - y_1 = m(x - x_1)$
Replace y with p, and replace x with t.
$p - p_1 = m(t - t_1)$
Substitute: $p_1 = 8.2$, $m = 0.6$, and $t_1 = 0$
$p - 8.2 = 0.6(t - 0)$
$p - 8.2 = 0.6t$
$p = 0.6t + 8.2$
The linear equation above represents the relation.

b) The year 2010 is 5 years since 2005.
Substitute $t = 5$ in the equation: $p = 0.6t + 8.2$
$p = 0.6(5) + 8.2$
$p = 3 + 8.2$
$p = 11.2$
In 2010, the sales of potash will be 11.2 million tonnes.

The year 2015 is 10 years since 2005.
Substitute $t = 10$ in the equation: $p = 0.6t + 8.2$
$p = 0.6(10) + 8.2$
$p = 6 + 8.2$
$p = 14.2$
In 2015, the sales of potash will be 14.2 million tonnes.
I assume that the relation continues for times beyond 2007 and remains linear.

18. a) The number of students is a linear function of the time in years since 2001.
The year 2003 is 2 years after 2001.
So, an ordered pair that satisfies this function is: (2, 3470)
The increase in student population is 198 students per year. This is the slope of a graph of this function.
Let t represent the time in years after 2001.
Let p represent the student population.
Use the slope-point form of the equation: $y - y_1 = m(x - x_1)$
Replace y with p, and replace x with t.
$p - p_1 = m(t - t_1)$
Substitute: $p_1 = 3470$, $m = 198$, and $t_1 = 2$

$p - 3470 = 198(t - 2)$
The linear equation above represents the relation.

b) The year 2005 is 4 years after 2001.
Substitute $t = 4$ in the equation: $p - 3470 = 198(t - 2)$
$p - 3470 = 198(4 - 2)$
$p - 3470 = 198(2)$
$p = 396 + 3470$
$p = 3866$
In January 2005, there were approximately 3866 students in francophone schools.
To check the answer, I know that the number of students increases by 198 a year, so, in the 2 years from January 2003 to January 2005, the number of students will increase by:
$2(198) = 396$
I add this number to the number of students in January 2003: $3470 + 396 = 3866$
This agrees with the answer I obtained by substitution.

19. a) Given G(–3, 11) and H(4, –3)

$$\text{Slope of GH} = \frac{\text{change in } y\text{-coordinates}}{\text{change in } x\text{-coordinates}}$$

$$\text{Slope of GH} = \frac{-3 - 11}{4 - (-3)}$$

$$\text{Slope of GH} = \frac{-14}{7}$$

Slope of GH $= -2$

b) Use the slope-point form of the equation: $y - y_1 = m(x - x_1)$
Substitute the coordinates of G and the slope; that is, substitute: $y_1 = 11$, $m = -2$, and $x_1 = -3$
$y - 11 = -2(x - (-3))$
$y - 11 = -2(x + 3)$

c) Use the slope-point form of the equation: $y - y_1 = m(x - x_1)$
Substitute the coordinates of H and the slope; that is, substitute: $y_1 = -3$, $m = -2$, and $x_1 = 4$
$y - (-3) = -2(x - 4)$
$y + 3 = -2(x - 4)$

d) Write each equation in slope-intercept form.
$y - 11 = -2(x + 3)$ becomes
$y - 11 = -2x - 6$
$y = -2x - 6 + 11$
$y = -2x + 5$

$y + 3 = -2(x - 4)$ becomes
$y + 3 = -2x + 8$
$y = -2x + 8 - 3$
$y = -2x + 5$
Since both equations have the same slope-intercept form, the equations are equivalent.

Another strategy I could have used is to graph both equations to see if the graphs are the same line.

20. a) i) A line that is parallel to the line with equation $y = -\frac{4}{3}x + 1$ has the same slope; that is, its slope is $-\frac{4}{3}$.
Use the slope-point form of the equation: $y - y_1 = m(x - x_1)$
Substitute the coordinates of D(–5, –3) and the slope; that is, substitute: $y_1 = -3$, $m = -\frac{4}{3}$, and $x_1 = -5$

$$y - (-3) = -\frac{4}{3}(x - (-5))$$
$$y + 3 = -\frac{4}{3}(x + 5)$$

ii) A line that is perpendicular to the line with equation $y = -\frac{4}{3}x + 1$ has a slope that is the negative reciprocal of $-\frac{4}{3}$; that is, its slope is $\frac{3}{4}$.
Use the slope-point form of the equation: $y - y_1 = m(x - x_1)$
Substitute the coordinates of D(–5, –3) and the slope; that is, substitute: $y_1 = -3$, $m = \frac{3}{4}$, and $x_1 = -5$

$$y - (-3) = \frac{3}{4}(x - (-5))$$
$$y + 3 = \frac{3}{4}(x + 5)$$

b) The two equations have the same constant terms on each side of the equation. The equations have different values for the coefficient of x.

21. a) i) A line that is parallel to the line with equation $y = 2x + 3$ has the same slope; that is, its slope is 2.
Use the slope-point form of the equation: $y - y_1 = m(x - x_1)$
Substitute the coordinates of C(1, –2) and the slope; that is, substitute: $y_1 = -2$, $m = 2$, and $x_1 = 1$

$$y - (-2) = 2(x - 1)$$
$$y + 2 = 2(x - 1)$$

ii) A line that is perpendicular to the line with equation $y = 2x + 3$ has a slope that is the negative reciprocal of 2; that is, its slope is $-\frac{1}{2}$.
Use the slope-point form of the equation: $y - y_1 = m(x - x_1)$
Substitute the coordinates of C(1, –2) and the slope; that is, substitute: $y_1 = -2$, $m = -\frac{1}{2}$, and $x_1 = 1$

$$y-(-2) = -\frac{1}{2}(x-1)$$
$$y+2 = -\frac{1}{2}(x-1)$$

22. a) A line that is parallel to the line with equation $y-3 = -\frac{5}{2}(x+2)$ has the same slope; that is, its slope is $-\frac{5}{2}$.

Use the slope-point form of the equation: $y - y_1 = m(x - x_1)$

Substitute the coordinates of E(2, 6) and the slope; that is, substitute: $y_1 = 6$, $m = -\frac{5}{2}$, and $x_1 = 2$

$$y-6 = -\frac{5}{2}(x-2)$$

b) A line that is perpendicular to the line with equation $y-3 = -\frac{5}{2}(x+2)$ has a slope that is the negative reciprocal of $-\frac{5}{2}$; that is, its slope is $\frac{2}{5}$.

Use the slope-point form of the equation: $y - y_1 = m(x - x_1)$

Substitute the coordinates of E(2, 6) and the slope; that is, substitute: $y_1 = 6$, $m = \frac{2}{5}$, and $x_1 = 2$

$$y-6 = \frac{2}{5}(x-2)$$

I know that each equation is correct because I can identify the coordinates of the given point and the given slope from each equation.

23. a) The point at an x-intercept of 4 has coordinates (4, 0). The given equation is $y = \frac{3}{5}x - 7$; so, the equation of a parallel line has the same slope; that is, its slope is $\frac{3}{5}$.

Use the slope-point form of the equation: $y - y_1 = m(x - x_1)$

Substitute the coordinates (4, 0) and the slope; that is, substitute: $y_1 = 0$, $m = \frac{3}{5}$, and $x_1 = 4$

$$y-0 = \frac{3}{5}(x-4)$$
$$y = \frac{3}{5}(x-4)$$

b) The line with x-intercept –3 and y-intercept 6 passes through the points with coordinates (–3, 0) and (0, 6). To determine the slope of this line, use:

$$\text{Slope} = \frac{\text{change in } y\text{-coordinates}}{\text{change in } x\text{-coordinates}}$$

$$\text{Slope} = \frac{6-0}{0-(-3)}$$

$$\text{Slope} = \frac{6}{3}\text{, or } 2$$

A perpendicular line has a slope that is the negative reciprocal of 2; that is, the slope of a perpendicular line is $-\frac{1}{2}$.

Use the slope-point form of the equation: $y - y_1 = m(x - x_1)$

Substitute the coordinates of F(4, –1) and the slope; that is, substitute: $y_1 = -1$, $m = -\frac{1}{2}$, and $x_1 = 4$

$$y-(-1) = -\frac{1}{2}(x-4)$$

$$y+1 = -\frac{1}{2}(x-4)$$

24. Determine the y-intercept of the line with equation: $y - 3 = \frac{2}{9}(x+5)$

Substitute $x = 0$.

$y - 3 = \frac{2}{9}(0+5)$ Solve for y.

$$y-3 = \frac{2}{9}(5)$$

$$y-3 = \frac{10}{9}$$

$$y = \frac{10}{9} + \frac{27}{9}$$

$$y = \frac{37}{9}$$

The other line has y-intercept $\frac{37}{9}$ and a slope that is the negative reciprocal of $\frac{2}{9}$; that is, its slope is $-\frac{9}{2}$.

Use the slope-point form of the equation: $y - y_1 = m(x - x_1)$

Substitute the coordinates of the intercept $\left(0, \frac{37}{9}\right)$ and the slope; that is, substitute: $y_1 = \frac{37}{9}$, $m = -\frac{9}{2}$, and $x_1 = 0$

$$y - \frac{37}{9} = -\frac{9}{2}(x-0)$$

$$y - \frac{37}{9} = -\frac{9}{2}x$$

$$y = -\frac{9}{2}x + \frac{37}{9}$$

25. The given equation is: $y = -\frac{5}{3}x - \frac{25}{3}$

The slope of a graph of the equation above is $-\frac{5}{3}$.

The slope of the graph of a perpendicular line is the negative reciprocal of $-\frac{5}{3}$; that is, the slope of a perpendicular line is $\frac{3}{5}$.

The perpendicular line passes through K(–2, –5).
Use the slope-point form of the equation: $y - y_1 = m(x - x_1)$
Substitute: $y_1 = -5$, $m = \frac{3}{5}$, and $x_1 = -2$

$$y - (-5) = \frac{3}{5}(x - (-2))$$
$$y + 5 = \frac{3}{5}(x + 2)$$

C

26. If two perpendicular lines intersect at a point, there are many possible equations for the lines; too many equations to count. The slope of one line could be any positive or negative real number. The slope of the perpendicular line would be the negative reciprocal of the real number.
Suppose the slope of one line is m.
The line passes through the point M(3, 5).
Use the slope-point form of the equation: $y - y_1 = m(x - x_1)$
Substitute: $y_1 = 5$ and $x_1 = 3$
$y - 5 = m(x - 3)$

The slope of the perpendicular line is the negative reciprocal of m; that is, the slope is $-\frac{1}{m}$.

The line passes through the point M(3, 5).
Use the slope-point form of the equation: $y - y_1 = m(x - x_1)$
Substitute: $y_1 = 5$, $m = -\frac{1}{m}$ and $x_1 = 3$

$$y - 5 = -\frac{1}{m}(x - 3)$$

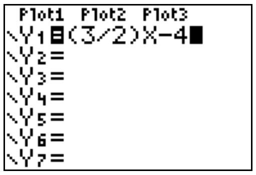
The two equations have the form: $y - 5 = m(x - 3)$ and $y - 5 = -\frac{1}{m}(x - 3)$, for all real values of m, except 0. When $m = 0$, the two equations are $y = 5$ and $x = 3$.

27. Use the slope-point form of the equation: $y - y_1 = m(x - x_1)$
The coordinates of the point at the y-intercept b are: $(0, b)$
Substitute: $y_1 = b$ and $x_1 = 0$ in the equation above.
$y - b = m(x - 0)$ Solve for y.
$y - b = mx$
$y = mx + b$

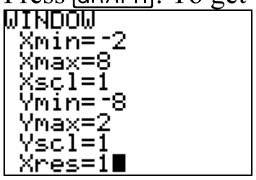
Checkpoint 2 **Assess Your Understanding (page 376)**

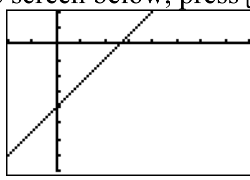
6.3

1. a) On a TI-83 calculator, press Y=, then input the equation $y = \frac{3}{2}x - 4$ with no spaces between the terms or symbols. The screen should look like this:



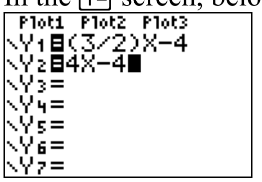
Press GRAPH. To get the screen below, press WINDOW and set the window as follows:





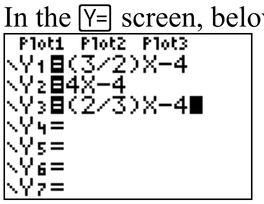
b) The line will have a greater slope when the x-coefficient increases. For example, use an x-coefficient of 4 and graph $y = 4x - 4$ on the same screen.

In the Y= screen, below \Y1=(3/2)X–4, input \Y2=4X–4.

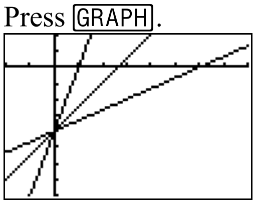


The line will have a lesser slope when the x-coefficient decreases. For example, use an x-coefficient of $\frac{2}{3}$ and graph the equation $y = \frac{2}{3}x - 4$ on the same screen.

In the Y= screen, below \Y2=4X–4, input \Y3=(2/3)X–4.

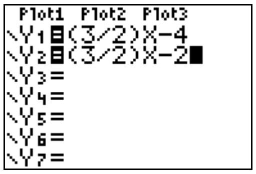


Press GRAPH.



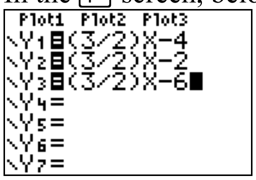
c) The line will have a greater y-intercept when the constant term increases. For example, use a constant term of –2 and graph $y = \frac{3}{2}x - 2$ on the same screen.

In the Y= screen, below \Y1=(3/2)X–4, input \Y2=(3/2)X–2.

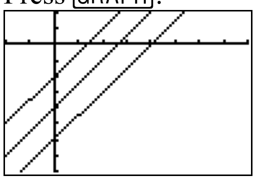


The line will have a lesser y-intercept when the constant term decreases. For example, use a constant term of –2 and graph $y = \frac{3}{2}x - 6$ on the same screen.

In the Y= screen, below \Y2=(3/2)X–2, input \Y3=(3/2)X–6.



Press GRAPH.



6.4

2. a) From the graph, two points have coordinates (0, 10) and (2, 60).

$\text{Slope} = \frac{\text{rise}}{\text{run}}$

$\text{Slope} = \frac{60 - 10}{2 - 0}$

$\text{Slope} = \frac{50}{2}$

Slope = 25

Since the slope is the quotient of distance in kilometres and time in hours, the slope is 25 km/h, which is the average speed of Eric's snowmobile.

From the graph, the d-intercept is 10 km, which is how far Eric was from home when he began his snowmobile ride.

b) An equation has the form $d = mt + b$, where m is the slope and b is the vertical intercept. So, an equation is: $d = 25t + 10$

c) i) Substitute $t = 2\frac{1}{4}$ in the equation, then solve for d.

Write $2\frac{1}{4}$ as 2.25.

$d = 25t + 10$
$d = 25(2.25) + 10$
$d = 56.25 + 10$
$d = 66.25$

Eric was 66.25 km from home after he had travelled $2\frac{1}{4}$ hours.

ii) Substitute $d = 45$, then solve for t.

$45 = 25t + 10$ Subtract 10 from each side.

$35 = 25t$ Divide each side by 25.

$\frac{35}{25} = t$

$t = 1.4$

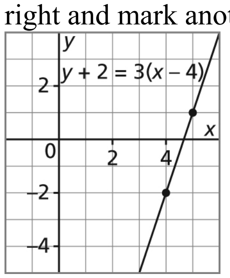
1.4 h is 1 h and 0.4×60 min, or 1 h 24 min.
Eric took 1 h 24 min to travel 45 km from home.

6.5

3. a) $y + 2 = 3(x - 4)$

The equation is in slope-point form, so I shall use the coordinates of a point and the slope to graph the equation.
From the equation, a point on the line has coordinates (4, –2) and the slope of the line is 3.

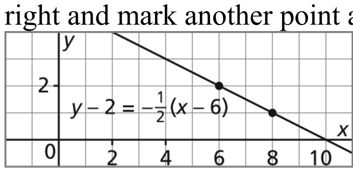
On a grid, plot the point (4, –2), then write the slope as $\frac{3}{1}$ and move 3 units up, 1 unit right and mark another point at (5, –1). Draw a line through the points.



b) $y - 2 = -\frac{1}{2}(x - 6)$

The equation is in slope-point form, so I will use the coordinates of a point and the slope to graph the equation.
From the equation, a point on the line has coordinates (6, 2) and the slope of the line is $-\frac{1}{2}$.

On a grid, plot the point (6, 2), then write the slope as $\frac{-1}{2}$ and move 1 unit down, 2 units right and mark another point at (8, 1). Draw a line through the points.



c) Determine the equation of the line that passes through (–4, 7) and (6, –1).

The slope of the line is: $\frac{\text{rise}}{\text{run}}$

Slope $= \frac{-1 - 7}{6 - (-4)}$

Slope = $\dfrac{-8}{10}$

Slope = $\dfrac{-4}{5}$

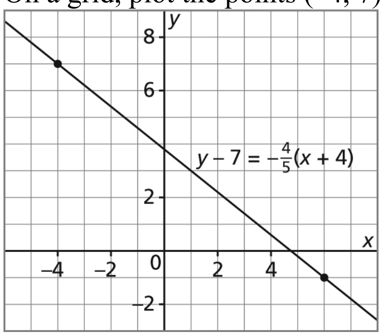
Use the slope-point form of the equation of a line: $y - y_1 = m(x - x_1)$

Substitute: $y_1 = 7$, $m = -\dfrac{4}{5}$, and $x_1 = -4$

$y - 7 = -\dfrac{4}{5}(x - (-4))$

$y - 7 = -\dfrac{4}{5}(x + 4)$

On a grid, plot the points (–4, 7) and (6, –1), then draw a line through them.



d) The required line is perpendicular to the line with equation $y + 4 = 2(x + 2)$.
This line has slope 2.

The slope of a perpendicular line is the negative reciprocal of 2; that is, the slope is $-\dfrac{1}{2}$.

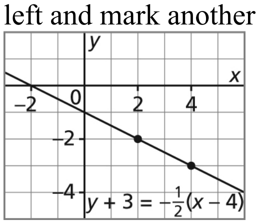
A line with this slope passes through the point with coordinates (4, –3), so use the slope-point form of the equation of a line: $y - y_1 = m(x - x_1)$

Substitute: $y_1 = -3$, $m = -\dfrac{1}{2}$, and $x_1 = 4$

$y - (-3) = -\dfrac{1}{2}(x - 4)$

$y + 3 = -\dfrac{1}{2}(x - 4)$

On a grid, plot the point (4, –3), then write the slope as $\dfrac{1}{-2}$ and move 1 unit up, 2 units left and mark another point at (2, –2). Draw a line through the points.



e) The point at an x-intercept of 5 has coordinates (5, 0).
The point at a y-intercept of 3 has coordinates (0, 3).

The slope of this line is: $\dfrac{\text{rise}}{\text{run}}$

Slope = $\dfrac{0-3}{5-0}$

Slope = $\dfrac{-3}{5}$, or $-\dfrac{3}{5}$

A line parallel to this line has the same slope of $-\dfrac{3}{5}$.

The required line passes through (–7, –2) and has slope $-\dfrac{3}{5}$.

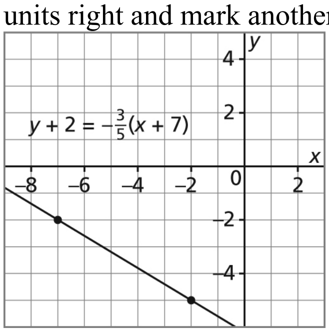
Use the slope-point form of the equation of a line: $y - y_1 = m(x - x_1)$

Substitute: $y_1 = -2$, $m = -\dfrac{3}{5}$, and $x_1 = -7$

$y - (-2) = -\dfrac{3}{5}(x - (-7))$

$y + 2 = -\dfrac{3}{5}(x + 7)$

On a grid, plot the point (–7, –2), then write the slope as $\dfrac{-3}{5}$ and move 3 units down, 5 units right and mark another point at (–2, –5). Draw a line through the points.



4. a) A line has slope 2 and y-intercept 3.
The slope-intercept form of an equation is: $y = mx + b$
Substitute $m = 2$ and $b = 3$.
The equation is: $y = 2x + 3$

b) Determine the coordinates of a point on the line in part a.
Substitute $x = 1$.
$y = 2(1) + 3$
$y = 2 + 3$
$y = 5$
A point on the line has coordinates (1, 5).
Use the slope-point form of the equation of a line: $y - y_1 = m(x - x_1)$
Substitute: $y_1 = 5$, $m = 2$, and $x_1 = 1$
$y - 5 = 2(x - 1)$

c) Both equations indicate that the slope of the line is 2.
The slope-intercept form indicates that the y-intercept is 3.

The slope-point form indicates that the line passes through the point with coordinates (1, 5).

Lesson 6.6 **General Form of the Equation for a Linear Relation** **Exercises (pages 384–385)**

A

4. Compare each given equation with each form given below:
Slope-intercept form: $y = mx + b$
Slope-point form: $y - y_1 = m(x - x_1)$
Standard form: $Ax + By = C$
General form: $Ax + By + C = 0$

a) $8x - 3y = 52$ is in standard form.

b) $9x + 4y + 21 = 0$ is in general form.

c) $y = 4x + 7$ is in slope-intercept form.

d) $y - 3 = 5(x + 7)$ is in slope-point form.

5. **a)** $8x - 3y = 24$
For the x-intercept, substitute $y = 0$.
$8x - 3(0) = 24$ Solve for x.
$8x = 24$ Divide each side by 8.
$x = 3$
For the y-intercept, substitute $x = 0$.
$8(0) - 3y = 24$ Solve for y.
$-3y = 24$ Divide each side by –3.
$y = -8$
The x-intercept is 3 and the y-intercept is –8.

b) $7x + 8y = 56$
For the x-intercept, substitute $y = 0$.
$7x + 8(0) = 56$ Solve for x.
$7x = 56$ Divide each side by 7.
$x = 8$
For the y-intercept, substitute $x = 0$.
$7(0) + 8y = 56$ Solve for y.
$8y = 56$ Divide each side by 8.
$y = 7$
The x-intercept is 8 and the y-intercept is 7.

c) $4x - 11y = 88$
For the x-intercept, substitute $y = 0$.
$4x - 11(0) = 88$ Solve for x.
$4x = 88$ Divide each side by 4.
$x = 22$
For the y-intercept, substitute $x = 0$.
$4(0) - 11y = 88$ Solve for y.
$-11y = 88$ Divide each side by –11.
$y = -8$
The x-intercept is 22 and the y-intercept is –8.

d) $2x - 9y = 27$
For the x-intercept, substitute $y = 0$.
$2x - 9(0) = 27$ Solve for x.
$2x = 27$ Divide each side by 2.
$x = 13.5$
For the y-intercept, substitute $x = 0$.
$2(0) - 9y = 27$ Solve for y.
$-9y = 27$ Divide each side by –9.
$y = -3$
The x-intercept is 13.5 and the y-intercept is –3.

6. An equation is in general form when it is written as $Ax + By + C = 0$, where A is a whole number, and B and C are integers.

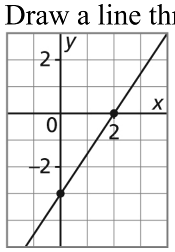
a) $4x + 3y = 36$ Subtract 36 from each side.
$4x + 3y - 36 = 0$

b) $2x - y = 7$ Subtract 7 from each side.
$2x - y - 7 = 0$

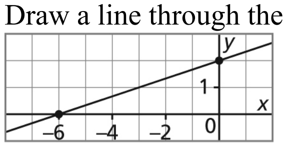
c) $y = -2x + 6$ Add $2x$ to each side and subtract 6 from each side.
$y + 2x - 6 = 0$ Rearrange the terms on the left side so the x-term is first.
$2x + y - 6 = 0$

d) $y = 5x - 1$ Subtract $5x$ from each side and add 1 to each side.
$y - 5x + 1 = 0$ Rearrange the terms on the left side so the x-term is first.
$-5x + y + 1 = 0$ Multiply each term by –1 so the x-term is positive.
$5x - y - 1 = 0$

7. a) Mark a point at 2 on the x-axis, and mark a point at –3 on the y-axis.
Draw a line through the points.



b) Mark a point at –6 on the x-axis, and mark a point at 2 on the y-axis.
Draw a line through the points.



B

8. a) i) The equation $-2x + 3y + 42 = 0$ is not in general form because the coefficient of x is not a whole number.

ii) The equation $4y - 5x = 100$ is not in general form because all the terms are not on the left side and the first term is not an x-term with a whole number coefficient.

iii) The equation $\frac{1}{2}x - \frac{1}{2}y + 1 = 0$ is not in general form because the coefficient of x is not a whole number and the coefficient of y is not an integer.

iv) The equation $5y + 9x - 20 = 0$ is not in general form because the x-term is not first.

b) i) $-2x + 3y + 42 = 0$ Multiply each term by –1.
$2x - 3y - 42 = 0$

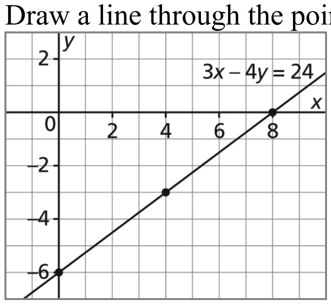
ii) $4y - 5x = 100$ Subtract 100 from each side.
$4y - 5x - 100 = 0$ Rearrange the terms on the left side so the x-term is first.
$-5x + 4y - 100 = 0$ Multiply each term by –1 so the x-term is positive.
$5x - 4y + 100 = 0$

iii) $\frac{1}{2}x - \frac{1}{2}y + 1 = 0$ Multiply each term by 2.
$x - y + 2 = 0$

iv) $5y + 9x - 20 = 0$ Interchange the first two terms.
$9x + 5y - 20 = 0$

9. a) i) $3x - 4y = 24$
For the x-intercept, substitute $y = 0$.
$3x - 4(0) = 24$ Solve for x.
$3x = 24$ Divide each side by 3.
$x = 8$
For the y-intercept, substitute $x = 0$.
$3(0) - 4y = 24$ Solve for y.
$-4y = 24$ Divide each side by –4.
$y = -6$
The x-intercept is 8 and the y-intercept is –6.

ii) Mark a point at 8 on the x-axis, and mark a point at –6 on the y-axis.
Draw a line through the points.



iii) A point on the graph appears to have coordinates (4, –3). Substitute these coordinates into the equation of the line.
Substitute $x = 4$ and $y = -3$ in $3x - 4y = 24$.
L.S. $= 3x - 4y$ R.S. $= 24$
$= 3(4) - 4(-3)$
$= 12 + 12$

$= 24$
Since the left side is equal to the right side, the point satisfies the equation, and the graph is correct.

b) i) $6x - 5y = -60$
For the x-intercept, substitute $y = 0$.

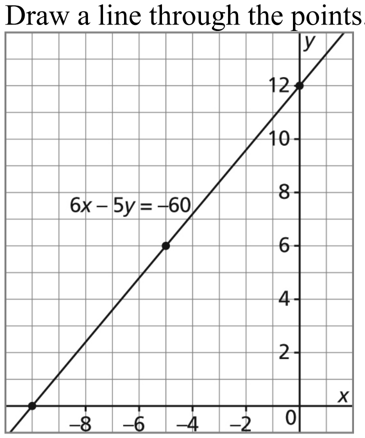
$6x - 5(0) = -60$	Solve for x.
$6x = -60$	Divide each side by 6.
$x = -10$	

For the y-intercept, substitute $x = 0$.

$6(0) - 5y = -60$	Solve for y.
$-5y = -60$	Divide each side by –5.
$y = 12$	

The x-intercept is –10 and the y-intercept is 12.

ii) Mark a point at –10 on the x-axis, and mark a point at 12 on the y-axis.
Draw a line through the points.



iii) A point on the graph appears to have coordinates (–5, 6). Substitute these coordinates into the equation of the line.
Substitute $x = -5$ and $y = 6$ in $6x - 5y = -60$.

L.S. $= 6x - 5y$ R.S. $= -60$
$= 6(-5) - 5(6)$
$= -30 - 30$
$= -60$

Since the left side is equal to the right side, the point satisfies the equation, and the graph is correct.

c) i) $3x - 2y = 24$
For the x-intercept, substitute $y = 0$.

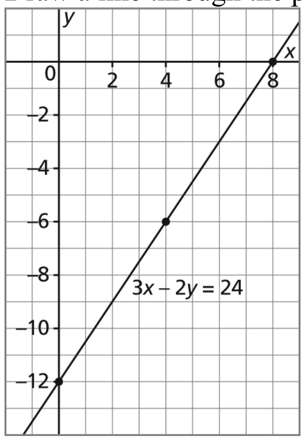
$3x - 2(0) = 24$	Solve for x.
$3x = 24$	Divide each side by 3.
$x = 8$	

For the y-intercept, substitute $x = 0$.

$3(0) - 2y = 24$	Solve for y.
$-2y = 24$	Divide each side by –2.
$y = -12$	

The x-intercept is 8 and the y-intercept is –12.

ii) Mark a point at 8 on the x-axis, and mark a point at –12 on the y-axis.
Draw a line through the points.



iii) A point on the graph appears to have coordinates (4, –6). Substitute these coordinates into the equation of the line.

Substitute $x = 4$ and $y = -6$ in $3x - 2y = 24$.

L.S. $= 3x - 2y$ R.S. $= 24$
$= 3(4) - 2(-6)$
$= 12 + 12$
$= 24$

Since the left side is equal to the right side, the point satisfies the equation, and the graph is correct.

d) **i)** $5x - y = 10$

For the x-intercept, substitute $y = 0$.

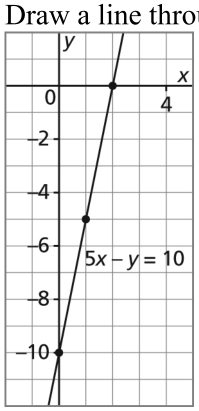
$5x - (0) = 10$ Solve for x.
$5x = 10$ Divide each side by 5.
$x = 2$

For the y-intercept, substitute $x = 0$.

$5(0) - y = 10$ Solve for y.
$-y = 10$ Multiply each side by –1.
$y = -10$

The x-intercept is 2 and the y-intercept is –10.

ii) Mark a point at 2 on the x-axis, and mark a point at –10 on the y-axis.
Draw a line through the points.



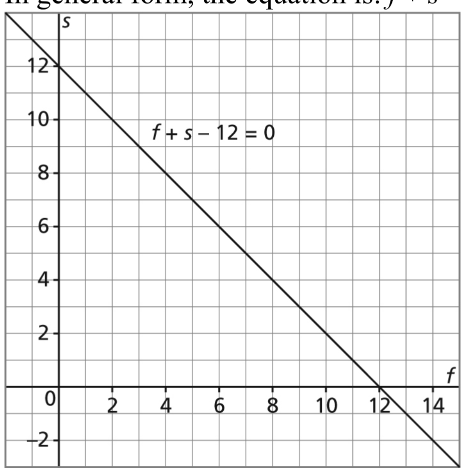
iii) A point on the graph appears to have coordinates (1, –5). Substitute these coordinates into the equation of the line.

Substitute $x = 1$ and $y = –5$ in $5x – y = 10$.
L.S. = $5x – y$ R.S. = 10
= $5(1) – (–5)$
= $5 + 5$
= 10
Since the left side is equal to the right side, the point satisfies the equation, and the graph is correct.

10. a) Determine pairs of numbers that have a sum of 12.

f	–2	0	4	10	12	15
s	14	12	8	2	0	–3

b), c) Since the numbers are real numbers, I may join the points because all numbers with a sum of 12 are permissible.
The sum of the numbers is 12, so an equation is: $f + s = 12$
To write the equation in general form, subtract 12 from each side.
In general form, the equation is: $f + s – 12 = 0$



d) From the graph:
When $f = –1$, $s = 13$
When $f = 1$, $s = 11$
When $f = 3$, $s = 9$
When $f = 7$, $s = 5$
When $f = 14$, $s = –2$
So, 6 pairs of integers with a sum of 12 are: –1, 13; 1, 11; 3, 9; 7, 5; 14, –2

11. a), b) When Rebecca uses only pans that hold 12 bars, she needs: $\frac{504}{12}$, or 42 pans

When Rebecca uses only pans that hold 36 bars, she needs: $\frac{504}{36}$, or 14 pans

When Rebecca uses 2 pans that hold 36 bars, she has $504 – 36 \times 2$, or 432 bars to put in pans that hold 12 bars. So, she needs $\frac{432}{12}$, or 36 pans that hold 12 bars.

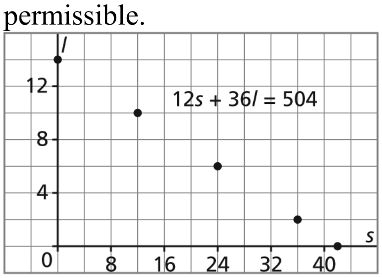
When Rebecca uses 6 pans that hold 36 bars, she has 504 – 36 × 6, or 288 bars to put in pans that hold 12 bars. So, she needs $\frac{288}{12}$, or 24 pans that hold 12 bars.

When Rebecca uses 10 pans that hold 36 bars, she has 504 – 36 × 10, or 144 bars to put in pans that hold 12 bars. So, she needs $\frac{144}{12}$, or 12 pans that hold 12 bars.

Let s represent the number of pans that hold 12 bars.
Let l represent the number of pans that hold 36 bars.
Write the data above in a table of values.

s	l
0	14
12	10
24	6
36	2
42	0

Plot the points on a grid. Do not join the points because only whole numbers of pans are permissible.



s pans hold $12s$ bars.
l pans hold $36l$ bars.
The total number of bars is 504.
So, an equation is: $12s + 36l = 504$

12. a) $4x + 3y - 24 = 0$ Solve for y. Subtract $4x$ from each side. Add 24 to each side.

$3y = -4x + 24$ Divide each side by 3.

$y = -\frac{4}{3}x + 8$ This equation is in slope-intercept form.

b) $3x - 8y + 12 = 0$ Solve for y. Subtract $3x$ and 12 from each side.

$-8y = -3x - 12$ Divide each side by –8.

$y = \frac{-3}{-8}x - \frac{12}{-8}$ Simplify.

$y = \frac{3}{8}x - \frac{3}{-2}$

$y = \frac{3}{8}x + \frac{3}{2}$ This equation is in slope-intercept form.

c) $2x - 5y - 15 = 0$ Solve for y. Subtract $2x$ from each side. Add 15 to each side.

$-5y = -2x + 15$ Divide each side by –5.

$y = \frac{-2}{-5}x + \frac{15}{-5}$ Simplify.

$y = \frac{2}{5}x + (-3)$

$y = \frac{2}{5}x - 3$ This equation is in slope-intercept form.

d) $7x + 3y + 10 = 0$ Solve for y. Subtract $7x$ and 10 from each side.

$3y = -7x - 10$ Divide each side by 3.

$y = -\frac{7}{3}x - \frac{10}{3}$ This equation is in slope-intercept form.

13. I wrote each equation in slope-intercept form, then I identified the slope of the line from the equation. When an equation is in slope-intercept form, the coefficient of x is the slope of the line.

a) $4x + y - 10 = 0$ Solve for y. Subtract $4x$ from each side. Add 10 to each side.

$y = -4x + 10$

The coefficient of x is –4, so the slope of the line is –4.

b) $3x - y + 33 = 0$ Solve for y. Subtract $3x$ and 33 from each side.

$-y = -3x - 33$ Multiply each side by –1.

$y = 3x + 33$

The coefficient of x is 3, so the slope of the line is 3.

c) $5x - y + 45 = 0$ Solve for y. Subtract $5x$ and 45 from each side.

$-y = -5x - 45$ Multiply each side by –1.

$y = 5x + 45$

The coefficient of x is 5, so the slope of the line is 5.

d) $10x + 2y - 16 = 0$ Solve for y. Subtract $10x$ from each side. Add 16 to each side.

$2y = -10x + 16$ Divide each side by 2.

$y = -5x + 8$

The coefficient of x is –5, so the slope of the line is –5.

14. a) $x - 2y + 10 = 0$

Since the coefficients of x and y are factors of the constant term 10, I will use intercepts to graph the equation.

For the x-intercept, substitute $y = 0$.

$x - 2(0) + 10 = 0$ Solve for x. Subtract 10 from each side.

$x = -10$

For the y-intercept, substitute $x = 0$.

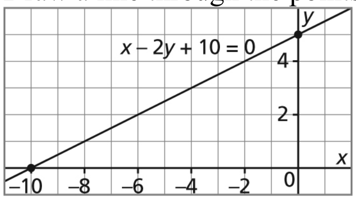
$0 - 2y + 10 = 0$ Solve for y. Subtract 10 from each side.

$-2y = -10$ Divide each side by –2.

$y = 5$

On a grid, mark a point at –10 on the x-axis and mark a point at 5 on the y-axis.

Draw a line through the points.



b) $2x + 3y - 15 = 0$

Since the coefficient of y is a factor of the constant term 15, I will use the y-intercept as one point to graph the equation. I will determine the coordinates of another point by substituting a value for x that is a multiple of 3, so that the corresponding y-coordinate is a whole number.

For the y-intercept, substitute $x = 0$.

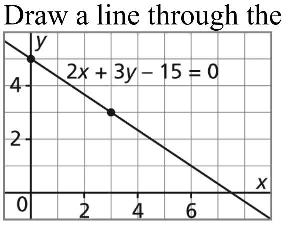
$2(0) + 3y - 15 = 0$	Solve for y. Add 15 to each side.
$3y = 15$	Divide each side by 3.
$y = 5$	

Substitute $x = 3$.

$2(3) + 3y - 15 = 0$	Simplify.
$6 + 3y - 15 = 0$	
$3y - 9 = 0$	Solve for y. Add 9 to each side.
$3y = 9$	Divide each side by 3.
$y = 3$	

On a grid, mark a point at 5 on the y-axis and plot the point (3, 3).

Draw a line through the points.



c) $7x + 4y + 4 = 0$

Since the coefficient of y is a factor of the constant term 4, I will use the y-intercept as one point to graph the equation. I will determine the coordinates of another point by substituting a value for x that is a multiple of 4, so that the corresponding y-coordinate is a whole number.

For the y-intercept, substitute $x = 0$.

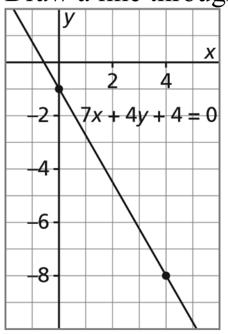
$7(0) + 4y + 4 = 0$	Solve for y. Subtract 4 from each side.
$4y = -4$	Divide each side by 4.
$y = -1$	

Substitute $x = 4$.

$7(4) + 4y + 4 = 0$	Simplify.
$28 + 4y + 4 = 0$	
$4y + 32 = 0$	Solve for y. Subtract 32 from each side.
$4y = -32$	Divide each side by 4.
$y = -8$	

On a grid, mark a point at –1 on the y-axis and plot the point (4, –8).

Draw a line through the points.



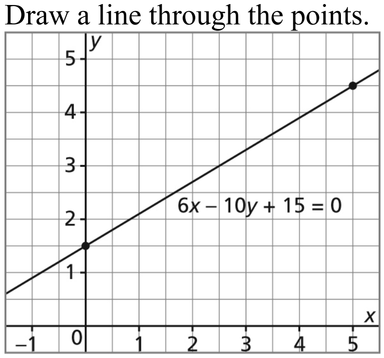
d) $6x - 10y + 15 = 0$
Since neither coefficient is a factor of the constant term, I will determine the coordinates of two points by substituting values for x. Whatever whole number values of x I choose, the y-coordinate will be a fraction.
Substitute $x = 0$.

$6(0) - 10y + 15 = 0$	Solve for y. Subtract 15 from each side.
$-10y = -15$	Divide each side by –10.
$y = 1.5$	

Substitute $x = 5$.

$6(5) - 10y + 15 = 0$	Simplify.
$30 - 10y + 15 = 0$	
$45 - 10y = 0$	Solve for y. Subtract 45 from each side.
$-10y = -45$	Divide each side by –10.
$y = 4.5$	

On a grid, use a scale of 2 squares to 1 unit, so the points can be plotted accurately. Plot points at (0, 1.5) and plot the point (5, 4.5).
Draw a line through the points.



15. a) The pipe is 96 ft. long.
Four pieces of 6-ft. pipe are: 4 × 6 ft. = 24 ft.
So, 96 ft. – 24 ft. = 72 ft. must be made from 8-ft. pieces.

The number of 8-ft. pieces is: $\frac{72}{8} = 9$

Nine 8-ft. pieces are needed.

b) The pipe is 96 ft. long.
Three pieces of 8-ft. pipe are: 3 × 8 ft. = 24 ft.

So, 96 ft. – 24 ft. = 72 ft. must be made from 6-ft. pieces.

The number of 6-ft. pieces is: $\frac{72}{6} = 12$

Twelve 6-ft. pieces are needed.

c) The pipe is 96 ft. long.
Three pieces of 6-ft. pipe are: 3×6 ft. = 18 ft.
So, 96 ft. – 18 ft. = 78 ft. must be made from 8-ft. pieces.

The number of 8-ft. pieces is: $\frac{78}{8} = 9.75$

Since this is not a whole number, 3 pieces of 6-ft. pipe could not be used.

d) The pipe is 96 ft. long.
Four pieces of 8-ft. pipe are: 4×8 ft. = 32 ft.
So, 96 ft. – 32 ft. = 64 ft. must be made from 6-ft. pieces.

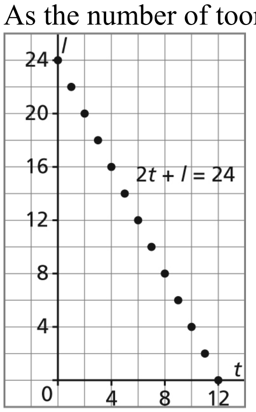
The number of 6-ft. pieces is: $\frac{64}{6} = 10.\overline{6}$

Since this is not a whole number, 4 pieces of 8-ft. pipe could not be used.

16. a) When Pascal saves all toonies, he has $\frac{24}{2}$, or 12 toonies.

When Pascal saves all loonies, he has 24 loonies.
When Pascal saves 1 toonie, he has \$24 – \$2, or \$22 in loonies.
Some data are: 12 toonies, 0 loonies; 0 toonies, 24 loonies; and 1 toonie, 22 loonies

b), c) Let t represent the number of toonies.
Let l represent the number of loonies.
Plot these points on a grid: (12, 0), (0, 24), (1, 22)
Do not join the points. Use a straightedge placed through the points to plot more points.
As the number of toonies increases by 1, the number of loonies decreases by 2.



The value of toonies in dollars + the value of loonies in dollars = \$24
Since 1 toonie is \$2, the value of toonies in dollars is: $2t$
Since 1 loonie is \$1, the value of loonies in dollars is: l
So, an equation is: $2t + l = 24$

d) i) Using the graph:
If Pascal had 6 toonies and 8 loonies, then the point (6, 8) would lie on the line in the graph above.

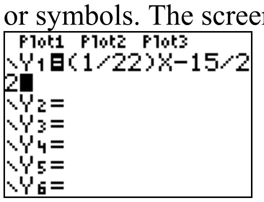
From the graph, the point with t-coordinate 6 has l-coordinate 12.
So, the point (6, 8) does not lie on the line and Pascal cannot have 6 toonies and 8 loonies.
Using the equation:
If Pascal had 6 toonies and 8 loonies, then the point (6, 8) would satisfy the equation:
$2t + l = 24$
Substitute $t = 6$ and $l = 8$.
L.S. $= 2t + l$ R.S. $= 24$
L.S. $= 2(6) + (8)$
L.S. $= 12 + 8$
L.S. $= 20$
Since the left side does not equal the right side, the point (6, 8) does not satisfy the equation and Pascal cannot have 6 toonies and 8 loonies.

ii) Using the graph:
If Pascal had 8 toonies and 6 loonies, then the point (8, 6) would lie on the line in the graph above.
From the graph, the point with t-coordinate 8 has l-coordinate 8.
So, the point (8, 6) does not lie on the line and Pascal cannot have 8 toonies and 6 loonies.
Using the equation:
If Pascal had 8 toonies and 6 loonies, then the point (8, 6) would satisfy the equation:
$2t + l = 24$
Substitute $t = 8$ and $l = 6$.
L.S. $= 2t + l$ R.S. $= 24$
L.S. $= 2(8) + (6)$
L.S. $= 16 + 6$
L.S. $= 22$
Since the left side does not equal the right side, the point (8, 6) does not satisfy the equation and Pascal cannot have 8 toonies and 6 loonies.

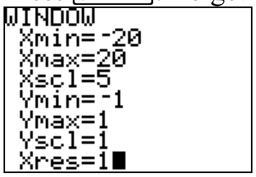
17. I used a graphing TI-83 calculator. I wrote each equation in function form, so I could input the equation using the $\boxed{\text{Y=}}$ key.

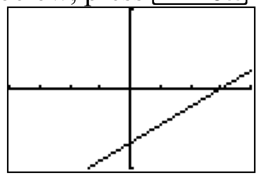
a) $x - 22y - 15 = 0$ Solve for y. Subtract x from each side. Add 15 to each side.
$-22y = -x + 15$ Divide each side by -22.
$$y = \frac{1}{22}x - \frac{15}{22}$$

On a TI-83 calculator, press $\boxed{\text{Y=}}$, then input the equation with no spaces between the terms or symbols. The screen should look like this:



Press GRAPH. To get the screen below, press WINDOW and set the window as follows:

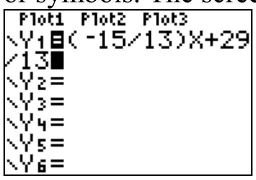




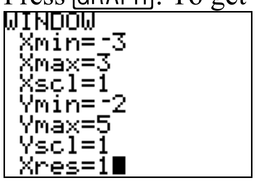
b) $15x + 13y - 29 = 0$ Solve for y. Subtract $15x$ from each side. Add 29 to each side.
$13y = -15x + 29$ Divide each side by 13.

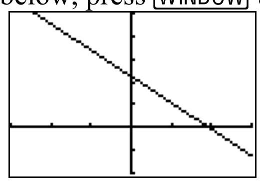
$$y = \frac{-15}{13}x + \frac{29}{13}$$

On a TI-83 calculator, press Y=, then input the equation with no spaces between the terms or symbols. The screen should look like this:



Press GRAPH. To get the screen below, press WINDOW and set the window as follows:

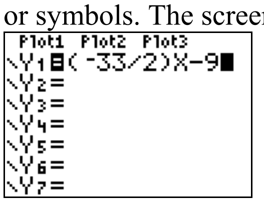




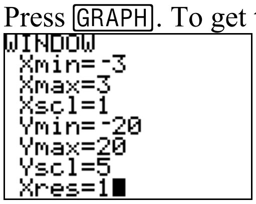
c) $33x + 2y + 18 = 0$ Solve for y. Subtract $33x$ and 18 from each side.
$2y = -33x - 18$ Divide each side by 2.

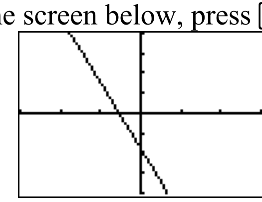
$$y = \frac{-33}{2}x - 9$$

On a TI-83 calculator, press Y=, then input the equation with no spaces between the terms or symbols. The screen should look like this:



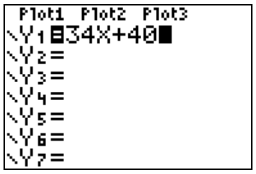
Press GRAPH. To get the screen below, press WINDOW and set the window as follows:



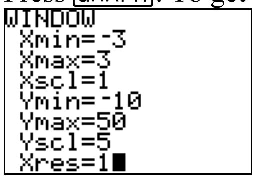


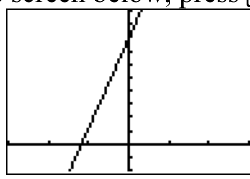
d) $34x - y + 40 = 0$ Solve for y. Subtract $34x$ and 40 from each side.
$-y = -34x - 40$ Multiply each side by -1.
$y = 34x + 40$

On a TI-83 calculator, press Y=, then input the equation with no spaces between the terms or symbols. The screen should look like this:



Press GRAPH. To get the screen below, press WINDOW and set the window as follows:





18. a)

$y = \frac{1}{3}x - 4$	Multiply each side by 3 to remove the fraction.
$3y = 3\left(\frac{1}{3}x\right) - 3(4)$	Simplify.
$3y = x - 12$	Subtract $3y$ from each side.
$0 = x - 3y - 12$	Write the terms on the left side.
$x - 3y - 12 = 0$	This equation is in general form.

b)

$y - 2 = \frac{1}{3}(x + 5)$	Multiply each side by 3 to remove the fraction.
$3(y - 2) = 3\left(\frac{1}{3}\right)(x + 5)$	Simplify.
$3y - 6 = x + 5$	Subtract $3y$ from each side. Add 6 to each side.
$0 = x - 3y + 11$	Write the terms on the left side.
$x - 3y + 11 = 0$	This equation is in general form.

c)

$y + 3 = -\frac{1}{4}(x - 1)$	Multiply each side by 4 to remove the fraction.
$4(y + 3) = 4\left(-\frac{1}{4}\right)(x - 1)$	Simplify.
$4y + 12 = -x + 1$	Add x to each side. Subtract 1 from each side.
$x + 4y + 11 = 0$	This equation is in general form.

d)

$y = -\frac{3}{2}x + \frac{4}{3}$	Multiply each side by the common denominator: $2 \times 3 = 6$
$6y = 6\left(-\frac{3}{2}x\right) + 6\left(\frac{4}{3}\right)$	Simplify.
$6y = -9x + 8$	Add $9x$ to each side. Subtract 8 from each side.
$9x + 6y - 8 = 0$	This equation is in general form.

19. a) In slope-intercept form, the equation is: $y = \frac{1}{3}x - 4$

To graph this line, mark a point at the y-intercept –4, then use the slope $\frac{1}{3}$ to move 1 unit up and 3 units right to mark a point at (3, –3).

From question 18a, in general form, the equation is: $x - 3y - 12 = 0$
Use intercepts to graph this line.
For the x-intercept, substitute $y = 0$.

$x - 3(0) - 12 = 0$ Solve for x. Add 12 to each side.
$x = 12$

For the y-intercept, substitute $x = 0$.

$0 - 3y - 12 = 0$ Solve for y. Add 12 to each side.
$-3y = 12$ Divide each side by –3.
$y = -4$

To graph this line, mark a point at 12 on the x-axis and mark a point at –4 on the y-axis.

To write the equation in slope-point form, determine the coordinates of a point on the line.

Substitute $x = 3$ in $x - 3y - 12 = 0$.

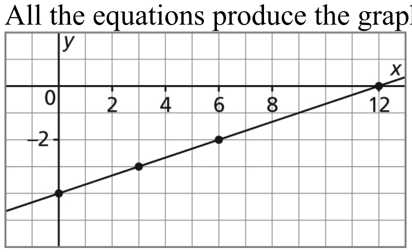
$3 - 3y - 12 = 0$
$-3y - 9 = 0$ Add 9 to each side.
$-3y = 9$ Divide each side by –3.
$y = -3$

A point on the line has coordinates (3, –3). The slope of the line is $\frac{1}{3}$.

So, in slope-point form, the equation is: $y - (-3) = \frac{1}{3}(x - 3)$; or $y + 3 = \frac{1}{3}(x - 3)$

To graph this line, mark a point at (3, –3), then use the slope $\frac{1}{3}$ to move 1 unit up and 3 units right to mark a point at (6, –2).

All the equations produce the graph below.



b) In slope-point form, the equation is: $y - 2 = \frac{1}{3}(x + 5)$

From this equation, the slope of the line is $\frac{1}{3}$ and the coordinates of a point on the line are (–5, 2).

To graph this line, mark a point at (–5, 2), then use the slope $\frac{1}{3}$ to move 1 unit up and 3 units right to mark a point at (–2, 3).

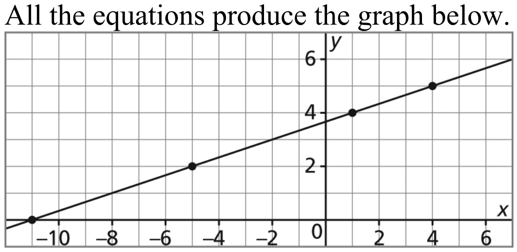
From question 18b, in general form, the equation is: $x - 3y + 11 = 0$
Use the x-intercept and one other point to graph this line.
For the x-intercept, substitute $y = 0$.

$x - 3(0) + 11 = 0$ Solve for x. Subtract 11 from each side.

$x = -11$
Substitute $x = 1$.
$1 - 3y + 11 = 0$
$-3y + 12 = 0$ Solve for y. Subtract 12 from each side.
$-3y = -12$ Divide each side by -3.
$y = 4$
To graph this line, mark a point at -11 on the x-axis and plot the point (1, 4).

To write the equation in slope-intercept form, solve this equation for y:
$y - 2 = \frac{1}{3}(x + 5)$ Add 2 to each side.
$y = 2 + \frac{1}{3}(x + 5)$ Remove the brackets.
$y = 2 + \frac{1}{3}x + \frac{5}{3}$ Simplify.
$y = \frac{1}{3}x + \frac{6}{3} + \frac{5}{3}$
$y = \frac{1}{3}x + \frac{11}{3}$
Since the y-intercept is not a whole number, determine the coordinates of another point on the line.
Substitute $x = 1$.
$y = \frac{1}{3}(1) + \frac{11}{3}$
$y = \frac{12}{3}$
$y = 4$
To graph this line, plot the point (1, 4), then use the slope $\frac{1}{3}$ to move 1 unit up and 3 units right to mark a point at (4, 5).

All the equations produce the graph below.



c) In slope-point form, the equation is: $y + 3 = -\frac{1}{4}(x - 1)$

From this equation, the slope of the line is $-\frac{1}{4}$ and the coordinates of a point on the line are (1, –3).
To graph this line, mark a point at (1, –3), then use the slope $-\frac{1}{4}$ to move 1 unit down and 4 units right to mark a point at (5, –4).

From question 18c, in general form, the equation is: $x + 4y + 11 = 0$
Use the x-intercept and one other point to graph this line.
For the x-intercept, substitute $y = 0$.

$x + 4(0) + 11 = 0$ Solve for x. Subtract 11 from each side.
$x = -11$

Substitute $x = 1$.

$1 + 4y + 11 = 0$
$4y + 12 = 0$ Solve for y. Subtract 12 from each side.
$4y = -12$ Divide each side by 4.
$y = -3$

To graph this line, mark a point at –11 on the x-axis and plot the point (1, –3).

To write the equation in slope-intercept form, solve this equation for y:

$y + 3 = -\frac{1}{4}(x - 1)$ Subtract 3 from each side.

$y = -3 - \frac{1}{4}(x - 1)$ Remove the brackets.

$y = -3 - \frac{1}{4}x + \frac{1}{4}$ Simplify.

$y = -\frac{1}{4}x - \frac{12}{4} + \frac{1}{4}$

$y = -\frac{1}{4}x - \frac{11}{4}$

Since the y-intercept is not a whole number, determine the coordinates of another point on the line.
Substitute $x = 1$.

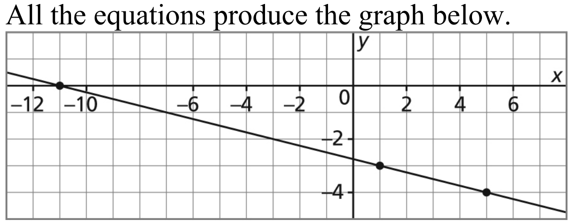
$y = -\frac{1}{4}(1) - \frac{11}{4}$

$y = -\frac{12}{4}$

$y = -3$

To graph this line, plot the point (1, –3), then use the slope $-\frac{1}{4}$ to move 1 down up and 4 units right to mark a point at (5, –4).

All the equations produce the graph below.



d) In slope-intercept form, the equation is: $y = -\frac{3}{2}x + \frac{4}{3}$

Since the y-intercept is not a whole number, determine the coordinates of two other points on the line.

Substitute $x = 0$.

$y = -\frac{3}{2}(0) + \frac{4}{3}$

$y = \frac{4}{3}$

Substitute $x = 2$.

$y = -\frac{3}{2}(2) + \frac{4}{3}$

$y = -3 + \frac{4}{3}$

$y = \frac{-9}{3} + \frac{4}{3}$

$y = \frac{-5}{3}$

Since no points appear to have integer coordinates, use a scale on the y-axis of 3 squares to 1 unit.

To graph this line, plot points at $\left(0, \frac{4}{3}\right)$ and $\left(2, -\frac{5}{3}\right)$.

From question 18d, in general form, the equation is: $9x + 6y - 8 = 0$
Determine the coordinates of two points on this line.
Substitute $x = 0$.

$9(0) + 6y - 8 = 0$	Solve for y. Add 8 to each side.
$6y = 8$	Divide by 6.
$y = \frac{8}{6}$, or $\frac{4}{3}$	

Substitute $x = 2$.

$9(2) + 6y - 8 = 0$	Simplify.
$10 + 6y = 0$	Solve for y. Subtract 10 from each side.
$6y = -10$	Divide by 6.
$y = -\frac{10}{6}$, or $-\frac{5}{3}$	

To graph this line, plot the points at $\left(0, \frac{4}{3}\right)$ and $\left(2, -\frac{5}{3}\right)$.

To write the equation in slope-point form, use the slope $-\frac{3}{2}$ and the coordinates of a point on the line, such as $\left(2, -\frac{5}{3}\right)$.

In slope-point form, the equation is: $y - \left(-\frac{5}{3}\right) = -\frac{3}{2}(x - 2)$, or $y + \frac{5}{3} = -\frac{3}{2}(x - 2)$

To graph this line, determine the coordinates of another point on the line.
Substitute $x = 0$.

$y + \frac{5}{3} = -\frac{3}{2}(0 - 2)$

$y + \frac{5}{3} = 3$ Subtract $\frac{5}{3}$ from each side.

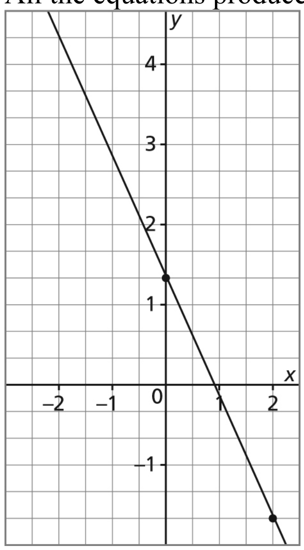
$y = 3 - \frac{5}{3}$

$y = \frac{9}{3} - \frac{5}{3}$

$y = \frac{4}{3}$

Plot the points $\left(2, -\frac{5}{3}\right)$ and $\left(0, \frac{4}{3}\right)$.

All the equations produce the graph below.



20. When $C = 0$, the equation $Ax + By + C = 0$ becomes $Ax + By = 0$.
Solve for y.

$Ax + By = 0$ Subtract Ax from each side.

$By = -Ax$ Divide each side by B.

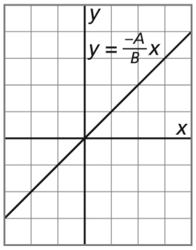
$y = \frac{-A}{B}x$

The slope of this line is $\frac{-A}{B}$ and its y-intercept is 0.

This line passes through the origin and has slope $\frac{-A}{B}$.

The slope is the opposite of the quotient of the coefficients of the x-term and the y-term.
When A and B have the same sign, the slope is negative.

When A and B have opposite signs, the slope is positive.



21. a) The x-intercept is 4 and the y-intercept is 6.
The coordinates of G are: (0, 6)
The coordinates of H are: (4, 0)

Slope of GH = $\frac{\text{rise}}{\text{run}}$

Slope of GH = $\frac{6-0}{0-4}$

Slope of GH = $\frac{6}{-4}$, or $-\frac{6}{4}$

The slope of the line is the opposite of the quotient of the y-intercept and the x-intercept; that is,

Slope $= -\frac{y\text{-intercept}}{x\text{-intercept}}$

b) This relationship is true for all lines. Suppose the x-intercept is a and the y-intercept is b. Then the coordinates of the point at the x-intercept is $(a, 0)$ and the coordinates of the point at the y-intercept is $(0, b)$.
The slope of the line through the points at the intercepts is:

$$\frac{b-0}{0-a} = -\frac{b}{a}$$

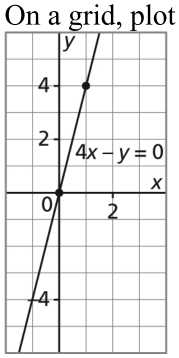
22. a) $2x + 3y - 6 = 0$
Determine the intercepts of the line with this equation.
For the x-intercept, substitute $y = 0$.
$2x + 3(0) - 6 = 0$
$2x - 6 = 0$ Add 6 to each side.
$2x = 6$ Divide each side by 2.
$x = 3$
For the y-intercept, substitute $x = 0$.
$2(0) + 3y - 6 = 0$
$3y - 6 = 0$ Add 6 to each side.
$3y = 6$ Divide each side by 3.
$y = 2$
The required graph has an x-intercept of 3 and a y-intercept of 2.
So, the required graph is Graph B.

b) $2x - 3y + 6 = 0$
Determine the intercepts of the line with this equation.
For the x-intercept, substitute $y = 0$.

$2x - 3(0) + 6 = 0$
$2x + 6 = 0$ Subtract 6 from each side.
$2x = -6$ Divide each side by 2.
$x = -3$
For the y-intercept, substitute $x = 0$.
$2(0) - 3y + 6 = 0$
$-3y + 6 = 0$ Subtract 6 from each side.
$-3y = -6$ Divide each side by -3.
$y = 2$
The required graph has an x-intercept of -3 and a y-intercept of 2.
So, the required graph is Graph A.

23. a) Write $4x - y = 0$ in slope-intercept form.
$4x - y = 0$ Solve for y.
$y = 4x$
The graph of this line has y-intercept 0, so it passes through the origin.
The x- and y-intercepts are at the same point so we cannot draw a line through them.

b) Determine the coordinates of another point.
Substitute $x = 1$.
$y = 4(1)$
$y = 4$
On a grid, plot points at (0, 0) and (1, 4), then draw a line through the points.



24. Write each equation in slope-intercept form, then look for equations that are the same.

a) $y = 3x + 6$ is in slope-intercept form.

b) $2x - 3y - 3 = 0$ Solve for y. Subtract $2x$ from each side. Add 3 to each side.
$-3y = -2x + 3$ Divide each side by -3.
$y = \frac{2}{3}x - 1$ This equation is in slope-intercept form.

c) $y - 2 = \frac{2}{3}(x - 2)$ Solve for y. Add 2 to each side.
$y = \frac{2}{3}(x - 2) + 2$ Remove brackets.
$y = \frac{2}{3}x - \frac{2}{3}(2) + 2$ Simplify.

$y = \frac{2}{3}x - \frac{4}{3} + 2$

$y = \frac{2}{3}x - \frac{4}{3} + \frac{6}{3}$

$y = \frac{2}{3}x + \frac{2}{3}$ This equation is in slope-intercept form.

d) $3x - y - 6 = 0$ Solve for y. Subtract $3x$ from each side. Add 6 to each side.
$-y = -3x + 6$ Multiply each side by –1.
$y = 3x - 6$ This equation is in slope-intercept form.

e) $y = \frac{2}{3}x - 1$ This equation is in slope-intercept form.

f) $y - 3 = 3(x - 3)$ Solve for y. Add 3 to each side.
$y = 3(x - 3) + 3$ Remove brackets.
$y = 3x - 9 + 3$ Simplify.
$y = 3x - 6$

g) $y - 1 = \frac{2}{3}(x - 3)$ Solve for y. Add 1 to each side.

$y = \frac{2}{3}(x - 3) + 1$ Remove brackets.

$y = \frac{2}{3}x - \frac{2}{3}(3) + 1$ Simplify.

$y = \frac{2}{3}x - 2 + 1$

$y = \frac{2}{3}x - 1$

h) $y + 3 = 3(x - 1)$ Solve for y. Subtract 3 from each side.
$y = 3(x - 1) - 3$ Remove brackets.
$y = 3x - 3 - 3$ Simplify.
$y = 3x - 6$
The equations in parts b, e, and g have the same slope-intercept form, so these equations are equivalent: $2x - 3y - 3 = 0$; $y = \frac{2}{3}x - 1$; and $y - 1 = \frac{2}{3}(x - 3)$
The equations in parts d, f, and h have the same slope-intercept form, so these equations are equivalent: $3x - y - 6 = 0$; $y - 3 = 3(x - 3)$; and $y + 3 = 3(x - 1)$

25. a) For an equation in general form to be difficult to graph using intercepts, the intercepts could be fractions; that is, the coefficients of the x- and y-terms are not factors of the constant term.
For example, $3x + 4y + 5 = 0$

b) To graph this equation, determine the coordinates of two points that satisfy the equation. Choose integer values of x so that when they are substituted in the equation, the values of y are also integers. Since the coefficient of the y-term is 4, find values of x so that when

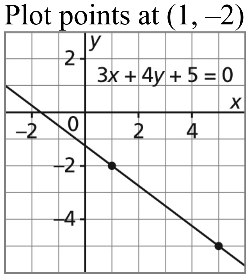
the value is substituted, then the sum of the $3x$ term and 5 is a multiple of 4.
For example, substitute $x = 1$.

$$\begin{aligned} 3(1) + 4y + 5 &= 0 \\ 3 + 4y + 5 &= 0 \\ 4y + 8 &= 0 \\ 4y &= -8 \\ y &= -2 \end{aligned}$$

Substitute $x = 5$.

$$\begin{aligned} 3(5) + 4y + 5 &= 0 \\ 15 + 4y + 5 &= 0 \\ 4y + 20 &= 0 \\ 4y &= -20 \\ y &= -5 \end{aligned}$$

Plot points at (1, –2) and (5, –5), then draw a line through the points.



C

26. a) $\dfrac{x}{4} + \dfrac{y}{3} = 1$ Multiply each side by the common denominator: $4 \times 3 = 12$

$12\left(\dfrac{x}{4}\right) + 12\left(\dfrac{y}{3}\right) = 12(1)$

$3x + 4y = 12$

$3x + 4y - 12 = 0$

Since this equation is in general form, it represents a linear function.

b) $y = \dfrac{10}{x}$ Multiply each side by x.

$xy = x\left(\dfrac{10}{x}\right)$

$xy = 10$

Since this equation cannot be written in general form, it does not represent a linear function.

c) $y = 2x(x + 4)$ Remove brackets.

$y = 2x^2 + 8x$

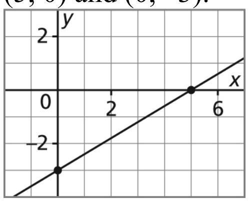
Since this equation cannot be written in general form, it does not represent a linear function.

d) $y = \dfrac{x + y}{4} + 2$ Multiply each side by 4.

$4y = 4\left(\dfrac{x + y}{4}\right) + 4(2)$ Simplify.

$4y = x + y + 8$ Collect like terms.
$0 = x - 3y + 8$
$x - 3y + 8 = 0$
Since this equation is in general form, it represents a linear function.

27. The x-intercept of a line is 5 and its y-intercept is –3. The coordinates of these points are (5, 0) and (0, –3).



Use the formula for the equation of a line given two points on the line.

$$\frac{y-y_1}{x-x_1} = \frac{y_2-y_1}{x_2-x_1}$$

Substitute: $y_1 = 0$, $x_1 = 5$, $y_2 = -3$, and $x_2 = 0$

$$\frac{y-0}{x-5} = \frac{-3-0}{0-5}$$

$$\frac{y}{x-5} = \frac{3}{5}$$ Multiply each side by $(x - 5)$.

$$y = \frac{3}{5}(x-5)$$

28. a) $Ax + By + C = 0$
Write the equation in slope-intercept form.
$Ax + By + C = 0$ Solve for y. Subtract Ax and C from each side.
$By = -Ax - C$ Divide each side by B.

$$y = \frac{-A}{B}x - \frac{C}{B}$$

The slope of the line is the coefficient of x, which is $-\frac{A}{B}$.

b) From part a, the y-intercept is the constant term in the equation in slope-intercept form; that is, $-\frac{C}{B}$

Review **(pages 388–390)**

6.1

1. a) From the graph, two points on the line have coordinates (–2, 3) and (4, –1).

$$\text{Slope} = \frac{\text{change in } y\text{-coordinates}}{\text{change in } x\text{-coordinates}}$$

$$\text{Slope} = \frac{-1-3}{4-(-2)}$$

$$\text{Slope} = \frac{-4}{6}$$

$$\text{Slope} = \frac{-2}{3}, \text{ or } -\frac{2}{3}$$

b) From the graph, two points on the line have coordinates (–5, 0) and (0, 4).

$$\text{Slope} = \frac{\text{change in } y\text{-coordinates}}{\text{change in } x\text{-coordinates}}$$

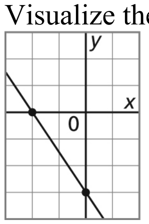
$$\text{Slope} = \frac{4-0}{0-(-5)}$$

$$\text{Slope} = \frac{4}{5}$$

2. a) The change in x is positive, and the change in y is negative.

So, the slope $= \dfrac{\text{change in } y\text{-coordinates}}{\text{change in } x\text{-coordinates}}$ is a negative number divided by a positive number, which is a negative number.
So, the slope is negative.

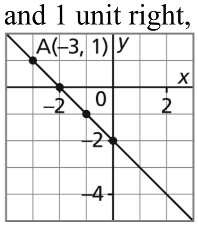
b) Visualize the intercepts or sketch a diagram.



The line goes down to the right so its slope is negative.

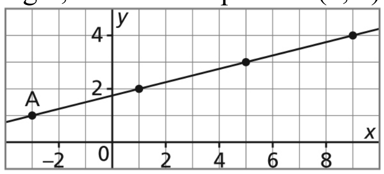
c) If a line does not have an x-intercept, it is a horizontal line, which has a slope of 0.

3. a) i) Plot point A(–3, 1). Write the slope of –1 as $\dfrac{-1}{1}$. From point A, move 1 unit down and 1 unit right, then mark a point at (–2, 0). Draw a line through these points.



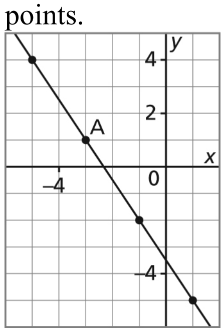
ii) Identify the coordinates of three other points from the line: (–2, 0), (–1, –1), and (0, –2)

b) **i)** Plot point A(–3, 1). The slope is $\frac{1}{4}$, so from point A, move 1 unit up and 4 units right, then mark a point at (1, 2). Draw a line through these points.



ii) Identify the coordinates of three other points from the line: (1, 2), (5, 3), and (9, 4)

c) **i)** Plot point A(–3, 1). Write the slope of $-\frac{3}{2}$ as $\frac{-3}{2}$ and from point A, move 3 units down and 2 units right, then mark a point at (–1, –2). Draw a line through these points.



ii) Identify the coordinates of three other points from the line: (–5, 4), (–1, –2), and (1, –5)

4. Since I know the coordinates of two points on each line, I use the formula:

$$\text{Slope} = \frac{\text{change in } y\text{-coordinates}}{\text{change in } x\text{-coordinates}}$$

a) The two points have coordinates: (–6, 8) and (–1, –2)

$$\text{Slope} = \frac{-2-8}{-1-(-6)}$$

$$\text{Slope} = \frac{-10}{5}$$

$$\text{Slope} = -2$$

b) The two points have coordinates: (–3, 7) and (5, –5)

$$\text{Slope} = \frac{-5-7}{5-(-3)}$$

$$\text{Slope} = \frac{-12}{8}$$
$$\text{Slope} = -\frac{3}{2}$$

5. **a)** Two points on the graph have coordinates: (2, 320) and (5, 800)

$$\text{Slope} = \frac{\text{change in vertical coordinates}}{\text{change in horizontal coordinates}}$$
$$\text{Slope} = \frac{800 - 320}{5 - 2}$$
$$\text{Slope} = \frac{480}{3}$$

Slope = 160

The vertical axis represents distance in metres and the horizontal axis represents time in minutes.

So the slope is 160 m/min, and this is Gabrielle's average speed.

b) The slope is equal to the rate of change.

c) **i)** From the graph, Gabrielle jogs 320 m in 2 min, so in 4 min, she will jog:
2×320 m = 640 m

ii) Gabrielle jogs 160 m in 1 min.
So, to jog 1000 m, she will take: $\frac{1000}{160}$ min = 6.25 min

6.2

6. Parallel lines have the same slope.
Perpendicular lines have slopes that are negative reciprocals; that is, the product of their slopes is –1.

a) The slope of FG is 3.

i) A parallel line also has slope 3.

ii) A perpendicular line has slope $-\frac{1}{3}$.

b) The slope of FG is $-\frac{6}{5}$.

i) A parallel line also has slope $-\frac{6}{5}$.

ii) A perpendicular line has slope $\frac{5}{6}$.

c) The slope of FG is $\frac{11}{8}$.

i) A parallel line also has slope $\frac{11}{8}$.

ii) A perpendicular line has slope $-\frac{8}{11}$.

d) The slope of FG is 1.

i) A parallel line also has slope 1.

ii) A perpendicular line has slope –1.

7. Use the formula:

$$\text{Slope} = \frac{\text{change in } y\text{-coordinates}}{\text{change in } x\text{-coordinates}}$$

to determine the slope of each line, then compare the slopes.
Parallel lines have the same slope.
Perpendicular lines have slopes that are negative reciprocals; that is, the product of their slopes is –1.

a) The points are: H(–3, 3) and J(–1, 7)

$$\text{Slope of HJ} = \frac{7-3}{-1-(-3)}$$

$$\text{Slope of HJ} = \frac{4}{2}$$

Slope of HJ = 2
The points are: K(–1, 2) and M(5, –1)

$$\text{Slope of KM} = \frac{-1-2}{5-(-1)}$$

$$\text{Slope of KM} = \frac{-3}{6}$$

$$\text{Slope of KM} = -\frac{1}{2}$$

Since 2 and $-\frac{1}{2}$ are negative reciprocals, then HJ and KM are perpendicular.

b) The points are: N(–4, –2) and P(–1, 7)

$$\text{Slope of NP} = \frac{7-(-2)}{-1-(-4)}$$

$$\text{Slope of NP} = \frac{9}{3}$$

Slope of NP = 3

The points are: Q(2, 5) and R(4, –1)

Slope of QR = $\dfrac{-1-5}{4-2}$

Slope of QR = $\dfrac{-6}{2}$

Slope of QR = –3

Since the slopes of 3 and –3 are neither equal nor negative reciprocals, then the lines are neither parallel nor perpendicular.

8. From the diagram, I can see that STUV is not a parallelogram because the opposite sides are not parallel. To justify my answer, I determine the slopes of two opposite sides, SV and TU.

Slope = $\dfrac{\text{rise}}{\text{run}}$

From the diagram, count squares to determine the rise and run from V to S.

Slope of SV = $\dfrac{5}{2}$

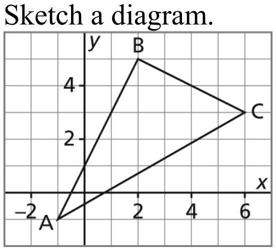
From the diagram, count squares to determine the rise and run from U to T.

Slope of TU = $\dfrac{6}{2}$

Slope of TU = 3

Since the slopes of SV and TU are not equal, these sides are not parallel, and the quadrilateral is not a parallelogram.

9. Sketch a diagram.



From the diagram, BC and AB might be perpendicular.
Determine their slopes to check.

Slope = $\dfrac{\text{rise}}{\text{run}}$

From the diagram, count squares to determine the rise and run from A to B.

Slope of AB = $\dfrac{6}{3}$, or 2

From the diagram, count squares to determine the rise and run from C to B.

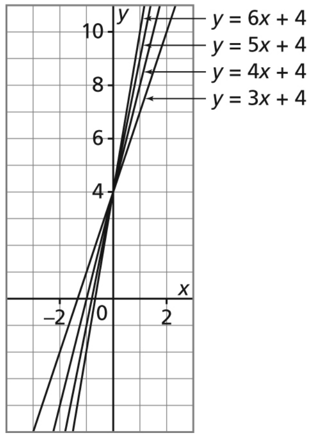
Slope of BC = $\dfrac{2}{-4}$, or $-\dfrac{1}{2}$

Since the slopes of AB and BC are negative reciprocals, these sides are perpendicular, and ΔABC is a right triangle, with $\angle$B = 90°.

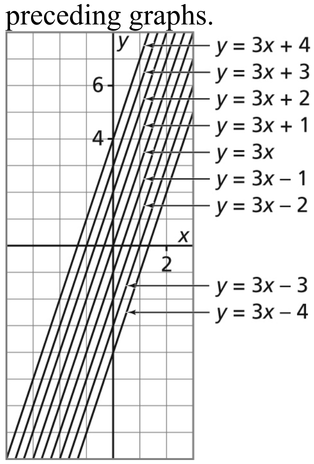
6.3

10. a) The graph of $y = 3x + 4$ has slope 3 and y-intercept 4.
When the coefficient of x increases by 1, the new equation is: $y = 4x + 4$; the slope of the graph is now 4 and the y-intercept is unchanged.

When the coefficient of x increases by 1 again, the new equation is: $y = 5x + 4$; the slope of the graph is now 5 and the y-intercept is unchanged.
When the coefficient of x increases by 1 again, the new equation is: $y = 6x + 4$; the slope of the graph is now 6 and the y-intercept is unchanged.
As the coefficient of x increases, the graphs are steeper.



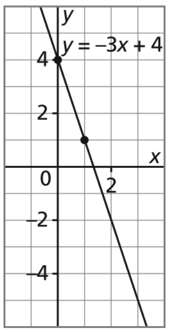
b) The graph of $y = 3x + 4$ has slope 3 and y-intercept 4.
When the constant term decreases by 1, the new equation is: $y = 3x + 3$; the slope of the graph is unchanged, and the y-intercept is now 3.
When the constant term decreases by 1 again, the new equation is: $y = 3x + 2$; the slope of the graph is unchanged, and the y-intercept is now 2.
When the constant term decreases by 1 again, the new equation is: $y = 3x + 1$; the slope of the graph is unchanged, and the y-intercept is now 1.
When the constant term decreases by 1 again, the new equation is: $y = 3x$; the slope of the graph is unchanged, and the y-intercept is now 0.
When the constant term decreases by 1 again, the new equation is: $y = 3x – 1$; the slope of the graph is unchanged, and the y-intercept is now –1.
When the constant term decreases by 1 again, the new equation is: $y = 3x – 2$; the slope of the graph is unchanged, and the y-intercept is now –2.
When the constant term decreases by 1 again, the new equation is: $y = 3x – 3$; the slope of the graph is unchanged, and the y-intercept is now –3.
When the constant term decreases by 1 again, the new equation is: $y = 3x – 4$; the slope of the graph is unchanged, and the y-intercept is now –4.
As the constant term decreases, the graph moves down the grid but remains parallel to the preceding graphs.



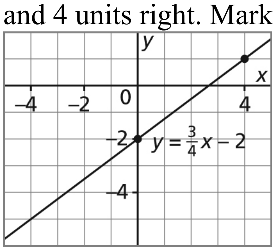
6.4

11. Compare each equation to the slope-intercept form: $y = mx + b$, where m is the slope of the graph of the equation and b is its y-intercept.

a) $y = -3x + 4$
$y = mx + b$
The slope, m, is –3 and the y-intercept, b, is 4.
Mark a point at the y-intercept. Write the slope as $\frac{-3}{1}$, then from the y-intercept, move 3 units down and 1 unit right. Mark a point. Draw a line through the two points.



b) $y = \frac{3}{4}x - 2$
$y = mx + b$
The slope, m, is $\frac{3}{4}$ and the y-intercept, b, is –2.
Mark a point at the y-intercept. Use the slope $\frac{3}{4}$; from the y-intercept, move 3 units up and 4 units right. Mark a point. Draw a line through the two points.



12. a) **i)** From the graph, two points on the graph have coordinates: (0, 1) and (3, 6)
To determine the slope, use:

$$\text{Slope} = \frac{\text{change in } y\text{-coordinates}}{\text{change in } x\text{-coordinates}}$$

$$\text{Slope} = \frac{6-1}{3-0}$$

$$\text{Slope} = \frac{5}{3}$$

From the graph, the y-intercept is 1.

ii) Use the slope-intercept form of the equation of a line: $y = mx + b$

Substitute: $m = \frac{5}{3}$ and $b = 1$

$y = \frac{5}{3}x + 1$

iii) From the graph, another point on the graph has coordinates (–3, –4)

Substitute $x = -3$ and $y = -4$ in the equation $y = \frac{5}{3}x + 1$.

L.S. $= y$	R. S. $= \frac{5}{3}x + 1$
$= -4$	$= \frac{5}{3}(-3) + 1$
	$= -5 + 1$
	$= -4$

Since the left side is equal to the right side, the equation is correct.

b) **i)** From the graph, two points on the graph have coordinates: (–2, 2) and (0, –1)
To determine the slope, use:

$$\text{Slope} = \frac{\text{change in } y\text{-coordinates}}{\text{change in } x\text{-coordinates}}$$

$$\text{Slope} = \frac{-1-2}{0-(-2)}$$

$$\text{Slope} = \frac{-3}{2}, \text{ or } -\frac{3}{2}$$

From the graph, the y-intercept is –1.

ii) Use the slope-intercept form of the equation of a line: $y = mx + b$

Substitute: $m = -\frac{3}{2}$ and $b = -1$

$y = -\frac{3}{2}x - 1$

iii) From the graph, another point on the graph has coordinates (2, –4).

Substitute $x = 2$ and $y = -4$ in the equation $y = -\frac{3}{2}x - 1$.

L.S. $= y$	R.S. $= -\frac{3}{2}x - 1$
$= -4$	$= -\frac{3}{2}(2) - 1$
	$= -3 - 1$
	$= -4$

Since the left side is equal to the right side, the equation is correct.

13. Identify the slope and y-intercept of each graph.

To determine each slope, count squares then use the formula: Slope = $\frac{\text{rise}}{\text{run}}$

Graph A has slope $\frac{2}{1}$, or 2; and y-intercept 3.

Graph B has slope $\frac{-2}{1}$, or –2; and y-intercept 3.

Graph C has slope $\frac{1}{2}$; and y-intercept –3.

Graph D has slope $\frac{-3}{1}$, or –3; and y-intercept –2.

a) $y = \frac{1}{2}x - 3$

The graph of this equation has slope $\frac{1}{2}$ and y-intercept –3, which is Graph C.

b) $y = -3x - 2$
The graph of this equation has slope –3 and y-intercept –2, which is Graph D.

c) $y = 2x + 3$
The graph of this equation has slope 2 and y-intercept 3, which is Graph A.

d) $y = -2x + 3$
The graph of this equation has slope –2 and y-intercept 3, which is Graph B.

14. a) The total amount in the account, A dollars, is the sum of the amount saved in w weeks plus the initial amount, \$40.
Every week Mason saves \$15, so in w weeks he will have saved $15w$ dollars.
So, an equation is: $A = 15w + 40$

b) Substitute $A = 355$, then solve for w.

$A = 15w + 40$
$355 = 15w + 40$ Subtract 40 from each side.
$315 = 15w$ Divide each side by 15.
$\frac{315}{15} = w$
$w = 21$

Mason will have \$355 in his account after 21 weeks.

c) On a graph of $A = 15w + 40$:
The slope is 15, which is the rate of change of the amount in the account; that is, Mason saves \$15/week.
The vertical intercept is 40, which is the amount, \$40, that was in the account when Mason began saving regularly.

15. The graph of $y = \frac{4}{7}x - 5$ has slope $\frac{4}{7}$ and y-intercept –5.

a) A line that is parallel to $y = \frac{4}{7}x - 5$ has the same slope of $\frac{4}{7}$ but a different y-intercept; so its equation could be: $y = \frac{4}{7}x + 5$ or $y = \frac{4}{7}x - 3$
I know that all 3 lines are parallel because they have the same slope.

b) A line that is perpendicular to $y = \frac{4}{7}x - 5$ has a slope that is the negative reciprocal of $\frac{4}{7}$; that is, its slope is $-\frac{7}{4}$. Its y-intercept may be the same as that of the original graph or it may be different. The equation of a perpendicular line could be: $y = -\frac{7}{4}x - 5$ or
$y = -\frac{7}{4}x + 6$
I know that each of the two new lines is perpendicular to the original line because the product of their slopes, $\left(-\frac{7}{4}\right)\left(\frac{4}{7}\right)$, is equal to –1.

6.5

16. The line with equation $y = 2x + 1$ has slope 2.
A perpendicular line DE has a slope that is the negative reciprocal of 2; that is, its slope is $-\frac{1}{2}$.
Line DE passes through F(–2, 3).
To determine the equation of line DE, use the slope-point form of the equation of a line:
$y - y_1 = m(x - x_1)$
Substitute: $y_1 = 3$, $m = -\frac{1}{2}$, and $x_1 = -2$

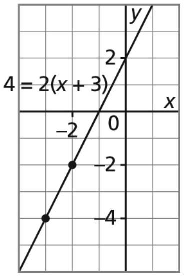
$$y - 3 = -\frac{1}{2}(x - (-2))$$

$$y - 3 = -\frac{1}{2}(x + 2)$$

An equation for line DE is: $y - 3 = -\frac{1}{2}(x + 2)$

17. a) i) $y + 4 = 2(x + 3)$
Write the equation as: $y - (-4) = 2(x - (-3))$
Compare the equation to the slope-point form: $y - y_1 = m(x - x_1)$
The slope of the graph is: $m = 2$
The coordinates of a point on the graph are: $(x_1, y_1) = (-3, -4)$

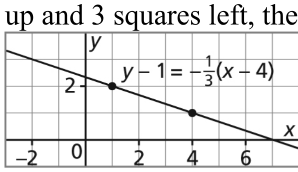
ii) Plot the point (–3, –4). From this point, use the slope, written as $\frac{2}{1}$, to move 2 squares up and 1 square right, then mark another point. Draw a line through the points.



iii) From the graph, another point on the graph has coordinates (–2, –2).
Use the slope-point form of the equation: $y - y_1 = m(x - x_1)$
Substitute: $y_1 = -2$, $m = 2$, and $x_1 = -2$
$y - (-2) = 2(x - (-2))$
$y + 2 = 2(x + 2)$ This is a different way to write the equation.

b) **i)** $y - 1 = -\frac{1}{3}(x - 4)$
Compare the equation to the slope-point form: $y - y_1 = m(x - x_1)$
The slope of the graph is: $m = -\frac{1}{3}$
The coordinates of a point on the graph are: $(x_1, y_1) = (4, 1)$

ii) Plot the point (4, 1). From this point, use the slope, written as $\frac{1}{-3}$, to move 1 square up and 3 squares left, then mark another point. Draw a line through the points.



iii) From the graph, another point on the graph has coordinates (1, 2).
Use the slope-point form of the equation: $y - y_1 = m(x - x_1)$
Substitute: $y_1 = 2$, $m = -\frac{1}{3}$, and $x_1 = 1$
$y - 2 = -\frac{1}{3}(x - 1)$ This is a different way to write the equation.

18. On each graph, I can identify the coordinates of two points, so I will write each equation in slope-point form, for which I need the slope and the coordinates of a point on the graph.

a) Two points on the graph have coordinates (2, 0) and (–1, –2).
To determine the slope of the graph, use the formula:
$$\text{Slope} = \frac{\text{change in } y\text{-coordinates}}{\text{change in } x\text{-coordinates}}$$

$$\text{Slope} = \frac{-2-0}{-1-2}$$

$$\text{Slope} = \frac{-2}{-3}\text{, or } \frac{2}{3}$$

The slope is $\frac{2}{3}$, and one point on the line has coordinates (2, 0).

Use the slope-point form of the equation: $y - y_1 = m(x - x_1)$

Substitute: $y_1 = 0$, $m = \frac{2}{3}$, and $x_1 = 2$

$$y - 0 = \frac{2}{3}(x - 2)$$

$$y = \frac{2}{3}(x - 2)$$

To verify that this equation is correct, check that the coordinates of another point on the line satisfy the equation.

Another point on the line has coordinates (–1, –2).

Substitute $x = -1$ and $y = -2$ in the equation $y = \frac{2}{3}(x - 2)$.

L.S. $= y$	R.S. $= \frac{2}{3}(x - 2)$
$= -2$	$= \frac{2}{3}(-1 - 2)$
	$= \frac{2}{3}(-3)$
	$= -2$

Since the left side is equal to the right side, the equation is correct.

b) Two points on the graph have coordinates (–3, 2) and (2, –1).

To determine the slope of the graph, use the formula:

$$\text{Slope} = \frac{\text{change in } y\text{-coordinates}}{\text{change in } x\text{-coordinates}}$$

$$\text{Slope} = \frac{-1-2}{2-(-3)}$$

$$\text{Slope} = \frac{-3}{5}\text{, or } -\frac{3}{5}$$

The slope is $-\frac{3}{5}$, and one point on the line has coordinates (–3, 2).

Use the slope-point form of the equation: $y - y_1 = m(x - x_1)$

Substitute: $y_1 = 2$, $m = -\frac{3}{5}$, and $x_1 = -3$

$$y - 2 = -\frac{3}{5}(x - (-3))$$

$$y - 2 = -\frac{3}{5}(x + 3)$$

To verify that this equation is correct, check that the coordinates of another point on the

line satisfy the equation.
Another point on the line has coordinates (2, –1).

Substitute $x = 2$ and $y = -1$ in the equation $y - 2 = -\frac{3}{5}(x + 3)$.

$$\begin{aligned} \text{L.S.} &= y - 2 \\ &= -1 - 2 \\ &= -3 \end{aligned} \qquad \begin{aligned} \text{R.S.} &= -\frac{3}{5}(x + 3) \\ &= -\frac{3}{5}(2 + 3) \\ &= -\frac{3}{5}(5) \\ &= -3 \end{aligned}$$

Since the left side is equal to the right side, the equation is correct.

19. I know the coordinates of two points on a line, so I will write each equation in slope-point form, for which I need the slope and the coordinates of a point on the line.

a) i) Two points on the line have coordinates (–3, –7) and (1, 5).
To determine the slope of the line, use the formula:

$$\text{Slope} = \frac{\text{change in } y\text{-coordinates}}{\text{change in } x\text{-coordinates}}$$

$$\text{Slope} = \frac{5 - (-7)}{1 - (-3)}$$

$$\text{Slope} = \frac{12}{4}, \text{ or } 3$$

The slope is 3, and one point on the line has coordinates (–3, –7).
Use the slope-point form of the equation: $y - y_1 = m(x - x_1)$
Substitute: $y_1 = -7$, $m = 3$, and $x_1 = -3$
$y - (-7) = 3(x - (-3))$
$y + 7 = 3(x + 3)$

b) i) To determine the coordinates of another point on the line, substitute a value for x, such as $x = 2$, in the equation, then solve for y.
$y + 7 = 3(x + 3)$
$y + 7 = 3(2 + 3)$
$y + 7 = 3(5)$
$y + 7 = 15$
$y = 8$
Another point on the line has coordinates (2, 8).

a) ii) Two points on the line have coordinates (–3, 3) and (5, –1).
To determine the slope of the line, use the formula:

$$\text{Slope} = \frac{\text{change in } y\text{-coordinates}}{\text{change in } x\text{-coordinates}}$$

$$\text{Slope} = \frac{-1 - 3}{5 - (-3)}$$

$$\text{Slope} = \frac{-4}{8}, \text{ or } -\frac{1}{2}$$

The slope is $-\frac{1}{2}$, and one point on the line has coordinates (–3, 3).

Use the slope-point form of the equation: $y - y_1 = m(x - x_1)$

Substitute: $y_1 = 3$, $m = -\frac{1}{2}$, and $x_1 = -3$

$$y - 3 = -\frac{1}{2}(x - (-3))$$

$$y - 3 = -\frac{1}{2}(x + 3)$$

b) ii) To determine the coordinates of another point on the line, substitute a value for x, such as $x = 1$, in the equation, then solve for y.

$$y - 3 = -\frac{1}{2}(x + 3)$$

$$y - 3 = -\frac{1}{2}(1 + 3)$$

$$y - 3 = -\frac{1}{2}(4)$$

$y - 3 = -2$

$y = 1$

Another point on the line has coordinates (1, 1).

20. a) Let p represent the number of people on a trip, and let C represent the cost of the trip in dollars. If a graph was drawn of C as a function of p, then two points on the graph would have coordinates (5, 220) and (3, 132).

Determine the slope of the graph.

$$\text{Slope} = \frac{\text{change in } C\text{-coordinates}}{\text{change in } p\text{-coordinates}}$$

$$\text{Slope} = \frac{220 - 132}{5 - 3}$$

$$\text{Slope} = \frac{88}{2}\text{, or } 44$$

The equation of the graph has the form:

$C = mp + b$

Substitute $C = 220$, $m = 44$, and $p = 5$, then solve for b.

$220 = 44 \times 5 + b$

$220 = 220 + b$

$b = 0$

An equation for the function is: $C = 44p$

b) The cost per person is \$44; this is the coefficient of p in the equation.

c) Substitute $C = 264$ in the equation, then solve for p.

$C = 44p$

$264 = 44p$ Divide each side by 44.

$$\frac{264}{44} = p$$

$p = 6$
Six people went on the trip.

6.6

21. a) i) $4y - 5x - 40 = 0$
This equation is not in general form because the first term on the left side is not the x-term.

ii) $\frac{1}{3}x + y = 4$
This equation is not in general form because the coefficient of the x-term is not a whole number and the constant term is not on the left side of the equation.

iii) $y - 2 = \frac{1}{3}(x + 4)$
This equation is in slope-point form.

iv) $y = \frac{1}{5}x + 3$
This equation is in slope-intercept form.

b) i) $4y - 5x - 40 = 0$
Multiply each side by –1 because the x-term must have a positive coefficient.
$-4y + 5x + 40 = 0$
Rearrange the first two terms so the x-term is first.
In general form: $5x - 4y + 40 = 0$

ii) $\frac{1}{3}x + y = 4$
Multiply each side by 3 to remove the fraction.
$3\left(\frac{1}{3}x\right) + 3y = 12$
$x + 3y = 12$
Subtract 12 from each side.
In general form: $x + 3y - 12 = 0$

iii) $y - 2 = \frac{1}{3}(x + 4)$
Multiply each side by 3.
$3(y - 2) = 3\left(\frac{1}{3}\right)(x + 4)$
$3y - 6 = x + 4$
Subtract $3y$ from each side. Add 6 to each side.
$0 = x - 3y + 10$
In general form: $x - 3y + 10 = 0$

iv) $y = \frac{1}{5}x + 3$

Multiply each side by 5.

$5y = 5\left(\frac{1}{5}\right)x + 5(3)$

$5y = x + 15$

Subtract $5y$ from each side.

$0 = x - 5y + 15$

In general form: $x - 5y + 15 = 0$

22. a) i) $3x - 4y - 24 = 0$

Since the coefficients of x and y are factors of the constant term, I will use intercepts to graph the equation.

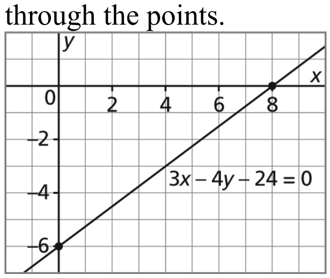
For the x-intercept, substitute $y = 0$.

$3x - 4(0) - 24 = 0$ Solve for x.
$3x - 24 = 0$ Add 24 to each side.
$3x = 24$ Divide each side by 3.
$x = 8$

For the y-intercept, substitute $x = 0$.

$3(0) - 4y - 24 = 0$ Solve for y.
$-4y - 24 = 0$ Add 24 to each side.
$-4y = 24$ Divide each side by –4.
$y = -6$

On a grid, mark a point at 8 on the x-axis and at –6 on the y-axis, then draw a line through the points.



ii) $x - 3y + 12 = 0$

Since the coefficients of x and y are factors of the constant term, I will use intercepts to graph the equation.

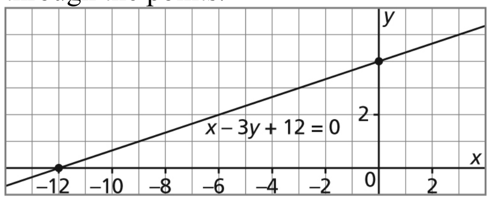
For the x-intercept, substitute $y = 0$.

$x - 3(0) + 12 = 0$ Solve for x.
$x + 12 = 0$ Subtract 12 from each side.
$x = -12$

For the y-intercept, substitute $x = 0$.

$(0) - 3y + 12 = 0$ Solve for y.
$-3y + 12 = 0$ Subtract 12 from each side.
$-3y = -12$ Divide each side by –3.
$y = 4$

On a grid, mark a point at –12 on the x-axis and at 4 on the y-axis, then draw a line through the points.



b) Use slope = $\frac{\text{rise}}{\text{run}}$

On each graph, count squares from the y-intercept to the origin to determine the rise, and count squares from the origin to the x-intercept to determine the run.

i) For the graph of $3x - 4y - 24 = 0$:

Slope = $\frac{6}{8}$, or $\frac{3}{4}$

ii) For the graph of $x - 3y + 12 = 0$:

Slope = $\frac{-4}{-12}$, or $\frac{1}{3}$

23. I cannot easily graph a line from its equation in general form when the coefficients of the x-term and y-term are not factors of the constant term; for example, $6x - 4y + 3 = 0$.
I write this equation in slope-intercept to determine the slope and y-intercept of the graph.

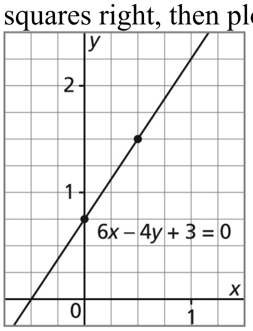
$6x - 4y + 3 = 0$ Subtract $6x$ and 3 from each side.

$-4y = -6x - 3$ Divide each side by -4.

$$y = \frac{-6}{-4}x - \frac{3}{-4}$$

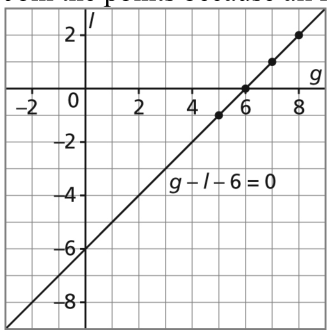
$$y = \frac{3}{2}x + \frac{3}{4}$$

To graph this equation, I use a scale of 4 squares to 1 unit so I can accurately plot a point at the y-intercept, which is $\frac{3}{4}$. From this point, I use the slope $\frac{3}{2}$ to move 3 squares up and 2 squares right, then plot another point. I draw a line through the points.



24. a) Choose a number, then subtract 6 from it; these two numbers differ by 6.
For example:
8, then $8 - 6 = 2$; the numbers are 8 and 2
7, then $7 - 6 = 1$; the numbers are 7 and 1
6, then $6 - 0 = 0$; the numbers are 6 and 0
5, then $5 - 6 = -1$; the numbers are 5 and -1

Plot these numbers as ordered pairs, with the greater number first and g plotted on the horizontal axis: (8, 2), (7, 1), (6, 0), (5, –1)
Join the points because all real numbers are possible values.



b) Assume that g represents the greater number and l represents the lesser number.
Then an equation is: $g - l = 6$
Subtract 6 from each side:
$g - l - 6 = 0$ This equation is in general form.

c) From the graph, 5 other pairs of numbers are the coordinates of 5 points on the graph; for example, 4 and –2; 3 and –3, 2 and –4, 1 and –5, 0 and –6

25. Write each equation in slope-intercept form, then compare the equations to determine which have the same slopes and the same y-intercepts.

a) $y = \frac{2}{5}x + 1$

This equation is in slope-intercept form; the slope of its graph is $\frac{2}{5}$ and its y-intercept is 1.

b) $y - 3 = \frac{2}{5}(x - 4)$ Add 3 to each side.

$y = \frac{2}{5}(x - 4) + 3$ Remove brackets.

$y = \frac{2}{5}x - \frac{2}{5}(4) + 3$ Simplify.

$y = \frac{2}{5}x - \frac{8}{5} + 3$

$y = \frac{2}{5}x - \frac{8}{5} + \frac{15}{5}$

$y = \frac{2}{5}x + \frac{7}{5}$

This equation is in slope-intercept form; the slope of its graph is $\frac{2}{5}$ and its y-intercept is $\frac{7}{5}$.

c) $y - 1 = \frac{2}{5}(x - 1)$ Add 1 to each side.

$y = \frac{2}{5}(x - 1) + 1$ Remove brackets.

$y = \frac{2}{5}x - \frac{2}{5}(1) + 1$ Simplify.

$y = \frac{2}{5}x - \frac{2}{5} + 1$

$y = \frac{2}{5}x - \frac{2}{5} + \frac{5}{5}$

$y = \frac{2}{5}x + \frac{3}{5}$

This equation is in slope-intercept form; the slope of its graph is $\frac{2}{5}$ and its y-intercept is $\frac{3}{5}$.

d) $y - 3 = \frac{2}{5}(x - 5)$ Add 3 to each side.

$y = \frac{2}{5}(x - 5) + 3$ Remove brackets.

$y = \frac{2}{5}x - \frac{2}{5}(5) + 3$ Simplify.

$y = \frac{2}{5}x - 2 + 3$

$y = \frac{2}{5}x + 1$

This equation is in slope-intercept form; the slope of its graph is $\frac{2}{5}$ and its y-intercept is 1.

e) $2x - 5y + 7 = 0$ Subtract $2x$ and 7 from each side.

$-5y = -2x - 7$ Divide each side by –5.

$y = \frac{-2}{-5}x - \frac{7}{-5}$

$y = \frac{2}{5}x + \frac{7}{5}$

This equation is in slope-intercept form; the slope of its graph is $\frac{2}{5}$ and its y-intercept is $\frac{7}{5}$.

f) $2x - 5y - 5 = 0$ Subtract $2x$ from each side. Add 5 to each side.

$-5y = -2x + 5$ Divide each side by –5.

$$y = \frac{-2}{-5}x + \frac{5}{-5}$$

$$y = \frac{2}{5}x - 1$$

This equation is in slope-intercept form; the slope of its graph is $\frac{2}{5}$ and its y-intercept is –1.

Equations $y = \frac{2}{5}x + 1$ and $y - 3 = \frac{2}{5}(x - 5)$ are equivalent.

Equations $y - 3 = \frac{2}{5}(x - 4)$ and $2x - 5y + 7 = 0$ are equivalent.

26. Determine the slope of each graph and identify its y-intercept where possible.

For Graph A:
Two points have coordinates (–1, 5) and (3, 0).

Use slope $= \dfrac{\text{change in } y\text{-coordinates}}{\text{change in } x\text{-coordinates}}$

$$\text{Slope} = \frac{0 - 5}{3 - (-1)}$$

$$\text{Slope} = \frac{-5}{4}, \text{ or } -\frac{5}{4}$$

For Graph B:
Its y-intercept is 3.
Two points have coordinates (5, –1) and (0, 3).

Use slope $= \dfrac{\text{change in } y\text{-coordinates}}{\text{change in } x\text{-coordinates}}$

$$\text{Slope} = \frac{3 - (-1)}{0 - 5}$$

$$\text{Slope} = \frac{4}{-5}, \text{ or } -\frac{4}{5}$$

For Graph C:
Two points have coordinates (–3, 3) and (2, –1).

Use slope $= \dfrac{\text{change in } y\text{-coordinates}}{\text{change in } x\text{-coordinates}}$

$$\text{Slope} = \frac{-1 - 3}{2 - (-3)}$$

$$\text{Slope} = \frac{-4}{5}, \text{ or } -\frac{4}{5}$$

Write each equation in slope-intercept form.

a) $y = -\frac{4}{5}x + 3$

This equation is in slope-intercept form; its graph has slope $-\frac{4}{5}$ and y-intercept 3. This matches Graph B.

b)

$y - 3 = -\frac{4}{5}(x + 3)$	Add 3 to each side.
$y = -\frac{4}{5}(x + 3) + 3$	Remove brackets.
$y = -\frac{4}{5}x - \frac{4}{5}(3) + 3$	Simplify.
$y = -\frac{4}{5}x - \frac{12}{5} + \frac{15}{5}$	
$y = -\frac{4}{5}x + \frac{3}{5}$	

This equation is in slope-intercept form; its graph has slope $-\frac{4}{5}$ and y-intercept $\frac{3}{5}$.
This slope matches Graph C. Check that a point on Graph C satisfies this equation.
Use the point (2, –1).

Substitute $x = 2$ and $y = -1$ in the equation $y - 3 = -\frac{4}{5}(x + 3)$.

L.S. $= y - 3$	R.S. $= -\frac{4}{5}(x + 3)$
$= -1 - 3$	$= -\frac{4}{5}(2 + 3)$
$= -4$	$= -\frac{4}{5}(5)$
	$= -4$

Since the left side is equal to the right side, the equation is correct and matches Graph C.

c)

$5x + 4y - 15 = 0$	Subtract $5x$ from each side. Add 15 to each side.
$4y = -5x + 15$	Divide each side by 4.
$y = \frac{-5}{4}x + \frac{15}{4}$	

This equation is in slope-intercept form; its graph has slope $-\frac{5}{4}$ and y-intercept $\frac{15}{4}$.

This slope matches Graph A. Check that a point on Graph A satisfies this equation.
Use the point (3, 0).
Substitute $x = 3$ and $y = 0$ in the equation $5x + 4y - 15 = 0$.

L.S. $= 5x + 4y - 15$	R.S. $= 0$
$= 5(3) + 4(0) - 15$	
$= 15 - 15$	
$= 0$	

Since the left side is equal to the right side, the equation is correct and matches Graph A.

27. a) Let a represent the time in hours Max works for family A who pays \$5/h.
Let b represent the time in hours Max works for family B who pays \$4/h.

When Max works only for family A, the time in hours he works is: $\frac{60}{5} = 12$

When Max works only for family B, the time in hours he works is: $\frac{60}{4} = 15$

When Max works for 4 h for family A, he earns: $4 \times \$5 = \$20$
So, he earns $\$60 - \$20 = \$40$ from family B.

So, Max works $\frac{40}{4}$ h, or 10 h for family B.

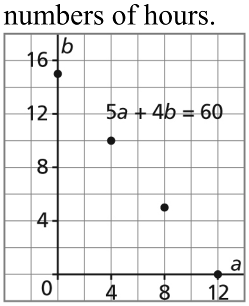
When Max works for 8 h for family A, he earns: $8 \times \$5 = \$40$
So, he earns $\$60 - \$40 = \$20$ from family B.

So, Max works $\frac{20}{4}$ h, or 5 h for family B.

Make a table for these data.

a hours	b hours
12	0
0	15
4	10
8	5

Plot these data on a grid. Do not join the points because we assume Max works for whole numbers of hours.



b) When Max works a hours at \$5/h, he earns: $5a$ dollars
When Max works b hours at \$4/h, he earns: $4b$ dollars
His total earnings are: \$60
So, an equation is: $5a + 4b = 60$

28. a) Let n represent the number of new releases Kylie rents, at \$5 each.
Let m represent the number of older movies Kylie rents, at \$3 each.

When Kylie rents only new releases, the number she rents is: $\frac{45}{5} = 9$

When Kylie rents only older movies, the number she rents is: $\frac{45}{3} = 15$

When Kylie rents 6 new releases, it costs: $6 \times \$5 = \$30$
So, Kylie spends $\$45 - \$30 = \$15$ on older movies.

So, Kylie rents $\frac{15}{3}$, or 5 older movies.

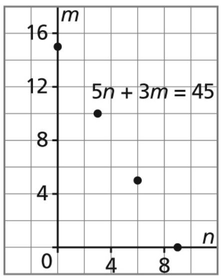
When Kylie rents 3 new releases, it costs: $3 \times \$5 = \$15$
So, Kylie spends $\$45 - \$15 = \$30$ on older movies.

So, Kylie rents $\frac{30}{3}$, or 10 older movies.

Make a table for these data.

n	m
9	0
0	15
6	5
3	10

Plot these data on a grid. Do not join the points because the number of movies is a whole number.



When Kylie rents n movies at \$5 each, she pays: $5n$ dollars
When Kylie rents m movies at \$3 each, she pays: $3m$ dollars
The total Kylie spent is: \$45
So, an equation is: $5n + 3m = 45$

b) **i)** Five new releases cost: $5 \times \$5 = \$25$
Six older movies cost: $6 \times \$3 = \$18$
Total cost is: $\$25 + \$18 = \$43$
Since this amount is not \$45, Kylie could not have rented these numbers of movies.

ii) Six new releases cost: $6 \times \$5 = \$30$
Five older movies cost: $5 \times \$3 = \$15$
Total cost is: $\$30 + \$15 = \$45$
Since this amount is \$45, Kylie could have rented these numbers of movies.

Practice Test (page 391)

1. For each line, count squares on the grid to determine the rise and the run.

Use slope = $\frac{\text{rise}}{\text{run}}$

Slope of AB = $\frac{-4}{6}$, or $-\frac{2}{3}$

Slope of CD = $\frac{6}{9}$, or $\frac{2}{3}$

Slope of EF = $\frac{-6}{4}$, or $-\frac{3}{2}$

Slope of GH = $\frac{6}{4}$, or $\frac{3}{2}$

The line with slope $-\frac{3}{2}$ is EF, which is choice C.

2. Write the equation $2x - 3y + 2 = 0$ in slope-intercept form.

$2x - 3y + 2 = 0$ Subtract $2x$ and 2 from each side.

$-3y = -2x - 2$ Divide each side by –3.

$$y = \frac{-2}{-3}x - \frac{2}{-3}$$

$$y = \frac{2}{3}x + \frac{2}{3}$$

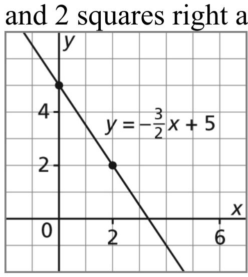
The graph of this equation has slope $\frac{2}{3}$.

From question 1, the line with slope $\frac{2}{3}$ is CD, which is choice B.

3 a) i) $y = -\frac{3}{2}x + 5$

Since the equation is in slope-intercept form, I use the slope $-\frac{3}{2}$ and the y-intercept 5 to graph the line.

I mark a point at 5 on the y-axis. I write the slope as $\frac{-3}{2}$, then move 3 squares down and 2 squares right and mark a point. I draw a line through the points.



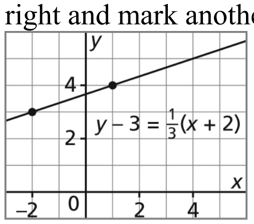
ii) $y - 3 = \frac{1}{3}(x + 2)$

This equation is in slope-point form.
To identify the coordinates of the point, I write the equation as:

$y - 3 = \frac{1}{3}(x - (-2))$

A point on the line has coordinates (–2, 3), and the slope of the line is $\frac{1}{3}$.

I mark a point at (–2, 3), then use the slope $\frac{1}{3}$ to move 1 square up and 3 squares right and mark another point. I draw a line through the points.



iii) $3x - 4y - 12 = 0$

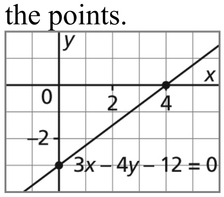
Since the coefficients of x and y in this equation are factors of 12, I can use intercepts to graph the equation.
For the x-intercept, substitute $y = 0$.

$3x - 4(0) - 12 = 0$
$3x - 12 = 0$ Solve for x. Add 12 to each side.
$3x = 12$ Divide each side by 3.
$x = 4$

For the y-intercept, substitute $x = 0$.

$3(0) - 4y - 12 = 0$
$-4y - 12 = 0$ Solve for y. Add 12 to each side.
$-4y = 12$ Divide each side by –4.
$y = -3$

I mark a point at 4 on the x-axis and a point at –3 on the y-axis. I draw a line through the points.



b) The graph of the equation $y = -\frac{3}{2}x + 5$ has slope $-\frac{3}{2}$, so any line parallel to this line will also have slope $-\frac{3}{2}$.

The required line has slope $-\frac{3}{2}$ and passes through the point with coordinates (6, 2).

Use the slope-point form of the equation:
$y - y_1 = m(x - x_1)$

Substitute: $y_1 = 2$, $m = -\frac{3}{2}$, and $x_1 = 6$

$y - 2 = -\frac{3}{2}(x - 6)$

I know my equation is correct because the slope of the graph of this equation is equal to the slope of the given parallel line, and the coordinates of the point on the graph given by this equation are equal to the given coordinates.

c) The graph of the equation $y - 3 = \frac{1}{3}(x + 2)$ has slope $\frac{1}{3}$. The perpendicular line has a slope that is the negative reciprocal of $\frac{1}{3}$; that is, its slope is –3.

The required line has slope –3 and passes through the point with coordinates (–1, 2).
Use the slope-point form of the equation:
$y - y_1 = m(x - x_1)$
Substitute: $y_1 = 2$, $m = -3$, and $x_1 = -1$

$y - 2 = -3(x - (-1))$	
$y - 2 = -3(x + 1)$	Write this equation in general form. Remove brackets.
$y - 2 = -3x - 3$	Add 3x and 3 to each side.
$3x + y + 1 = 0$	This equation is in general form.

d) To determine the coordinates of a point P on the graph of $3x - 4y - 12 = 0$, substitute a value for x, then solve the equation for y.
Since the coefficient of y is 4 and the constant term, 12, is a multiple of 4, choose a value of x that is a multiple of 4 so the value of y is an integer.
Substitute $x = 8$ in the equation:

$3x - 4y - 12 = 0$	
$3(8) - 4y - 12 = 0$	Simplify.
$24 - 4y - 12 = 0$	Collect like terms.
$12 - 4y = 0$	Subtract 12 from each side of the equation.
$-4y = -12$	Divide each side by –4.
$y = 3$	

The coordinates of P are (8, 3).
The line passes through P and Q(1, 5).
Determine the slope of PQ.

Use slope $= \frac{\text{change in } y\text{-coordinates}}{\text{change in } x\text{-coordinates}}$

Slope $= \frac{5 - 3}{1 - 8}$

Slope $= \frac{2}{-7}$, or $-\frac{2}{7}$

Substitute the coordinates of Q and the slope in the slope-point form of the equation of a line:
$y - y_1 = m(x - x_1)$

Substitute: $y_1 = 5$, $m = -\frac{2}{7}$, and $x_1 = 1$

$y - 5 = -\frac{2}{7}(x - 1)$	Solve for y. Add 5 to each side.
$y = -\frac{2}{7}(x - 1) + 5$	Remove brackets.

$$y = -\frac{2}{7}x + \frac{2}{7} + 5$$

$$y = -\frac{2}{7}x + \frac{2}{7} + \frac{35}{7}$$

$$y = -\frac{2}{7}x + \frac{37}{7}$$

This equation is in slope-intercept form.

4. a) Since I can identify the slope of the line and its y-intercept from the graph, I will write the equation in slope-intercept form: $y = mx + b$
I count squares on the grid to determine the rise and the run.

Slope = $\frac{\text{rise}}{\text{run}}$

Slope = $\frac{-4}{2}$, or -2

From the graph, the y-intercept is -2.
So, an equation is: $y = -2x - 2$

b) The line is horizontal, so its equation is: $y = -1$
In general form, this is: $y + 1 = 0$

c) Since I can calculate the slope and identify the coordinates of a point on the line, I shall use the slope-point form: $y - y_1 = m(x - x_1)$
The coordinates of two points on the line are $(-1, -2)$ and $(3, 1)$. I count squares on the grid between these points to determine the rise and the run.

Slope = $\frac{\text{rise}}{\text{run}}$

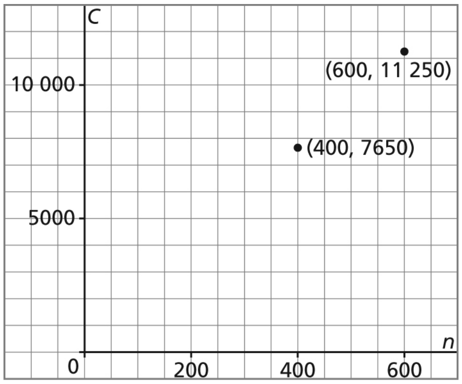
Slope = $\frac{3}{4}$

Substitute the coordinates of the point (3, 1) and the slope $\frac{3}{4}$ in the equation

$y - y_1 = m(x - x_1)$

So, an equation is: $y - 1 = \frac{3}{4}(x - 3)$

5. The cost, C dollars, is a function of the number of people, n, who attend.
Use the given information as the coordinates of two points on a sketch of a graph of the function.
Plot these points: (600, 11 250) and (400, 7650)



The cost per person is the slope of the line through the points on the graph.

$$\text{Slope} = \frac{\text{change in } C\text{-coordinates}}{\text{change in } n\text{-coordinates}}$$

$$\text{Slope} = \frac{11\,250 - 7650}{600 - 400}$$

$$\text{Slope} = \frac{3600}{200}$$

Slope = 18

The cost per person is \$18.

Use the slope-point form of the equation:

$C - C_1 = m(n - n_1)$

Substitute: $C_1 = 7650$, $m = 18$, and $n_1 = 400$

$C - 7650 = 18(n - 400)$ This is an equation of the function.

a), c) To determine the cost for 340 people to attend, substitute $n = 340$ in the equation:

$C - 7650 = 18(n - 400)$

$C - 7650 = 18(340 - 400)$ Solve for C.

$C - 7650 = 18(-60)$

$C - 7650 = -1080$

$C = 7650 - 1080$

$C = 6570$

The cost for 340 people to attend is \$6570.

b), c) To determine the number of people who can attend for a cost of \$9810, substitute $C = 9810$ in the equation:

$C - 7650 = 18(n - 400)$

$9810 - 7650 = 18(n - 400)$ Solve for n. Remove brackets and simplify.

$2160 = 18n - 7200$

$9360 = 18n$

$n = 520$

For a cost of \$9810, 520 people can attend.