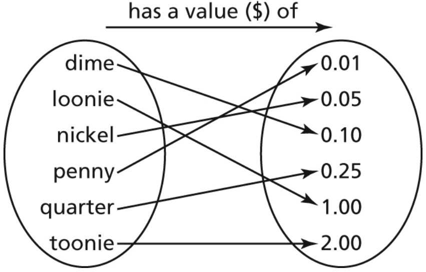
Lesson 5.1 Representing Relations Exercises (pages 262–263)

A

3. **a)** **i)** The relation shows the association "has a value, in dollars, of" from a set of coins to a set of numbers. For example, a penny has a value of $0.01.

ii) The relation as a set of ordered pairs:
{(penny, 0.01), (nickel, 0.05), (dime, 0.10), (quarter, 0.25), (loonie, 1.00), (toonie, 2.00)}

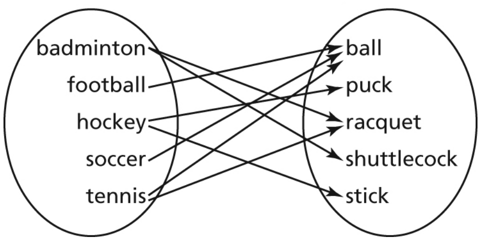
The relation as an arrow diagram:



b) **i)** The relation shows the association "is played with a" from a set of sports to a set of equipment. For example, tennis is played with a racquet.

ii) The relation as a set of ordered pairs:
{(badminton, shuttlecock), (badminton, racquet), (hockey, puck), (hockey, stick), (tennis, ball), (tennis, racquet), (soccer, ball)}

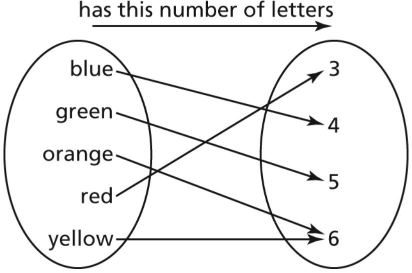
The relation as an arrow diagram:



4. **a)** The relation as a table:

Word	Number of letters
blue	4
green	5
orange	6
red	3
yellow	6

b) The relation as an arrow diagram:

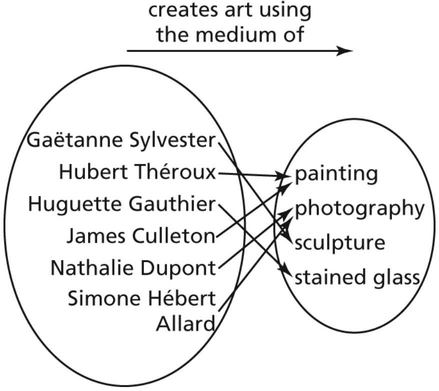


B

5. **a)** The relation shows the association "creates art using the medium of" from a set of francophone artists from Manitoba to a set of artistic media. For example, Nathalie Dupont creates art using the medium of photography.

b) **i)** The relation as a set of ordered pairs:
{(Gaëtanne Sylvester, sculpture), (Hubert Théroux, painting),
(Huguette Gauthier, stained glass), (James Culleton, painting),
(Nathalie Dupont, photography), (Simone Hébert Allard, photography)}

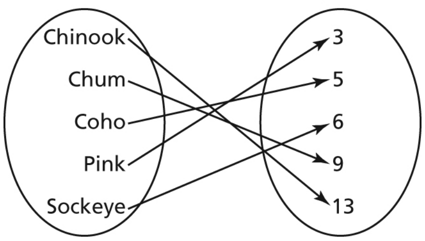
ii) The relation as an arrow diagram:



6. a) The relation shows the association "has a typical mass, in kilograms, of" from a set of salmon species to a set of masses. For example, a coho salmon has a typical mass of 5 kg.

b) In the ordered pairs, the first item is the type of salmon. The second item is the typical mass of the salmon.
The relation as a set of ordered pairs:
{(Chinook, 13), (Chum, 9), (Coho, 5), (Pink, 3), (Sockeye, 6)}

c) The relation as an arrow diagram:



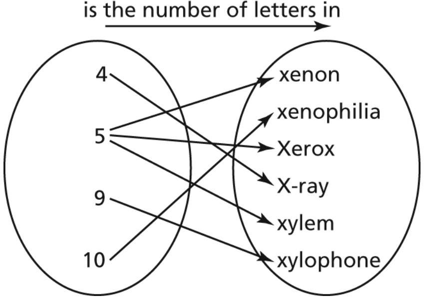
7. **a)** The arrow diagram shows a relation with the association "is the number of letters in" from a set of numbers to a set of words beginning with the letter Z. For example, 6 is the number of letters in *zombie*.

b) Sample response:
The relation as a set of ordered pairs:
{(3, Zen), (4, zany), (4, zero), (5, zebra), (6, zombie), (7, Zamboni), (8, zeppelin)}

The relation as a table:

Number of letters	**Word beginning with Z**
3	Zen
4	zany
4	zero
5	zebra
6	zombie
7	Zamboni
8	zeppelin

c) Sample response:
The arrow diagram is:



The relation as a set of ordered pairs:
{(4, X-ray), (5, xenon), (5, Xerox), (5, xylem), (9, xylophone), (10, xenophilia)}

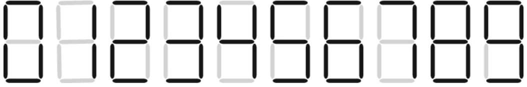
The relation as a table:

Number of letters	**Word beginning with X**
4	X-ray
5	xenon
5	Xerox
5	xylem
9	xylophone
10	xenophilia

8. **a)** The arrow diagram shows a relation with the association “translates to” from a set of French words to a set of English words.

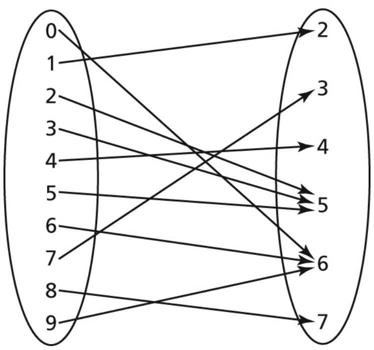
b) Two ordered pairs that belong to the relation are: (oui, yes) and (et, and)

9. **a)** Sketch each digit:



The ordered pairs are:
{(0, 6), (1, 2), (2, 5), (3, 5), (4, 4), (5, 5), (6, 6), (7, 3), (8, 7), (9, 6)}

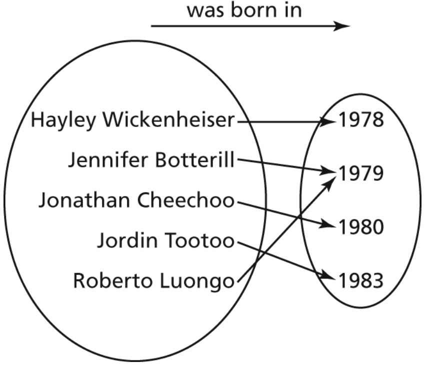
b) Sample response:
The relation as an arrow diagram:



The relation as a table of values:

Digit	Number of lit segments
0	6
1	2
2	5
3	5
4	4
5	5
6	6
7	3
8	7
9	6

10. a) The relation as an arrow diagram is:

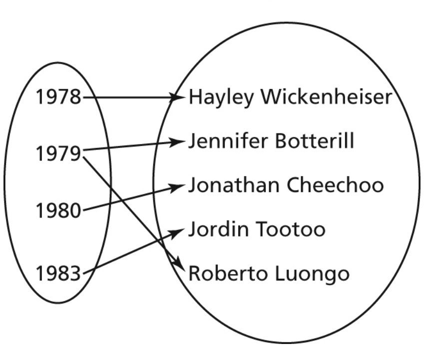


The relation as a set of ordered pairs:
{(Hayley Wickenheiser, 1978), (Jennifer Botterill, 1979), (Jonathan Cheechoo, 1980), (Jordin Tootoo, 1983), (Roberto Luongo, 1979)}

The relation as a table:

Hockey player	**Birth year**
Hayley Wickenheiser	1978
Jennifer Botterill	1979
Jonathan Cheechoo	1980
Jordin Tootoo	1983
Roberto Luongo	1979

b) The relation as an arrow diagram is:



The relation as a set of ordered pairs:
{(1978, Hayley Wickenheiser), (1979, Jennifer Botterill), (1979, Roberto Luongo), (1980, Jonathan Cheechoo), (1983, Jordin Tootoo)}

The relation as a table:

Birth year	**Hockey player**
1978	Hayley Wickenheiser
1979	Jennifer Botterill
1979	Roberto Luongo
1980	Jonathan Cheechoo
1983	Jordin Tootoo

11. Answers will vary.

a) Ordered pairs should be in the form: (older person, younger person)

b) Other associations include:
"is taller than"
"is involved in more school groups than"
"usually wakes up earlier than"

C

12. a) i) Create a table to organize the possible dice rolls.
Highlight the even sums:

Sums						
Roll	**1**	**2**	**3**	**4**	**5**	**6**
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

The set of ordered pairs is:
{(1, 1), (1, 3), (1, 5), (2, 2), (2, 4), (2, 6), (3, 1), (3, 3), (3, 5), (4, 2), (4, 4), (4, 6), (5, 1), (5, 3), (5, 5), (6, 2), (6, 4), (6, 6)}

ii) Use the table from part i. Subtract the lesser number from the greater number.
Highlight the differences that are prime numbers:

Differences						
Roll	**1**	**2**	**3**	**4**	**5**	**6**
1	0	1	2	3	4	5
2	1	0	1	2	3	4
3	2	1	0	1	2	3
4	3	2	1	0	1	2
5	4	3	2	1	0	1
6	5	4	3	2	1	0

The set of ordered pairs is:
{(1, 3), (1, 4), (1, 6), (2, 4), (2, 5), (3, 1), (3, 5), (3, 6), (4, 1), (4, 2), (4, 6), (5, 2), (5, 3), (6, 1), (6, 3), (6, 4)}

b) In part a, the order doesn't matter because the sum of 2 numbers is the same regardless of the order of the numbers, and the difference doesn't matter for 2 dice because I subtract the lesser number from the greater number since there is no way of telling the dice apart.

13. a) Each arrow goes from a parent to a child.
Arrows point to 6 different dots, so 6 children are shown.

b) Arrows come from 4 different dots, so 4 parents are shown.

c) Grandparents are parents of parents. There are 2 parents of parents, so 2 grandparents are shown.

14. a) Each arrow goes from a sister to her sibling.
Arrows go from 2 different dots, so 2 females are shown.

b) The dots that are not female are male. There are 5 dots in total and 2 of them are female, so the remaining 3 dots are male.

Lesson 5.2 Properties of Functions Exercises (pages 270–273)

A

4. a) This relation is a function because each number in the first set associates with exactly 1 number in the second set.

b) This relation is not a function because the number 4 in the first set associates with 4 numbers in the second set.

c) This relation is a function because each number in the first set associates with exactly 1 number in the second set.

5. a) Each ordered pair has a different first element, so for every first element there is exactly one second element. So, the relation is a function.
The domain is: {1, 2, 3, 4}
The range is: {3, 6, 9, 12}

b) The ordered pairs (0, 1) and (0, –1) have the same first element, 0. So, the relation is not a function.
The domain is: {1, 0, –1}
The range is: {0, 1, –1}

c) Each ordered pair has a different first element, so for every first element there is exactly one second element. So, the relation is a function.
The domain is: {2, 4, 6, 8}
The range is: {3, 5, 7, 9}

d) The ordered pairs (0, 1), (0, 2), and (0, 3) have the same first element, 0; and the ordered pairs (1, 2) and (1, 3) have the same first element, 1. So, the relation is not a function.
The domain is: {0, 1, 2}
The range is: {1, 2, 3}

6. a) $C(n) = 20n + 8$

b) $P(n) = n - 3$

c) $t(d) = 5d$

d) Use f to name the function when the variable is y:
$f(x) = -x$

7. a) $d = 3t - 5$

b) Use y for the variable when the function name is f and the other variable is x:
$y = -6x + 4$

c) $C = 5n$

d) $P = 2n - 7$

B

8. **a)** Each ordered pair has a different first element, so for every first element there is exactly one second element. So, the relation is a function.
The domain is the set of first elements:
{1, 2, 3, 4}
The range is the set of second elements:
{1, 8, 27, 64}

b) The ordered pairs (3, 4), (3, 5), (3, 6), and (3, 7) have the same first element, 3.
So, the relation is not a function.
The domain is: {3}
The range is: {4, 5, 6, 7}

9. **a)** **i)** For each number in the first column, there is only one number in the second column. So, the relation is a function.

ii) From an understanding of the situation, the cost, C, depends on the number of cans of juice purchased, n. So, C is the dependent variable and n is the independent variable.

iii) The domain is the set of numbers in the first column of the table:
{1, 2, 3, 4, 5, 6, …}
The range is the set of numbers in the second column of the table:
{2.39, 4.00, 6.39, 8.00, 10.39, 12.00, …}

b) **i)** For each number in the first column, there is only one number in the second column. So, the relation is a function.

ii) From an understanding of the situation, the temperature, T, depends on the altitude, A. So, T is the dependent variable and A is the independent variable.

iii) The domain is the set of numbers in the first column of the table:
{610, 1220, 1830, 2440, 3050, 3660, …}
The range is the set of numbers in the second column of the table:
{15.0, 11.1, 7.1, 3.1, –0.8, –4.8, …}

10. a) The ordered pairs (3, isosceles triangle), (3, equilateral triangle), (3, right triangle), and (3, scalene triangle) have the same first element, 3; and the ordered pairs (4, square), (4, rectangle), (4, rhombus), (4, trapezoid), and (4, parallelogram) have the same first element, 4. So, the relation is not a function.

b) Write the new relation as a set of ordered pairs and list the first elements in alphabetical order: {(equilateral triangle, 3), (hexagon, 6), (isosceles triangle, 3), (parallelogram, 4), (pentagon, 5), (rectangle, 4), (rhombus, 4), (right triangle, 3), (scalene triangle, 3), (square, 4), (trapezoid, 4)}
Each ordered pair has a different first element, so for every first element there is exactly one second element. So, the new relation is a function.

c) For the relation in part a:
The domain is the set of first elements: {3, 4, 5, 6}
The range is the set of second elements: {equilateral triangle, isosceles triangle, right triangle, scalene triangle, square, rectangle, rhombus, trapezoid, parallelogram, pentagon, hexagon}
For the function in part b:
The domain is the set of first elements:
{equilateral triangle, hexagon, isosceles triangle, parallelogram, pentagon, rectangle, rhombus, right triangle, scalene triangle, square, trapezoid}
The range is the set of second elements:
{3, 4, 5, 6}

11. a) For a function, each element in the first column is associated with exactly one element in the second column. So, each element in the first column only occurs once in that column. The only column with no repetition is: "Name"
So, any relation written as a table where the first column is "Name" will be a function.
For example, these two relations are functions:

Name	**From**
Marie	Edmonton
Gabriel	Falher
Élise	Bonnyville
Christophe	Calgary
Jean	Edmonton
Mélanie	Edmonton
Nicole	Red Deer
Marc	Légal

Name	**Age**
Marie	13
Gabriel	16
Élise	14
Christophe	13
Jean	15
Mélanie	15
Nicole	17
Marc	13

b) Any relation written as a table where the first column is not "Name" will be a relation that is not a function. For example, these two relations are not functions:

Age	**Name**
13	Marie
16	Gabriel
14	Élise
13	Christophe
15	Jean
15	Mélanie
17	Nicole
13	Marc

From	**Age**
Edmonton	13
Falher	16
Bonnyville	14
Calgary	13
Edmonton	15
Edmonton	15
Red Deer	17
Légal	13

12. The statement in part a is true. A function is a special type of relation, so it is still a relation. For example, {(1, 2), (2, 3), (3, 4)} is a function because each first element in the ordered pairs occurs exactly once. It is also a relation because it is a set of ordered pairs. However, a relation such as {(0, 1), (0, 2), (0, 3)} is not a function because the first element 0 occurs in each ordered pair.

13. a) Create one table for a relation with the association "is worth this number of points" from a set of letters to a set of numbers. Create a second table for a relation with the association "points are associated with the letter" from a set of numbers to a set of letters.

Letter	Number
A	1
D	2
F	4
G	2
M	3
Q	10
T	1
X	8
Z	10

Number	Letter
1	A
1	T
2	D
2	G
3	M
4	F
8	X
10	Q
10	Z

b) The first table represents a function; each letter in the first column is associated with exactly one number in the second column.
In the second table, the numbers 1, 2, and 10 in the first column are associated with more than 1 letter in the second column, so this does not represent a function.

14. a) To determine $f(1)$, use:

$f(x) = -5x + 11$ Substitute: $x = 1$

$f(1) = -5(1) + 11$

$f(1) = -5 + 11$

$f(1) = 6$

b) To determine $f(-3)$, use:

$f(x) = -5x + 11$ Substitute: $x = -3$

$f(-3) = -5(-3) + 11$

$f(-3) = 15 + 11$

$f(-3) = 26$

c) To determine $f(0)$, use:

$f(x) = -5x + 11$ Substitute: $x = 0$

$f(0) = -5(0) + 11$

$f(0) = 0 + 11$

$f(0) = 11$

d) To determine $f(1.2)$, use:

$f(x) = -5x + 11$ Substitute: $x = 1.2$

$f(1.2) = -5(1.2) + 11$

$f(1.2) = -6 + 11$

$f(1.2) = 5$

15. a) i) To determine the value of n when $f(n) = 11$, use:

$f(n) = 2n - 7$ Substitute: $f(n) = 11$

$11 = 2n - 7$ Solve for n.

$11 + 7 = 2n$

$18 = 2n$

$n = 9$

ii) To determine the value of n when $f(n) = -6$, use:

$f(n) = 2n - 7$ Substitute: $f(n) = -6$

$-6 = 2n - 7$ Solve for n.

$-6 + 7 = 2n$

$1 = 2n$

$n = \frac{1}{2}$, or 0.5

b) **i)** To determine the value of x when $g(x) = 41$, use:

$g(x) = -5x + 1$ Substitute: $g(x) = 41$

$41 = -5x + 1$ Solve for x.

$41 - 1 = -5x$

$40 = -5x$

$x = -8$

ii) To determine the value of x when $g(x) = -16$, use:

$g(x) = -5x + 1$ Substitute: $g(x) = -16$

$-16 = -5x + 1$ Solve for x.

$-16 - 1 = -5x$

$-17 = -5x$

$x = \frac{17}{5}$, or 3.4

16. a) A measure in centimetres is a function of the measure in inches.
In two variables, $C = 2.54i$

b) To determine $C(12)$, use:

$C(i) = 2.54i$ Substitute: $i = 12$

$C(12) = 2.54(12)$

$C(12) = 30.48$

This means that a length of 12 in. is 30.48 cm.

c) To determine the value of i when $C(i) = 100$, use:

$C(i) = 2.54i$ Substitute: $C(i) = 100$

$100 = 2.54i$ Solve for i.

$\frac{100}{2.54} = i$

$i = 39.3700\ldots$

This means that a length of 100 cm is approximately 39 in.

17. a) The distance to Meadow Lake is a function of the travelling time.
In function notation, $D(t) = -80t + 300$

b) At the start of the journey, 0 h have passed, so $t = 0$.
Determine $D(0)$.

$D(t) = -80t + 300$ Substitute: $t = 0$

$D(0) = -80(0) + 300$

$D(0) = 300$

The car was 300 km from Meadow Lake at the start its journey.

18. a) **i)** To determine *f*(15), use:

$f(l) = 2.754l + 71.475$ Substitute: $l = 15$

$f(15) = 2.754(15) + 71.475$

$f(15) = 112.785$

This means that a female with humerus 15 cm long is approximately 113 cm tall.

ii) To determine *m*(20), use:

$m(l) = 2.894l + 70.641$ Substitute: $l = 20$

$m(20) = 2.894(20) + 70.641$

$m(20) = 128.521$

This means that a male with humerus 20 cm long is approximately 129 cm tall.

b) **i)** To determine the value of *l* when *f*(*l*) = 142, use:

$f(l) = 2.754l + 71.475$ Substitute: $f(l) = 142$

$142 = 2.754l + 71.475$

$142 - 71.475 = 2.754l$

$70.525 = 2.754l$

$\frac{70.525}{2.754} = l$

$l = 25.6082\ldots$

This means that a female 142 cm tall has a humerus approximately 26 cm long.

ii) To determine the value of *l* when *m*(*l*) = 194, use:

$m(l) = 2.894l + 70.641$ Substitute: $m(l) = 194$

$194 = 2.894l + 70.641$

$194 - 70.641 = 2.894l$

$123.359 = 2.894l$

$\frac{123.359}{2.894} = l$

$l = 42.6257\ldots$

This means that a male 194 cm tall has a humerus approximately 43 cm long.

c) Sample response:
My humerus is approximately 33 cm long.
Use the formula for a female:

$f(l) = 2.754l + 71.475$ Substitute: $l = 33$

$f(15) = 2.754(33) + 71.475$

$f(15) = 162.357$

The formula estimates my height as approximately 162 cm.
I am approximately 164 cm tall, so the height given by the formula is close to my actual height.

19. a) i) To determine *C*(50), use:

$$C(f)=\frac{5}{9}(f-32) \qquad \text{Substitute: } f=50$$

$$C(50)=\frac{5}{9}(50-32)$$

$$C(50)=\frac{5}{9}(18)$$

$$C(50)=10$$

ii) To determine *C*(–13), use:

$$C(f)=\frac{5}{9}(f-32) \qquad \text{Substitute: } f=-13$$

$$C(-13)=\frac{5}{9}(-13-32)$$

$$C(-13)=\frac{5}{9}(-45)$$

$$C(-13)=-25$$

b) i) To determine the value of *f* when *C*(*f*) = 20, use:

$$C(f)=\frac{5}{9}(f-32) \qquad \text{Substitute: } C(f)=20$$

$$20=\frac{5}{9}(f-32)$$

$$\frac{9}{5}(20)=f-32$$

$$36=f-32$$

$$32+36=f$$

$$f=68$$

ii) To determine the value of *f* when *C*(*f*) = –35, use:

$$C(f)=\frac{5}{9}(f-32) \qquad \text{Substitute: } C(f)=-35$$

$$-35=\frac{5}{9}(f-32)$$

$$\frac{9}{5}(-35)=f-32$$

$$-63=f-32$$

$$32-63=f$$

$$f=-31$$

c) **i)** $C(32) = 0$
ii) $C(212) = 100$
iii) $C(356) = 180$

C

20. The temperature in degrees Fahrenheit, *F*, is determined from the temperature in degrees Celsius, *c*. So, *F* is the dependent variable and *c* is the independent variable.

Multiply the Celsius temperature by $\frac{9}{5}$: $\frac{9}{5}c$

Add 32: $\frac{9}{5}c+32$

So, the equation in function notation is:

$$F(c)=\frac{9}{5}c+32$$

21. An equation for the area, *A*, of the rectangle is:

$A = lw$ Substitute: $A = 9$

$9 = lw$ Solve for *w*.

$w=\frac{9}{l}$

An equation for the perimeter, *P*, of the rectangle is:

$P = 2(l+w)$ Substitute: $w=\frac{9}{l}$.

$P=2\left(l+\frac{9}{l}\right)$

$P=2l+\frac{18}{l}$

In function notation:

$P(l)=2l+\frac{18}{l}$

22. An equation for the perimeter, *P*, of the rectangle is:

$P = 2(l+w)$ Substitute: $P = 12$

$12 = 2(l+w)$ Solve for *l*.

$\frac{12}{2}=l+w$

$6 = l+w$

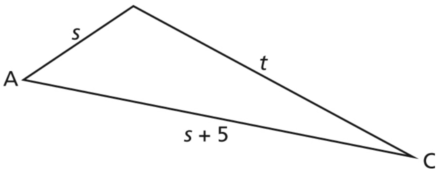
$l = 6-w$

In function notation:
$l(w) = 6-w$

The domain is:
$0 < w < 6$

The range is:
$0 < l < 6$

23. Sketch and label the triangle:



An equation for the perimeter, P, of the triangle is:

$P = s + (s+5) + t$ Substitute: $P = 16$

$16 = 2s + 5 + t$ Solve for l.

$16 - 2s - 5 = t$

$t = 11 - 2s$

In function notation:
$t(s) = 11 - 2s$

In a triangle, the length of a side is less than the sum of the lengths of the other two sides.
So, we get these inequalities:

$\text{AB} < \text{AC} + \text{BC}$		$\text{AC} < \text{AB} + \text{BC}$		$\text{BC} < \text{AB} + \text{AC}$
$s < (s+5) + t$	and	$s + 5 < s + t$	and	$t < s + (s+5)$
$-5 < t$		$5 < t$		$t < 2s + 5$

So:
$t > 5$ and $t < 2s + 5$

Use $t > 5$ to determine a corresponding inequality for s:

$t > 5$ Substitute: $t = 11 - 2s$

$11 - 2s > 5$

$2s < 6$

$s < 3$

Use $t < 2s + 5$ to determine inequalities for s and t:

$t < 2s + 5$ Substitute: $t = 11 - 2s$

$11 - 2s < 2s + 5$

$11 - 5 < 2s + 2s$

$6 < 4s$

$$s > \frac{6}{4}$$

$s > 1.5$

Write the equation $t = 11 - 2s$ in terms of s:

$$t = 11 - 2s$$
$$2s = 11 - t$$
$$s = \frac{1}{2}(11 - t)$$

So, when $s > 1.5$:

$s > 1.5$ Substitute: $s = \frac{1}{2}(11 - t)$

$$\frac{1}{2}(11 - t) > 1.5$$
$$11 - t > 2(1.5)$$
$$11 - t > 3$$
$$t < 11 - 3$$
$$t < 8$$

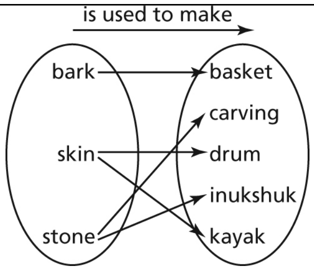
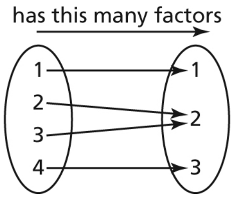
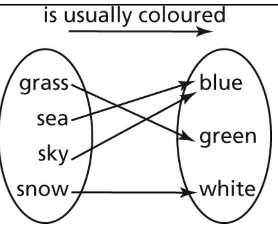
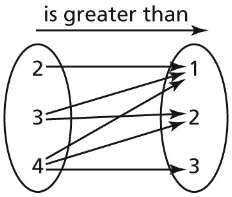
So, $t > 5$, $s < 3$, $s > 1.5$, and $t < 8$
Combine these inequalities to determine the domain and range of the function:

The domain is: $1.5 < s < 3$
The range is: $5 < t < 8$

Checkpoint 1 **Assess Your Understanding (page 275)**

5.1

1. Sample response:

<table>
<tr><th></th><th>Description in words</th><th>Set of ordered pairs</th><th>Arrow diagram</th><th>Table or graph</th></tr>
<tr><td>a)</td><td>This relation shows the association “is used to make” from a set of materials to a set of objects.</td><td>{(skin, drum), (skin, kayak), (bark, basket), (stone, inukshuk), (stone, carving)}</td><td> </td><td><table><tr><th>Material</th><th>Object</th></tr><tr><td>bark</td><td>basket</td></tr><tr><td>skin</td><td>drum</td></tr><tr><td>skin</td><td>kayak</td></tr><tr><td>stone</td><td>carving</td></tr><tr><td>stone</td><td>inukshuk</td></tr></table></td></tr>
<tr><td>b)</td><td>This relation shows the association “has this many factors” from the natural numbers from 1 to 4 to a set of natural numbers.</td><td>{(1, 1), (2, 2), (3, 2), (4, 3)}</td><td> </td><td><table><tr><th>Number</th><th>Number of Factors</th></tr><tr><td>1</td><td>1</td></tr><tr><td>2</td><td>2</td></tr><tr><td>3</td><td>2</td></tr><tr><td>4</td><td>3</td></tr></table></td></tr>
<tr><td>c)</td><td>This relation shows the association “is usually coloured” from a set of objects to a set of colours.</td><td>{(grass, green), (sea, blue), (sky, blue), (snow, white)}</td><td> </td><td><table><tr><th>Object</th><th>Colour</th></tr><tr><td>grass</td><td>green</td></tr><tr><td>sea</td><td>blue</td></tr><tr><td>sky</td><td>blue</td></tr><tr><td>snow</td><td>white</td></tr></table></td></tr>
<tr><td>d)</td><td>For the numbers 1 to 4, the first number in an ordered pair is greater than the second number.</td><td>{(2, 1), (3, 1), (3, 2), (4, 1), (4, 2), (4, 3)}</td><td> </td><td><table><tr><th>Number</th><th>Number</th></tr><tr><td>2</td><td>1</td></tr><tr><td>3</td><td>1</td></tr><tr><td>3</td><td>2</td></tr><tr><td>4</td><td>1</td></tr><tr><td>4</td><td>2</td></tr><tr><td>4</td><td>3</td></tr></table></td></tr>
</table>

5.2

2. a) Justifications may vary. For example:
The relation in part a is not a function because two different ordered pairs have skin as a first element and two different ordered pairs have stone as a first element.

The relation in part b is a function because the ordered pairs have different first elements.

The relation in part c is a function because the ordered pairs have different first elements.

The relation in part d is not a function because two different ordered pairs have 3 as a first element and three different ordered pairs have 4 as a first element.

b) In part b,
the domain is: {1, 2, 3, 4}
the range is: {1, 2, 3}

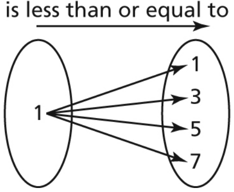
In part c,
the domain is: {grass, sea, sky, snow}
the range is: {blue, green, white}

3. Sample response:

a) i) A relation that is not a function has at least two different ordered pairs with the same first element.
So, this relation is not a function:
{(1, 1), (1, 3), (1, 5), (1, 7)}

ii) For a function, each first element is associated with exactly one second element.
So, this is a function:
{(1, 1), (3, 3), (5, 5), (7, 7)}

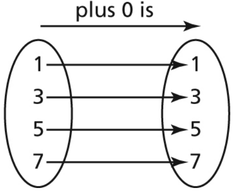
b) i) Represent the relation as an arrow diagram:



Represent the relation as a table of values:

Number	Number
1	1
1	3
1	5
1	7

ii) Represent the relation as an arrow diagram:



Represent the relation as a table of values:

Number	Number
1	1
3	3
5	5
7	7

4. a) From an understanding of the situation, the temperature, *T*, depends on the distance below Earth's surface, *d*. So, *T* is the dependent variable and *d* is the independent variable.

b) In two variables, $T = 10d + 20$

c) To determine $T(5)$, use:

$T(d) = 10d + 20$ Substitute: $d = 5$

$T(5) = 10(5) + 20$

$T(5) = 70$

This means that at a depth of 5 km below Earth's surface, the temperature is 70°C.

d) To determine the value of *d* when $T(d) = 50$, use:

$T(d) = 10d + 20$ Substitute: $T(d) = 50$

$50 = 10d + 20$

$50 - 20 = 10d$

$30 = 10d$

$\frac{30}{10} = d$

$d = 3$

This means that, at a depth of 3 km below Earth's surface, the temperature is 50°C.

Lesson 5.3 Interpreting and Sketching Graphs Exercises (pages 281–283)

A

3. a) Bear F has the greatest mass because it is represented by the point on the graph farthest to the right and the horizontal axis represents mass. Its mass is approximately 650 kg.

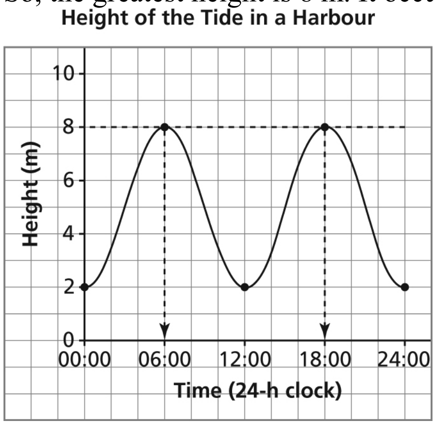
b) Bear A is the shortest because it is represented by the lowest point on the graph and the vertical axis represents height. Its height is approximately 0.7 m.

c) Bears D and E have the same mass because the points that represent them lie on the same vertical line that passes through 400 on the *Mass* axis. The mass is 400 kg.

d) Bears D and H have the same height because the points that represent them lie on the same horizontal line that passes through approximately 2.25 on the *Height* axis. The height is approximately 2.25 m.

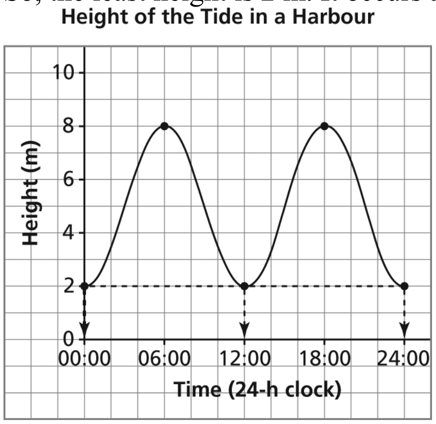
B

4. a) Draw a horizontal line through the highest points on the graph. This line intersects the *Height* axis at 8 m. Draw vertical lines from the points where the horizontal line intersects the graph. These lines intersect the *Time* axis at 06:00 and 18:00.
So, the greatest height is 8 m. It occurs at 06:00 and 18:00.

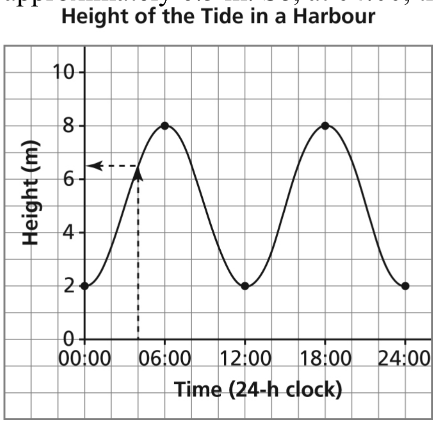


b) Draw a horizontal line through the lowest points on the graph. This line intersects the *Height* axis at 2 m. Draw vertical lines from the points where the horizontal line intersects the graph. These lines intersect the *Time* axis at 00:00 (midnight), 12:00 (noon), and 24:00 (midnight).

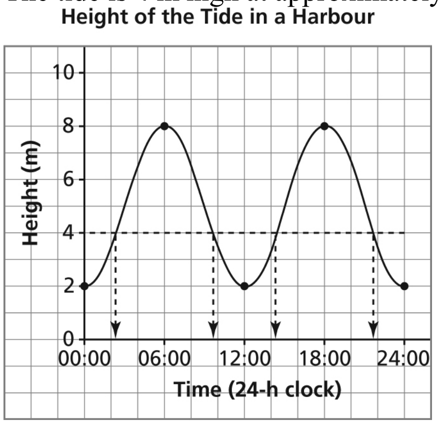
So, the least height is 2 m. It occurs at 00:00, 12:00, and 24:00.



c) Draw a vertical line through 04:00 on the *Time* axis. From the point where this line intersects the graph, draw a horizontal line, which intersects the *Height* axis at approximately 6.5 m. So, at 04:00, the tide is approximately 6.5 m.



d) Draw a horizontal line through 4 m on the *Height* axis. From the point where this line intersects the graph, draw vertical lines to intersect the *Time* axis.
The tide is 4 m high at approximately 02:20, 09:40, 14:20, and 21:40.



5. When Sepideh pulls the rope, the height of the flag increases. As she moves her hands up the rope, the height of the flag does not change. So, Graph B best represents the situation because it has segments that go up to the right and horizontal segments.

6.

Segment	**Graph**	**Run**
OA	The graph goes up to the right, so as time increases, Gill's distance from home increases.	In the first 5 min, Gill leaves home and runs 1 km.
AB	The graph is horizontal, so as time increases the distance stays the same.	Gill stops running for 5 min.
BC	The graph goes up to the right, so as time increases, Gill's distance from home increases.	Gill runs 1 km farther from home during the next 10 min.
CD	The graph goes down to the right, so as time increases, Gill's distance from home decreases.	Gill runs 2 km back home during the next 10 min.

7.

Segment	**Graph**	**Dive**
OA	The graph goes up to the right, so as time increases, Katanya's depth increases.	In the first 4 min, Katanya dives down 15 m below the surface of the water.
AB	The graph is horizontal, so as time increases Katanya's depth stays the same.	Katanya explores her environment at a depth of 15 m below the surface of the water for 6 min.
BC	The graph goes up to the right, so as time increases, Katanya's depth increases.	Katanya descends another 10 m during the next 4 min.
CD	The graph is horizontal, so as time increases Katanya's depth stays the same.	Katanya explores her environment at a depth of 25 m below the surface of the water for 4 min.
DE	The graph goes down to the right, so as time increases, Katanya's depth decreases.	Katanya ascends 25 m in 10 min to reach the surface of the water.

8. **a)** Look at the graph of Cost against Age.
Helicopter B is older because it is represented by the point on the graph farther to the right and the horizontal axis represents age.
Helicopter B costs less to operate because it is represented by the lower point on the graph and the vertical axis represents cost to operate.
So, the statement "The older helicopter is cheaper to operate" is true.

b) Look at the graph of Maximum Speed against Number of Seats.
Helicopter A has more seats because it is represented by the point on the graph farther to the right and the horizontal axis represents the number of seats.
Helicopter A is faster because it is represented by the higher point on the graph and the vertical axis represents maximum speed.
So, the statement "The helicopter with more seats has the lower maximum speed" is false.

c) Look at the graph of Maximum Speed against Number of Seats.
Helicopter B is slower because it is represented by the lower point on the graph and the vertical axis represents maximum speed.
Look at the graph of Cost against Age.
Helicopter B is cheaper to operate because it is represented by the lower point on the graph and the vertical axis represents cost to operate.
So, the statement "The helicopter with the lower maximum speed is cheaper to operate" is true.

d) Look at the graph of Maximum Speed against Number of Seats.
Helicopter A has the greater maximum speed because it is represented by the higher point on the graph and the vertical axis represents maximum speed.
Look at the graph of Cost against Age.
Helicopter A is newer because it is represented by the point on the graph farther to the left and the horizontal axis represents age.
So, the statement "The helicopter with the greater maximum speed is older" is false.

e) Look at the graph of Maximum Speed against Number of Seats.
Helicopter B has fewer seats because it is represented by the point on the graph farther to the left and the horizontal axis represents the number of seats.
Look at the graph of Cost against Age.
Helicopter B is older because it is represented by the point on the graph farther to the right and the horizontal axis represents age.
So, the statement "The helicopter with fewer seats is newer" is false.

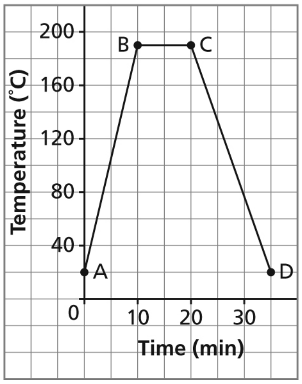
9. a)

Segment	**Graph**	**Journey**
AB	The graph goes down to the right, so as time increases, the volume of gas decreases.	At the start of the journey, there is 25 L of gas in the snowmobile. After 2 h of travel, there is 15 L of gas remaining in the tank.
BC	The graph goes up steeply to the right, so as time increases, the volume of gas increases.	The tank is filled until it contains 30 L of gas.
CD	The graph goes down to the right, so as time increases, the volume of gas decreases.	After nearly 2 h of travel, there is 20 L of gas remaining in the tank.
DE	The graph is horizontal, so as time increases the volume of gas in the tank stays the same.	The snowmobile stops for 2 h.
EF	The graph goes down to the right, so as time increases, the volume of gas decreases.	After 3 h of travel, there is 5 L of gas remaining in the tank.
FG	The graph goes up steeply to the right, so as time increases, the volume of gas increases.	The tank is filled until it contains 30 L of gas.

b) At the start of the journey, there was 25 L of gas in the snowmobile because the journey starts at point A with coordinates (0, 25). At two points during the journey, the tank contained 30 L of gas, so the tank was not full at the start of the journey.

10. Draw and label axes on a grid. The horizontal axis represents time in minutes, and the vertical axis represents temperature in degrees Celsius.

Temperature of an Oven

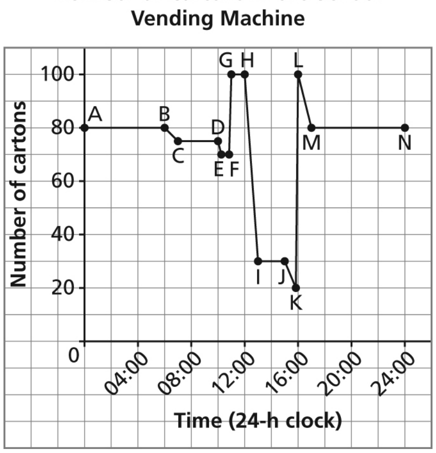


Segment	**Oven**
AB	The oven is turned on and increases in temperature from 20°C to 190°C in 10 min, so the segment goes up to the right.
BC	The cookies are placed in the oven and baked for 10 min. Since the temperature doesn't change, the segment is horizontal.
CD	The oven is turned off and the temperature decreases to 20°C in 15 min, so the segment goes down to the right.

11.

Distance from home	**Journey**
0 km to 0.5 km	The car leaves home and accelerates to 50 km/h.
0.5 km to 1.5 km	The car travels at a constant speed of 50 km/h.
1.5 km to 2 km	The car decelerates to 40 km/h, then accelerates to 50 km/h.
2 km to 3 km	The car travels at a constant speed of 50 km/h.
3 km to 3.5 km	The car accelerates to 80 km/h.
3.5 km to 5.5 km	The car travels at a constant speed of 80 km/h.
5.5 km to 5.6 km	The car decelerates to a stop.
5.6 km to 6 km	The car accelerates to 60 km/h.
6 km to 7.4 km	The car travels at a constant speed of 60 km/h.
7.4 km to 7.5 km	The car decelerates to a stop at the person's work.

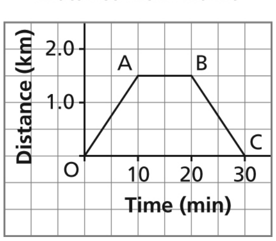
12. Graphs may vary. For example:
Draw and label axes on a grid. The horizontal axis represents time on a 24-h clock, and the vertical axis represents the number of cartons in the machine.



Segment	Vending machine
AB	The number of cartons remains constant until approximately 6 A.M., when students and teachers begin arriving at school.
BC	From 6 A.M. to 7 A.M., 5 cartons are purchased, so the graph goes down to the right.
CD	No cartons are purchased from 7 A.M. to 8 A.M. or while students are in class from 8 A.M. to 10 A.M., so the number of cartons in the machine remains constant. There are 75 cartons in the machine during this time period.
DE	From 10 A.M. to 10:15 A.M., 5 cartons are purchased, so the graph goes down to the right.
EF	From 10:15 A.M. to 10:55 A.M., students are in class, so no cartons are purchased and the number of cartons in the machine remains constant.
FG	From 10:55 A.M. to 11 A.M., the machine is filled to 100 cartons.
GH	From 11 A.M. to noon, students are in class, so no cartons are purchased and the number of cartons in the machine remains constant.
HI	From noon to 1 P.M., students purchase 70 cartons, so the graph goes down to the right.
IJ	From 1 P.M. to 3 P.M., students are in class, so no cartons are purchased and the number of cartons in the machine remains constant.
JK	From 3 P.M. to 3:55 P.M., 10 cartons are purchased, so the graph goes down to the right.
KL	From 3:55 P.M. to 4 P.M., the machine is filled to 100 cartons.
LM	From 4 P.M. to 5 P.M., students purchase 20 cartons, so the graph goes down to the right.
MN	No cartons are purchased after 5 P.M., so the number of cartons in the machine remains constant.

13. There are 2 errors in the graph.
When Jonah's mom asks him a question, he turns the volume down. So, the graph should lie below a volume level of 40 for the time period from 3 min to 4 min.
When Jonah presses the mute button, the volume level should drop to 0 immediately, so the graph should show a vertical line segment from a volume level of 80 to 0 at 9 min.

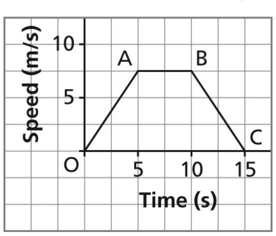
14. a) Copy and label the graph.



Situation: A person jogs from home to a park 1.5 km away in 10 min. He sits on a park bench and reads for 10 min. Then he jogs 1.5 km home in 10 min.

Justification: Line segment OA goes up to the right, so the person's distance from home is increasing with time.
Segment AB is horizontal, so the person's distance from home remains constant.
Segment BC goes down to the right, so the person's distance from home decreases with time.

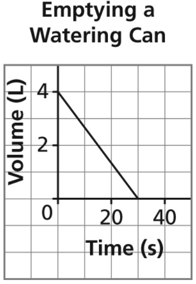
b) Copy and label the graph.



Situation: A person sprints down a track starting from a standstill. It takes the person 5 s to reach a speed of 7.5 m/s. After 5 s of running at 7.5 m/s, the person slows down and stops after another 5 s.

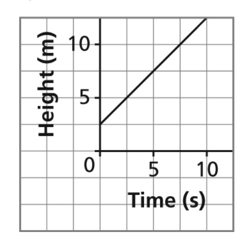
Justification: Line segment OA goes up to the right, so the person's speed is increasing with time.
Segment AB is horizontal, so the person's speed remains constant.
Segment BC goes down to the right, so the person's speed decreases with time.

15. a) Copy and label the graph. The dependent variable is the volume in litres.



Situation: A watering can contains 4 L of water. It is poured at a steady rate so the watering can is empty after 30 s.

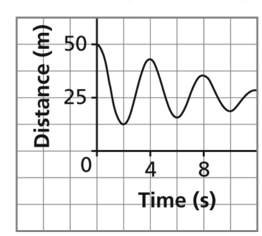
b) Copy and label the graph. The dependent variable is height in metres.



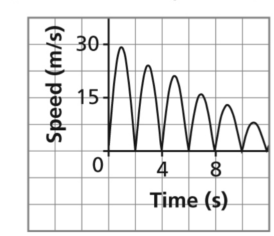
Situation: A person releases a helium balloon. It starts at a height of 2.5 m above the ground. After 10 s, it is at a height of 12.5 m above the ground.

C

16. a) i) Sketch a graph. The distance decreases until the bungee chord is fully extended, then increases; this pattern continues until the person stops bouncing.



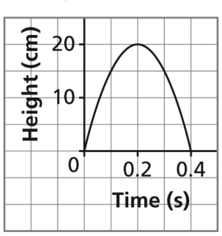
ii) Sketch a graph. The speed increases and decreases during the time a person descends to come to an instantaneous stop. This pattern continues for the ascent, and each repeated cycle of descent and ascent.



b) **Similarities:** Both graphs are curves because on the first graph, speed is not constant, and on the second graph, acceleration is not constant.

Differences: The top graph is a continuous curve whose highest and lowest points get closer together. The bottom graph is a series of upside down U-shaped curves, whose highest points decrease.
The curves in the bottom graph always start and end at a speed of 0 m/s.

17. a) This graph could represent the height of a grasshopper during one hop.
The independent variable is time, and the dependent variable is the height of the grasshopper.

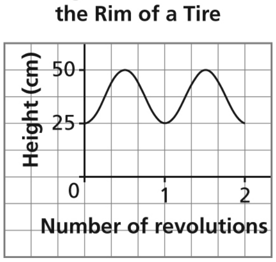


It takes 0.2 s for a grasshopper to jump 20 cm high, and another 0.2 s for it to return to the ground.

b) The graph could represent the cost of parking in a parking garage.
The independent variable is time, and the dependent variable is the cost.

It costs $1 to park for up to 30 min, $2 to park from 30 min to 60 min, and $3 to park from 60 min to 90 min.

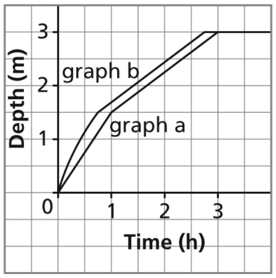
c) This graph could represent the height of a point on the rim of a tire on a truck over time.
The independent variable is time, and the dependent variable is the height of the point.



The point starts at the lowest point on the rim, 25 cm above the ground. As the wheel goes around, the point moves up to a maximum height of 50 cm, then down, then up again.

18. Sketch the graphs on the same grid.

Depth of Water in Two Pools

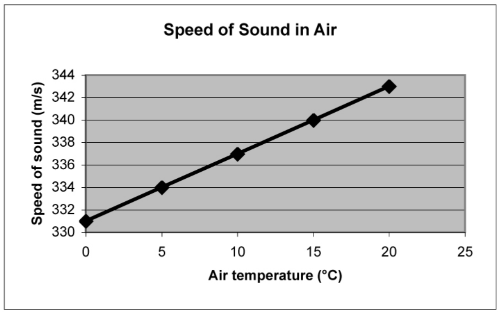


In part a, the water will fill the deeper depression at a constant rate, then the second depression at a slower rate since it is wider. When the pool is full, the tap will be shut off, and the volume of water in the pool will remain constant.

In part b, as the pool width increases, the water will fill the pool less and less rapidly, so the graph is a curve. When the width is constant, the water will fill the pool at a slower, but constant rate. When the pool is full, the tap will be shut off, and the volume of water in the pool will remain constant.

Lesson 5.4 Math Lab: Graphing Data Assess Your Understanding (page 286)

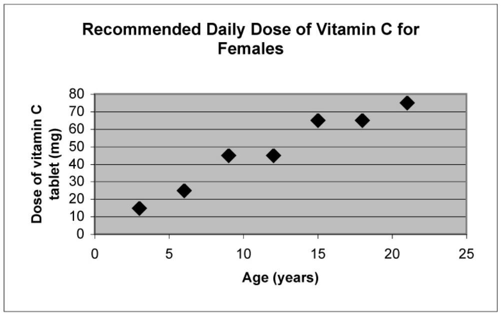
1. a) i) Graph the data using a computer spreadsheet program. Label the axes.



The points on the graph are joined because the air temperature could be any number of degrees Celsius between 0°C and 20°C and the speed of sound could be any number of metres per second between 331 m/s and 343 m/s.

ii) Every number in the first column of the table appears exactly once, so the relation is a function.

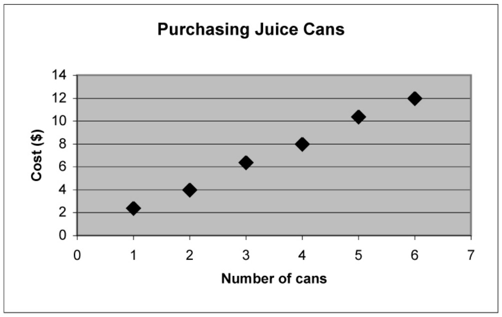
b) i) Graph the data using a computer spreadsheet program. Label the axes.



The points on the graph are not joined because the ages are whole numbers of years and it is not clear whether any number is permissible for the dose of vitamin C in a tablet.

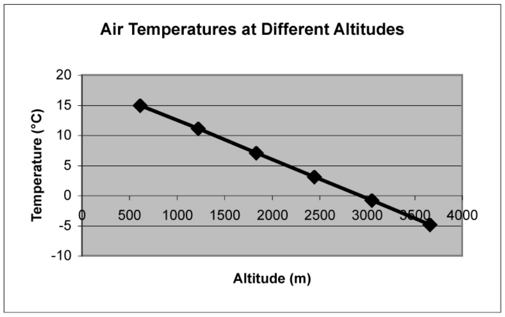
ii) Every number in the first column of the table appears exactly once, so the relation is a function.

2. a) Graph the data using a computer spreadsheet program. Label the axes.



The points are not joined because only whole numbers are permissible for the number of juice cans purchased.
The relation is a function because a vertical line drawn on the graph passes through at most 1 point on the graph.

b) Graph the data using a computer spreadsheet program. Label the axes.



The points on the graph are joined because the altitude could be any number of metres between 610 m and 3660 m and the temperature could be any number of degrees Celsius between –5°C and 15°C.
The relation is a function because a vertical line drawn on the graph passes through at most 1 point on the graph.

Lesson 5.5 Graphs of Relations and Functions Exercises (pages 294–297)

A

4. **a)** The domain is the set of x-coordinates of the points on the graph:
$\{-2, -1, 0, 1, 2\}$
The range is the set of y-coordinates of the points on the graph:
$\{-4, -2, 0, 2, 4\}$

b) The domain is the set of x-coordinates of the points on the graph:
$\{-3, -1, 0, 2, 3\}$
The range is the set of y-coordinates of the points on the graph:
$\{-2, 0, 1, 2, 3\}$

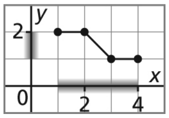
c) The domain is the set of x-coordinates of the points on the graph:
$\{-3, -2, -1, 0, 1, 2, 3\}$
The range is the set of y-coordinates of the points on the graph:
$\{2\}$

5. A vertical line drawn on each graph intersects the graph at 0 points or 1 point.

6. **a)** The graph of $y = 1$ represents a function because each point on the line has a different x-coordinate.

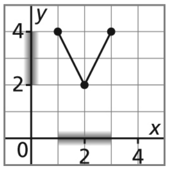
b) The graph of $x = 1$ does not represent a function because each point on the line has the same x-coordinate, 1.

7. **a)** Visualize the shadow of the graph on the x- and y-axes.



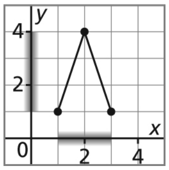
The domain is the set of x-values of the graph:
$1 \le x \le 4$
The range is the set of y-values of the graph:
$1 \le y \le 2$
So, match part a with part iv.

b) Visualize the shadow of the graph on the x- and y-axes.



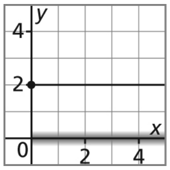
The domain is the set of x-values of the graph:
$1 \le x \le 3$
The range is the set of y-values of the graph:
$2 \le y \le 4$
So, match part b with part i.

c) Visualize the shadow of the graph on the x- and y-axes.



The domain is the set of x-values of the graph:
$1 \le x \le 3$
The range is the set of y-values of the graph:
$1 \le y \le 4$
So, match part c with part ii.

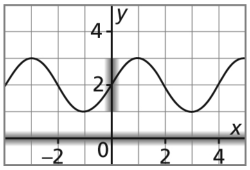
d) Visualize the shadow of the graph on the x- and y-axes.



The domain is the set of x-values of the graph:
$x \ge 0$
The range is the set of y-values of the graph:
$y = 2$
So, match part d with part iii.

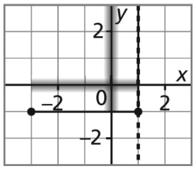
B

8. a) This graph is a function.
Any vertical line drawn on the graph passes through exactly 1 point.
Visualize the shadow of the graph on the *x*- and *y*-axes.

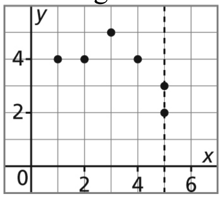


The domain is the set of *x*-values of the graph, which is the set of all real numbers.
The range is the set of *y*-values of the graph:
$1 \le y \le 3$

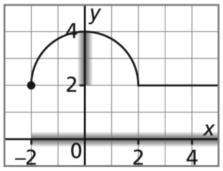
b) This graph is not a function. The vertical line $x = 1$ passes through many points on the graph. Visualize the shadow of the graph on the *x*- and *y*-axes.
The domain is the set of *x*-values of the graph: $-3 \le x \le 1$
The range is the set of *y*-values of the graph: $y \ge -1$



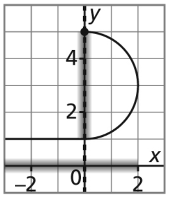
c) This graph is not a function. The vertical line $x = 5$ passes through the points (5, 2) and (5, 3) on the graph.
The domain is the set of *x*-values of the graph: {1, 2, 3, 4, 5}
The range is the set of *y*-values of the graph: {2, 3, 4, 5}



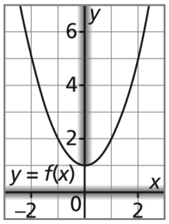
d) This graph is a function. It is not possible to draw a vertical line on the graph that passes through 2 points on the graph.
Visualize the shadow of the graph on the *x*- and *y*-axes.
The domain is the set of *x*-values of the graph:
$x \ge -2$
The range is the set of *y*-values of the graph:
$2 \le y \le 4$



e) This graph is not a function. The vertical line $x = 0$ passes through the points (0, 1) and (0, 5) on the graph. Visualize the shadow of the graph on the *x*- and *y*-axes.
The domain is the set of *x*-values of the graph: $x \le 2$
The range is the set of *y*-values of the graph: $1 \le y \le 5$

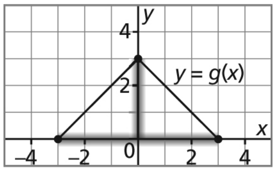


9. **a)** Visualize the shadow of the graph on the x- and y-axes.



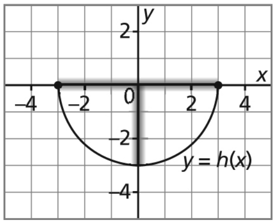
The domain is the set of x-values of the graph, which is the set of all real numbers.
The range is the set of y-values of the graph:
$y \geq 1$

b) Visualize the shadow of the graph on the x- and y-axes.



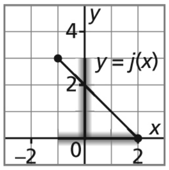
The domain is the set of x-values of the graph:
$-3 \leq x \leq 3$
The range is the set of y-values of the graph:
$0 \leq y \leq 3$

c) Visualize the shadow of the graph on the x- and y-axes.



The domain is the set of x-values of the graph:
$-3 \leq x \leq 3$
The range is the set of y-values of the graph:
$-3 \leq y \leq 0$

d) Visualize the shadow of the graph on the x- and y-axes.



The domain is the set of x-values of the graph:
$-1 \leq x \leq 2$
The range is the set of y-values of the graph:
$0 \leq y \leq 3$

10. a) The points on the graph should not be connected because the price is calculated per letter, and only whole numbers of letters are possible.

b) The points on the graph should be connected because the plane's altitude and travel time could be any positive number.

c) The points on the graph should be connected because the baby's mass and age could be any positive number.

d) The points on the graph should be connected because both a number and its cube root can be any real number.

11. a) i) The data in the graph represent the distance of a school bus from the school from 8:00 to 9:00.

ii) The data in the graph represent the number of students on a school bus from 8:00 to 9:00.

b) i) The independent variable is the time.
The dependent variable is the distance from the school.

ii) The independent variable is the time.
The dependent variable is the number of students.

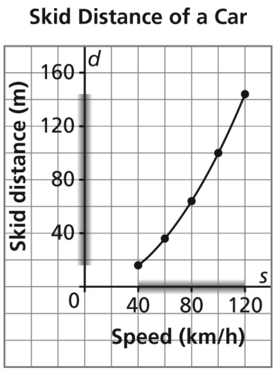
c) The points on graph A are connected because all values of time and distance are permissible between the indicated plotted points.
The points on graph B are not connected because it is impossible to have only part of a student on a bus.

12. a) The points on the graph are connected because the car's speed in kilometres per hour and the skid length in metres can be any positive number between the plotted points.

b) Visualize the shadow of the graph on the x- and y-axes.

The domain is the set of s-values of the graph:
$40 \le s \le 120$
The range is the set of d-values of the graph:
approximately $16 \le d \le 144$

The domain and range cannot contain negative numbers because it is impossible to have a negative skid distance or a negative speed. The domain is also restricted because the relationship shown on the graph may not be true for speeds less than 40 km/h and greater than 120 km/h.



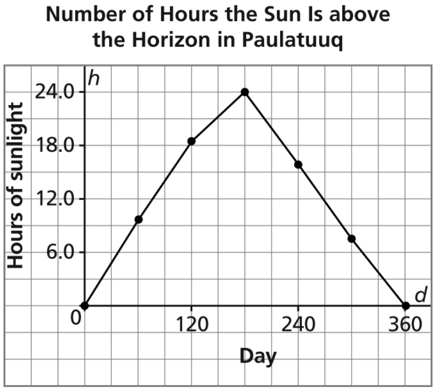
13. a) The number of cars in the parking lot depends on the time of day. So, *t* is the independent variable and *n* is the dependent variable.

b) The points are not connected because it is impossible to have part of a car in a parking lot.

c) The domain is the set of *x*-coordinates of the points:
{8:00, 10:00, 12:00, 14:00, 16:00}
The range is the set of *y*-coordinates of the points, approximately:
{4, 25, 31, 64, 65}
The domain can be any time between 00:00 and 24:00, all the possible times in one day.
The range can be any whole number up to the number of parking spaces in the lot.

14. a) The hours of sunlight depends on the time of year, or the day. So, the number of days after January 1 is the independent variable and the number of hours the sun is above the horizon, *h*, is the dependent variable.

b) Graph the data.



I connected the points because the relationship shown on the graph is true for days represented by points between the ones plotted.

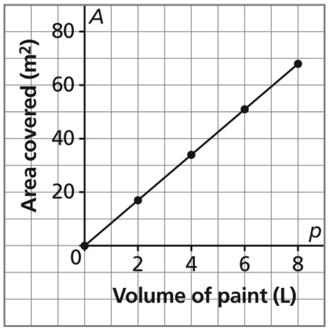
c) The relation is a function because each number in the first column of the table is associated with exactly one number in the second column in the table. In the graph, any vertical line passes through at most one point.

15. a) Copy and complete the table.

Volume of paint, *p* (L)	0	2	4	6	8
Cost, *c* ($)	0	24	48	72	96
Area covered, *A* (m^2)	0	17	34	51	68

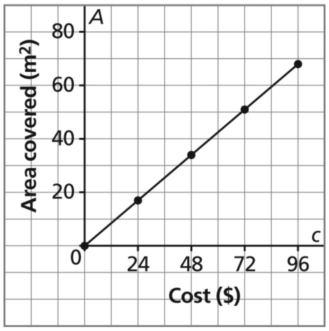
b) Graph the data.

Area Covered by Paint



c) Graph the data.

Area that Can Be
Covered for a Given Cost



d) In part b:
The domain is the set of *p*-values of the graph:
$0 \le p \le 8$
The range is the set of *A*-values of the graph:
$0 \le A \le 68$

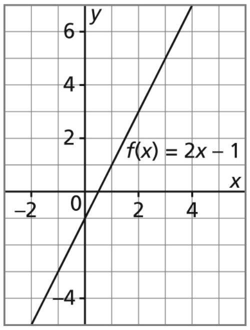
In part c:
The domain is the set of *c*-values of the graph:
$0 \le c \le 96$
The range is the set of *A*-values of the graph:
$0 \le A \le 68$

16. The domain value is a value of x. The range value is a value of $f(x)$.

a) To determine the value of $f(x)$ when $x = 0$:

When $x = 0$, $f(x)$ is the y-intercept, which is –1.

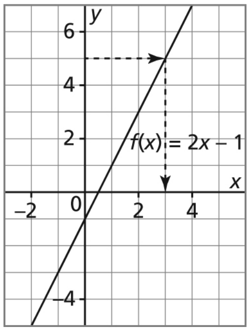
When the domain value is 0, the range value is –1.



b) To determine the value of x when $f(x) = 5$:

Begin at $f(x) = 5$ on the y-axis.
Draw a horizontal line to the graph,
then a vertical line to the x-axis.
The line intersects the x-axis at 3.
So, when $f(x) = 5$, $x = 3$

When the range value is 5, the domain value is 3.

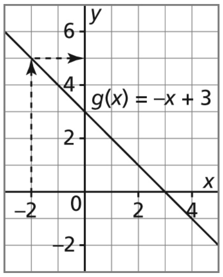


17. The domain value is a value of x. The range value is a value of $g(x)$.

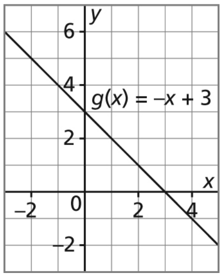
a) To determine the value of $g(x)$ when $x = –2$:

Begin at $x = –2$ on the x-axis.
Draw a vertical line to the graph,
then a horizontal line to the y-axis.
The line intersects the y-axis at 5.
So, $g(–2) = 5$

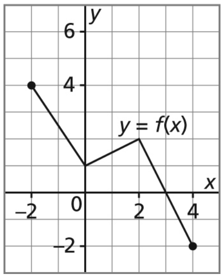
When the domain value is –2, the range value is 5.



b) To determine the value of x when $g(x) = 0$:
When $g(x) = 0$, x is the x-intercept, which is 3.
When the range value is 0, the domain value is 3.



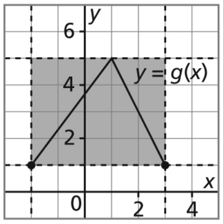
18. Sample response:
A possible function is:



The domain of this function is:
$-2 \leq x \leq 4$
The range of this function is:
$-2 \leq y \leq 4$

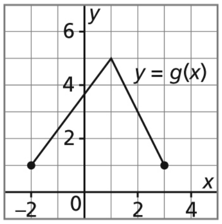
19. Answers will vary. For example:

a) On a grid, draw faint vertical lines through $x = -2$ and $x = 3$; and faint horizontal lines through $y = 1$ and $y = 5$.

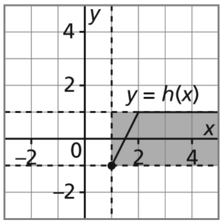


Draw line segments inside the region enclosed by the faint lines so that there is exactly one point on the graph for every x-value between –2 and 3, and there is at least one point on the graph for every y-value between 1 and 5.

A function is:

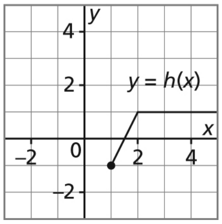


b) On a grid, draw a faint vertical line through $x = 1$; and faint horizontal lines through $y = -1$ and $y = 1$.

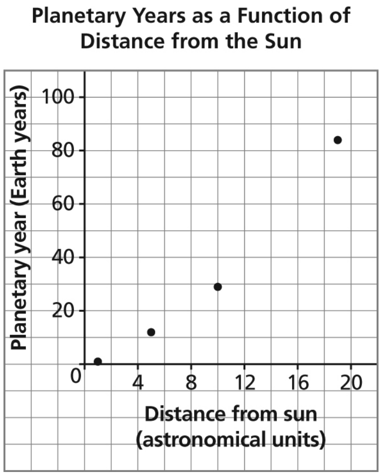


Draw line segments inside the region between the horizontal lines and to the right of the vertical line so that there is exactly one point on the graph for every x-value greater than or equal to 1, and there is at least one point on the graph for every y-value between –1 and 1.

A function is:



20. a) Sketch a graph.

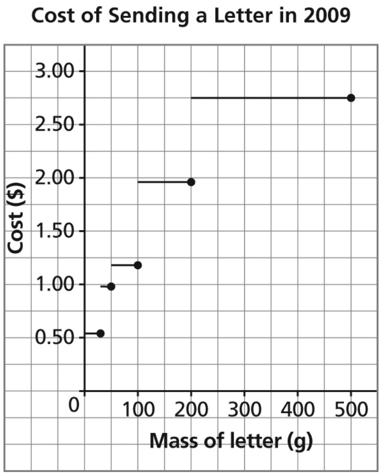


I didn't connect the points because there are only a few planets between the ones shown and I am not sure if the relationship shown in the graph is true for them.

b) The domain of this function is the distances from the sun:
{1, 5, 10, 19}
The range of this function is the numbers of planetary years:
{1, 12, 29, 84}

C

21. a) Sketch a graph.



I connected the points in each interval, but not the points at the endpoints of the intervals because otherwise my graph would have vertical line segments and would not have been a function.

b) The domain of this function is the masses of letters:
All real numbers between 0 and 500.
The range of this function is the costs to send the letters:
{0.54, 0.98, 1.18, 1.96, 2.75}

22. Yes, the points should have been connected because the patient has a temperature at all times, even if it is not measured, and the patient's temperature at a certain time was most likely between the temperatures measured immediately before and after that time.

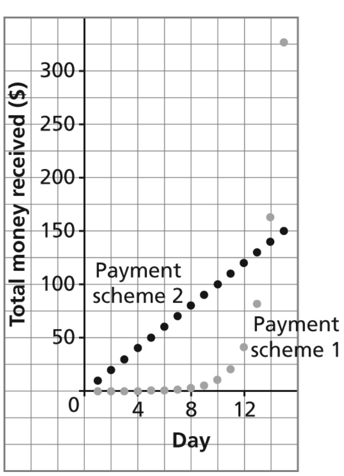
23. A graph with time as its independent variable and a discrete measure as its dependent variable should not have its points connected. For example, a graph of the number of people on a bus over 4 h should not have its points connected because it's impossible to have only part of a person on the bus. So, the statement is false.

24. a) Tables of values:

Payment Scheme 1		
Day	**Money each day ($)**	**Total money received ($)**
1	0.01	0.01
2	0.02	0.01 + 0.02 = 0.03
3	0.04	0.03 + 0.04 = 0.07
4	0.08	0.07 + 0.08 = 0.15
5	0.16	0.15 + 0.16 = 0.31
6	0.32	0.31 + 0.32 = 0.63
7	0.64	0.63 + 0.64 = 1.27
8	1.28	1.27 + 1.28 = 2.55
9	2.56	2.55 + 2.56 = 5.11
10	5.12	5.11 + 5.12 = 10.23
11	10.24	10.23 + 10.24 = 20.47
12	20.48	20.47 + 20.48 = 40.95
13	40.96	40.95 + 40.96 = 81.91
14	81.92	81.91 + 81.92 = 163.83
15	163.84	163.83 + 163.84 = 327.67

Payment Scheme 2	
Day	**Total money received ($)**
1	10
2	20
3	30
4	40
5	50
6	60
7	70
8	80
9	90
10	100
11	110
12	120
13	130
14	140
15	150

b) Graph the data.

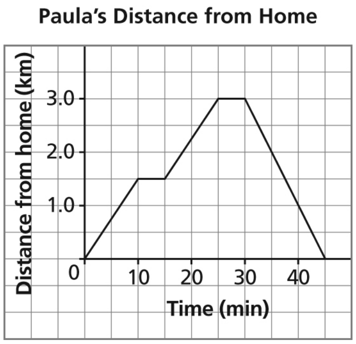


c) I would choose Payment Scheme 1 because after 13 days, the money received is greater and increases at a faster rate.

Checkpoint 2 **Assess Your Understanding (page 299)**

5.3

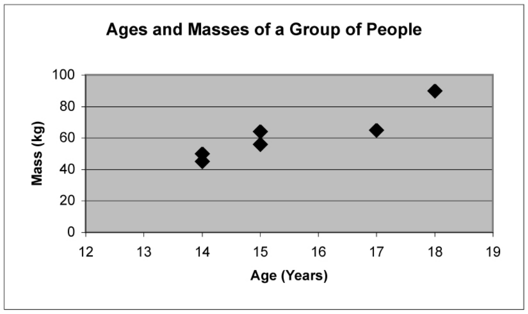
1. Copy and label the graph.



Situation: Paula jogs from her house to a store 1.5 km away in 10 min. She spends 5 min in the store, then jogs another 1.5 km away from her house to another store. It takes Paula 10 min to get to the second store. She spends 5 min in the second store, then returns home. It takes Paula 15 min to travel 3 km to her home.

5.4

2. **a)** Graph the data using a computer spreadsheet program. Label the axes.



b) The points should not be joined because each person's age was measured in a whole number of years.

c) The domain is the set of numbers in the first column of the table of values:
{14, 15, 17, 18}
The range is the set of numbers in the second column of the table of values:
{45, 50, 56, 64, 65, 90}

d) The domain is restricted to ages in whole numbers of years from 0 to 18 years. The range will then be the corresponding masses in kilograms.

5.5

3. **a)** The vertical line $x = 0$ passes through the points (0, 1) and (0, 5) on the graph.
So, the graph does not represent a function.
Its domain is:
$0 \le x \le 2$
Its range is:
$1 \le y \le 5$

b) Any vertical line drawn on the graph passes through at most 1 point.
So, the graph is a function.
Its domain is:
$x \ge -3$
Its range is:
$y \ge 0$

c) Any vertical line drawn on the graph passes through at most 1 point.
So, the graph is a function.
Its domain is:
$-2 \le x \le 2$
Its range is:
$-8 \le y \le 8$

Lesson 5.6 **Properties of Linear Relations** **Exercises (pages 308–310)**

A

3. a) The terms in the first column are in numerical order.
So, calculate the change in each variable.

Time (min)	Change in Time (min)	Distance (m)	Change in Distance (m)
0		10	
2	2 – 0 = 2	50	50 – 10 = 40
4	4 – 2 = 2	90	90 – 50 = 40
6	6 – 4 = 2	130	130 – 90 = 40

Since the changes in both variables are constant, the table of values represents a linear relation.

b) The terms in the first column are in numerical order.
So, calculate the change in each variable.

Time (s)	Change in Time (s)	Speed (m/s)	Change in Speed (m/s)
0		10	
1	1 – 0 = 1	20	20 – 10 = 10
2	2 – 1 = 1	40	40 – 20 = 20
3	3 – 2 = 1	80	80 – 40 = 40

The changes in time are constant, but the changes in speed are not constant.
So, the table of values does not represent a linear relation.

c) Arrange the terms in the first column in numerical order.
Then calculate the change in each variable.

Speed (m/s)	Change in Speed (m/s)	Time (s)	Change in Time (s)
0		0	
5	5 – 0 = 5	2.5	2.5 – 0 = 2.5
10	10 – 5 = 5	5	5 – 2.5 = 2.5
15	15 – 10 = 5	7.5	7.5 – 5 = 2.5

Since the changes in both variables are constant, the table of values represents a linear relation.

d) Arrange the terms in the first column in numerical order.
Then calculate the change in each variable.

Distance (m)	Change in Distance (m)	Speed (m/s)	Change in Speed (m/s)
1		1	
4	4 – 1 = 3	2	2 – 1 = 1
9	9 – 4 = 5	3	3 – 2 = 1
16	16 – 9 = 7	4	4 – 3 = 1

The changes in speed are constant, but the changes in distance are not constant.
So, the table of values does not represent a linear relation.

4. The first elements are in numerical order.
So, calculate the change in each variable.

a) +2 +2 +2

{(3, 11), (5, 9), (7, 7), (9, 5)}

−2 −2 −2

Since the changes in both elements are constant, the set of ordered pairs represents a linear relation.

b) +2 +2 +2

{(−2, 3), (0, 1), (2, −3), (4, −7)}

−2 −4 −4

The changes in the first elements are constant, but the changes in the second elements are not constant. So, the set of ordered pairs does not represent a linear relation.

c) This relation has two ordered pairs with first element 1 and two ordered pairs with first element 2.
So, the changes in the first elements are not constant, and the set of ordered pairs does not represent a linear relation.

5. a) The graph is a line, so it represents a linear relation.

b) The graph is a line, so it represents a linear relation.

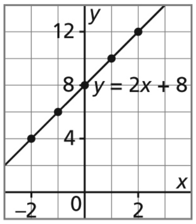
c) All the points on the graph do not lie on the same line, so the graph does not represent a linear relation.

d) The graph is a curve, so it does not represent a linear relation.

B

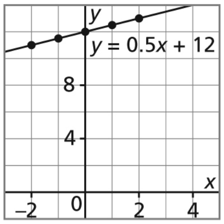
6. a) i) $y = 2x + 8$

x	y
–2	4
–1	6
0	8
1	10
2	12



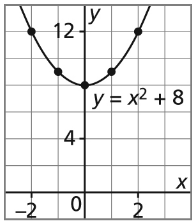
ii) $y = 0.5x + 12$

x	y
–2	11
–1	11.5
0	12
1	12.5
2	13



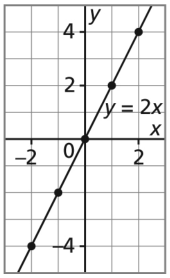
iii) $y = x^2 + 8$

x	y
–2	12
–1	9
0	8
1	9
2	12

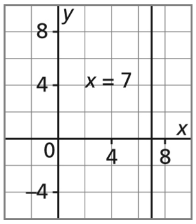


iv) $y = 2x$

x	y
–2	–4
–1	–2
0	0
1	2
2	4

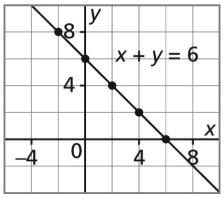


v) $x = 7$
This is a vertical line that passes through (7, 0).



vi) $x + y = 6$

x	y
–2	8
0	6
2	4
4	2
6	0



b) The relations in part a, i, ii, iv, v, and vi have graphs that are straight lines, so the relations are linear.
The relation in part a, iii has a graph that is a curve, so the relation is not linear.

7. a) i) From an understanding of the situation, the braking distance, *d*, depends on the speed of the car, *s*. So, *d* is the dependent variable and *s* is the independent variable.

ii) Look at the change in each variable.

	s	*d*	
	50	13	
+10	60	20	+7
+10	70	27	+7
+10	80	35	+8

The change in the first variable is constant, but the change in the second variable is not constant. So, the relation is not linear.

b) i) From an understanding of the situation, the altitude of the plane, *a*, depends on the time, *t*, that has elapsed since it started its descent. So, *a* is the dependent variable and *t* is the independent variable.

ii) Look at the change in each variable.

	t	*a*	
	0	12 000	
+2	2	11 600	–400
+2	4	11 200	–400
+2	6	10 800	–400
+2	8	10 400	–400

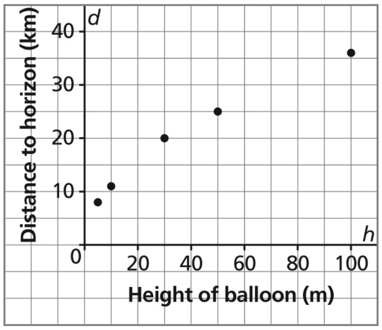
The changes in both variables are constant, so the relation is linear.

iii) The rate of change is:

$$\frac{-400\text{ m}}{2\text{ min}} = -200\text{ m/min}$$

8. a) Graph the data.

Distance to the Horizon for a Given Height in a Hot-Air Balloon



b) The relation is not linear because the points on the graph do not lie on a straight line.

9. I could examine the changes in the first and second coordinates. If both changes are constant, the relation is linear.
I could graph the ordered pairs. If the points lie on a straight line, the relation is linear.

10. Create a table of values, then check to see if the relation is linear.

	Number of people	Cost ($)	
	2	95	
+1	3	105	+10
+1	4	115	+10
+1	5	125	+10

There is a constant change of 1 in the first column, and a constant change of 10 in the second column. So, the relation is linear.

11. The first elements are in numerical order. So, calculate the change in each variable.

+4 +4 +4

{(0, 3600), (4, 3526), (8, 3353.5), (12, 3147.5)}

–74 –172.5 –206

The change in the first elements is constant, but the change in the second elements is not constant. So, this set of ordered pairs does not represent a linear relation.

+6 +6 +6

{(15, 2988.5), (21, 2670.5), (27, 2352.5), (33, 2034.5)}

–318 –318 –318

The changes in both elements are constant, so this set of ordered pairs represents a linear relation.

12. a) The equation represents a linear relation because it equates the dependent variable *C* to the sum of a constant and the product of the rate of change and the independent variable.

b) The rate of change is 15. It represents the cost per guest, $15, after the fixed cost of $550 has been paid.

13. Sample response: I could create a table of values for the relation. Then I could either check the differences in the numbers in each column or I could plot the points in the table. If the differences are constant or the points lie on a line, the relation is linear. Otherwise, it is not linear.

14. a) From an understanding of the situation, the cost of the phone call, *C*, depends on the time, *t*, that has elapsed. So, *C* is the dependent variable and *t* is the independent variable.

b) Sample response:
Choose two points on the line: (5, 0.40) and (15, 1.20)
Calculate the change in each variable from one point to the other.
The change in time is:
15 min – 5 min = 10 min
The change in cost is:
$1.20 – $0.40 = $0.80
The rate of change is:

$$\frac{\$0.80}{10 \text{ min}} = \$0.08/\text{min}$$

The rate of change is positive, so the cost is increasing with time.
Every minute, the cost of the phone call increases by \$0.80.

15. Sample response:
Choose two points on the line: (1, 9.20) and (3, 7.60)
Calculate the change in each variable from one point to the other.
The change in the number of toll booths is:
3 booths – 1 booth = 2 booths
The change in the amount Kashala has in change is:
\$7.60 – \$9.20 = –\$1.60
The rate of change is:

$$\frac{-\$1.60}{2\text{ booths}} = -\$0.80/\text{booth}$$

The rate of change is negative, so the amount Kashala has in change decreases with time.
At every toll booth, Kashala pays \$0.80.

16. a) A person who works 0 h earns \$0.
So, substitute $x = 0$ into the equations:

Equation 1:	Equation 2:	Equation 3:
$y = 500 + 40x$	$y = 35 - 0.06x$	$y = 20x$
$y = 500 + 40(0)$	$y = 35 - 0.06(0)$	$y = 20(0)$
$y = 500$	$y = 35$	$y = 0$

So, Equation 3 seems to be the best match. It could represent an hourly wage of \$20/h.
So, after 1 h, the person earns \$20. Look for a set of ordered pairs that contains (1, 20).
Set B contains (1, 20), so it seems to be the best match.
So, Equation 3 and Set B match this situation.

b) The cost of a banquet is the sum of a fixed cost and a variable cost, so look for an equation that has a constant plus the product of a constant and a variable.
Equation 1 seems to be the best match. It could represent a flat fee of \$500 plus \$40/person.
So, if 100 people attend, the cost is \$500 + \$4000 = \$4500. Look for a set of ordered pairs that contains (100, 4500).
Set C contains (100, 4500), so it seems to be the best match.
So, Equation 1 and Set C match this situation.

c) The volume of gas in the tank decreases as the car is driven, so look for an equation with a negative rate of change.
Equation 2 seems to be the best match. It could represent a car with a tank size of 35 L that uses 0.06 L of gas per kilometre driven.
So, if the car is driven for 100 km, the volume of gas remaining in the tank is:
35 L – 6 L = 29 L
So, look for a set with the ordered pair (100, 29).
Set A contains (100, 29), so it seems to be the best match.
So, Equation 2 and Set A match this situation.

17. a) i) A constant speed is a constant rate of change; that is, a measure of how the distance changes with time.
The change in altitude is 500 m in 10 m. So, the rate of change is:

$$\frac{-500\text{ m}}{10\text{ min}} = -50\text{ m/min}$$

So, the relation is linear.

ii) Create a table of values for the situation.

	Time (h)	**Number of bacteria**	
	0	100	
+1	1	300	+200
+1	2	900	+600
+1	3	2700	+1800
+1	4	8100	+5400

The change in time is constant, but the change in the number of bacteria is not constant. So, the situation does not represent a linear relation.

iii) Create a table of values for the situation.

	Distance (km)	**Cost ($)**	
	0	5	
+1	1	7	+2
+1	2	9	+2
+1	3	11	+2
+1	4	13	+2

Since the change in each variable is constant, the situation represents a linear relation.

iv) Create a table of values for the situation.

	Number of yearbooks	**Cost ($)**	
	0	500	
+100	100	(100)(5) + 500 = 1000	+500
+100	200	(200)(5) + 500 = 1500	+500
+100	300	(300)(5) + 500 = 2000	+500
+100	400	(400)(5) + 500 = 2500	+500

Since the change in each variable is constant, the situation represents a linear relation.

v) Create a table of values for the situation.

	Time (years)	**Value of investment ($)**	
	0	1000.00	
+1	1	1000(1.12) = 1120.00	+120.00
+1	2	1120(1.12) = 1254.40	+134.40
+1	3	1254.40(1.12) = 1404.93	+150.53
+1	4	1404.93(1.12) = 1573.52	+168.59

The change in time is constant, but the change in the value of the investment is not constant. So, the situation does not represent a linear relation.

b) i) The altitude of the hang glider depends on the time since she started her descent. So, altitude is the dependent variable and time is the independent variable.
From part a, the rate of change is –50 m/min
The rate of change is negative because the hang glider's altitude is decreasing over time.
Every minute, the altitude decreases by 50 m.

iii) The taxi cost depends on the distance travelled. So, the cost is the dependent variable and the distance travelled is the independent variable.
The rate of change is: $\frac{\$2}{1\text{ km}} = \$2\text{/km}$
The rate of change is positive because the cost increases with the distance travelled.
Every kilometre, the cost increases by $2.

iv) The printing cost depends on the number of yearbooks to be printed. So, the cost is the dependent variable and the number of yearbooks to be printed is the independent variable.
The rate of change is: $5/yearbook
The rate of change is positive because the cost increases with the number of yearbooks printed.
For every yearbook to be printed, the cost increases by $5.

C

18. a) The formula represents a relation with P as the dependent variable and s as the independent variable. The variable s is raised to the exponent 1, so the relation is linear.

b) The formula represents a relation with A as the dependent variable and s as the independent variable. The variable s is raised to the exponent 2, so the relation is not linear.

c) The formula represents a relation with V as the dependent variable and r as the independent variable. The variable r is raised to the exponent 3, so the relation is not linear.

d) The formula represents a relation with C as the dependent variable and d as the independent variable. The variable d is raised to the exponent 1, so the relation is linear.

e) The formula represents a relation with A as the dependent variable and r as the independent variable. The variable r is raised to the exponent 2, so the relation is not linear.

19. a) Each equation represents a relation with V as the dependent variable and n as the independent variable.
In the first equation, $V = 24\,000 - 2000n$, the variable n is raised to the exponent 1, so the relation is linear.
In the second equation, $V = 24\,000(0.2^n)$, the variable n is the exponent of a power, so the relation is not linear.

b) The rate of change for the first equation is –\$2000/year.
The rate of change is negative, so the value of the truck decreases with time.
Every year, the value of the truck depreciates by \$2000.

20. The speed of sound is constant, and since speed is the rate of change of distance over time, the relation is linear.

21. Suppose the pickers pick berries at the same rate and 1 picker could harvest the patch in 8 h.
So, 2 pickers could harvest the patch in 4 h.
4 pickers could harvest the patch in 2 h.
8 pickers could harvest the patch in 1 h.
So, a set of ordered pairs for the relation is: {(1, 8), (2, 4), (4, 2), (8, 1)}
The rate of change calculated from the first 2 ordered pairs is:

$$\frac{4-8}{2-1} = -4$$

The rate of change calculated from the 2nd and 3rd ordered pairs is:

$$\frac{2-4}{4-2} = \frac{-2}{2}, \text{ or } -1$$

Since the rate of change is not constant, the relation is not linear.

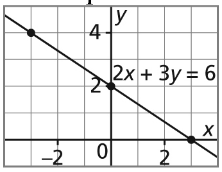
22. a) The change in each variable will be constant because there is only one pair of points to consider.
For example, this relation has exactly 2 ordered pairs: {(0, 1), (4, 22)}

$$\begin{tikzcd}[column sep=tiny] \{(0, \arrow[rr, bend left, "+4"] & 1), \arrow[rr, bend right, "+21"'] & (4, & 22)\} \end{tikzcd}$$

The change in each variable is constant.
So, the statement is true.

b) The statement is true. An equation of the form $Ax + By = C$ represents a linear function because the terms in x and y are added, and the exponents of x and y are 1. The only linear relation that is not a function is a vertical line. A vertical line has the form $x = D$, for a constant D. Since B is a non-zero constant, it is impossible to write an equation that is equivalent to the equation $Ax + By = C$ and does not contain a term involving y. So, it does not represent a vertical line.
For example, this equation has the form $Ax + By = C$, where A, B, and C are non-zero constants: $2x + 3y = 6$
Create a table of values for this equation and graph them on a coordinate grid.

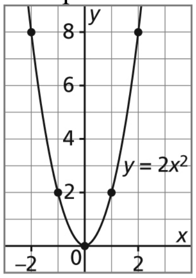
x	y
–3	4
0	2
3	0



This graph represents a linear function.

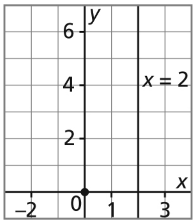
c) This equation has the form $y = Cx^2$, where C is a non-zero constant: $y = 2x^2$
Create a table of values for this equation then graph it on a coordinate grid.

x	y
–2	8
–1	2
0	0
1	2
2	8



The graph is a curve.
So, the statement is false.

d) The graph of any equation of the form $x = C$, where C is a constant, is a vertical line.
A line always represents a linear relation.
For example, $x = 2$ has the form $x = C$, where C is the constant 2.
The graph of $x = 2$ is a vertical line through (2, 0).
So, the statement is true.



e) A linear relation is only a linear function when the relation does not contain two different ordered pairs with the same first element. From part d, any vertical line, such as $x = 2$, is a linear relation. However, every ordered pair has the same first element, 2. So, the statement is false.

Lesson 5.7 **Interpreting Graphs of Linear Functions** **Exercises (pages 319–323)**

A

4. a) i) On the vertical axis, the point of intersection has coordinates (0, 0).
The vertical intercept is 0.
On the horizontal axis, the point of intersection has coordinates (0, 0).
The horizontal intercept is 0.

ii) Two points on the graph are: (0, 0) and (3, 120)
The rate of change is:

$$\begin{aligned}\frac{\text{Change in } d}{\text{Change in } t} &= \frac{120\text{ km} - 0\text{ km}}{3\text{ h} - 0\text{ h}}\\ &= \frac{120\text{ km}}{3\text{ h}}\\ &= 40\text{ km/h}\end{aligned}$$

iii) The domain is all possible *t*-values:
$0 \le t \le 3$
The range is all possible *d*-values:
$0 \le d \le 120$

b) i) On the vertical axis, the point of intersection has coordinates (0, 100).
The vertical intercept is 100.
On the horizontal axis, the point of intersection has coordinates (4, 0).
The horizontal intercept is 4.

ii) Use the coordinates in part i.
The rate of change is:

$$\begin{aligned}\frac{\text{Change in } d}{\text{Change in } t} &= \frac{0\text{ km} - 100\text{ km}}{4\text{ h} - 0\text{ h}}\\ &= \frac{-100\text{ km}}{4\text{ h}}\\ &= -25\text{ km/h}\end{aligned}$$

iii) The domain is all possible *t*-values:
$0 \le t \le 4$
The range is all possible *d*-values:
$0 \le d \le 100$

5. a) i) On the vertical axis, the point of intersection has coordinates (0, 400).
The vertical intercept is 400.

ii) Two points on the graph have coordinates: (0, 400) and (8, 1200)
The rate of change is:

$$\begin{aligned}\frac{\text{Change in } A}{\text{Change in } t} &= \frac{1200 \text{ ft.} - 400 \text{ ft.}}{8 \text{ min} - 0 \text{ min}}\\ &= \frac{800 \text{ ft.}}{8 \text{ min}}\\ &= 100 \text{ ft./min}\end{aligned}$$

iii) The domain is all possible t-values:
$0 \le t \le 8$
The range is all possible A-values:
$400 \le A \le 1200$

b) i) On the vertical axis, the point of intersection has coordinates (0, 1000).
The vertical intercept is 1000.

ii) Two points on the graph have coordinates: (0, 1000) and (8, 600)
The rate of change is:

$$\begin{aligned}\frac{\text{Change in } A}{\text{Change in } t} &= \frac{600 \text{ ft.} - 1000 \text{ ft.}}{8 \text{ min} - 0 \text{ min}}\\ &= \frac{-400 \text{ ft.}}{8 \text{ min}}\\ &= -50 \text{ ft./min}\end{aligned}$$

iii) The domain is all possible t-values:
$0 \le t \le 8$
The range is all possible A-values:
$600 \le A \le 1000$

B

6. Since each function is linear, its graph is a straight line.

a) $f(x) = 4x + 3$

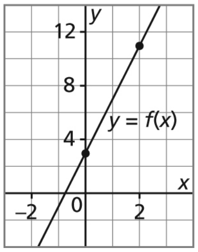
Determine the y-intercept:
When $x = 0$,
$f(0) = 4(0) + 3$
$f(0) = 3$

Determine the x-intercept:
When $f(x) = 0$,
$0 = 4x + 3$
$-3 = 4x$
$x = -\frac{3}{4}$

Determine the coordinates of a third point on the graph.
When $x = 2$,
$f(2) = 4(2) + 3$
$f(2) = 11$

Plot the points (0, 3) and (2, 11), then draw a line through them.

Check that the graph passes through $(-\frac{3}{4}, 0)$.



b) $g(x) = -3x + 5$

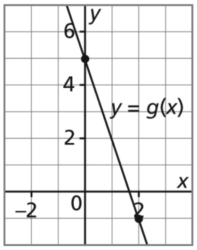
Determine the y-intercept:
When $x = 0$,
$g(0) = -3(0) + 5$
$g(0) = 5$

Determine the x-intercept:
When $g(x) = 0$,
$0 = -3x + 5$
$3x = 5$
$x = \frac{5}{3}$

Determine the coordinates of a third point on the graph.
When $x = 2$,
$g(2) = -3(2) + 5$
$g(2) = -1$

Plot the points (0, 5) and (2, –1), then draw a line through them.

Check that the graph passes through $(\frac{5}{3}, 0)$.



c) $h(x) = 9x - 2$

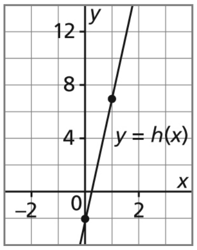
Determine the y-intercept:
When $x = 0$,
$h(0) = 9(0) - 2$
$h(0) = -2$

Determine the x-intercept:
When $h(x) = 0$,
$h(x) = 9x - 2$
$0 = 9x - 2$
$2 = 9x$
$x = \frac{2}{9}$

Determine the coordinates of a third point on the graph.
When $x = 1$,
$h(1) = 9(1) - 2$
$h(1) = 7$

Plot the points (0, –2) and (1, 7), then draw a line through them.

Check that the graph passes through ($\frac{2}{9}$, 0).



d) $k(x) = -5x - 2$

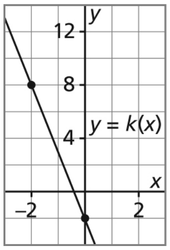
Determine the y-intercept:
When $x = 0$,
$k(0) = -5(0) - 2$
$k(0) = -2$

Determine the x-intercept:
When $k(x) = 0$,
$0 = -5x - 2$
$5x = -2$
$x = -\frac{2}{5}$

Determine the coordinates of a third point on the graph.
When $x = -2$,
$k(-2) = -5(-2) - 2$
$k(-2) = 8$

Plot the points (0, –2) and (–2, 8), then draw a line through them.

Check that the graph passes through ($-\frac{2}{5}$, 0).



7. a) Choose two points on the graph: (0, 0) and (2, 18)
The rate of change is:

$$\frac{\text{Change in } A}{\text{Change in } V} = \frac{18\text{ m}^2 - 0\text{ m}^2}{2\text{ L} - 0\text{ L}}$$

$$= 9\text{ m}^2/\text{L}$$

Every litre of paint covers an area of 9 m^2.

b) Since 1 L of paint covers 9 m^2, then 6 L of paint will cover: 6(9 m^2) = 54 m^2
Six litres of paint cover an area of 54 m^2.

c) Since 9 m^2 is covered by 1 L, then 45 m^2 will be covered by: $\frac{45}{9}$ L = 5 L

Five litres of paint would cover 45 m^2.

8. Determine the rate of change and vertical intercept for each graph.

i) Two points on the graph are: (0, 20) and (10, 0)
The rate of change is:

$$\begin{aligned}\frac{\text{Change in } T}{\text{Change in } t} &= \frac{0°\text{C} - 20°\text{C}}{10\text{ h} - 0\text{ h}}\\ &= \frac{-20°\text{C}}{10\text{ h}}\\ &= -2°\text{C/h}\end{aligned}$$

On the vertical axis, the point of intersection has coordinates (0, 20).
The vertical intercept is 20.

ii) Two points on the graph are: (2, 0) and (4, 10)
The rate of change is:

$$\begin{aligned}\frac{\text{Change in } T}{\text{Change in } t} &= \frac{10°\text{C} - 0°\text{C}}{4\text{ h} - 2\text{ h}}\\ &= \frac{10°\text{C}}{2\text{ h}}\\ &= 5°\text{C/h}\end{aligned}$$

On the vertical axis, the point of intersection has coordinates (0, –10).
The vertical intercept is –10.

iii) Two points on the graph are: (0, 20) and (2, 0)
The rate of change is:

$$\begin{aligned}\frac{\text{Change in } T}{\text{Change in } t} &= \frac{0°\text{C} - 20°\text{C}}{2\text{ h} - 0\text{ h}}\\ &= \frac{-20°\text{C}}{2\text{ h}}\\ &= -10°\text{C/h}\end{aligned}$$

On the vertical axis, the point of intersection has coordinates (0, 20).
The vertical intercept is 20.

iv) Two points on the graph are: (0, –10) and (10, 0)
The rate of change is:

$$\begin{aligned}\frac{\text{Change in } T}{\text{Change in } t} &= \frac{0°\text{C} - (-10°\text{C})}{10\text{ h} - 0\text{ h}}\\ &= \frac{10°\text{C}}{10\text{ h}}\\ &= 1°\text{C/h}\end{aligned}$$

On the vertical axis, the point of intersection has coordinates (0, –10).
The vertical intercept is –10.

a) The graph in part ii has a rate of change of 5°C/h and a vertical intercept of –10°C.

b) The graph in part iii has a rate of change of –10°C/h and a vertical intercept of 20°C.

9. a) On the vertical axis, the point of intersection has coordinates (0, 0). The vertical intercept is 0.
On the horizontal axis, the point of intersection has coordinates (0, 0). The horizontal intercept is 0.
The point where the graph intersects the axes, (0, 0), represents the cost of running the backhoe for 0 h: $0

b) Choose two points on the graph: (0, 0) and (5, 400)
The rate of change is:

$$\begin{aligned}\frac{\text{Change in } C}{\text{Change in } t} &= \frac{\$400-\$0}{5\text{ h}-0\text{ h}}\\ &= \frac{\$400}{5\text{ h}}\\ &= \$80/\text{h}\end{aligned}$$

It costs $80 to run the backhoe for 1 h.

c) The domain is all possible *t*-values:
$0 \le t \le 10$
The range is all possible *C*-values:
$0 \le C \le 800$

d) From part a, it costs $80 to run the backhoe for 1 h.
So, the cost for 7 h is: 7($80) = $560
It costs $560 to run the backhoe for 7 h.

e) From part a, it costs $80 to run the backhoe for 1 h.
So, the time for a cost of $360 is:

$$\frac{360}{80}\text{ h} = 4.5\text{ h}$$

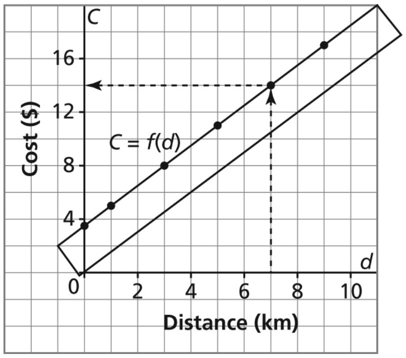
When the cost is $360, the backhoe is run for 4.5 h.

10. a) Choose two points on the graph: (3, 8) and (7, 14)
The rate of change is:

$$\frac{\text{Change in } C}{\text{Change in } d} = \frac{\$14 - \$8}{7\text{ km} - 3\text{ km}}$$
$$= \frac{\$6}{4\text{ km}}$$
$$= \$1.50/\text{km}$$

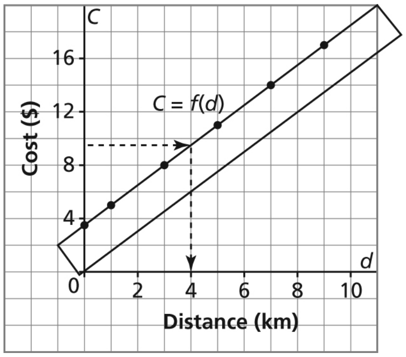
Every kilometre driven costs an additional $1.50.

b) To estimate the cost when the distance is 7 km, use the graph.
From 7 on the *d*-axis, draw a vertical line to the graph, then a horizontal line to the *C*-axis.



From the graph, when the distance is 7 km, the cost is approximately $14.

c) To estimate the distance when the cost is $9.50, use the graph.
From 9.5 on the *C*-axis, draw a horizontal line to the graph, then a vertical line to the *d*-axis.



From the graph, when the cost is $9.50, the distance is approximately 4 km.

11. Choose two points on the Smart car graph: (0, 35) and (500, 5)
The rate of change for the graph of the fuel consumption of a Smart car is:

$$\begin{aligned}\frac{\text{Change in } V}{\text{Change in } d} &= \frac{5\text{ L} - 35\text{ L}}{500\text{ km} - 0\text{ km}}\\ &= \frac{-30\text{ L}}{500\text{ km}}\\ &= -0.06\text{ L/km}\end{aligned}$$

The Smart car uses 0.06 L of fuel to drive 1 km.

Choose two points on the SUV graph: (0, 80) and (550, 10)
The rate of change for the graph of the fuel consumption of an SUV is:

$$\begin{aligned}\frac{\text{Change in } V}{\text{Change in } d} &= \frac{10\text{ L} - 80\text{ L}}{550\text{ km} - 0\text{ km}}\\ &= \frac{-70\text{ L}}{550\text{ km}}\\ &= -0.1\overline{27}\text{ L/km}\end{aligned}$$

An SUV uses $0.1\overline{27}$ L of fuel to drive 1 km.

The SUV uses more than twice as much fuel to drive 1 km. So, a driver of an SUV will pay more than twice as much for gas as a driver of a Smart car.

12. a) The dogsled finished the race when its distance from the finish line was 0 km.
From the graph, the dogsled finished the race after 2.5 h.

b) The average speed is the rate of change of distance over time.
Choose two points: (0, 60) and (2.5, 0)
The rate of change is:

$$\begin{aligned}\frac{\text{Change in } d}{\text{Change in } t} &= \frac{0\text{ km} - 60\text{ km}}{2.5\text{ h} - 0\text{ h}}\\ &= \frac{-60\text{ km}}{2.5\text{ h}}\\ &= -24\text{ km/h}\end{aligned}$$

The rate of change is negative, which means that the distance to the finish line is decreasing with time.
The average speed of the dogsled was 24 km/h.

c) The length of the race is the distance of the dogsled from the finish line at 0 h.
From the graph, the race was 60 km long.

d) When the dogsled had completed $\frac{2}{3}$ of the race, it had travelled: $\frac{2}{3}(60\text{ km}) = 40\text{ km}$
So, its distance from the finish line was:
60 km – 40 km = 20 km
From part b, the dogsled travels 24 km/h.

So, to travel 40 km, it takes: $\frac{40}{24}$ h $=1.\overline{6}$ h, or $1\frac{2}{3}$ h

The dogsled completed $\frac{2}{3}$ of the race in $1\frac{2}{3}$ h, or 1 h 40 min.

13. a) From Graph A, it takes 50 min to fill the empty tank to 100 m^3.
From Graph B, it takes 25 min to empty the tank that starts with 100 m^3 of fuel.
So, it takes longer to fill the empty tank.

b) When the tank in Graph B was half empty, it contained 50 m^3 of fuel and 12.5 min had elapsed.
From Graph A, after 12.5 min, the tank that was being filled would contain 25 m^3 of fuel.

14. a) The points on the graph are joined because the scale on the axes is so small that it would be impossible to distinguish every point on the graph.

b) i) The vertical intercept of the graph is 200, so the fixed cost is \$200.
Choose two points on the graph: (0, 200) and (40, 800)
The rate of change is:

$$\frac{\text{Change in } C}{\text{Change in } n} = \frac{\$800 - \$200}{40 \text{ sweatshirts} - 0 \text{ sweatshirts}}$$
$$= \$15/\text{sweatshirt}$$

An equation that represents this situation is: $C = 200 + 15n$
Substitute: $C = 700$

$$700 = 200 + 15n \qquad \text{Solve for } n.$$
$$500 = 15n$$
$$\frac{500}{15} = n$$
$$n = 33.\overline{3}$$

So, approximately 33 sweatshirts can be bought for \$700.

ii) From part a, the rate of change is \$15/sweatshirt.
So, one more sweatshirt would increase the cost by \$15.

15. Since each function is linear, its graph is a straight line.

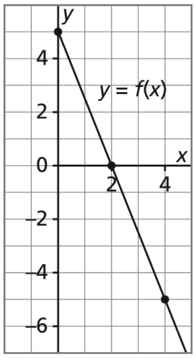
a) $f(x) = 5 - 2.5x$

Determine the y-intercept:
When $x = 0$,
$f(0) = 5 - 2.5(0)$
$f(0) = 5$

Determine the x-intercept:
When $f(x) = 0$,
$0 = 5 - 2.5x$
$2.5x = 5$
$x = \frac{5}{2.5}$
$x = 2$

Determine the coordinates of a third point on the graph.
When $x = 4$,
$f(x) = 5 - 2.5x$
$f(4) = 5 - 2.5(4)$
$f(4) = -5$

Plot the points (0, 5), (2, 0), and (4, –5), then draw a line through them.



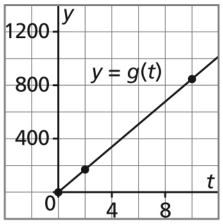
b) $g(t) = 85t$

Determine the vertical intercept:
When $t = 0$,
$g(0) = 85(0)$
$g(0) = 0$

Determine the coordinates of a second point on the graph:
When $t = 2$,
$g(2) = 85(2)$
$g(2) = 190$

Determine the coordinates of a third point on the graph.
When $t = 10$,
$g(10) = 85(10)$
$g(10) = 850$

Plot the points (0, 0), (2, 190), and (10, 850), then draw a line through them.



c) $h(n) = 750 + 55n$

Determine the vertical intercept:
When $n = 0$,
$h(0) = 750 + 55(0)$
$h(0) = 750$

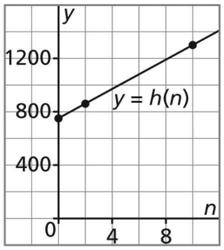
Determine the horizontal intercept:
When $h(n) = 0$,
$0 = 750 + 55n$
$-750 = 55n$
$-\frac{750}{55} = n$
$n = -\frac{150}{11}$, or $-13.\overline{63}$

Since the horizontal intercept is outside the domain, determine the coordinates of two other points on the graph.

When $n = 2$,
$h(2) = 750 + 55(2)$
$h(2) = 860$

When $n = 10$,
$h(10) = 750 + 55(10)$
$h(10) = 1300$

Plot the points (0, 750), (2, 860), and (10, 1300), then draw a line through them.



d) $V(d) = 55 - 0.08d$

Determine the vertical intercept:
When $d = 0$,
$V(0) = 55 - 0.08(0)$
$V(0) = 55$

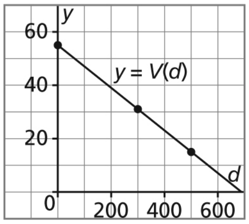
Determine the *d*-intercept:
When $V(d) = 0$,
$0 = 55 - 0.08d$
$0.08d = 55$
$d = \frac{55}{0.08}$
$d = 687.5$

The horizontal intercept is not easy to graph, so determine the coordinates of two other points on the graph.

When $d = 300$,
$V(300) = 55 - 0.08(300)$
$V(300) = 31$

When $d = 500$,
$V(500) = 55 - 0.08(500)$
$V(500) = 15$

Plot the points (0, 55), (300, 31) and (500, 15), then draw a line through them.



16. a) The profit on each bar sold is the rate of change of the graph.
Choose two points on the graph: (0, –40) and (300, 200)

$$\frac{\text{Change in } P}{\text{Change in } n} = \frac{\$200 - (-\$40)}{300 \text{ bars} - 0 \text{ bars}}$$
$$= \frac{\$240}{300 \text{ bars}}$$
$$= \$0.80/\text{bar}$$

So, the profit is $0.80 per bar sold.

b) On the vertical axis, the point of intersection has coordinates (0, –40). The vertical intercept is –40. This point of intersection represents the loss when 0 bars are sold: $40; that is, the cost price of the power bars.

On the horizontal axis, the point of intersection has coordinates (50, 0). The horizontal intercept is 50. This point of intersection represents the number of bars that must be sold to reach the break-even point, when no profit is made and there is no loss: 50 bars

c) The domain of the function is:
All whole numbers up to 300
The range of the function is:
All multiples of 0.80 between and including –40 to 200.
I wouldn't want to list all the values in the range because there are 301 of them.

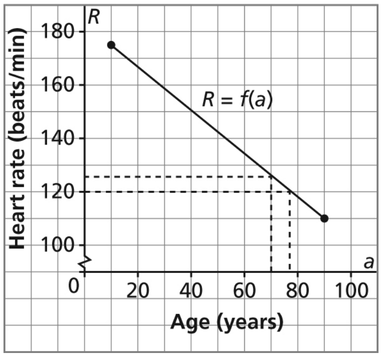
17. a) There are no intercepts on the graph because it is unlikely that people less than 10 years old or more than 90 years old will be doing a stress test.

b) Choose two points on the graph: (10, 175) and (90, 110)
The rate of change is:

$$\begin{aligned}\frac{\text{Change in } R}{\text{Change in } a} &= \frac{110 \text{ beats/min} - 175 \text{ beats/min}}{90 \text{ years} - 10 \text{ years}} \\ &= \frac{-65 \text{ beats/min}}{80 \text{ years}} \\ &= -0.8125(\text{beats/min})/\text{year}\end{aligned}$$

The rate of change is negative, so the recommended heart rate decreases with age.
For every additional year of age, the recommended heart rate decreases by approximately 1 beat/min.

c) Use the graph. From 120 on the vertical axis, draw a horizontal line to intersect the graph, then a vertical line to intersect the horizontal axis at approximately 77.



The recommended maximum heart rate is 120 beats/min for someone aged 77.

d) Use the graph. From 70 on the horizontal axis, draw a vertical line to intersect the graph, then a horizontal line to intersect the vertical axis at approximately 125.
A person aged 70 has a recommended heart rate of approximately 125 beats/min.

C

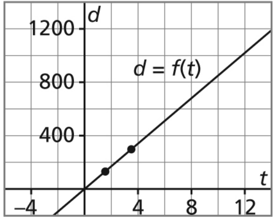
18. a) **i)** The line intersects the x-axis at (5, 0), so the x-intercept is 5.
The line intersects the y-axis at (0, 5), so the y-intercept is 5.

ii) The coordinates of each point on the line have the sum of 5.
Since the intercepts are the same, the relation is described by this equation:
$x + y = 5$

b) **i)** The line intersects the x-axis at (5, 0), so the x-intercept is 5.
The line intersects the y-axis at (0, –5), so the y-intercept is –5.

ii) The coordinates of each point on the line have a difference of 5, with $x > y$.
Since the intercepts are opposites, the relation is described by this equation:
$x - y = 5$

19. a) Plot the points (1.5, 127.5) and (3.5, 297.5).
Draw a line through the points.



b) The line passes through these points: (1.5, 127.5) and (3.5, 297.5)
The rate of change is:

$$\frac{\text{Change in } d}{\text{Change in } t} = \frac{297.5 - 127.5}{3.5 - 1.5}$$
$$= 85$$

For each increase of $t = 1$, d increases by 85.

For each increase of $t = 1.5$, d increases by $85\left(\frac{3}{2}\right) = 127.5$

$f(3.5) = 297.5$
So, $f(5) = 297.5 + 127.5$
$= 425$

c) Let the coordinates of the point for which $f(t) = 212.5$ be (t, 212.5).
Use this point and the point (1.5, 127.5) to write an expression for the rate of change.
Equate this expression to 85.

$$\frac{212.5-127.5}{t-1.5}=85$$
$$\frac{85}{t-1.5}=85$$
$$\frac{85}{85}=t-1.5$$
$$1=t-1.5$$
$$t=2.5$$

Use the equation: $f(t) = 85t$
When $f(t) = 212.5$,

$$f(t)=85t$$
$$212.5=85t$$
$$\frac{212.5}{85}=t$$
$$t=2.5$$

d) For each decrease in t = 1.5, *d* decreases by 127.5. Since $f(1.5)=127.5$, then $f(0)=0$. The graph passes through the origin. So, a possible context is a car's distance from home as it travels away at a constant speed of 85 km/h.

20. a) The vertical intercept indicates the person's distance from Duke Point when starting the journey at Parksville.
The horizontal intercept indicates the person's distance from Parksville after completing the journey and reaching Duke Point.

The distance between the two locations doesn't change, so the intercepts have the same value.

b) Use the points: (0, 50) and (50, 0)
The rate of change is:

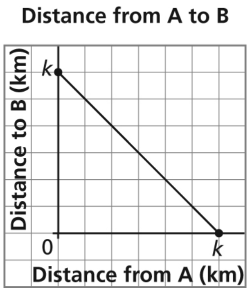
$$\frac{\text{Change in } d}{\text{Change in } p} = \frac{0\text{ km} - 50\text{ km}}{50\text{ km} - 0\text{ km}}$$
$$= -1$$

The rate of change has no units because it is the quotient of two measurements in kilometres.

For every 1 km the ferry moves away from Parksville, it moves 1 km closer to Duke Point.

c) Interchanging the dependent and independent variables would interchange the labels on the axes, but the line on the graph would stay the same.

d) The graph will go down to the right.
Both the horizontal and vertical intercepts will be k.
Both the domain and range will be all real numbers from 0 to k.
The rate of change will be –1.



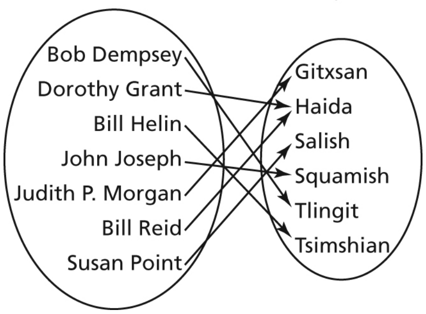
Review (pages 326–328)

5.1

1. a) The table shows a relation with the association "has this cultural heritage" from a set of artists to a set of First Nations heritages.

b) i) The relation as a set of ordered pairs:
{(Bob Dempsey, Tlingit), (Dorothy Grant, Haida), (Bill Helin, Tsimshian), (John Joseph, Squamish), (Judith P. Morgan, Gitxsan), (Bill Reid, Haida), (Susan Point, Salish)}

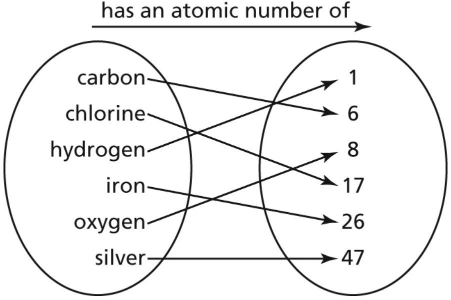
ii) The relation as an arrow diagram:



2. a) The relation in a table:

Element	Atomic Number
carbon	6
chlorine	17
hydrogen	1
iron	26
oxygen	8
silver	47

The relation as an arrow diagram:

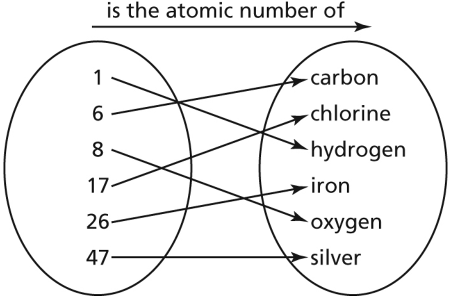


The relation as a set of ordered pairs:
{(carbon, 6), (chlorine, 17), (hydrogen, 1), (iron, 26), (oxygen, 8), (silver, 47)}

b) The relation in a table:

Atomic Number	Element
1	hydrogen
6	carbon
8	oxygen
17	chlorine
26	iron
47	silver

The relation as an arrow diagram:



The relation as a set of ordered pairs:
{(1, hydrogen), (6, carbon), (8, oxygen), (17, chlorine), (26, iron), (47, silver)}

5.2

3. a) {(4, 3), (4, 2), (4, 1), (4, 0)}
This relation is not a function because the number 4 is a first element in 4 different ordered pairs.

b) {(2, 4), (–2, 4), (3, 9), (–3, 9)}
This relation is a function because the ordered pairs have different first elements.

c) {(2, 8), (3, 12), (4, 16), (5, 20)}
This relation is a function because the ordered pairs have different first elements.

d) {(5, 5), (5, –5), (–5, 5), (–5, –5)}
This relation is not a function because each of the numbers 5 and –5 appears as the first element in 2 different ordered pairs.

4. a) $f(x) = -4x + 9$

b) $C(n) = 12n + 75$

c) $D(t) = -20t + 150$

d) $P(s) = 4s$

5. a) As an equation in 2 variables, $P(n) = 5n - 300$ is:
$P = 5n - 300$

b) The profit depends on the number of students who attend. So, n is the independent variable and P is the dependent variable.

c) To determine $P(150)$, use:

$$\begin{aligned} P(n) &= 5n - 300 \qquad \text{Substitute: } n = 150 \\ P(150) &= 5(150) - 300 \\ P(150) &= 450 \end{aligned}$$

This means that if 150 students attend the dance, the profit will be $450.

d) To determine the value of n when $P(n) = 700$, use:

$$\begin{aligned} P(n) &= 5n - 300 \qquad \text{Substitute: } P(n) = 700 \\ 700 &= 5n - 300 \\ 1000 &= 5n \\ n &= 200 \end{aligned}$$

This means that the profit is $700 when 200 students attend the dance.

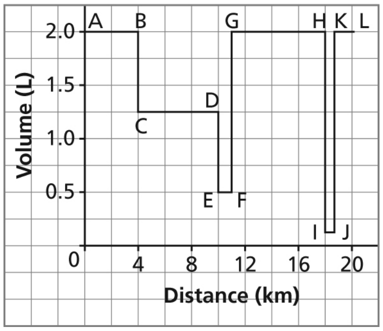
5.3

6. a) The distance to Laura’s house from school is a positive number, so the vertical intercept of the graph should be a positive number. When Laura cycles home, the graph should show a steep decrease because her distance from home is decreasing. When Laura reaches home, her distance from home is 0. When she walks back to school, the graph should show a moderate increase because her walking speed is slower than her cycling speed and her distance from home is increasing. So, Graph A best matches the situation.

b) Graph D could represent Laura’s journey to school to pick up her bike. She walks to school, then picks up her bicycle and rides home.

7. a) Label the segments of the graph.

Water in Liam’s Flask



Segment	Graph	Journey
AB	The graph is horizontal, so as distance increases the volume of water in the flask stays the same.	Liam walks 4 km with 2.0 L of water in his flask.
BC	The graph is vertical below point B, so the volume of water in the flask decreases.	Liam drinks or spills 0.75 L of water.
CD	The graph is horizontal, so as distance increases the volume of water in the flask stays the same.	Liam walks 6 km with 1.25 L of water in his flask.
DE	The graph is vertical below point D, so the volume of water in the flask decreases.	Liam drinks or spills 0.75 L of water from his flask.
EF	The graph is horizontal, so as distance increases the volume of water in the flask stays the same.	Liam walks 1 km with 0.5 L of water in his flask.
FG	The graph is vertical above point F, so the volume of water in the flask increases.	Liam fills his flask so that it contains 2.0 L of water.
GH	The graph is horizontal, so as distance increases the volume of water in the flask stays the same.	Liam walks 7 km with 2.0 L of water in his flask.
HI	The graph is vertical below point H, so the volume of water in the flask decreases.	Liam drinks or spills approximately 1.875 L of water from his flask.
IJ	The graph is horizontal, so as distance increases the volume of water in the flask stays the same.	Liam walks approximately 0.7 km with 0.125 L of water in his flask.
JK	The graph is vertical above point J, so the volume of water in the flask increases.	Liam fills his flask so that it contains 2.0 L of water.
KL	The graph is horizontal, so as distance increases the volume of water in the flask stays the same.	Liam walks approximately 1.3 km with 2.0 L of water remaining in his flask.

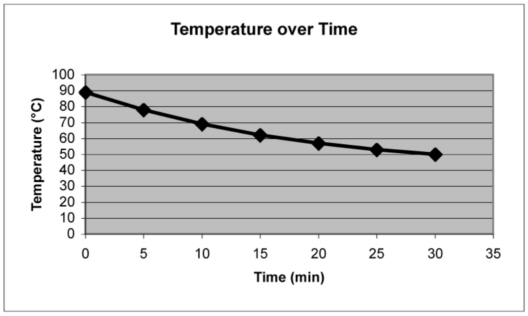
b) Liam filled his flask two times as shown on the graph by line segments FG and JK.

c) The vertical intercept of the graph is 2.0, so Liam started the hike with 2.0 L of water in his flask.

d) The independent variable is the distance Liam hikes. The dependent variable is the volume of water in his flask.

5.4

8. a) Graph the data using a computer spreadsheet program. Label the axes.



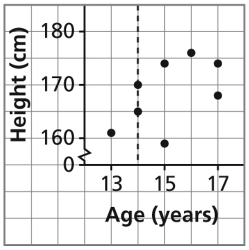
I joined the points because all times between 0 min and 30 min are permissible and all temperatures between 50°C and 89°C are permissible.

b) The graph represents a function because a vertical line drawn on the graph passes through at most one point.

5.5

9. a) This graph is not a function. A vertical line can be drawn through the points (14, 165) and (14, 170) on the graph.

Heights and Ages of 8 Students



The domain is the set of ages: {13, 14, 15, 16, 17}
The domain is the set of heights: approximately {159, 161, 165, 168, 170, 174, 176}

b) This graph is a function. Any vertical line drawn on the graph would pass through 0 points or 1 point.

The domain is the set of times:
{08:00, 10:00, 12:00, 14:00, 16:00, 18:00}
The range is the set of the numbers of bicycles:
{2, 5, 10, 20, 25}

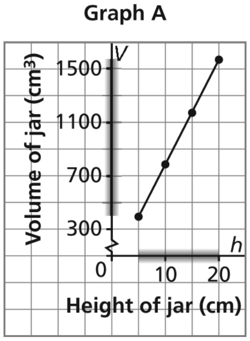
10. a) i) Graph A represents the volume of a jar, V cubic centimetres, as a function of its height, h centimetres.

ii) Graph B represents the number of marbles in a jar, n, as a function of the jar's height, h centimetres.

b) i) For Graph A, the independent variable is the height of the jar, h. The dependent variable is the volume of the jar, V.

ii) For Graph B, the independent variable is the height of the jar, h. The dependent variable is the number of marbles in the jar, n.

c) i) Visualize the shadow of the graph on the axes.



The domain is the set of heights:
$5 \le h \le 20$
The range is the set of volumes:
approximately $400 \le V \le 1575$

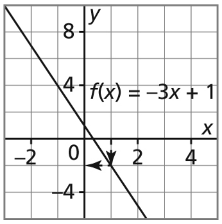
ii) For Graph B: The domain is the set of heights.
{5, 10, 15, 20}
The range is the set of the numbers of marbles:
{14, 28, 42, 56}

d) The points are joined in Graph A because it is possible for a jar to have any height between 5 cm and 20 cm and any volume between 400 cm^3 and 1575 cm^3.
The points are not joined in Graph B because only whole numbers of marbles are permissible.

11. The domain value is a value of x. The range value is a value of $f(x)$.

a) To determine the value of $f(x)$ when $x = 1$:

Begin at $x = 1$ on the x-axis.
Draw a vertical line to the graph, then a horizontal line to the y-axis.

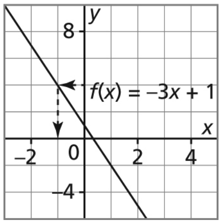


The line intersects the y-axis at –2.
So, $f(1) = –2$

When the domain value is 1, the range value is –2.

b) To determine the value of x when $f(x) = 4$:

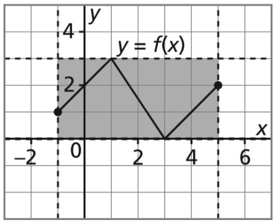
Begin at $f(x) = 4$ on the y-axis.
Draw a horizontal line to the graph, then a vertical line to the x-axis.



The line intersects the x-axis at –1.
So, $f(–1) = 4$

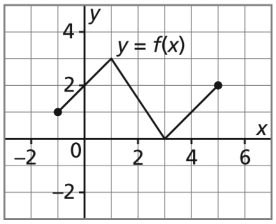
When the range value is 4, the domain value is –1.

12. a) On a grid, draw faint vertical lines through $x = –1$ and $x = 5$; and faint horizontal lines through $y = 0$ and $y = 3$.

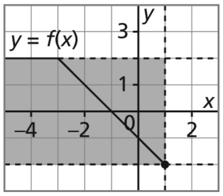


Draw line segments inside the region enclosed by the faint lines so that there is exactly one point on the graph for every x-value between –1 and 5, and there is at least one point on the graph for every y-value between 0 and 3.

A function is:

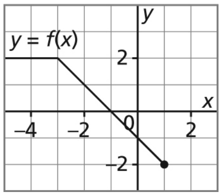


b) On a grid, draw a faint vertical line through $x = 1$; and faint horizontal lines through $y = –2$ and $y = 2$.



Draw line segments inside the region between the horizontal lines and to the left of the vertical line so that there is exactly one point on the graph for every x-value less than or equal to 1, and there is at least one point on the graph for every y-value between –2 and 2.

A function is:



5.6

13. a) {(1, 5), (5, 5), (9, 5), (13, 5)}
Since every second element in the ordered pairs is 5, the points lie on a horizontal line.
So, the set of ordered pairs represents a linear relation.

b) {(1, 2), (1, 4), (1, 6), (1, 8)}
Since every first element in the ordered pairs is 1, the points lie on a vertical line.
So, the set of ordered pairs represents a linear relation.

c) The first elements are in numerical order.
So, calculate the change in each variable.

+1 +3 +2

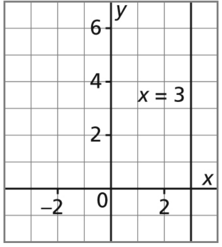
{(–2, –3), (–1, –2), (2, 1), (4, –3)}

+1 +3 –4

There is no constant change in the first elements or the second elements. For the last two pairs, as the first element increases by 2, the second element decreases by 4. So, the set of ordered pairs does not represent a linear relation.

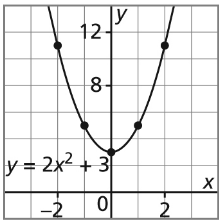
14. a) i) $x = 3$

This is a vertical line that passes through (3, 0).



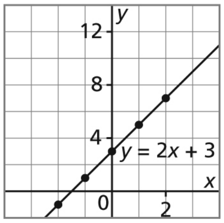
ii) $y = 2x^2 + 3$

x	y
–2	11
–1	5
0	3
1	5
2	11



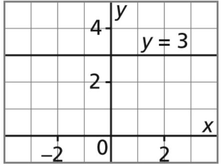
iii) $y = 2x + 3$

x	y
–2	–1
–1	1
0	3
1	5
2	7



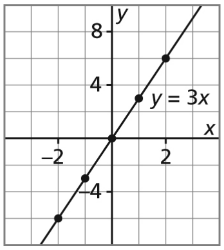
iv) $y = 3$

This is a horizontal line that passes through (0, 3).



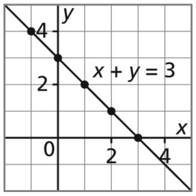
v) $y = 3x$

x	y
–2	–6
–1	–3
0	0
1	3
2	6



vi) $x + y = 3$

x	y
–1	4
0	3
1	2
2	1
3	0



b) The equations in parts i, iii, iv, v, and vi represent linear relations because their graphs are straight lines.

15. a) The equation represents a linear relation because, when g changes by 1, N changes by $\frac{1}{15}$.

b) The rate of change is $\frac{1}{15}$. For every 1 g of carbohydrates that Isabelle consumes, she gives herself $\frac{1}{15}$ of a unit of insulin.

5.7

16. a) Distance is shown on the vertical axis of the graph.
Jadan travelled 6000 m from Whitehorse to the Grey Mountain Road lookout, so at the start of her bicycle trip she was 6000 m, or 6 km, from the lookout.

b) The domain is all possible values of the number of revolutions: $0 \le n \le 2800$
The range is all possible values of the distance: $0 \le d \le 6000$

c) Choose two points on the graph: (0, 0) and (2800, 6000)
The rate of change is:

$$\frac{\text{Change in } d}{\text{Change in } n} = \frac{6000\text{ m} - 0\text{ m}}{2800\text{ revolutions} - 0\text{ revolutions}}$$
$$= 2.1428\ldots\text{ m/revolution}$$

In one revolution of the wheel, the bicycle covers a distance of approximately 2 m.

d) The distance the bicycle travels in one revolution of its wheel is the circumference of the wheel.
An equation that relates the circumference, C, of a circle to its diameter, d, is:
$C = \pi d$

$$C = \pi d \qquad \text{Substitute: } C \doteq 2.1428$$
$$2.1428 \doteq \pi d$$
$$\frac{2.1428}{\pi} \doteq d$$
$$d \doteq 0.6820$$

So, the bicycle wheel has a diameter of approximately 0.68 m, or 68 cm.

17. Determine the vertical intercept and the rate of change for each graph.

a) When $t = 0$, $T = 3$, so the vertical intercept is 3°C.
Choose 2 points on the graph: (0, 3) and (1, 0)
Calculate the rate of change:

$$\begin{aligned}\frac{\text{Change in } T}{\text{Change in } t} &= \frac{0°\text{C} - 3°\text{C}}{1\text{ h} - 0\text{ h}}\\ &= -3°\text{C/h}\end{aligned}$$

So, this graph matches part ii.

b) When $t = 0$, $T = -3$, so the vertical intercept is –3°C.
Choose 2 points on the graph: (0, –3) and (1, 0)
Calculate the rate of change:

$$\begin{aligned}\frac{\text{Change in } T}{\text{Change in } t} &= \frac{0°\text{C} - (-3°\text{C})}{1\text{ h} - 0\text{ h}}\\ &= 3°\text{C/h}\end{aligned}$$

So, this graph matches part iii.

c) When $t = 0$, $T = -3$, so the vertical intercept is –3°C.

Choose 2 points on the graph: (0, –3) and (9, 0)
Calculate the rate of change:

$$\begin{aligned}\frac{\text{Change in } T}{\text{Change in } t} &= \frac{0°\text{C} - (-3°\text{C})}{9\text{ h} - 0\text{ h}}\\ &= \frac{3°\text{C}}{9\text{ h}}\\ &= \frac{1}{3}°\text{C/h}\end{aligned}$$

So, this graph matches part i.

18. a) On the graph, the company makes a profit when the profit, P, is a positive number. When $P = 0$, $n = 200$; this is the break-even point, when the company does not suffer a loss or make a profit.
The company begins to make a profit when 201 baseball caps have been sold.

b) The profit on the sale of each baseball cap is the rate of change.
Choose two points on the graph: (200, 0) and (300, 400)

$$\begin{aligned}\frac{\text{Change in } P}{\text{Change in } n} &= \frac{\$400 - \$0}{300 \text{ caps} - 200 \text{ caps}}\\ &= \frac{\$400}{100 \text{ caps}}\\ &= \$4/\text{cap}\end{aligned}$$

So, the profit on each baseball cap is \$4.

c) The vertical intercept is –800.
The rate of change is 4.
So, an equation to represent the situation is: $P = 4n - 800$

i) $P = 4n - 800$ Substitute: $P = 600$

$$\begin{aligned}600 &= 4n - 800\\ 1400 &= 4n\\ n &= 350\end{aligned}$$

350 caps must be sold to make a profit of \$600.

ii) $P = 4n - 800$ Substitute: $P = 1200$

$$\begin{aligned}1200 &= 4n - 800\\ 2000 &= 4n\\ n &= 500\end{aligned}$$

500 caps must be sold to make a profit of \$1200.

d) The profit doesn't double because it depends on the sale of caps and the initial cost of \$800 to buy or make the caps. So, doubling the number of caps does not double the profit.

Practice Test (page 329)

1. To determine $f(-3)$, use:

$$f(x) = 3 - 6x \qquad \text{Substitute: } x = -3$$

$$f(-3) = 3 - 6(-3)$$

$$f(-3) = 21$$

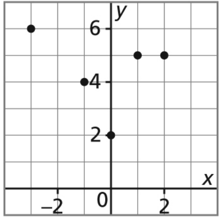
So, answer B is correct.

2. Equations A and D represent horizontal lines.
Equation B represents a line with a rate of change of 5 that passes through the origin.
Equation C has x raised to the exponent 2, so it is not a linear function.
So, answer C is correct.

3. a) i) {(2, 5), (–3, 6), (1, 5), (–1, 4), (0, 2)}
The relation is a function because each ordered pair has a different first element.

ii) The domain is the set of first elements:
{–3, –1, 0, 1, 2}
The range is the set of second elements:
{2, 4, 5, 6}

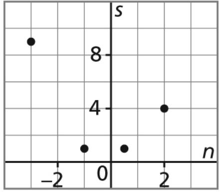
Represent the function as a graph.
Plot the points:



The function is not linear because the points on the graph do not lie on a straight line.

b) i) The relation is a function because no number appears more than once in the first column of the table of values.

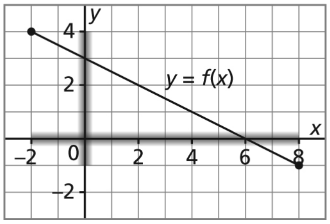
ii) The domain is the set of numbers in the first column of the table:
{–3, –1, 1, 2, …}
The range is the set of numbers in the second column of the table:
{1, 4, 9, …}
Represent the function as a graph.
Plot the points:



The function is not linear because the points do not lie on a straight line.

c) **i)** The relation is a function because any vertical line passes through at most 1 point on the graph.

ii) Visualize the shadow of the graph on the x- and y-axes.



The domain is the set of x-values of the graph:
$-2 \le x \le 8$
The range is the set of y-values of the graph:
$-1 \le y \le 4$

Represent the relation as a table of values.

x	–2	0	2	4	6	8
y	4	3	2	1	0	–1

Choose two points on the graph: (0, 3) and (–1, 8)

The rate of change is $-\frac{1}{2}$.

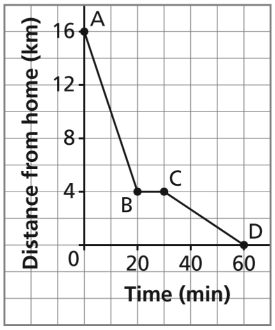
The function is linear because the graph is a non-vertical line.

iii) The independent variable is x. The dependent variable is y.

4. Situation: Jamie’s school is 16 km from her house. Jamie rides her friend’s bike from school to her friend’s house, which is 4 km from her own house. She arrives at her friend’s house 20 min after she left school. She talks to her friend for 10 min, then jogs the remaining 4 km home in 30 min.

Justification: Line segment AB goes down to the right, so the person’s distance from home is decreasing with time.
Segment BC is horizontal, so the person’s distance from home remains constant.
Segment CD goes down to the right, so the person’s distance from home decreases with time.

Jamie’s Journey Home

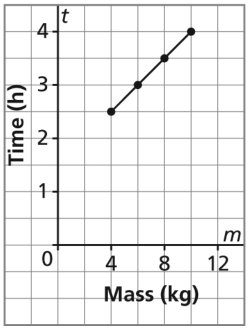


5. a) The relation is a function because no number is repeated in the first row.

b) The time, t, needed to cook the turkey depends on its mass, m. So, time is the dependent variable and mass is the independent variable.

c) Graph the data.
Connect the points because it is possible for a turkey to have any mass between 4 kg and 10 kg, in which case, its cooking time is likely to be between 2.5 h and 4.0 h.

Time Needed to Cook a Turkey



d) The domain is all possible values of the mass: $4 \le m \le 10$
The range is all possible values of the cooking time: $2.5 \le t \le 4.0$
The graph could be extended for turkeys with greater or lesser mass.
The domain and range are restricted to positive real numbers because it is impossible for a turkey to have a negative mass or to cook a turkey for a negative length of time.

e) Choose 2 points on the graph: (4, 2.5) and (6, 3.0)
The rate of change is:

$$\begin{aligned}\frac{\text{Change in } t}{\text{Change in } m} &= \frac{3.0\text{ h} - 2.5\text{ h}}{6\text{ kg} - 4\text{ kg}}\\ &= \frac{0.5\text{ h}}{2\text{ kg}}\\ &= 0.25\text{ h/kg}\end{aligned}$$

For every additional 1 kg, the time needed to cook the turkey increases by 0.25 h.

f) Use the rate of change:
7 kg is 1 kg more than 6 kg, and a turkey with mass 6 kg should be cooked for 3.0 h.

So, a turkey with mass 7 kg should be cooked for:
3.0 h + 0.25 h = 3.25 h