Lesson 4.1 Math Lab: Estimating Roots — Assess Your Understanding (page 206)

1. **a)** Answers may vary; for example, $\sqrt{34}$, $\sqrt[3]{21}$, $\sqrt[4]{45}$, $\sqrt[5]{12}$

b) For $\sqrt{34}$, the radicand is 34 and the index is 2.
For $\sqrt[3]{21}$, the radicand is 21 and the index is 3.
For $\sqrt[4]{45}$, the radicand is 45 and the index is 4.
For $\sqrt[5]{12}$, the radicand is 12 and the index is 5.

c) When the index is 2, I take the square root of the number.
When the index is 3, I take the cube root of the number.
When the index is 4, I take the fourth root of the number.
When the index is 5, I take the fifth root of the number.

2. **a)** $$\sqrt{36} = \sqrt{6 \cdot 6} \\ = 6$$

b) $$\sqrt[3]{8} = \sqrt[3]{2 \cdot 2 \cdot 2} \\ = 2$$

c) $$\sqrt[4]{10\,000} = \sqrt[4]{10 \cdot 10 \cdot 10 \cdot 10} \\ = 10$$

d) $$\sqrt[5]{-32} = \sqrt[5]{(-2) \cdot (-2) \cdot (-2) \cdot (-2) \cdot (-2)} \\ = -2$$

e) $$\sqrt[3]{\frac{27}{125}} = \sqrt[3]{\frac{3}{5} \cdot \frac{3}{5} \cdot \frac{3}{5}} \\ = \frac{3}{5}$$

f) $$\sqrt{2.25} = \sqrt{1.5 \cdot 1.5} \\ = 1.5$$

g) $$\sqrt[3]{0.125} = \sqrt[3]{0.5 \cdot 0.5 \cdot 0.5} \\ = 0.5$$

h) $$\sqrt[4]{625} = \sqrt[4]{5 \cdot 5 \cdot 5 \cdot 5} \\ = 5$$

3. Use benchmarks with guess and check.

a) 8 is between the perfect squares 4 and 9, but closer to 9.
So, $\sqrt{8}$ is between 2 and 3, but closer to 3.
Estimate to 1 decimal place: $\sqrt{8} \doteq 2.9$
Square the estimate: $2.9^2 = 8.41$ (too large, but close)
Revise the estimate: $\sqrt{8} \doteq 2.8$
Square the estimate: $2.8^2 = 7.84$ (very close)
7.84 is closer to 8, so $\sqrt{8}$ is approximately 2.8.

b) 9 is between the perfect cubes 8 and 27, but closer to 8.
So, $\sqrt[3]{9}$ is between 2 and 3, but closer to 2.
Estimate to 1 decimal place: $\sqrt[3]{9} \doteq 2.2$
Cube the estimate: $2.2^3 = 10.648$ (too large)
Revise the estimate: $\sqrt[3]{9} \doteq 2.1$
Cube the estimate: $2.1^3 = 9.261$ (very close)
9.261 is closer to 9, so $\sqrt[3]{9}$ is approximately 2.1.

c) 10 is between the perfect fourth powers 1 and 16, but closer to 16.
So, $\sqrt[4]{10}$ is between 1 and 2, but closer to 2.
Estimate to 1 decimal place: $\sqrt[4]{10} \doteq 1.7$
Raise the estimate to the fourth power: $1.7^4 = 8.3521$ (too small)
Revise the estimate: $\sqrt[4]{10} \doteq 1.8$
Raise the estimate to the fourth power: $1.8^4 = 10.4976$ (very close)
10.4976 is closer to 10, so $\sqrt[4]{10}$ is approximately 1.8.

d) 13 is between the perfect squares 9 and 16, but closer to 16.
So, $\sqrt{13}$ is between 3 and 4, but closer to 4.
Estimate to 1 decimal place: $\sqrt{13} \doteq 3.6$
Square the estimate: $3.6^2 = 12.96$ (very close)
So, $\sqrt{13}$ is approximately 3.6.

e) 15 is between the perfect cubes 8 and 27, but closer to 8.
So, $\sqrt[3]{15}$ is between 2 and 3, but closer to 2.
Estimate to 1 decimal place: $\sqrt[3]{15} \doteq 2.4$
Cube the estimate: $2.4^3 = 13.824$ (too small)
Revise the estimate: $\sqrt[3]{15} \doteq 2.5$
Cube the estimate: $2.5^3 = 15.625$ (close)
15.625 is closer to 15, so $\sqrt[3]{15}$ is approximately 2.5.

f) 17 is between the perfect fourth powers 16 and 81, but closer to 16.
So, $\sqrt[4]{17}$ is between 2 and 3, but closer to 2.
Estimate to 1 decimal place: $\sqrt[4]{17} \doteq 2.1$
Raise the estimate to the fourth power: $2.1^4 = 19.4481$ (too large)
Revise the estimate: $\sqrt[4]{17} \doteq 2.0$
Raise the estimate to the fourth power: $2.0^4 = 16.0$ (close)
16 is closer to 17, so $\sqrt[4]{17}$ is approximately 2.0.

g) 19 is between the perfect squares 16 and 25, but closer to 16.
So, $\sqrt{19}$ is between 4 and 5, but closer to 4.
Estimate to 1 decimal place: $\sqrt{19} \doteq 4.3$
Square the estimate: $4.3^2 = 18.49$ (too small, but close)
Revise the estimate: $\sqrt{19} \doteq 4.4$
Square the estimate: $4.4^2 = 19.36$ (very close)
19.36 is closer to 19, so $\sqrt{19}$ is approximately 4.4.

h) 20 is between the perfect cubes 8 and 27, but closer to 27.
So, $\sqrt[3]{20}$ is between 2 and 3, but closer to 3.
Estimate to 1 decimal place: $\sqrt[3]{20} \doteq 2.8$
Cube the estimate: $2.8^3 = 21.952$ (too large)
Revise the estimate: $\sqrt[3]{20} \doteq 2.7$
Cube the estimate: $2.7^3 = 19.683$ (close)
19.683 is closer to 20, so $\sqrt[3]{20}$ is approximately 2.7.

4. a) When I try to determine the square root of –4 using a calculator, I get an error message. This makes sense because I cannot write a negative number as the product of two equal factors.

b) I get the same result with a negative radicand when the index is an even number.

c) **i)** When a radicand is negative, I can evaluate or estimate the value of the radical when the index is an odd number. This is because the product of an odd number of negative factors is negative.

ii) When a radicand is negative, I cannot evaluate or estimate the value of the radical when the index is an even number. This is because the product of an even number of negative factors is positive.

5. a) **i)** $2^2 = 4$, so $2 = \sqrt{4}$
ii) $2^3 = 8$, so $2 = \sqrt[3]{8}$
iii) $2^4 = 16$, so $2 = \sqrt[4]{16}$

b) **i)** $3^2 = 9$, so $3 = \sqrt{9}$

ii) $3^3 = 27$, so $3 = \sqrt[3]{27}$

iii) $3^4 = 81$, so $3 = \sqrt[4]{81}$

c) **i)** $4^2 = 16$, so $4 = \sqrt{16}$

ii) $4^3 = 64$, so $4 = \sqrt[3]{64}$

iii) $4^4 = 256$, so $4 = \sqrt[4]{256}$

d) **i)** $10^2 = 100$, so $10 = \sqrt{100}$

ii) $10^3 = 1000$, so $10 = \sqrt[3]{1000}$

iii) $10^4 = 10\,000$, so $10 = \sqrt[4]{10\,000}$

e) **i)** $0.9^2 = 0.81$, so $0.9 = \sqrt{0.81}$

ii) $0.9^3 = 0.729$, so $0.9 = \sqrt[3]{0.729}$

iii) $0.9^4 = 0.6561$, so $0.9 = \sqrt[4]{0.6561}$

f) **i)** $0.2^2 = 0.04$, so $0.2 = \sqrt{0.04}$

ii) $0.2^3 = 0.008$, so $0.2 = \sqrt[3]{0.008}$

iii) $0.2^4 = 0.0016$, so $0.2 = \sqrt[4]{0.0016}$

6. Answers may vary.

a) For $\sqrt[n]{x}$ to be a whole number, the radical must be a square root of a whole number that is a perfect square, the cube root of a whole number that is a perfect cube, the fourth root of a whole number that is a perfect fourth power, and so on.

For example, $\sqrt[4]{1296}$

$$\begin{aligned}\sqrt[4]{1296} &= \sqrt[4]{6 \cdot 6 \cdot 6 \cdot 6} \\ &= 6\end{aligned}$$

b) For $\sqrt[n]{x}$ to be a negative integer, the radical must be a cube root of a negative integer that is a perfect cube, the fifth root of a negative integer that is a perfect fifth power, and so on.

For example, $\sqrt[5]{-243}$

$$\begin{aligned}\sqrt[5]{-243} &= \sqrt[5]{(-3)(-3)(-3)(-3)(-3)} \\ &= -3\end{aligned}$$

c) For $\sqrt[n]{x}$ to be a rational number, the radical must be a square root of a rational number that is a perfect square, the cube root of a rational number that is a perfect cube, the fourth root of a rational number that is a perfect fourth power, and so on.

For example, $\sqrt[3]{0.64}$

$$\begin{aligned}\sqrt[3]{0.064} &= \sqrt[3]{(0.4)(0.4)(0.4)} \\ &= 0.4\end{aligned}$$

d) For $\sqrt[n]{x}$ to be an approximate decimal, the radical must be a square root of a number that is not a perfect square, the cube root of a number that is not a perfect cube, the fourth root of a number that is not a perfect fourth power, and so on.

For example, $\sqrt[3]{13}$

$\sqrt[3]{13} \doteq 2.4$

Lesson 4.2 Irrational Numbers Exercises (pages 211–212)

A

3. a) $\sqrt{12}$ is irrational because 12 is not a perfect square.
The decimal form of $\sqrt{12}$ neither terminates nor repeats.

b) $\sqrt[4]{16}$ is rational because 16 is a perfect fourth power.
Its decimal form is 2.0, which terminates.

c) $\sqrt[3]{-100}$ is irrational because –100 is not a perfect cube.
The decimal form of $\sqrt[3]{-100}$ neither terminates nor repeats.

d) $\sqrt{\frac{4}{9}}$ is rational because $\frac{4}{9}$ is a perfect square.
$\sqrt{\frac{4}{9}} = \frac{2}{3}$ or $0.\overline{6}$, which is a repeating decimal

e) $\sqrt{1.25}$ is irrational because 1.25 is not a perfect square.
The decimal form of $\sqrt{1.25}$ neither terminates nor repeats.

f) 1.25 is rational because it is a terminating decimal.

4. Natural numbers are the set of numbers 1, 2, 3, 4, 5, …
Integers are the set of numbers …–3, –2, –1, 0, 1, 2, 3, …
A rational number is a number that can be written as the quotient of two integers; its decimal form either terminates or repeats.
An irrational number is a number that cannot be written as the quotient of two integers; its decimal form neither terminates nor repeats.

$\frac{4}{3}$ is a quotient of integers, so $\frac{4}{3}$ is a rational number.
$0.3\overline{4}$ is a repeating decimal, so $0.3\overline{4}$ is a rational number.
–5 is both an integer and a rational number.
$\sqrt[4]{9}$ is an irrational number because 9 is not a perfect fourth power. The decimal form of $\sqrt[4]{9}$ neither terminates nor repeats.
–2.1538 is a terminating decimal, so –2.1538 is a rational number.
$\sqrt[3]{27}$ is a rational number because 27 is a perfect cube. $\sqrt[3]{27} = 3$, which is a natural number and an integer
7 is an integer, a natural number, and a rational number.

a) 7, $\sqrt[3]{27}$

b) –5, $\sqrt[3]{27}$, 7

c) $\frac{4}{3}$, $0.3\overline{4}$, –5, –2.1538, $\sqrt[3]{27}$, 7

d) $\sqrt[4]{9}$

B

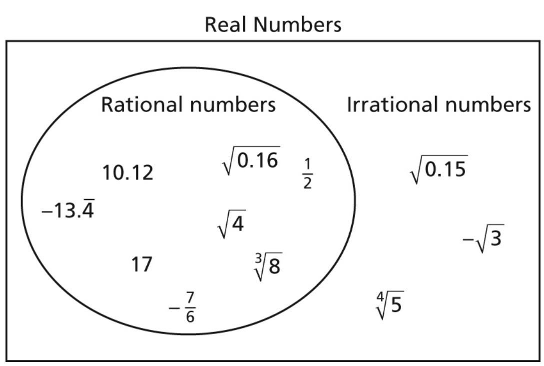
5. a) $\sqrt{49}$ is a rational number because 49 is a perfect square. $\sqrt{49}$ = 7, which is a terminating decimal
$\sqrt[4]{16}$ is a rational number because 16 is a perfect fourth power. $\sqrt[4]{16}$ = 2, which is a terminating decimal

b) $\sqrt{21}$ is an irrational number because 21 is not a perfect square.
The decimal form of $\sqrt{21}$ neither terminates nor repeats.
$\sqrt[3]{36}$ is an irrational number because 36 is not a perfect cube.
The decimal form of $\sqrt[3]{36}$ neither terminates nor repeats.

6. a) 12.247 448 71 is a terminating decimal, so 12.247 448 71 is a rational number.

b) Since 150 is not a perfect square, I know that $\sqrt{150}$ is an irrational number and that its decimal form neither terminates nor repeats. The calculator screen indicates that the decimal value of $\sqrt{150} = 12.247\ 448\ 71$. But, when I enter 12.247 448 71 in my calculator and square it, I get $149.\overline{9}$ instead of exactly 150. So, the value shown on the screen is a close approximation of $\sqrt{150}$, but not an exact value.

7. a)



b) $\frac{1}{2}$ and $-\frac{7}{6}$ are rational because each is a quotient of integers.
$-\sqrt{3}$, $\sqrt{0.15}$, and $\sqrt[4]{5}$ are irrational because 3 and 0.15 are not perfect squares and 5 is not a perfect fourth power.
$\sqrt{4}$, $\sqrt{0.16}$, and $\sqrt[3]{8}$ are rational because 4 and 0.16 are perfect squares and 8 is a perfect cube.
10.12 is rational because it is a terminating decimal.
$-13.\overline{4}$ is rational because it is a repeating decimal.
17 is rational because it can be written as a quotient of integers, $\frac{17}{1}$.

8. a) $\sqrt[3]{8}$ is rational because 8 is a perfect cube.

I used a calculator: $\sqrt[3]{8} = 2$ and 2 can be written as a quotient of integers; $\frac{2}{1}$.

b) $\sqrt[3]{64}$ is rational because 64 is a perfect cube.

I used a calculator: $\sqrt[3]{64} = 4$ and 4 can be written as a quotient of integers; $\frac{4}{1}$.

c) $\sqrt[3]{30}$ is irrational because 30 is not a perfect cube.
I used a calculator: the decimal form of $\sqrt[3]{30}$ appears to neither terminate nor repeat.

d) $\sqrt[3]{300}$ is irrational because 300 is not a perfect cube.
I used a calculator: the decimal form of $\sqrt[3]{300}$ appears to neither terminate nor repeat.

9. a) 5 is between the perfect squares 4 and 9, and is closer to 4.

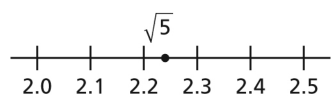
$\sqrt{4}$ $\sqrt{5}$ $\sqrt{9}$

↓ ↓ ↓

2 ? 3

Use a calculator.

$\sqrt{5} = 2.2360\ldots$



b) 12 is between the perfect cubes 8 and 27, and is closer to 8.

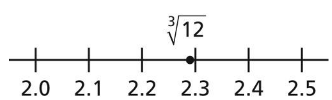
$\sqrt[3]{8}$ $\sqrt[3]{12}$ $\sqrt[3]{27}$

↓ ↓ ↓

2 ? 3

Use a calculator.

$\sqrt[3]{12} = 2.2894\ldots$



c) 25 is between the perfect fourth powers 16 and 81, and is closer to 16.

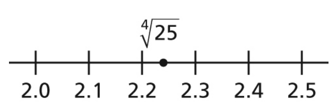
$\sqrt[4]{16}$ $\sqrt[4]{25}$ $\sqrt[4]{81}$

↓ ↓ ↓

2 ? 3

Use a calculator.

$\sqrt[4]{25} = 2.2360\ldots$

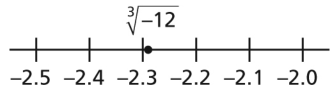


d) –12 is between the perfect cubes –8 and –27, and is closer to –8.

$\sqrt[3]{-8}$ $\sqrt[3]{-12}$ $\sqrt[3]{-27}$

↓ ↓ ↓

–2 ? –3



Use a calculator.

$\sqrt[3]{-12} = -2.2894\ldots$

10. a) 70 is between the perfect cubes 64 and 125, and is closer to 64.

$\sqrt[3]{64}$ $\sqrt[3]{70}$ $\sqrt[3]{125}$

↓ ↓ ↓

4 ? 5

Use a calculator.

$\sqrt[3]{70} = 4.1212\ldots$

50 is between the perfect squares 49 and 64, and is closer to 49.

$\sqrt{49}$ $\sqrt{50}$ $\sqrt{64}$

↓ ↓ ↓

7 ? 8

Use a calculator.

$\sqrt{50} = 7.0710\ldots$

100 is between the perfect fourth powers 81 and 256, and is closer to 81.

$\sqrt[4]{81}$ $\sqrt[4]{100}$ $\sqrt[4]{256}$

↓ ↓ ↓

3 ? 4

Use a calculator.

$\sqrt[4]{100} = 3.1622\ldots$

400 is between the perfect cubes 343 and 512, and is closer to 343.

$\sqrt[3]{343}$ $\sqrt[3]{400}$ $\sqrt[3]{512}$

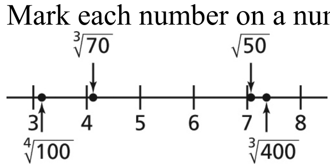
↓ ↓ ↓

7 ? 8

Use a calculator.

$\sqrt[3]{400} = 7.3680\ldots$

Mark each number on a number line.



From greatest to least: $\sqrt[3]{400}$, $\sqrt{50}$, $\sqrt[3]{70}$, $\sqrt[4]{100}$

b) 89 is between the perfect squares 81 and 100, and is closer to 81.

$\sqrt{81}$ $\sqrt{89}$ $\sqrt{100}$

↓ ↓ ↓

9 ? 10

Use a calculator.

$\sqrt{89} = 9.4339\ldots$

250 is between the perfect fourth powers 81 and 256, and is closer to 256.

$\sqrt[4]{81}$ $\sqrt[4]{250}$ $\sqrt[4]{256}$

↓ ↓ ↓

3 ? 4

Use a calculator.

$\sqrt[4]{250} = 3.9763\ldots$

–150 is between the perfect cubes –125 and –216, and is closer to –125.

$\sqrt[3]{-125}$ $\sqrt[3]{-150}$ $\sqrt[3]{-216}$

↓ ↓ ↓

–5 ? –6

Use a calculator.

$\sqrt[3]{-150} = -5.3132\ldots$

150 is between the perfect cubes 125 and 216, and is closer to 125.

$\sqrt[3]{125}$ $\sqrt[3]{150}$ $\sqrt[3]{216}$

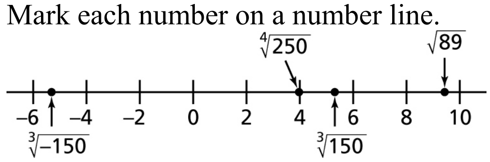
↓ ↓ ↓

5 ? 6

Use a calculator.

$\sqrt[3]{150} = 5.3132\ldots$

Mark each number on a number line.



From greatest to least: $\sqrt{89}$, $\sqrt[3]{150}$, $\sqrt[4]{250}$, $\sqrt[3]{-150}$

11. 40 is between the perfect squares 36 and 49, and is closer to 36.

$\sqrt{36}$ → 6, $\sqrt{40}$ → ?, $\sqrt{49}$ → 7

Use a calculator.

$\sqrt{40} = 6.3245\ldots$

500 is between the perfect cubes 343 and 512, and is closer to 512.

$\sqrt[3]{343}$ → 7, $\sqrt[3]{500}$ → ?, $\sqrt[3]{512}$ → 8

Use a calculator.

$\sqrt[3]{500} = 7.9370\ldots$

98 is between the perfect squares 81 and 100, and is closer to 100.

$\sqrt{81}$ → 9, $\sqrt{98}$ → ?, $\sqrt{100}$ → 10

Use a calculator.

$\sqrt{98} = 9.8994\ldots$

98 is between the perfect cubes 64 and 125, and is closer to 125.

$\sqrt[3]{64}$ → 4, $\sqrt[3]{98}$ → ?, $\sqrt[3]{125}$ → 5

Use a calculator.

$\sqrt[3]{98} = 4.6104\ldots$

75 is between the perfect squares 64 and 81, and is closer to 81.

$\sqrt{64}$ → 8, $\sqrt{75}$ → ?, $\sqrt{81}$ → 9

Use a calculator.

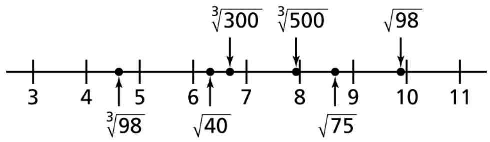
$\sqrt{75} = 8.6602\ldots$

300 is between the perfect cubes 216 and 343, and is closer to 343.

$\sqrt[3]{216}$ → 6, $\sqrt[3]{300}$ → ?, $\sqrt[3]{343}$ → 7

Use a calculator.
$\sqrt[3]{300} = 6.6943\ldots$

Mark each number on a number line.



From least to greatest: $\sqrt[3]{98}$, $\sqrt{40}$, $\sqrt[3]{300}$, $\sqrt[3]{500}$, $\sqrt{75}$, $\sqrt{98}$
To check, I can subtract the number on the left from the number to its right each time and I should always get a positive difference.

12. Use a calculator.
$\frac{-14}{5} = -2.8$

Use a calculator.
$\frac{123}{99} = 1.\overline{24}$

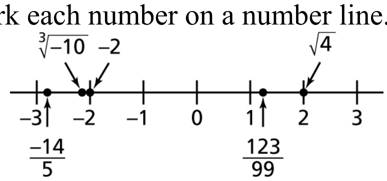
–10 is between the perfect cubes –8 and –27, and is closer to –8.

$\sqrt[3]{-8}$ → –2
$\sqrt[3]{-10}$ → ?
$\sqrt[3]{-27}$ → –3

Use a calculator.
$\sqrt[3]{-10} = -2.1544\ldots$

$\sqrt{4} = 2$

Mark each number on a number line.



From least to greatest: $\frac{-14}{5}$, $\sqrt[3]{-10}$, –2, $\frac{123}{99}$, $\sqrt{4}$

$\frac{-14}{5}$ and $\frac{123}{99}$ are rational because each is a quotient of integers.

–2 is rational because it can be written as a quotient of integers, $\frac{-2}{1}$.

$\sqrt[3]{-10}$ is irrational because –10 is not a perfect cube.

$\sqrt{4}$ is rational because 4 is a perfect square.

13. Use the Pythagorean Theorem to determine the length of the hypotenuse.
In a right triangle with hypotenuse length h and legs of lengths a and b,

$h^2 = a^2 + b^2$ Substitute: $a = 5$ and $b = 3$

$h^2 = 5^2 + 3^2$

$h^2 = 25 + 9$

$h^2 = 34$

$h = \sqrt{34}$

Because 34 is not a perfect square, $\sqrt{34}$ is an irrational number.
The length of the hypotenuse is $\sqrt{34}$ cm.

14. a) i) Natural numbers are the counting numbers: 1, 2, 3, 4, …
Integers are the set of numbers … –3, –2, –1, 0, 1, 2, 3, …
So, all natural numbers are integers. The statement is true.

ii) All integers are rational numbers because any integer n can be written as $\frac{n}{1}$.
The statement is true.

iii) The set of whole numbers is the set of natural numbers with the number 0 included.
So, all whole numbers are not natural numbers. The statement is false.

iv) Assume that we consider the root of a rational number.
π is irrational, and π is not a root of a rational number.
So, not all irrational numbers are roots.
The statement is false.

Assume that we consider the root of an irrational number.
π is the nth root of π^n. So, all irrational numbers are roots.
The statement is true.

v) All rational numbers can be written as the quotient of two integers. Since all natural numbers can be written as a quotient with denominator 1, some rational numbers are natural numbers. The statement is true.

b) iii) The number 0 is a whole number, but it is not a natural number.

iv) π is irrational, and π is not a root of a rational number.

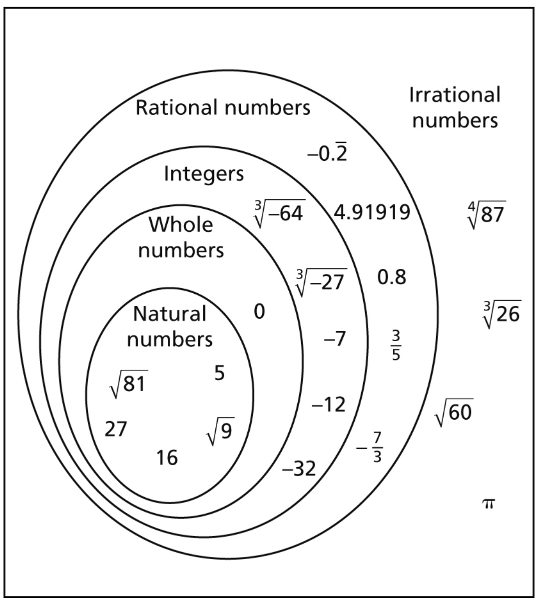
15. Answers may vary.

a) The decimal form of a rational number either terminates or repeats. So, write a terminating decimal that is not a natural number; for example, 1.5.

b) The set of whole numbers is the set of natural numbers with the number 0 included. So, the only whole number that is not a natural number is 0.

c) $\sqrt{21}$ is an irrational number because 21 is not a perfect square.

16. a) Some numbers belong to more than one set.
Place these numbers in the smallest set to which it belongs.
$\frac{3}{5}$ is a rational number because it is the quotient of two integers.
4.91919 is a rational number because it is a terminating decimal.
16 is a natural number.
$\sqrt[3]{-64} = -4$, which is an integer
$\sqrt{60}$ is an irrational number because 60 is not a perfect square.
$\sqrt{9} = 3$, which is a natural number
–7 is an integer.
0 is a whole number.



b) Answers may vary.
Natural numbers: I chose 2 counting numbers, 5 and 27, and the square root of a natural number that is a perfect square, $\sqrt{81}$.
Whole numbers: I cannot choose any more whole numbers that are not natural numbers.
Integers: I chose –12 and –32, and the cube root of a negative integer that is a perfect cube, $\sqrt[3]{-27}$.
Rational numbers: I chose a repeating decimal, $-0.\overline{2}$, a terminating decimal, 0.8, and a fraction, $-\frac{7}{3}$.
Irrational numbers: I chose the cube root of a number that is not a perfect cube, $\sqrt[3]{26}$, the fourth root of a number that is not a perfect fourth power, $\sqrt[4]{87}$, and π because its decimal form neither terminates nor repeats.

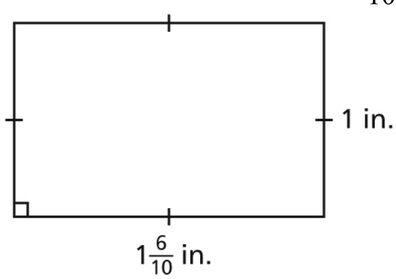
17. The formula for the volume, V, of a cube with edge length s units is:
$V = s^3$
To determine the value of s, take the cube root of each side.
$\sqrt[3]{V} = \sqrt[3]{s^3}$
$\sqrt[3]{V} = s$
Answers may vary.
a) For s to be irrational, V must be a positive number that is not a perfect cube.
For example, $V = 15$

b) For s to be rational, V must be a positive number that is a perfect cube.
For example, $V = 64$

18. a) $\frac{1+\sqrt{5}}{2} = 1.6180\ldots$
This value to the nearest tenth is 1.6.

b) Since the units are inches, write 1.6 as a fraction: $1\frac{6}{10}$

Draw a rectangle with length $1\frac{6}{10}$ in. and width 1 in.



c) Answers may vary.
I measured a book with length 21 cm and width 13 cm.
The ratio of length to width is: $\frac{21}{13} = 1.6153...$
Since the ratio of length to width is approximately 1.6 to 1, the book approximates a golden rectangle.

19. The pyramid has base side length 755 ft. and height 481 ft.

The ratio of base side length to its height is: $\frac{755}{481} = 1.5696...$

Since I can write 1.5696… as $\frac{1.5696...}{1}$, the ratio is approximately 1.6:1.

From question 18, I know the golden ratio $\frac{1+\sqrt{5}}{2}$:1 is approximately 1.6:1.

So, the ratio of the base side length of the pyramid to its height approximates the golden ratio.

20. The formula for the area, A, of a square with side length s units is:
$A = s^2$
To determine the value of s, take the square root of each side.
$\sqrt{A} = \sqrt{s^2}$
$\sqrt{A} = s$

a) Since 40 is not a perfect square, $\sqrt{40}$ is irrational.
So, the side length of the square is irrational.
The perimeter of a square is: $P = 4s$
Since the product of a rational number and an irrational number is irrational, the perimeter of the square is irrational.

b) Since 81 is a perfect square, $\sqrt{81}$ is rational.
So, the side length of the square is rational.
The perimeter of a square is: $P = 4s$
Since the product of two rational numbers is rational, the perimeter of the square is rational.

C

21. Since $\sqrt[n]{\frac{a}{b}}$ is rational, when $n = 2$, $\frac{a}{b}$ is a perfect square; when $n = 3$, $\frac{a}{b}$ is a perfect cube;

when $n = 4$, $\frac{a}{b}$ is a perfect fourth power; and so on.

So, each prime factor of a and b must occur a multiple of n times.
For example, when $n = 2$:
$$\sqrt{\frac{16}{9}} = \sqrt{\frac{(2)(2)(2)(2)}{(3)(3)}}$$
The prime factors of 16 occur 4 times; and 4 is a multiple of 2.
The prime factors of 9 occur 2 times; and 2 is a multiple of 2.

When $n = 3$:
$$\sqrt[3]{\frac{8}{125}} = \sqrt[3]{\frac{(2)(2)(2)}{(5)(5)(5)}}$$
Each prime factor of 8 and 125 occurs 3 times; and 3 is a multiple of 3.

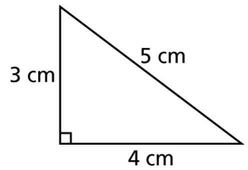
When $n = 4$:

$$\sqrt[4]{\frac{256}{81}} = \sqrt[4]{\frac{(2)(2)(2)(2)(2)(2)(2)(2)}{(3)(3)(3)(3)}}$$

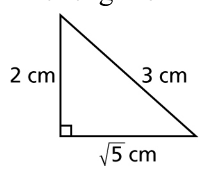
The prime factors of 256 occur 8 times; and 8 is a multiple of 4.
The prime factors of 81 occur 4 times; and 4 is a multiple of 4.

22. In a right triangle with hypotenuse length h and legs of lengths a and b, $h^2 = a^2 + b^2$.

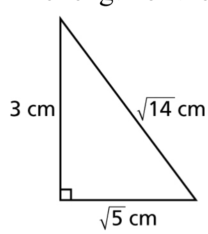
a) Since all natural numbers are rational numbers, I drew a triangle with side lengths that are natural numbers.



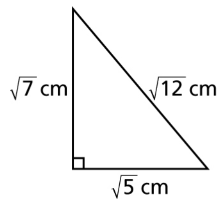
b) I started with the irrational number $\sqrt{5}$ as the length of one leg in centimetres. I then determined a value for b so that $\left(\sqrt{5}\right)^2 + b^2$ is a perfect square. Since $5 + 4 = 9$ and $2^2 = 4$, I know $b = 2$.
The length of the hypotenuse, in centimetres, is $\sqrt{9} = 3$.



c) I started with the irrational number $\sqrt{5}$ as the length of one leg, in centimetres. I then determined a value for b so that $\left(\sqrt{5}\right)^2 + b^2$ is not a perfect square, with b a natural number. I chose $b = 3$.
The length of the hypotenuse, in centimetres, is $\sqrt{5 + 3^2} = \sqrt{14}$.



d) I started with the irrational number $\sqrt{5}$ as the length of one leg, in centimetres. I then determined a value for b so that $\left(\sqrt{5}\right)^2 + b^2$ is not a perfect square, with b an irrational number. I chose $b = \sqrt{7}$. The length of the hypotenuse, in centimetres, is $\sqrt{5 + 7} = \sqrt{12}$.



23. a) Yes, when a rational number is not a perfect square, its square root is irrational. For example, 31 is a rational number because it can be written as the quotient of two integers, $\frac{31}{1}$. But, $\sqrt{31}$ is an irrational number because 31 is not a perfect square.

b) A rational number can be written in the form $\frac{a}{b}$, where a and b are integers. When a rational number is squared, the result is $\frac{a^2}{b^2}$. Since the square of an integer is another integer, it is impossible for the square of a rational number to be irrational. So, the square root of an irrational number cannot be rational.

24. A number is a perfect square when the exponent of each factor in its prime factorization is a multiple of 2; for example, $9 = 3^2$.
A number is a perfect cube when the exponent of each factor in its prime factorization is a multiple of 3; for example, $64 = 2^6$, or $2^{2(3)}$.
A number is a perfect fourth power when the exponent of each factor in its prime factorization is a multiple of 4; for example, $81 = 3^4$.
For a number to be a perfect square, a perfect cube, and a perfect fourth power, the exponent of each factor in its prime factorization must be a multiple of 2, 3, and 4.
List the multiples of 2, 3, and 4:
2: 2, 4, 6, 8, 10, 12, …
3: 3, 6, 9, 12, …
4: 4, 8, 12, …
The least common multiple of 2, 3, and 4 is 12.
So, to generate numbers with the property that their square roots, cube roots, and fourth roots are all rational numbers, raise the number to the 12th power; for example, $2^{12} = 4096$.
For example

$$\sqrt{4096} = \sqrt{2^{12}} = \sqrt{2^{2(6)}} = 2^6 = 64$$

$$\sqrt[3]{4096} = \sqrt[3]{2^{12}} = \sqrt[3]{2^{3(4)}} = 2^4 = 16$$

$$\sqrt[4]{4096} = \sqrt[4]{2^{12}} = \sqrt[4]{2^{4(3)}} = 2^3 = 8$$

Lesson 4.3 Mixed and Entire Radicals Exercises (pages 218–219)

A

3.

Square of Number	Perfect Square	Square Root
$1^2 = 1 \cdot 1$	1	1
$2^2 = 2 \cdot 2$	4	2
$3^2 = 3 \cdot 3$	9	3
$4^2 = 4 \cdot 4$	16	4
$5^2 = 5 \cdot 5$	25	5
$6^2 = 6 \cdot 6$	36	6
$7^2 = 7 \cdot 7$	49	7
$8^2 = 8 \cdot 8$	64	8
$9^2 = 9 \cdot 9$	81	9
$10^2 = 10 \cdot 10$	100	10
$11^2 = 11 \cdot 11$	121	11
$12^2 = 12 \cdot 12$	144	12
$13^2 = 13 \cdot 13$	169	13
$14^2 = 14 \cdot 14$	196	14
$15^2 = 15 \cdot 15$	225	15
$16^2 = 16 \cdot 16$	256	16
$17^2 = 17 \cdot 17$	289	17
$18^2 = 18 \cdot 18$	324	18
$19^2 = 19 \cdot 19$	361	19
$20^2 = 20 \cdot 20$	400	20

4. **a)**
$$\begin{aligned}\sqrt{8} &= \sqrt{2\cdot 4}\\ &= \sqrt{2\cdot 2\cdot 2}\\ &= \sqrt{(2\cdot 2)\cdot 2}\\ &= \sqrt{2\cdot 2}\cdot\sqrt{2}\\ &= 2\cdot\sqrt{2}\\ &= 2\sqrt{2}\end{aligned}$$

b)
$$\begin{aligned}\sqrt{12} &= \sqrt{4\cdot 3}\\ &= \sqrt{2\cdot 2\cdot 3}\\ &= \sqrt{(2\cdot 2)\cdot 3}\\ &= \sqrt{2\cdot 2}\cdot\sqrt{3}\\ &= 2\cdot\sqrt{3}\\ &= 2\sqrt{3}\end{aligned}$$

c) $\sqrt{32} = \sqrt{4 \cdot 8}$
$= \sqrt{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2}$
$= \sqrt{(2 \cdot 2) \cdot (2 \cdot 2) \cdot 2}$
$= \sqrt{2 \cdot 2} \cdot \sqrt{2 \cdot 2} \cdot \sqrt{2}$
$= 2 \cdot 2 \cdot \sqrt{2}$
$= 4\sqrt{2}$

d) $\sqrt{50} = \sqrt{25 \cdot 2}$
$= \sqrt{5 \cdot 5 \cdot 2}$
$= \sqrt{(5 \cdot 5) \cdot 2}$
$= \sqrt{5 \cdot 5} \cdot \sqrt{2}$
$= 5 \cdot \sqrt{2}$
$= 5\sqrt{2}$

e) $\sqrt{18} = \sqrt{9 \cdot 2}$
$= \sqrt{3 \cdot 3 \cdot 2}$
$= \sqrt{(3 \cdot 3) \cdot 2}$
$= \sqrt{3 \cdot 3} \cdot \sqrt{2}$
$= 3 \cdot \sqrt{2}$
$= 3\sqrt{2}$

f) $\sqrt{27} = \sqrt{9 \cdot 3}$
$= \sqrt{3 \cdot 3 \cdot 3}$
$= \sqrt{(3 \cdot 3) \cdot 3}$
$= \sqrt{3 \cdot 3} \cdot \sqrt{3}$
$= 3 \cdot \sqrt{3}$
$= 3\sqrt{3}$

g)
$$\begin{aligned}\sqrt{48} &= \sqrt{4 \cdot 12} \\ &= \sqrt{2 \cdot 2 \cdot 2 \cdot 2 \cdot 3} \\ &= \sqrt{(2 \cdot 2) \cdot (2 \cdot 2) \cdot 3} \\ &= \sqrt{2 \cdot 2} \cdot \sqrt{2 \cdot 2} \cdot \sqrt{3} \\ &= 2 \cdot 2 \cdot \sqrt{3} \\ &= 4\sqrt{3}\end{aligned}$$

h)
$$\begin{aligned}\sqrt{75} &= \sqrt{25 \cdot 3} \\ &= \sqrt{5 \cdot 5 \cdot 3} \\ &= \sqrt{(5 \cdot 5) \cdot 3} \\ &= \sqrt{5 \cdot 5} \cdot \sqrt{3} \\ &= 5 \cdot \sqrt{3} \\ &= 5\sqrt{3}\end{aligned}$$

5. **a)** Write 5 as: $\sqrt{5 \cdot 5} = \sqrt{25}$

$$\begin{aligned}5\sqrt{2} &= \sqrt{25} \cdot \sqrt{2} \\ &= \sqrt{25 \cdot 2} \\ &= \sqrt{50}\end{aligned}$$

b) Write 6 as: $\sqrt{6 \cdot 6} = \sqrt{36}$

$$\begin{aligned}6\sqrt{2} &= \sqrt{36} \cdot \sqrt{2} \\ &= \sqrt{36 \cdot 2} \\ &= \sqrt{72}\end{aligned}$$

c) Write 7 as: $\sqrt{7 \cdot 7} = \sqrt{49}$

$$\begin{aligned}7\sqrt{2} &= \sqrt{49} \cdot \sqrt{2} \\ &= \sqrt{49 \cdot 2} \\ &= \sqrt{98}\end{aligned}$$

d) Write 8 as: $\sqrt{8 \cdot 8} = \sqrt{64}$

$$\begin{aligned}8\sqrt{2} &= \sqrt{64} \cdot \sqrt{2} \\ &= \sqrt{64 \cdot 2} \\ &= \sqrt{128}\end{aligned}$$

e) Write 5 as: $\sqrt{5 \cdot 5} = \sqrt{25}$

$5\sqrt{3} = \sqrt{25} \cdot \sqrt{3}$

$= \sqrt{25 \cdot 3}$

$= \sqrt{75}$

f) Write 6 as: $\sqrt{6 \cdot 6} = \sqrt{36}$

$6\sqrt{3} = \sqrt{36} \cdot \sqrt{3}$

$= \sqrt{36 \cdot 3}$

$= \sqrt{108}$

g) Write 7 as: $\sqrt{7 \cdot 7} = \sqrt{49}$

$7\sqrt{3} = \sqrt{49} \cdot \sqrt{3}$

$= \sqrt{49 \cdot 3}$

$= \sqrt{147}$

h) Write 8 as: $\sqrt{8 \cdot 8} = \sqrt{64}$

$8\sqrt{3} = \sqrt{64} \cdot \sqrt{3}$

$= \sqrt{64 \cdot 3}$

$= \sqrt{192}$

6. a)

Cube of Number	Perfect Cube	Cube Root
$1^3 = 1 \cdot 1 \cdot 1$	1	1
$2^3 = 2 \cdot 2 \cdot 2$	8	2
$3^3 = 3 \cdot 3 \cdot 3$	27	3
$4^3 = 4 \cdot 4 \cdot 4$	64	4
$5^3 = 5 \cdot 5 \cdot 5$	125	5
$6^3 = 6 \cdot 6 \cdot 6$	216	6
$7^3 = 7 \cdot 7 \cdot 7$	343	7
$8^3 = 8 \cdot 8 \cdot 8$	512	8
$9^3 = 9 \cdot 9 \cdot 9$	729	9
$10^3 = 10 \cdot 10 \cdot 10$	1000	10

b)

Fourth Power of Number	Perfect Fourth Power	Fourth Root
$1^4 = 1 \cdot 1 \cdot 1 \cdot 1$	1	1
$2^4 = 2 \cdot 2 \cdot 2 \cdot 2$	16	2
$3^4 = 3 \cdot 3 \cdot 3 \cdot 3$	81	3
$4^4 = 4 \cdot 4 \cdot 4 \cdot 4$	256	4
$5^4 = 5 \cdot 5 \cdot 5 \cdot 5$	625	5

B

7. a) Use the Pythagorean Theorem in right ΔCBA.

$AC^2 + AB^2 = BC^2$ Substitute: AC = 1, AB = 2

$1^2 + 2^2 = BC^2$

$1 + 4 = BC^2$

$5 = BC^2$

$BC = \sqrt{5}$

Use the Pythagorean Theorem in right ΔDEA.

$AD^2 + AE^2 = DE^2$ Substitute: AD = 3, AE = 6

$3^2 + 6^2 = DE^2$

$9 + 36 = DE^2$

$45 = DE^2$

$DE = \sqrt{45}$

Each side of ΔDEA is 3 times the length of the corresponding side in ΔCBA.

So, DE = 3(CB)

$\sqrt{45} = 3\sqrt{5}$

b)
$$\begin{aligned}\sqrt{45} &= \sqrt{9 \cdot 5}\\ &= \sqrt{3 \cdot 3 \cdot 5}\\ &= \sqrt{(3 \cdot 3) \cdot 5}\\ &= \sqrt{3 \cdot 3} \cdot \sqrt{5}\\ &= 3 \cdot \sqrt{5}\\ &= 3\sqrt{5}\end{aligned}$$

8. a) Use the Pythagorean Theorem in right ΔQRS.

$QR^2 + RS^2 = QS^2$ Substitute: QR = 3, RS = 1

$3^2 + 1^2 = QS^2$

$9 + 1 = QS^2$

$10 = QS^2$

$QS = \sqrt{10}$

Use the Pythagorean Theorem in right ΔPRT.

$PR^2 + RT^2 = PT^2$ Substitute: PR = 6, RT = 2

$6^2 + 2^2 = PT^2$

$36 + 4 = PT^2$

$40 = PT^2$

$PT = \sqrt{40}$

Each side of ΔPRT is 2 times the length of the corresponding side in ΔQRS.

So, PT = 2(QS)

$\sqrt{40} = 2\sqrt{10}$

b) $\sqrt{40} = \sqrt{4 \cdot 10}$
$= \sqrt{2 \cdot 2 \cdot 10}$
$= \sqrt{(2 \cdot 2) \cdot 10}$
$= \sqrt{2 \cdot 2} \cdot \sqrt{10}$
$= 2 \cdot \sqrt{10}$
$= 2\sqrt{10}$

9. When I simplify $\sqrt{50}$, it helps to rewrite $\sqrt{50}$ as $\sqrt{25} \cdot \sqrt{2}$ because 25 is a perfect square. I can write $\sqrt{25}$ as 5. Then $\sqrt{50} = 5\sqrt{2}$.
When I simplify $\sqrt{50}$, it does not help to rewrite $\sqrt{50}$ as $\sqrt{10} \cdot \sqrt{5}$ because neither 10 nor 5 is a perfect square.

10. a) The factors of 90 are: 1, 2, 3, 5, 6, 9, 10, 15, 18, 30, 45, 90
The greatest perfect square is $9 = 3 \cdot 3$, so write 90 as $9 \cdot 10$.
$\sqrt{90} = \sqrt{9 \cdot 10}$
$= \sqrt{9} \cdot \sqrt{10}$
$= 3 \cdot \sqrt{10}$
$= 3\sqrt{10}$

b) The factors of 73 are 1 and 73.
There are no perfect square factors other than 1.
So, $\sqrt{73}$ cannot be simplified.

c) The factors of 108 are: 1, 2, 3, 4, 6, 9, 12, 18, 27, 36, 54, 108
The greatest perfect square is $36 = 6 \cdot 6$, so write 108 as $36 \cdot 3$.
$\sqrt{108} = \sqrt{36 \cdot 3}$
$= \sqrt{36} \cdot \sqrt{3}$
$= 6 \cdot \sqrt{3}$
$= 6\sqrt{3}$

d) The factors of 600 are: 1, 2, 3, 4, 5, 6, 8, 10, 12, 15, 20, 24, 25, 30, 40, 50, 60, 75, 100, 120, 150, 200, 300, 600
The greatest perfect square is $100 = 10 \cdot 10$, so write 600 as $100 \cdot 6$.
$\sqrt{600} = \sqrt{100 \cdot 6}$
$= \sqrt{100} \cdot \sqrt{6}$
$= 10 \cdot \sqrt{6}$
$= 10\sqrt{6}$

e) The factors of 54 are: 1, 2, 3, 6, 9, 18, 27, 54
The greatest perfect square is $9 = 3 \cdot 3$, so write 54 as $9 \cdot 6$.

$$\begin{aligned}\sqrt{54} &= \sqrt{9 \cdot 6}\\ &= \sqrt{9} \cdot \sqrt{6}\\ &= 3 \cdot \sqrt{6}\\ &= 3\sqrt{6}\end{aligned}$$

f) The factors of 91 are 1, 7, 13, and 91.
There are no perfect square factors other than 1.
So, $\sqrt{91}$ cannot be simplified.

g) The factors of 28 are: 1, 2, 4, 7, 14, 28
The greatest perfect square is $4 = 2 \cdot 2$, so write 28 as $4 \cdot 7$.

$$\begin{aligned}\sqrt{28} &= \sqrt{4 \cdot 7}\\ &= \sqrt{4} \cdot \sqrt{7}\\ &= 2 \cdot \sqrt{7}\\ &= 2\sqrt{7}\end{aligned}$$

h) The factors of 33 are 1, 3, 11, and 33.
There are no perfect square factors other than 1.
So, $\sqrt{33}$ cannot be simplified.

i) The factors of 112 are: 1, 2, 4, 7, 8, 14, 16, 28, 56, 112
The greatest perfect square is $16 = 4 \cdot 4$, so write 112 as $16 \cdot 7$.

$$\begin{aligned}\sqrt{112} &= \sqrt{16 \cdot 7}\\ &= \sqrt{16} \cdot \sqrt{7}\\ &= 4 \cdot \sqrt{7}\\ &= 4\sqrt{7}\end{aligned}$$

11. a) The factors of 16 are: 1, 2, 4, 8, 16
The greatest perfect cube is $8 = 2 \cdot 2 \cdot 2$, so write 16 as $8 \cdot 2$.

$$\begin{aligned}\sqrt[3]{16} &= \sqrt[3]{8 \cdot 2}\\ &= \sqrt[3]{8} \cdot \sqrt[3]{2}\\ &= 2 \cdot \sqrt[3]{2}\\ &= 2\sqrt[3]{2}\end{aligned}$$

b) The factors of 81 are: 1, 3, 9, 27, 81
The greatest perfect cube is 27 = $3 \cdot 3 \cdot 3$, so write 81 as $27 \cdot 3$.

$$\begin{aligned}\sqrt[3]{81} &= \sqrt[3]{27 \cdot 3}\\ &= \sqrt[3]{27} \cdot \sqrt[3]{3}\\ &= 3 \cdot \sqrt[3]{3}\\ &= 3\sqrt[3]{3}\end{aligned}$$

c) The factors of 256 are: 1, 2, 4, 8, 16, 32, 64, 128, 256
The greatest perfect cube is 64 = $4 \cdot 4 \cdot 4$, so write 256 as $64 \cdot 4$.

$$\begin{aligned}\sqrt[3]{256} &= \sqrt[3]{64 \cdot 4}\\ &= \sqrt[3]{64} \cdot \sqrt[3]{4}\\ &= 4 \cdot \sqrt[3]{4}\\ &= 4\sqrt[3]{4}\end{aligned}$$

d) The factors of 128 are: 1, 2, 4, 8, 16, 32, 64, 128
The greatest perfect cube is 64 = $4 \cdot 4 \cdot 4$, so write 128 as $64 \cdot 2$.

$$\begin{aligned}\sqrt[3]{128} &= \sqrt[3]{64 \cdot 2}\\ &= \sqrt[3]{64} \cdot \sqrt[3]{2}\\ &= 4 \cdot \sqrt[3]{2}\\ &= 4\sqrt[3]{2}\end{aligned}$$

e) The factors of 60 are: 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60
There are no perfect cube factors other than 1.
So, $\sqrt[3]{60}$ cannot be simplified.

f) The factors of 192 are: 1, 2, 3, 4, 6, 8, 12, 16, 24, 32, 48, 64, 96, 192
The greatest perfect cube is 64 = $4 \cdot 4 \cdot 4$, so write 192 as $64 \cdot 3$.

$$\begin{aligned}\sqrt[3]{192} &= \sqrt[3]{64 \cdot 3}\\ &= \sqrt[3]{64} \cdot \sqrt[3]{3}\\ &= 4 \cdot \sqrt[3]{3}\\ &= 4\sqrt[3]{3}\end{aligned}$$

g) The factors of 135 are: 1, 3, 5, 9, 15, 27, 45, 135
The greatest perfect cube is 27 = $3 \cdot 3 \cdot 3$, so write 135 as $27 \cdot 5$.

$$\begin{aligned}\sqrt[3]{135} &= \sqrt[3]{27 \cdot 5}\\ &= \sqrt[3]{27} \cdot \sqrt[3]{5}\\ &= 3 \cdot \sqrt[3]{5}\\ &= 3\sqrt[3]{5}\end{aligned}$$

h) The factors of 100 are: 1, 2, 4, 5, 10, 20, 25, 50, 100
There are no perfect cube factors other than 1.
So, $\sqrt[3]{100}$ cannot be simplified.

i) The factors of 500 are: 1, 2, 4, 5, 10, 20, 25, 50, 100, 125, 250, 500
The greatest perfect cube is $125 = 5 \cdot 5 \cdot 5$, so write 500 as $125 \cdot 4$.

$$\begin{aligned}\sqrt[3]{500} &= \sqrt[3]{125 \cdot 4}\\ &= \sqrt[3]{125} \cdot \sqrt[3]{4}\\ &= 5 \cdot \sqrt[3]{4}\\ &= 5\sqrt[3]{4}\end{aligned}$$

j) The factors of 375 are: 1, 3, 5, 15, 25, 75, 125, 375
The greatest perfect cube is $125 = 5 \cdot 5 \cdot 5$, so write 375 as $125 \cdot 3$.

$$\begin{aligned}\sqrt[3]{375} &= \sqrt[3]{125 \cdot 3}\\ &= \sqrt[3]{125} \cdot \sqrt[3]{3}\\ &= 5 \cdot \sqrt[3]{3}\\ &= 5\sqrt[3]{3}\end{aligned}$$

12. a) Write 3 as: $\sqrt{3 \cdot 3} = \sqrt{9}$

$$\begin{aligned}3\sqrt{2} &= \sqrt{9} \cdot \sqrt{2}\\ &= \sqrt{9 \cdot 2}\\ &= \sqrt{18}\end{aligned}$$

b) Write 4 as: $\sqrt{4 \cdot 4} = \sqrt{16}$

$$\begin{aligned}4\sqrt{2} &= \sqrt{16} \cdot \sqrt{2}\\ &= \sqrt{16 \cdot 2}\\ &= \sqrt{32}\end{aligned}$$

c) Write 6 as: $\sqrt{6 \cdot 6} = \sqrt{36}$

$$\begin{aligned}6\sqrt{5} &= \sqrt{36} \cdot \sqrt{5}\\ &= \sqrt{36 \cdot 5}\\ &= \sqrt{180}\end{aligned}$$

d) Write 5 as: $\sqrt{5 \cdot 5} = \sqrt{25}$

$$\begin{aligned}5\sqrt{6} &= \sqrt{25} \cdot \sqrt{6}\\ &= \sqrt{25 \cdot 6}\\ &= \sqrt{150}\end{aligned}$$

e) Write 7 as: $\sqrt{7 \cdot 7} = \sqrt{49}$

$7\sqrt{7} = \sqrt{49} \cdot \sqrt{7}$
$= \sqrt{49 \cdot 7}$
$= \sqrt{343}$

f) Write 2 as: $\sqrt[3]{2 \cdot 2 \cdot 2} = \sqrt[3]{8}$

$2\sqrt[3]{2} = \sqrt[3]{8} \cdot \sqrt[3]{2}$
$= \sqrt[3]{8 \cdot 2}$
$= \sqrt[3]{16}$

g) Write 3 as: $\sqrt[3]{3 \cdot 3 \cdot 3} = \sqrt[3]{27}$

$3\sqrt[3]{3} = \sqrt[3]{27} \cdot \sqrt[3]{3}$
$= \sqrt[3]{27 \cdot 3}$
$= \sqrt[3]{81}$

h) Write 4 as: $\sqrt[3]{4 \cdot 4 \cdot 4} = \sqrt[3]{64}$

$4\sqrt[3]{3} = \sqrt[3]{64} \cdot \sqrt[3]{3}$
$= \sqrt[3]{64 \cdot 3}$
$= \sqrt[3]{192}$

i) Write 5 as: $\sqrt[3]{5 \cdot 5 \cdot 5} = \sqrt[3]{125}$

$5\sqrt[3]{2} = \sqrt[3]{125} \cdot \sqrt[3]{2}$
$= \sqrt[3]{125 \cdot 2}$
$= \sqrt[3]{250}$

j) Write 2 as: $\sqrt[3]{2 \cdot 2 \cdot 2} = \sqrt[3]{8}$

$2\sqrt[3]{9} = \sqrt[3]{8} \cdot \sqrt[3]{9}$
$= \sqrt[3]{8 \cdot 9}$
$= \sqrt[3]{72}$

13. a) Yes, every mixed radical can be expressed as an entire radical. To express a mixed radical as an entire radical, I write the number in front of the radical as the square root of its square, or the cube root of its cube, or the fourth root of its perfect fourth power, and so on, depending on the index of the radical.

For example, to write $3\sqrt{3}$ as an entire radical, I write 3 as $\sqrt{3 \cdot 3} = \sqrt{9}$, then multiply:

$\sqrt{9} \cdot \sqrt{3} = \sqrt{27}$

To write $2\sqrt[4]{3}$ as an entire radical, I write 2 as $\sqrt[4]{2 \cdot 2 \cdot 2 \cdot 2} = \sqrt[4]{16}$, then multiply:

$\sqrt[4]{16} \cdot \sqrt[4]{3} = \sqrt[4]{48}$

Every mixed radical can be expressed as an entire radical.

b) No, every entire radical cannot be expressed as a mixed radical. An entire radical can be written as a mixed radical when one of the factors of the radicand is a perfect square, or a perfect cube, or a perfect fourth power, and so on, depending on the index of the radical.
For example, $\sqrt[3]{16}$ can be written as a mixed radical because $16 = 8 \cdot 2$, and 8 is a perfect cube; so $\sqrt[3]{16} = 2\sqrt[3]{2}$.
$\sqrt[3]{21}$ cannot be written as a mixed radical because the factors of 21 are 1, 3, 7, 21, and there are no cube factors other than 1.

14. The formula for the area, A, of a square with side length s units is:
$A = s^2$
To determine the value of s, take the square root of each side.
$\sqrt{A} = \sqrt{s^2}$
$\sqrt{A} = s$ Substitute: $A = 252$
$s = \sqrt{252}$
The factors of 252 are: 1, 2, 3, 4, 6, 7, 9, 12, 14, 18, 21, 28, 36, 42, 63, 84, 126, 252
The greatest perfect square is $36 = 6 \cdot 6$, so write 252 as $36 \cdot 7$.

$$\begin{aligned} s &= \sqrt{36 \cdot 7} \\ &= \sqrt{36} \cdot \sqrt{7} \\ &= 6 \cdot \sqrt{7} \\ &= 6\sqrt{7} \end{aligned}$$

The square has side length $6\sqrt{7}$ ft.

15. The formula for the volume, V, of a cube with edge length e units is:
$V = e^3$
To determine the value of e, take the cube root of each side.
$\sqrt[3]{V} = \sqrt[3]{e^3}$
$\sqrt[3]{V} = e$ Substitute: $V = 200$
$e = \sqrt[3]{200}$
The factors of 200 are: 1, 2, 4, 5, 8, 10, 20, 25, 40, 50, 100, 200
The greatest perfect cube is $8 = 2 \cdot 2 \cdot 2$, so write 200 as $8 \cdot 25$.

$$\begin{aligned} e &= \sqrt[3]{8 \cdot 25} \\ &= \sqrt[3]{8} \cdot \sqrt[3]{25} \\ &= 2 \cdot \sqrt[3]{25} \\ &= 2\sqrt[3]{25} \end{aligned}$$

The cube has edge length $2\sqrt[3]{25}$ cm.

16. The formula for the area, A, of a square with side length s units is:
$A = s^2$
To determine the value of s, take the square root of each side.

$\sqrt{A} = \sqrt{s^2}$

$\sqrt{A} = s$ Substitute: $A = 54$

$s = \sqrt{54}$

The factors of 54 are: 1, 2, 3, 6, 9, 18, 27, 54

The greatest perfect square is $9 = 3 \cdot 3$, so write 54 as $9 \cdot 6$.

$s = \sqrt{9 \cdot 6}$

$= \sqrt{9} \cdot \sqrt{6}$

$= 3 \cdot \sqrt{6}$

$= 3\sqrt{6}$

The formula for the perimeter, *P*, of a square with side length *s* units is:

$P = 4s$ Substitute: $s = 3\sqrt{6}$

$= 4\left(3\sqrt{6}\right)$

$= 12\sqrt{6}$

The perimeter of the square is $12\sqrt{6}$ in.

17. a) The factors of 48 are: 1, 2, 3, 4, 6, 8, 12, 16, 24, 48

The greatest perfect fourth power is $16 = 2 \cdot 2 \cdot 2 \cdot 2$, so write 48 as $16 \cdot 3$.

$\sqrt[4]{48} = \sqrt[4]{16 \cdot 3}$

$= \sqrt[4]{16} \cdot \sqrt[4]{3}$

$= 2 \cdot \sqrt[4]{3}$

$= 2\sqrt[4]{3}$

b) The factors of 405 are: 1, 3, 5, 9, 15, 27, 45, 81, 135, 405

The greatest perfect fourth power is $81 = 3 \cdot 3 \cdot 3 \cdot 3$, so write 405 as $81 \cdot 5$.

$\sqrt[4]{405} = \sqrt[4]{81 \cdot 5}$

$= \sqrt[4]{81} \cdot \sqrt[4]{5}$

$= 3 \cdot \sqrt[4]{5}$

$= 3\sqrt[4]{5}$

c) The factors of 1250 are: 1, 2, 5, 10, 25, 50, 125, 250, 625, 1250

The greatest perfect fourth power is $625 = 5 \cdot 5 \cdot 5 \cdot 5$, so write 1250 as $625 \cdot 2$.

$\sqrt[4]{1250} = \sqrt[4]{625 \cdot 2}$

$= \sqrt[4]{625} \cdot \sqrt[4]{2}$

$= 5 \cdot \sqrt[4]{2}$

$= 5\sqrt[4]{2}$

d) The factors of 176 are: 1, 2, 4, 8, 11, 16, 22, 44, 88, 176
The greatest perfect fourth power is $16 = 2 \cdot 2 \cdot 2 \cdot 2$, so write 176 as $16 \cdot 11$.

$$\begin{aligned}\sqrt[4]{176} &= \sqrt[4]{16 \cdot 11}\\ &= \sqrt[4]{16} \cdot \sqrt[4]{11}\\ &= 2 \cdot \sqrt[4]{11}\\ &= 2\sqrt[4]{11}\end{aligned}$$

18. a) Write 6 as: $\sqrt[4]{6 \cdot 6 \cdot 6 \cdot 6} = \sqrt[4]{1296}$

$$\begin{aligned}6\sqrt[4]{3} &= \sqrt[4]{1296} \cdot \sqrt[4]{3}\\ &= \sqrt[4]{1296 \cdot 3}\\ &= \sqrt[4]{3888}\end{aligned}$$

b) Write 7 as: $\sqrt[4]{7 \cdot 7 \cdot 7 \cdot 7} = \sqrt[4]{2401}$

$$\begin{aligned}7\sqrt[4]{2} &= \sqrt[4]{2401} \cdot \sqrt[4]{2}\\ &= \sqrt[4]{2401 \cdot 2}\\ &= \sqrt[4]{4802}\end{aligned}$$

c) Write 3 as: $\sqrt[5]{3 \cdot 3 \cdot 3 \cdot 3 \cdot 3} = \sqrt[5]{243}$

$$\begin{aligned}3\sqrt[5]{4} &= \sqrt[5]{243} \cdot \sqrt[5]{4}\\ &= \sqrt[5]{243 \cdot 4}\\ &= \sqrt[5]{972}\end{aligned}$$

d) Write 4 as: $\sqrt[5]{4 \cdot 4 \cdot 4 \cdot 4 \cdot 4} = \sqrt[5]{1024}$

$$\begin{aligned}4\sqrt[5]{3} &= \sqrt[5]{1024} \cdot \sqrt[5]{3}\\ &= \sqrt[5]{1024 \cdot 3}\\ &= \sqrt[5]{3072}\end{aligned}$$

19. a) Use the Pythagorean Theorem in each right triangle, moving counterclockwise.
Each hypotenuse becomes a leg of the next triangle.

$$\begin{aligned}(h_1)^2 &= 1^2 + 1^2\\ (h_1)^2 &= 1 + 1\\ (h_1)^2 &= 2\\ h_1 &= \sqrt{2}\end{aligned}$$

$$\begin{aligned}(h_2)^2 &= (h_1)^2 + 1^2\\ (h_2)^2 &= \left(\sqrt{2}\right)^2 + 1\\ (h_2)^2 &= 2 + 1\\ (h_2)^2 &= 3\\ h_2 &= \sqrt{3}\end{aligned}$$

$(h_3)^2 = (h_2)^2 + 1^2$
$(h_3)^2 = \left(\sqrt{3}\right)^2 + 1$
$(h_3)^2 = 3 + 1$
$(h_3)^2 = 4$
$h_3 = \sqrt{4}$

The pattern continues. The lengths of the hypotenuses, in units, are:
$\sqrt{2}$, $\sqrt{3}$, $\sqrt{4}$, $\sqrt{5}$, $\sqrt{6}$, $\sqrt{7}$, $\sqrt{8}$, $\sqrt{9}$, $\sqrt{10}$, $\sqrt{11}$, $\sqrt{12}$, $\sqrt{13}$, $\sqrt{14}$

b) **i)** The radicand starts at 2 and increases by 1 each time.

ii) The 1st triangle has an hypotenuse of length $\sqrt{2}$ units.
The 2nd triangle has an hypotenuse of length $\sqrt{3}$ units.
The 3rd triangle has an hypotenuse of length $\sqrt{4}$ units.
The radicand is always 1 greater than the number of the triangle.
So, the 50th triangle has an hypotenuse of length $\sqrt{50+1}$, or $\sqrt{51}$ units.

iii) The radicands of the hypotenuse lengths of the first 100 triangles start at 2 and end at 101.
To be able to write an entire radical as a mixed radical, the radicand must be divisible by a perfect square, and not be a perfect square.
• The radicand could be divisible by 4, and not be a perfect square. So, list the multiples of 4 and cross out the perfect squares.
~~4~~, 8, 12, ~~16~~, 20, 24, 28, 32, ~~36~~, 40, 44, 48, 52, 56, 60, ~~64~~, 68, 72, 76, 80, 84, 88, 92, 96, ~~100~~
OR
• The radicand could be divisible by 9, and not be a perfect square. So, list the multiples of 9 and cross out the perfect squares.
~~9~~, 18, 27, ~~36~~, 45, 54, 63, 72, ~~81~~, 90, 99 (72 is in both lists, so it only counts once)
OR
• The radicand could be divisible by 25, and not be a perfect square. So, list the multiples of 25 and cross out the perfect squares.
~~25~~, 50, 75, ~~100~~
OR
• The radicand could be divisible by 49, and not be a perfect square. So, list the multiples of 49 and cross out the perfect squares.
~~49~~, 98
We do not have to find radicands that are divisible by 16, 36, and 64 because they are divisible by 4.
Count how many numbers have not been crossed out.
So, 30 of the first 100 triangles have hypotenuse lengths that can be written as mixed radicals.

20. In the second line, to write 8 as the cube root of a perfect cube, the student took the cube root of 8 instead of cubing 8.
A correct solution is:

$$\begin{aligned}8\sqrt[3]{2} &= 8 \cdot \sqrt[3]{2}\\ &= \sqrt[3]{512} \cdot \sqrt[3]{2}\\ &= \sqrt[3]{512 \cdot 2}\\ &= \sqrt[3]{1024}\end{aligned}$$

21. In the first line, the student wrote $\sqrt{96}$ as $\sqrt{4} \cdot \sqrt{48}$ instead of $\sqrt{4} \cdot \sqrt{24}$. In the third line, the student wrote $\sqrt{48}$ as $\sqrt{8} \cdot \sqrt{6}$, which does not help in the simplification because neither 8 nor 6 is a perfect square. In the fourth line, the student wrote $\sqrt{8}$ as 4, which is not correct; 8 is not a perfect square.
A correct solution is:
The factors of 96 are: 1, 2, 3, 4, 6, 8, 12, 16, 24, 32, 48, 96
The greatest perfect square is $16 = 4 \cdot 4$, so write 96 as $16 \cdot 6$.

$$\begin{aligned}\sqrt{96} &= \sqrt{16 \cdot 6}\\ &= \sqrt{16} \cdot \sqrt{6}\\ &= 4 \cdot \sqrt{6}\\ &= 4\sqrt{6}\end{aligned}$$

22. Since all the radicals are square roots, I will rewrite each mixed radical as an entire radical, then order the entire radicals from the greatest radicand to the least radicand.

a) Write 9 as: $\sqrt{9 \cdot 9} = \sqrt{81}$

$$\begin{aligned}9\sqrt{2} &= \sqrt{81} \cdot \sqrt{2}\\ &= \sqrt{81 \cdot 2}\\ &= \sqrt{162}\end{aligned}$$

Write 2 as: $\sqrt{2 \cdot 2} = \sqrt{4}$

$$\begin{aligned}2\sqrt{6} &= \sqrt{4} \cdot \sqrt{6}\\ &= \sqrt{4 \cdot 6}\\ &= \sqrt{24}\end{aligned}$$

Write 8 as: $\sqrt{8 \cdot 8} = \sqrt{64}$

$$\begin{aligned}8\sqrt{3} &= \sqrt{64} \cdot \sqrt{3}\\ &= \sqrt{64 \cdot 3}\\ &= \sqrt{192}\end{aligned}$$

Write 4 as: $\sqrt{4 \cdot 4} = \sqrt{16}$

$4\sqrt{5} = \sqrt{16} \cdot \sqrt{5}$
$= \sqrt{16 \cdot 5}$
$= \sqrt{80}$

Write 6 as: $\sqrt{6 \cdot 6} = \sqrt{36}$
$6\sqrt{2} = \sqrt{36} \cdot \sqrt{2}$
$= \sqrt{36 \cdot 2}$
$= \sqrt{72}$

From greatest to least, the entire radicals are: $\sqrt{192}$, $\sqrt{162}$, $\sqrt{80}$, $\sqrt{72}$, $\sqrt{24}$
So, from greatest to least, the mixed radicals are: $8\sqrt{3}$, $9\sqrt{2}$, $4\sqrt{5}$, $6\sqrt{2}$, $2\sqrt{6}$

b) Write 4 as: $\sqrt{4 \cdot 4} = \sqrt{16}$
$4\sqrt{7} = \sqrt{16} \cdot \sqrt{7}$
$= \sqrt{16 \cdot 7}$
$= \sqrt{112}$

Write 8 as: $\sqrt{8 \cdot 8} = \sqrt{64}$
$8\sqrt{3} = \sqrt{64} \cdot \sqrt{3}$
$= \sqrt{64 \cdot 3}$
$= \sqrt{192}$

Write 2 as: $\sqrt{2 \cdot 2} = \sqrt{4}$
$2\sqrt{13} = \sqrt{4} \cdot \sqrt{13}$
$= \sqrt{4 \cdot 13}$
$= \sqrt{52}$

Write 6 as: $\sqrt{6 \cdot 6} = \sqrt{36}$
$6\sqrt{5} = \sqrt{36} \cdot \sqrt{5}$
$= \sqrt{36 \cdot 5}$
$= \sqrt{180}$

From greatest to least, the entire radicals are: $\sqrt{192}$, $\sqrt{180}$, $\sqrt{112}$, $\sqrt{52}$
So, from greatest to least, the mixed radicals are: $8\sqrt{3}$, $6\sqrt{5}$, $4\sqrt{7}$, $2\sqrt{13}$

c) Write 7 as: $\sqrt{7 \cdot 7} = \sqrt{49}$

$$7\sqrt{3} = \sqrt{49} \cdot \sqrt{3}$$
$$= \sqrt{49 \cdot 3}$$
$$= \sqrt{147}$$

Write 9 as: $\sqrt{9 \cdot 9} = \sqrt{81}$

$$9\sqrt{2} = \sqrt{81} \cdot \sqrt{2}$$
$$= \sqrt{81 \cdot 2}$$
$$= \sqrt{162}$$

Write 5 as: $\sqrt{5 \cdot 5} = \sqrt{25}$

$$5\sqrt{6} = \sqrt{25} \cdot \sqrt{6}$$
$$= \sqrt{25 \cdot 6}$$
$$= \sqrt{150}$$

$\sqrt{103}$ is an entire radical.

Write 3 as: $\sqrt{3 \cdot 3} = \sqrt{9}$

$$3\sqrt{17} = \sqrt{9} \cdot \sqrt{17}$$
$$= \sqrt{9 \cdot 17}$$
$$= \sqrt{153}$$

From greatest to least, the entire radicals are: $\sqrt{162}$, $\sqrt{153}$, $\sqrt{150}$, $\sqrt{147}$, $\sqrt{103}$
So, from greatest to least, the radicals are: $9\sqrt{2}$, $3\sqrt{17}$, $5\sqrt{6}$, $7\sqrt{3}$, $\sqrt{103}$

23. a) I know $\sqrt{4} = 2$.

$$\sqrt{400} = \sqrt{4 \cdot 100}$$
$$= \sqrt{4} \cdot \sqrt{100}$$
$$= 2 \cdot 10$$
$$= 20$$
$$\sqrt{40\ 000} = \sqrt{4 \cdot 10\ 000}$$
$$= \sqrt{4} \cdot \sqrt{10\ 000}$$
$$= 2 \cdot 100$$
$$= 200$$

In the entire radicals, the radicand starts at 4 and is multiplied by 100 each time.
The square root starts at 2, and is multiplied by 10 each time, which is the square root of 100.
The next two radicals are:

$\sqrt{40\,000 \cdot 100} = \sqrt{4\,000\,000}$

$\sqrt{4\,000\,000 \cdot 100} = \sqrt{400\,000\,000}$

b) I know $\sqrt[3]{27} = 3$.

$$\begin{aligned}\sqrt[3]{27\,000} &= \sqrt[3]{27 \cdot 1000}\\ &= \sqrt[3]{27} \cdot \sqrt[3]{1000}\\ &= 3 \cdot 10\\ &= 30\end{aligned}$$

$$\begin{aligned}\sqrt[3]{27\,000\,000} &= \sqrt[3]{27 \cdot 1\,000\,000}\\ &= \sqrt[3]{27} \cdot \sqrt[3]{1\,000\,000}\\ &= 3 \cdot 100\\ &= 300\end{aligned}$$

In the entire radicals, the radicand starts at 27 and is multiplied by 1000 each time.
The cube root starts at 3, and is multiplied by 10 each time, which is the cube root of 1000.
The next two radicals are:

$\sqrt[3]{27\,000\,000 \cdot 1000} = \sqrt[3]{27\,000\,000\,000}$

$\sqrt[3]{27\,000\,000\,000 \cdot 1000} = \sqrt[3]{27\,000\,000\,000\,000}$

c)

$$\begin{aligned}\sqrt{8} &= \sqrt{4 \cdot 2}\\ &= \sqrt{4} \cdot \sqrt{2}\\ &= 2\sqrt{2}\end{aligned}$$

$$\begin{aligned}\sqrt{800} &= \sqrt{400 \cdot 2}\\ &= \sqrt{400} \cdot \sqrt{2}\\ &= 20\sqrt{2}\end{aligned}$$

$$\begin{aligned}\sqrt{80\,000} &= \sqrt{40\,000 \cdot 2}\\ &= \sqrt{40\,000} \cdot \sqrt{2}\\ &= 200\sqrt{2}\end{aligned}$$

In the entire radicals, the radicand starts at 8 and is multiplied by 100 each time.
In the mixed radicals, the number in front of the radical starts at 2, and is multiplied by 10 each time, which is the square root of 100. The radicand is always 2.
The next two radicals are:

$\sqrt{80\,000 \cdot 100} = \sqrt{8\,000\,000}$

$\sqrt{8\,000\,000 \cdot 100} = \sqrt{800\,000\,000}$

d)

$$\sqrt[3]{24} = \sqrt[3]{8 \cdot 3}$$
$$= \sqrt[3]{8} \cdot \sqrt[3]{3}$$
$$= 2\sqrt[3]{3}$$

$$\sqrt[3]{24\,000} = \sqrt[3]{8000 \cdot 3}$$
$$= \sqrt[3]{8000} \cdot \sqrt[3]{3}$$
$$= 20\sqrt[3]{3}$$

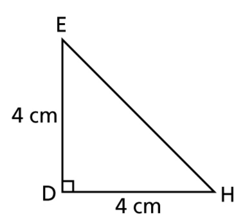
$$\sqrt[3]{24\,000\,000} = \sqrt[3]{8\,000\,000 \cdot 3}$$
$$= \sqrt[3]{8\,000\,000} \cdot \sqrt[3]{3}$$
$$= 200\sqrt[3]{3}$$

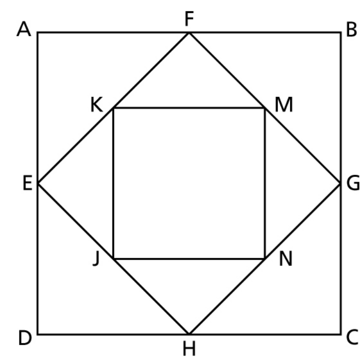
In the entire radicals, the radicand starts at 24 and is multiplied by 1000 each time.
In the mixed radicals, the number in front of the radical starts at 2, and is multiplied by 10 each time, which is the cube root of 1000.
The next two radicals are:

$$\sqrt[3]{24\,000\,000 \cdot 1000} = \sqrt[3]{24\,000\,000\,000}$$
$$\sqrt[3]{24\,000\,000\,000 \cdot 1000} = \sqrt[3]{24\,000\,000\,000\,000}$$

C

24. Label the vertices of the squares.





In ΔDEH, DE = DH

$$DE = \frac{1}{2}(DA)$$
$$= \frac{1}{2}(8)$$
$$= 4$$

Use the Pythagorean Theorem in right ΔDEH to determine EH, the side length of square EFGH.

$EH^2 = DE^2 + DH^2$
$EH^2 = 4^2 + 4^2$
$EH^2 = 16 + 16$
$EH^2 = 32$

$$EH = \sqrt{32}$$
$$= \sqrt{16 \cdot 2}$$
$$= \sqrt{16} \cdot \sqrt{2}$$
$$= 4\sqrt{2}$$

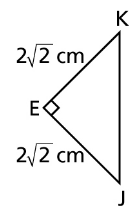
Square EFGH has side length $4\sqrt{2}$ cm.

Use the formula for the area, A, of a square with side length s:

$$
\begin{aligned}
A &= s^2 \\
&= \left(4\sqrt{2}\right)^2 \\
&= 4\sqrt{2} \cdot 4\sqrt{2} \\
&= 16 \cdot 2 \\
&= 32
\end{aligned}
$$

Square EFGH has area 32 cm^2.

In ΔJEK, JE = EK



$$
\begin{aligned}
JE &= \frac{1}{2}(EH) \\
&= \frac{1}{2}\left(4\sqrt{2}\right) \\
&= 2\sqrt{2}
\end{aligned}
$$

Use the Pythagorean Theorem in right ΔJEK to determine KJ, the side length of square JKMN.

$$
\begin{aligned}
KJ^2 &= JE^2 + EK^2 \\
KJ^2 &= \left(2\sqrt{2}\right)^2 + \left(2\sqrt{2}\right)^2 \\
KJ^2 &= (4 \cdot 2) + (4 \cdot 2) \\
KJ^2 &= 8 + 8 \\
KJ^2 &= 16 \\
KJ &= \sqrt{16} \\
KJ &= 4
\end{aligned}
$$

Square JKMN has side length 4 cm.

Use the formula for the area, A, of a square with side length s:

$$
\begin{aligned}
A &= s^2 \\
&= 4^2 \\
&= 16
\end{aligned}
$$

Square JKMN has area 16 cm^2.

25. a) i)

$$
\begin{aligned}
\sqrt{200} &= \sqrt{100 \cdot 2} \\
&= \sqrt{100} \cdot \sqrt{2} \\
&= 10\sqrt{2}
\end{aligned}
$$

Since $\sqrt{2} \doteq 1.4142$, $10\sqrt{2} \doteq 10(1.4142)$
$= 14.142$

So, $\sqrt{200} \doteq 14.142$

ii)
$$\begin{aligned}\sqrt{20\,000} &= \sqrt{10\,000 \cdot 2}\\ &= \sqrt{10\,000} \cdot \sqrt{2}\\ &= 100\sqrt{2}\end{aligned}$$

Since $\sqrt{2} \doteq 1.4142$, $100\sqrt{2} \doteq 100(1.4142)$
$= 141.42$

So, $\sqrt{20\,000} \doteq 141.42$

b) **i)**
$$\begin{aligned}\sqrt{8} &= \sqrt{4 \cdot 2}\\ &= \sqrt{4} \cdot \sqrt{2}\\ &= 2\sqrt{2}\end{aligned}$$

Since $\sqrt{2} \doteq 1.4142$, $2\sqrt{2} \doteq 2(1.4142)$
$= 2.8284$

So, $\sqrt{8} \doteq 2.8284$

ii)
$$\begin{aligned}\sqrt{18} &= \sqrt{9 \cdot 2}\\ &= \sqrt{9} \cdot \sqrt{2}\\ &= 3\sqrt{2}\end{aligned}$$

Since $\sqrt{2} \doteq 1.4142$, $3\sqrt{2} \doteq 3(1.4142)$
$= 4.2426$

So, $\sqrt{18} \doteq 4.2426$

iii)
$$\begin{aligned}\sqrt{32} &= \sqrt{16 \cdot 2}\\ &= \sqrt{16} \cdot \sqrt{2}\\ &= 4\sqrt{2}\end{aligned}$$

Since $\sqrt{2} \doteq 1.4142$, $4\sqrt{2} \doteq 4(1.4142)$
$= 5.6568$

So, $\sqrt{32} \doteq 5.6568$

iv)
$$\begin{aligned}\sqrt{50} &= \sqrt{25 \cdot 2}\\ &= \sqrt{25} \cdot \sqrt{2}\\ &= 5\sqrt{2}\end{aligned}$$

Since $\sqrt{2} \doteq 1.4142$, $5\sqrt{2} \doteq 5(1.4142)$

$$= 7.071$$

So, $\sqrt{50} \doteq 7.071$

Checkpoint 1 **Assess Your Understanding (page 221)**

4.1

1. a) $\sqrt{81} = \sqrt{9 \cdot 9}$
$= 9$

b) $\sqrt[3]{-125} = \sqrt[3]{(-5)(-5)(-5)}$
$= -5$

c) $\sqrt[4]{256} = \sqrt[4]{4 \cdot 4 \cdot 4 \cdot 4}$
$= 4$

d) $\sqrt[5]{243} = \sqrt[5]{3 \cdot 3 \cdot 3 \cdot 3 \cdot 3}$
$= 3$

I wrote the radicand as the product of the same number of equal factors as the index of the radical. For example, when the index was 3, I wrote the radicand as the product of 3 equal factors.

2. Instead of using the root keys on a calculator, I can use benchmarks with guess and check.

a) 10 is between the perfect squares 9 and 16, but closer to 9.
So, $\sqrt{10}$ is between 3 and 4, but closer to 3.
Estimate to 1 decimal place: $\sqrt{10} \doteq 3.1$
Square the estimate: $3.1^2 = 9.61$ (too small, but close)
Revise the estimate: $\sqrt{10} \doteq 3.2$
Square the estimate: $3.2^2 = 10.24$ (too large, but close)
$\sqrt{10}$ is between 3.1 and 3.2, but closer to 3.2.
Now, estimate to 2 decimal places.
Revise the estimate: $\sqrt{10} \doteq 3.16$
Square the estimate: $3.16^2 = 9.9856$ (too small, very close)
Revise the estimate: $\sqrt{10} \doteq 3.17$
Square the estimate: $3.17^2 = 10.0489$ (too large, very close)
9.9856 is closer to 10, so $\sqrt{10}$ is approximately 3.16.

b) 15 is between the perfect cubes 8 and 27, but closer to 8.
So, $\sqrt[3]{15}$ is between 2 and 3, but closer to 2.
Estimate to 1 decimal place: $\sqrt[3]{15} \doteq 2.4$
Cube the estimate: $2.4^3 = 13.824$ (too small)
Revise the estimate: $\sqrt[3]{15} \doteq 2.5$
Cube the estimate: $2.5^3 = 15.625$ (too large, but close)
$\sqrt[3]{15}$ is between 2.4 and 2.5, but closer to 2.5.
Now, estimate to 2 decimal places.

Revise the estimate: $\sqrt[3]{15} \doteq 2.46$

Cube the estimate: $2.46^3 = 14.886\ 936$ (too small, very close)

Revise the estimate: $\sqrt[3]{15} \doteq 2.47$

Cube the estimate: $2.47^3 = 15.069\ 223$ (too large, very close)

15.069 223 is closer to 15, so $\sqrt[3]{15}$ is approximately 2.47.

c) 9 is between the perfect fourth powers 1 and 16, but closer to 16.
So, $\sqrt[4]{9}$ is between 1 and 2, but closer to 2.

Estimate to 1 decimal place: $\sqrt[4]{9} \doteq 1.7$

Raise the estimate to the fourth power: $1.7^4 = 8.3521$ (too small)

Revise the estimate: $\sqrt[4]{9} \doteq 1.8$

Raise the estimate to the fourth power: $1.8^4 = 10.4976$ (too large)

$\sqrt[4]{9}$ is between 1.7 and 1.8, but closer to 1.7.
Now, estimate to 2 decimal places.
Revise the estimate: $\sqrt[4]{9} \doteq 1.73$

Raise the estimate to the fourth power: $1.73^4 = 8.957\ 450\ 41...$ (too small, very close)

Revise the estimate: $\sqrt[4]{9} \doteq 1.74$

Raise the estimate to the fourth power: $1.74^4 = 9.166\ 361\ 76...$ (too large, close)

8.957 450 41… is closer to 9, so $\sqrt[4]{9}$ is approximately 1.73.

d) 23 is between the perfect fifth powers 1 and 32, but closer to 32.
So, $\sqrt[5]{23}$ is between 1 and 2, but closer to 2.

Estimate to 1 decimal place: $\sqrt[5]{23} \doteq 1.8$

Raise the estimate to the fifth power: $1.8^5 = 18.895\ 68$ (too small)

Revise the estimate: $\sqrt[5]{23} \doteq 1.9$

Raise the estimate to the fifth power: $1.9^5 = 24.760\ 99$ (too large, but close)

$\sqrt[5]{23}$ is between 1.8 and 1.9, but closer to 1.9.
Now, estimate to 2 decimal places.
Revise the estimate: $\sqrt[5]{23} \doteq 1.88$

Raise the estimate to the fifth power: $1.88^5 = 23.484\ 928\ 72...$ (too large, close)

Revise the estimate: $\sqrt[5]{23} \doteq 1.87$

Raise the estimate to the fifth power: $1.87^5 = 22.866\ 938\ 97...$ (too small, very close)

22.866 938 97… is closer to 23, so $\sqrt[5]{23}$ is approximately 1.87.

3. I used my calculator. $\sqrt[4]{60} \doteq 2.783\ 157\ 684$
The decimal representation fills the calculator screen.
The decimal does not appear to repeat and it does not appear to terminate. The decimal representation appears to go on forever.

4.2

4. a) $\sqrt{11}$ is irrational because 11 is not a perfect square.
The decimal form of $\sqrt{11}$ neither terminates nor repeats.

b) $\sqrt[3]{16}$ is irrational because 16 is not a perfect cube.
The decimal form of $\sqrt[3]{16}$ neither terminates nor repeats.

c) $\sqrt[3]{-16}$ is irrational because –16 is not a perfect cube.
The decimal form of $\sqrt[3]{-16}$ neither terminates nor repeats.

d) $\sqrt{121}$ is rational because 121 is a perfect square.
Its decimal form is 11.0, which terminates.

e) $\sqrt{\frac{121}{16}}$ is rational because $\frac{121}{16}$ is a perfect square.
$\sqrt{\frac{121}{16}} = \frac{11}{4}$ or 2.75, which is a terminating decimal

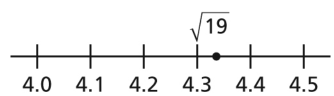
f) $\sqrt{12.1}$ is irrational because 12.1 is not a perfect square.
The decimal form of $\sqrt{12.1}$ neither terminates nor repeats.

5. I will use benchmarks to estimate the position of each irrational number, then use a calculator to refine the estimate.

a) 19 is between the perfect squares 16 and 25, and is closer to 16.

$\sqrt{16}$ → 4, $\sqrt{19}$ → ?, $\sqrt{25}$ → 5

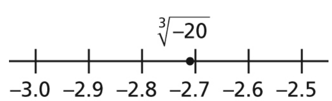
Use a calculator.
$\sqrt{19} = 4.3588\ldots$



b) –20 is between the perfect cubes –8 and –27, and is closer to –27.

$\sqrt[3]{-8}$ → –2, $\sqrt[3]{-20}$ → ?, $\sqrt[3]{-27}$ → –3

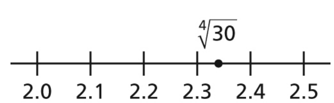
Use a calculator.
$\sqrt[3]{-20} = -2.7144\ldots$



c) 30 is between the perfect fourth powers 16 and 81, and is closer to 16.

$\sqrt[4]{16}$ → 2, $\sqrt[4]{30}$ → ?, $\sqrt[4]{81}$ → 3

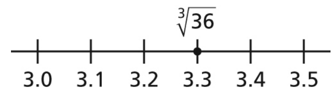
Use a calculator.



$\sqrt[4]{30} = 2.3403\ldots$

d) 36 is between the perfect cubes 27 and 64, and is closer to 27.

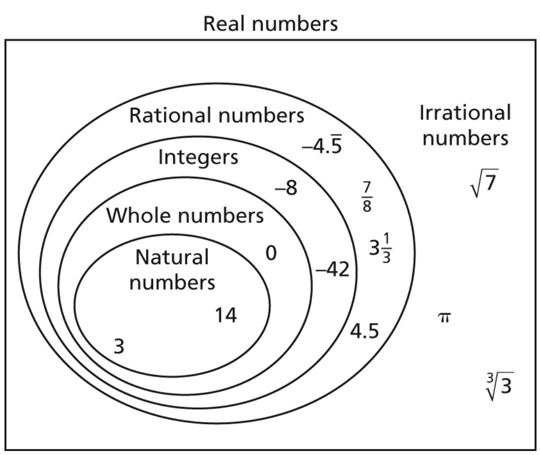
$\sqrt[3]{27}$ → 3 $\quad$ $\sqrt[3]{36}$ → ? $\quad$ $\sqrt[3]{64}$ → 4



Use a calculator.
$\sqrt[3]{36} = 3.3019\ldots$

6. a) Some numbers belong to more than one set.
Place each of these numbers in the smallest set to which it belongs.

i) $3\frac{1}{3}$ is a rational number because it can be written as the quotient of two integers: $\frac{10}{3}$
ii) –42 is an integer.
iii) 4.5 is a rational number because it is a terminating decimal.
iv) $-4.\overline{5}$ is a rational number because it is a repeating decimal.
v) 0 is a whole number.
vi) 14 is a natural number.
vii) $\sqrt{7}$ is an irrational number because 7 is not a perfect square.
viii) π is an irrational number because its decimal representation neither terminates nor repeats.



b) Answers may vary.
Natural number: I chose a counting number, 3.
Whole number: I cannot choose another whole number that is not a natural number.
Integer: I chose –8.
Rational number: I chose the fraction $\frac{7}{8}$ because it is the quotient of two integers.
Irrational number: I chose the cube root of a number that is not a perfect cube, $\sqrt[3]{3}$.

7. a) i) 32 is between the perfect squares 25 and 36, and is closer to 36.

$$\begin{array}{ccc} \sqrt{25} & \sqrt{32} & \sqrt{36} \\ \downarrow & \downarrow & \downarrow \\ 5 & ? & 6 \end{array}$$

Use a calculator.

$\sqrt{32} = 5.6568\ldots$

ii) 72 is between the perfect cubes 64 and 125, and is closer to 64.

$$\begin{array}{ccc} \sqrt[3]{64} & \sqrt[3]{72} & \sqrt[3]{125} \\ \downarrow & \downarrow & \downarrow \\ 4 & ? & 5 \end{array}$$

Use a calculator.

$\sqrt[3]{72} = 4.1601\ldots$

iii) 100 is between the perfect fourth powers 81 and 256, and is closer to 81.

$$\begin{array}{ccc} \sqrt[4]{81} & \sqrt[4]{100} & \sqrt[4]{256} \\ \downarrow & \downarrow & \downarrow \\ 3 & ? & 4 \end{array}$$

Use a calculator.

$\sqrt[4]{100} = 3.1622\ldots$

iv) 50 is between the perfect cubes 27 and 64, and is closer to 64.

$$\begin{array}{ccc} \sqrt[3]{27} & \sqrt[3]{50} & \sqrt[3]{64} \\ \downarrow & \downarrow & \downarrow \\ 3 & ? & 4 \end{array}$$

Use a calculator.

$\sqrt[3]{50} = 3.6840\ldots$

v) 65 is between the perfect squares 64 and 81, and is closer to 64.

$$\begin{array}{ccc} \sqrt{64} & \sqrt{65} & \sqrt{81} \\ \downarrow & \downarrow & \downarrow \\ 8 & ? & 9 \end{array}$$

Use a calculator.

$\sqrt{65} = 8.0622\ldots$

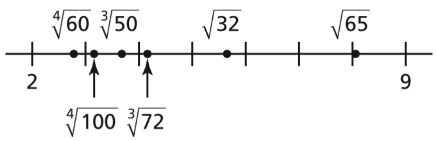
vi) 60 is between the perfect fourth powers 16 and 81, and is closer to 81.

$$\begin{array}{ccc} \sqrt[4]{16} & \sqrt[4]{60} & \sqrt[4]{81} \\ \downarrow & \downarrow & \downarrow \\ 2 & ? & 3 \end{array}$$

Use a calculator.

$\sqrt[4]{60} = 2.7831\ldots$

Mark each number on a number line.



b) From greatest to least, the numbers are: $\sqrt{65}$, $\sqrt{32}$, $\sqrt[3]{72}$, $\sqrt[3]{50}$, $\sqrt[4]{100}$, $\sqrt[4]{60}$

8. Answers may vary.

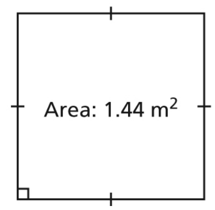
a) For the perimeter of the square to be rational,
the side length of the square must be rational.
Choose a rational number for the side length, 1.2 m.
Use the formula for the area, A, of a square
with side length s:

$A = s^2$ Substitute: $s = 1.2$

$A = 1.2^2$

$A = 1.44$

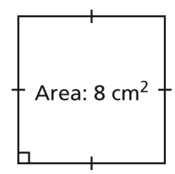
So, the square could have area 1.44 m^2.



b) For the perimeter of the square to be irrational,
the side length of the square must be irrational.
Since the side length, s, of a square with area A is:

$s = \sqrt{A}$, A cannot be a perfect square.

Since 8 is not a perfect square, the square could have area 8 cm^2.



4.3

9. a) The factors of 45 are: 1, 3, 5, 9, 15, 45
The greatest perfect square is $9 = 3 \cdot 3$, so write 45 as $9 \cdot 5$.

$$\begin{aligned}\sqrt{45} &= \sqrt{9 \cdot 5}\\ &= \sqrt{9} \cdot \sqrt{5}\\ &= 3 \cdot \sqrt{5}\\ &= 3\sqrt{5}\end{aligned}$$

b) The factors of 96 are: 1, 2, 3, 4, 6, 8, 12, 16, 24, 32, 48, 96
The greatest perfect cube is $8 = 2 \cdot 2 \cdot 2$, so write 96 as $8 \cdot 12$.

$$\begin{aligned}\sqrt[3]{96} &= \sqrt[3]{8 \cdot 12}\\ &= \sqrt[3]{8} \cdot \sqrt[3]{12}\\ &= 2 \cdot \sqrt[3]{12}\\ &= 2\sqrt[3]{12}\end{aligned}$$

c) The factors of 17 are: 1, 17
There are no perfect square factors other than 1.
So, $\sqrt{17}$ cannot be simplified.

d) The factors of 48 are: 1, 2, 3, 4, 6, 8, 12, 16, 24, 48
The greatest perfect fourth power is 16 = $2 \cdot 2 \cdot 2 \cdot 2$, so write 48 as $16 \cdot 3$.

$$\begin{aligned}\sqrt[4]{48} &= \sqrt[4]{16 \cdot 3} \\ &= \sqrt[4]{16} \cdot \sqrt[4]{3} \\ &= 2 \cdot \sqrt[4]{3} \\ &= 2\sqrt[4]{3}\end{aligned}$$

e) The factors of 80 are: 1, 2, 4, 5, 8, 10, 16, 20, 40, 80
The greatest perfect cube is 8 = $2 \cdot 2 \cdot 2$, so write 80 as $8 \cdot 10$.

$$\begin{aligned}\sqrt[3]{80} &= \sqrt[3]{8 \cdot 10} \\ &= \sqrt[3]{8} \cdot \sqrt[3]{10} \\ &= 2 \cdot \sqrt[3]{10} \\ &= 2\sqrt[3]{10}\end{aligned}$$

f) The factors of 50 are: 1, 2, 5, 10, 25, 50
There are no perfect fourth power factors other than 1.
So, $\sqrt[4]{50}$ cannot be simplified.

10. Answers may vary.
I chose $\sqrt{45}$.
List all the factors of 45.
Look for the greatest factor that is a perfect square.
Write the radicand as the product of two factors, one of which is the greatest perfect square factor.
Use the multiplication property of radicals to write the radical as the product of two radicals.
Evaluate the radical whose radicand is a perfect square.
The radical is now in simplest form.

11. a) Write 3 as: $\sqrt{3 \cdot 3} = \sqrt{9}$

$$\begin{aligned}3\sqrt{7} &= \sqrt{9} \cdot \sqrt{7} \\ &= \sqrt{9 \cdot 7} \\ &= \sqrt{63}\end{aligned}$$

b) Write 2 as: $\sqrt[3]{2 \cdot 2 \cdot 2} = \sqrt[3]{8}$

$$\begin{aligned}2\sqrt[3]{4} &= \sqrt[3]{8} \cdot \sqrt[3]{4} \\ &= \sqrt[3]{8 \cdot 4} \\ &= \sqrt[3]{32}\end{aligned}$$

c) Write 7 as: $\sqrt{7 \cdot 7} = \sqrt{49}$

$$7\sqrt{3} = \sqrt{49} \cdot \sqrt{3}$$
$$= \sqrt{49 \cdot 3}$$
$$= \sqrt{147}$$

d) Write 2 as: $\sqrt[4]{2 \cdot 2 \cdot 2 \cdot 2} = \sqrt[4]{16}$

$$2\sqrt[4]{12} = \sqrt[4]{16} \cdot \sqrt[4]{12}$$
$$= \sqrt[4]{16 \cdot 12}$$
$$= \sqrt[4]{192}$$

e) Write 3 as: $\sqrt[3]{3 \cdot 3 \cdot 3} = \sqrt[3]{27}$

$$3\sqrt[3]{10} = \sqrt[3]{27} \cdot \sqrt[3]{10}$$
$$= \sqrt[3]{27 \cdot 10}$$
$$= \sqrt[3]{270}$$

f) Write 6 as: $\sqrt{6 \cdot 6} = \sqrt{36}$

$$6\sqrt{11} = \sqrt{36} \cdot \sqrt{11}$$
$$= \sqrt{36 \cdot 11}$$
$$= \sqrt{396}$$

Lesson 4.4 Fractional Exponents and Radicals Exercises (pages 227–228)

A

3. The denominator of the exponent is the index of the radical.

a) $16^{\frac{1}{2}} = \sqrt{16}$
$= 4$

b) $36^{\frac{1}{2}} = \sqrt{36}$
$= 6$

c) $64^{\frac{1}{3}} = \sqrt[3]{64}$
$= 4$

d) $32^{\frac{1}{5}} = \sqrt[5]{32}$
$= 2$

e) $(-27)^{\frac{1}{3}} = \sqrt[3]{-27}$
$= -3$

f) $(-1000)^{\frac{1}{3}} = \sqrt[3]{-1000}$
$= -10$

4. Write each decimal exponent as a fraction.

a) $0.5 = \frac{1}{2}$

So, $100^{0.5} = 100^{\frac{1}{2}}$
$= \sqrt{100}$
$= 10$

b) $0.25 = \frac{1}{4}$

So, $81^{0.25} = 81^{\frac{1}{4}}$
$= \sqrt[4]{81}$
$= 3$

c) $0.2 = \frac{1}{5}$

So, $1024^{0.2} = 1024^{\frac{1}{5}}$
$= \sqrt[5]{1024}$
$= 4$

d) $0.2 = \frac{1}{5}$

So, $(-32)^{0.2} = (-32)^{\frac{1}{5}}$
$= \sqrt[5]{-32}$
$= -2$

5. Use the rule: $x^{\frac{1}{n}} = \sqrt[n]{x}$

a) $36^{\frac{1}{3}} = \sqrt[3]{36}$

b) $48^{\frac{1}{2}} = \sqrt{48}$

c) $(-30)^{\frac{1}{5}} = \sqrt[5]{-30}$

6. Use the rule: $\sqrt[n]{x} = x^{\frac{1}{n}}$

a) $\sqrt{39} = 39^{\frac{1}{2}}$

b) $\sqrt[4]{90} = 90^{\frac{1}{4}}$

c) $\sqrt[3]{29} = 29^{\frac{1}{3}}$

d) $\sqrt[5]{100} = 100^{\frac{1}{5}}$

7. a) Any number raised to the exponent 0 is 1.

$8^0 = 1$

b) $$\begin{aligned} 8^{\frac{1}{3}} &= \sqrt[3]{8} \\ &= 2 \end{aligned}$$

c) $$\begin{aligned} 8^{\frac{2}{3}} &= \left(8^{\frac{1}{3}}\right)^2 \\ &= \left(\sqrt[3]{8}\right)^2 \\ &= 2^2 \\ &= 4 \end{aligned}$$

d) $$\begin{aligned} 8^{\frac{3}{3}} &= 8^1 \\ &= 8 \end{aligned}$$

e) $$\begin{aligned} 8^{\frac{4}{3}} &= \left(8^{\frac{1}{3}}\right)^4 \\ &= \left(\sqrt[3]{8}\right)^4 \\ &= 2^4 \\ &= 16 \end{aligned}$$

f) $$\begin{aligned} 8^{\frac{5}{3}} &= \left(8^{\frac{1}{3}}\right)^5 \\ &= \left(\sqrt[3]{8}\right)^5 \\ &= 2^5 \\ &= 32 \end{aligned}$$

B

8. Use $a^{\frac{m}{n}} = \left(\sqrt[n]{a}\right)^m$ or $\sqrt[n]{a^m}$.

a) $4^{\frac{2}{3}} = \left(\sqrt[3]{4}\right)^2$ or $\sqrt[3]{4^2}$

b) $(-10)^{\frac{3}{5}} = \left(\sqrt[5]{-10}\right)^3$ or $\sqrt[5]{(-10)^3}$

c) $2.3^{\frac{3}{2}} = \left(\sqrt{2.3}\right)^3$ or $\sqrt{2.3^3}$

9. The formula for the volume, V, of a cube with edge length e units is:
$V = e^3$
To determine the value of e, take the cube root of each side.
$\sqrt[3]{V} = \sqrt[3]{e^3}$
$\sqrt[3]{V} = e$ Substitute: $V = 350$
$e = \sqrt[3]{350}$ or $e = 350^{\frac{1}{3}}$
The cube has edge length $\sqrt[3]{350}$ cm or $350^{\frac{1}{3}}$ cm.

10. Use $a^{\frac{m}{n}} = \left(\sqrt[n]{a}\right)^m$ or $\sqrt[n]{a^m}$.

a) $48^{\frac{2}{3}} = \left(\sqrt[3]{48}\right)^2$ or $\sqrt[3]{48^2}$

b) $(-1.8)^{\frac{5}{3}} = \left(\sqrt[3]{-1.8}\right)^5$ or $\sqrt[3]{(-1.8)^5}$

c) $2.5 = \frac{5}{2}$

So, $\left(\frac{3}{8}\right)^{2.5} = \left(\frac{3}{8}\right)^{\frac{5}{2}}$

$= \left(\sqrt{\frac{3}{8}}\right)^5$ or $\sqrt{\left(\frac{3}{8}\right)^5}$

d) $0.75 = \frac{3}{4}$

So, $0.75^{0.75} = (0.75)^{\frac{3}{4}}$

$= \left(\sqrt[4]{0.75}\right)^3$ or $\sqrt[4]{0.75^3}$

e) $\left(-\frac{5}{9}\right)^{\frac{2}{5}} = \left(\sqrt[5]{-\frac{5}{9}}\right)^2$ or $\sqrt[5]{\left(-\frac{5}{9}\right)^2}$

f) $1.5 = \frac{3}{2}$

So, $1.25^{1.5} = 1.25^{\frac{3}{2}}$

$= \left(\sqrt{1.25}\right)^3$ or $\sqrt{1.25^3}$

11. Use: $\sqrt[n]{a^m} = a^{\frac{m}{n}}$

a) $\sqrt{3.8^3} = 3.8^{\frac{3}{2}}$

Or, since $\frac{3}{2} = 1.5$, $\sqrt{3.8^3} = 3.8^{1.5}$

b) $\left(\sqrt[3]{-1.5}\right)^2 = (-1.5)^{\frac{2}{3}}$

c) $\sqrt[4]{\left(\frac{9}{5}\right)^5} = \left(\frac{9}{5}\right)^{\frac{5}{4}}$

Or, since $\frac{5}{4} = 1.25$, $\sqrt[4]{\left(\frac{9}{5}\right)^5} = \left(\frac{9}{5}\right)^{1.25}$

d) $\sqrt[3]{\left(\frac{3}{8}\right)^4} = \left(\frac{3}{8}\right)^{\frac{4}{3}}$

e) $\left(\sqrt{\frac{5}{4}}\right)^3 = \left(\frac{5}{4}\right)^{\frac{3}{2}}$

Or, since $\frac{3}{2} = 1.5$, $\left(\sqrt{\frac{5}{4}}\right)^3 = \left(\frac{5}{4}\right)^{1.5}$

f) $\sqrt[5]{(-2.5)^3} = (-2.5)^{\frac{3}{5}}$

Or, since $\frac{3}{5} = 0.6$, $\sqrt[5]{(-2.5)^3} = (-2.5)^{0.6}$

12. a) $9^{\frac{3}{2}} = \left(9^{\frac{1}{2}}\right)^3$

$= \left(\sqrt{9}\right)^3$

$= 3^3$

$= 27$

b) $\left(\frac{27}{8}\right)^{\frac{2}{3}} = \left[\left(\frac{27}{8}\right)^{\frac{1}{3}}\right]^2$

$= \left(\sqrt[3]{\frac{27}{8}}\right)^2$

$= \left(\frac{3}{2}\right)^2$

$= \frac{9}{4}$

c) $(-27)^{\frac{2}{3}} = \left[(-27)^{\frac{1}{3}}\right]^2$

$= \left(\sqrt[3]{-27}\right)^2$

$= (-3)^2$

$= 9$

d) The exponent $1.5 = \frac{3}{2}$

So, $0.36^{1.5} = 0.36^{\frac{3}{2}}$

$= \left(0.36^{\frac{1}{2}}\right)^3$

$= \left(\sqrt{0.36}\right)^3$

$= 0.6^3$

$= 0.216$

e) $(-64)^{\frac{2}{3}} = \left[(-64)^{\frac{1}{3}}\right]^2$

$= \left(\sqrt[3]{-64}\right)^2$

$= (-4)^2$

$= 16$

f) $\left(\frac{4}{25}\right)^{\frac{3}{2}} = \left[\left(\frac{4}{25}\right)^{\frac{1}{2}}\right]^3$

$= \left(\sqrt{\frac{4}{25}}\right)^3$

$= \left(\frac{2}{5}\right)^3$

$= \frac{8}{125}$

13. Raising a number to the exponent $\frac{1}{2}$ is equivalent to taking the square root of the number.

So, to write an equivalent form of each number using a power with exponent $\frac{1}{2}$, square the number, then write it as a square root.

a) $2^2 = 4$

So, $2 = 4^{\frac{1}{2}}$, or $\sqrt{4}$

b) $4^2 = 16$

So, $4 = 16^{\frac{1}{2}}$, or $\sqrt{16}$

c) $10^2 = 100$

So, $10 = 100^{\frac{1}{2}}$, or $\sqrt{100}$

d) $3^2 = 9$

So, $3 = 9^{\frac{1}{2}}$, or $\sqrt{9}$

e) $5^2 = 25$

So, $5 = 25^{\frac{1}{2}}$, or $\sqrt{25}$

14. Raising a number to the exponent $\frac{1}{3}$ is equivalent to taking the cube root of the number.

So, to write an equivalent form of each number using a power with exponent $\frac{1}{3}$, cube the number, then write it as a cube root.

a) $(-1)^3 = -1$

So, $-1 = (-1)^{\frac{1}{3}}$, or $\sqrt[3]{-1}$

b) $2^3 = 8$

So, $2 = 8^{\frac{1}{3}}$, or $\sqrt[3]{8}$

c) $3^3 = 27$

So, $3 = 27^{\frac{1}{3}}$, or $\sqrt[3]{27}$

d) $(-4)^3 = -64$

So, $-4 = (-64)^{\frac{1}{3}}$, or $\sqrt[3]{-64}$

e) $4^3 = 64$

So, $4 = 64^{\frac{1}{3}}$, or $\sqrt[3]{64}$

15. Since 4 is a perfect square, and $4^{\frac{3}{2}}$ and $\left(\frac{1}{4}\right)^{\frac{3}{2}}$ involve the square root of 4, I will evaluate these numbers without a calculator. I can evaluate 4^2 using mental math. Because 4 is not a perfect cube, I will use a calculator to evaluate $\sqrt[3]{4}$.

Use a calculator: $\sqrt[3]{4} = 1.5874...$

$$\begin{aligned} 4^{\frac{3}{2}} &= \left(4^{\frac{1}{2}}\right)^3 \\ &= \left(\sqrt{4}\right)^3 \\ &= 2^3 \\ &= 8 \end{aligned}$$

$4^2 = 16$

$$\begin{aligned} \left(\frac{1}{4}\right)^{\frac{3}{2}} &= \left[\left(\frac{1}{4}\right)^{\frac{1}{2}}\right]^3 \\ &= \left(\sqrt{\frac{1}{4}}\right)^3 \\ &= \left(\frac{1}{2}\right)^3 \\ &= \frac{1}{8} \end{aligned}$$

So, from least to greatest, the numbers are: $\left(\frac{1}{4}\right)^{\frac{3}{2}}$, $\sqrt[3]{4}$, $4^{\frac{3}{2}}$, 4^2

16. a) i) The exponent $1.5 = \frac{3}{2}$

$$\begin{aligned} \text{So, } 16^{1.5} &= 16^{\frac{3}{2}} \\ &= \left(16^{\frac{1}{2}}\right)^3 \\ &= \left(\sqrt{16}\right)^3 \\ &= 4^3 \\ &= 64 \end{aligned}$$

ii) The exponent $0.75 = \frac{3}{4}$

$$\begin{aligned}\text{So, } 81^{0.75} &= 81^{\frac{3}{4}} \\ &= \left(81^{\frac{1}{4}}\right)^3 \\ &= \left(\sqrt[4]{81}\right)^3 \\ &= 3^3 \\ &= 27\end{aligned}$$

iii) The exponent $0.8 = \frac{4}{5}$

$$\begin{aligned}\text{So, } (-32)^{0.8} &= (-32)^{\frac{4}{5}} \\ &= \left[(-32)^{\frac{1}{5}}\right]^4 \\ &= \left(\sqrt[5]{-32}\right)^4 \\ &= (-2)^4 \\ &= 16\end{aligned}$$

iv) The exponent $0.5 = \frac{1}{2}$

$$\begin{aligned}\text{So, } 35^{0.5} &= 35^{\frac{1}{2}} \\ &= \sqrt{35} \quad \text{Since 35 is not a perfect square, use a calculator.} \\ &= 5.9160...\end{aligned}$$

v) The exponent $1.5 = \frac{3}{2}$

$$\begin{aligned}\text{So, } 1.21^{1.5} &= 1.21^{\frac{3}{2}} \\ &= \left(1.21^{\frac{1}{2}}\right)^3 \\ &= \left(\sqrt{1.21}\right)^3 \\ &= 1.1^3 \\ &= 1.331\end{aligned}$$

vi) The exponent $0.6 = \frac{3}{5}$

So,

$$\left(\frac{3}{4}\right)^{0.6} = \left(\frac{3}{4}\right)^{\frac{3}{5}}$$

$$= \left(\sqrt[5]{\frac{3}{4}}\right)^3$$ Since $\frac{3}{4}$ is not a perfect fifth power, use a calculator.

$$= 0.8414...$$

b) I was able to evaluate the powers in parts i, ii, iii, and v without a calculator. I can tell before I evaluate because: in part i, 16 is a perfect square; in part ii, 81 is a perfect fourth power; in part iii, –32 is a perfect fifth power; and in part v, 1.21 is a perfect square.

17. Use the formula: $h = 35d^{\frac{2}{3}}$ Substitute: $d = 3.2$

$$h = 35(3.2)^{\frac{2}{3}}$$

$$= 35\left(\sqrt[3]{3.2}\right)^2$$ Since 3.2 is not a perfect cube, use a calculator.

$$= 76.0036...$$

A fir tree with base diameter 3.2 m is approximately 76 m tall.

18. The rule for a power with a rational exponent is:

$$x^{\frac{m}{n}} = \left(\sqrt[n]{x}\right)^m$$

In the first line, the student should have written $1.96^{\frac{3}{2}}$ as $\left(\sqrt{1.96}\right)^3$

The correct solution is:

$$1.96^{\frac{3}{2}} = \left(\sqrt{1.96}\right)^3$$

$$= (1.4)^3$$

$$= 2.744$$

19. Use the formula: $SA = 0.096m^{0.7}$ Substitute: $m = 40$

$$SA = 0.096(40)^{0.7}$$ Use a calculator.

$$= 1.2697\ldots$$

The surface area of a child with mass 40 kg is approximately 1.3 m^2.

20. a) Use the expression: $100(0.5)^{\frac{n}{5}}$

Substitute: $n = \frac{1}{2}$

$$\begin{aligned} 100(0.5)^{\frac{n}{5}} &= 100(0.5)^{\frac{\frac{1}{2}}{5}} \\ &= 100(0.5)^{\frac{1}{2}\cdot\frac{1}{5}} \\ &= 100(0.5)^{\frac{1}{10}} \quad \text{Use a calculator.} \\ &= 93.3032... \end{aligned}$$

After $\frac{1}{2}$ h, approximately 93% of caffeine remains in your body.

Use a calculator to evaluate the expression: $100(0.87)^{\frac{1}{2}} = 93.2737...$

After $\frac{1}{2}$ h, approximately 93% of caffeine remains in your body.

Both expressions give the same result, to the nearest whole number.

b) Use the expression: $100(0.5)^{\frac{n}{5}}$

Substitute: $n = 1.5$

$$\begin{aligned} 100(0.5)^{\frac{n}{5}} &= 100(0.5)^{\frac{1.5}{5}} \\ &= 100(0.5)^{0.3} \quad \text{Use a calculator.} \\ &= 81.2252... \end{aligned}$$

After 1.5 h, approximately 81% of caffeine remains in your body.

c) Use the expression: $100(0.5)^{\frac{n}{5}}$

Equate the expression to 50, the percent of caffeine that remains.

$$100(0.5)^{\frac{n}{5}} = 50 \quad \text{Solve for } n\text{. Divide both sides by 100.}$$

$$0.5^{\frac{n}{5}} = \frac{50}{100}$$

$$0.5^{\frac{n}{5}} = 0.5$$

$$0.5^{\frac{n}{5}} = 0.5^{1} \quad \text{Equate the exponents.}$$

$$\frac{n}{5} = 1$$

$$n = 5$$

So, after 5 h, 50% of caffeine remains in your body.

21. Use the formula: $T \doteq 0.2R^{\frac{3}{2}}$

To calculate the period for Earth, substitute: $R = 149$

$T \doteq 0.2(149)^{\frac{3}{2}}$ Use a calculator.

$\doteq 363.7553...$

It takes approximately 363.8 Earth days for Earth to orbit the sun.

To calculate the period for Mars, substitute: $R = 228$

$T \doteq 0.2(228)^{\frac{3}{2}}$ Use a calculator.

$\doteq 688.5449...$

It takes approximately 688.5 Earth days for Mars to orbit the sun.
So, Mars has the longer period.

C

22. Karen is correct. You can only multiply a number by itself a whole number of times.

Lesson 4.5 **Negative Exponents and Reciprocals** **Exercises (pages 233–234)**

A

3. **a)** Use the rule: $\frac{1}{x^n} = x^{-n}$

$\frac{1}{5^4} = 5^{-4}$

b) To write with a positive exponent, write the reciprocal of the fraction.

$\left(-\frac{1}{2}\right)^{-3} = (-2)^3$

c) Use the rule: $\frac{1}{x^{-n}} = x^n$

$\frac{1}{3^{-2}} = 3^2$

d) Use the rule: $\frac{1}{x^{-n}} = x^n$

$\frac{1}{4^{-2}} = 4^2$

4. Use: $x^{-n} = \frac{1}{x^n}$

a) $4^2 = 16$

So, $4^{-2} = \frac{1}{4^2}$

$= \frac{1}{16}$

b) $2^4 = 16$

So, $2^{-4} = \frac{1}{2^4}$

$= \frac{1}{16}$

c) $6^1 = 6$

So, $6^{-1} = \frac{1}{6^1}$

$= \frac{1}{6}$

d) $4^3 = 64$

So, $4^{-3} = \frac{1}{4^3}$

$= \frac{1}{64}$

Each pair of answers involves the same numbers. The number in the numerator of one answer is the number in the denominator of the other answer, and the number in the denominator of one answer is the number in the numerator of the other answer. The answers in each pair are different in that they are reciprocals.

5. Use: $x^{-n} = \frac{1}{x^n}$

Since $2^{10} = 1024$,

then $2^{-10} = \frac{1}{2^{10}}$

$= \frac{1}{1024}$

6. Use: $x^{-n} = \frac{1}{x^n}$

a) $2^{-3} = \frac{1}{2^3}$

b) $3^{-5} = \frac{1}{3^5}$

c) $(-7)^{-2} = \frac{1}{(-7)^2}$

Since the square of a negative number is positive, this can be written as:

$(-7)^{-2} = \frac{1}{7^2}$

7. To write each power with a positive exponent, write the reciprocal of the fraction.

a) $\left(\frac{1}{2}\right)^{-2} = 2^2$

b) $\left(\frac{2}{3}\right)^{-3} = \left(\frac{3}{2}\right)^3$

c) $\left(-\frac{6}{5}\right)^{-4} = \left(-\frac{5}{6}\right)^4$

Since a negative number raised to the fourth power is positive, this can be written as:

$\left(-\frac{6}{5}\right)^{-4} = \left(\frac{5}{6}\right)^4$

8. **a)** $3^{-2} = \frac{1}{3^2}$

$= \frac{1}{9}$

b) $2^{-4} = \frac{1}{2^4}$

$= \frac{1}{16}$

c) $(-2)^{-5} = \frac{1}{(-2)^5}$

$= \frac{1}{-32}$

$= -\frac{1}{32}$

d) $\left(\frac{1}{3}\right)^{-3} = \left(\frac{3}{1}\right)^3$

$= 3^3$

$= 27$

e) $\left(-\frac{2}{3}\right)^{-2} = \left(-\frac{3}{2}\right)^2$

$= \frac{9}{4}$

f) $\frac{1}{5^{-3}} = 5^3$

$= 125$

B

9. **a)** $4^{-\frac{1}{2}} = \frac{1}{4^{\frac{1}{2}}}$ Write with a positive exponent.

$= \frac{1}{\sqrt{4}}$ Take the square root.

$= \frac{1}{2}$

b) Write 0.09 as a fraction: $\frac{9}{100}$

Then,

$$0.09^{-\frac{1}{2}} = \left(\frac{9}{100}\right)^{-\frac{1}{2}}$$

$$= \left(\frac{100}{9}\right)^{\frac{1}{2}}$$ Write with a positive exponent.

$$= \sqrt{\frac{100}{9}}$$ Take the square root.

$$= \frac{10}{3}$$

c) $$27^{-\frac{1}{3}} = \frac{1}{27^{\frac{1}{3}}}$$ Write with a positive exponent.

$$= \frac{1}{\sqrt[3]{27}}$$ Take the cube root.

$$= \frac{1}{3}$$

d) $$(-64)^{-\frac{1}{3}} = \frac{1}{(-64)^{\frac{1}{3}}}$$ Write with a positive exponent.

$$= \frac{1}{\sqrt[3]{-64}}$$ Take the cube root.

$$= \frac{1}{-4}$$

$$= -\frac{1}{4}$$

e) Write –0.027 as a fraction: $-\frac{27}{1000}$

Then,

$$(-0.027)^{-\frac{2}{3}} = \left(-\frac{27}{1000}\right)^{-\frac{2}{3}}$$

$$= \left(-\frac{1000}{27}\right)^{\frac{2}{3}}$$ Write with a positive exponent.

$$= \left(\sqrt[3]{-\frac{1000}{27}}\right)^{2}$$ Take the cube root.

$$= \left(-\frac{10}{3}\right)^{2}$$ Square the result.

$$= \frac{100}{9}$$

f) $$32^{-\frac{2}{5}} = \frac{1}{32^{\frac{2}{5}}}$$ Write with a positive exponent.

$$= \frac{1}{\left(\sqrt[5]{32}\right)^{2}}$$ Take the fifth root.

$$= \frac{1}{2^{2}}$$ Square the result.

$$= \frac{1}{4}$$

g) $$9^{-\frac{3}{2}} = \frac{1}{9^{\frac{3}{2}}}$$ Write with a positive exponent.

$$= \frac{1}{\left(\sqrt{9}\right)^{3}}$$ Take the square root.

$$= \frac{1}{3^{3}}$$ Cube the result.

$$= \frac{1}{27}$$

h) Write 0.04 as a fraction: $\frac{4}{100}$

Then,

$$\begin{aligned}
0.04^{-\frac{3}{2}} &= \left(\frac{4}{100}\right)^{-\frac{3}{2}} \\
&= \left(\frac{100}{4}\right)^{\frac{3}{2}} && \text{Write with a positive exponent.} \\
&= \left(\sqrt{\frac{100}{4}}\right)^{3} && \text{Take the square root.} \\
&= \left(\frac{10}{2}\right)^{3} && \text{Cube the result.} \\
&= \frac{1000}{8} \\
&= 125
\end{aligned}$$

10. Answers may vary.

a) $\frac{1}{9} = \frac{1}{3^2}$ Use: $\frac{1}{x^n} = x^{-n}$

$= 3^{-2}$

b) $\frac{1}{5} = \frac{1}{\sqrt{25}}$

$= \frac{1}{25^{\frac{1}{2}}}$ Use: $\frac{1}{x^n} = x^{-n}$

$= 25^{-\frac{1}{2}}$

c) $4 = 2^2$

$= \left(\frac{2}{1}\right)^2$ To write with a negative exponent, write the reciprocal of the fraction.

$= \left(\frac{1}{2}\right)^{-2}$

d) $-3 = \sqrt[3]{-27}$

$= (-27)^{\frac{1}{3}}$

$= \left(\frac{-27}{1}\right)^{\frac{1}{3}}$ To write with a negative exponent, write the reciprocal of the fraction.

$= \left(\frac{1}{-27}\right)^{-\frac{1}{3}}$

11. Use a calculator to evaluate:

$P = 3000(1.025)^{-5}$

$= 2651.5628\ldots$

To have $3000 in 5 years, $2651.56 must be invested now.

12. In the first line of the solution, to write the power with a positive exponent, the student wrote the fraction inside the brackets as a positive fraction instead of writing the reciprocal of the fraction.

A correct solution is:

$$\left(-\frac{64}{125}\right)^{-\frac{5}{3}} = \left(-\frac{125}{64}\right)^{\frac{5}{3}}$$
$$= \left(\sqrt[3]{-\frac{125}{64}}\right)^{5}$$
$$= \left(-\frac{5}{4}\right)^{5}$$
$$= -\frac{3125}{1024}$$

13. a) $27^{-\frac{4}{3}} = \frac{1}{27^{\frac{4}{3}}}$ Write with a positive exponent.

$= \frac{1}{\left(\sqrt[3]{27}\right)^{4}}$ Take the cube root.

$= \frac{1}{3^4}$ Raise the result to the fourth power.

$= \frac{1}{81}$

b) The exponent $-1.5 = -\frac{3}{2}$

$$\begin{aligned} 16^{-1.5} &= 16^{-\frac{3}{2}} \\ &= \frac{1}{16^{\frac{3}{2}}} && \text{Write with a positive exponent.} \\ &= \frac{1}{\left(\sqrt{16}\right)^3} && \text{Take the square root.} \\ &= \frac{1}{4^3} && \text{Cube the result.} \\ &= \frac{1}{64} \end{aligned}$$

c) The exponent $-0.4 = -\frac{4}{10}$, or $-\frac{2}{5}$

$$\begin{aligned} 32^{-0.4} &= 32^{-\frac{2}{5}} \\ &= \frac{1}{32^{\frac{2}{5}}} && \text{Write with a positive exponent.} \\ &= \frac{1}{\left(\sqrt[5]{32}\right)^2} && \text{Take the fifth root.} \\ &= \frac{1}{2^2} && \text{Square the result.} \\ &= \frac{1}{4} \end{aligned}$$

d)

$$\begin{aligned} \left(-\frac{8}{27}\right)^{-\frac{2}{3}} &= \left(-\frac{27}{8}\right)^{\frac{2}{3}} && \text{Write with a positive exponent.} \\ &= \left(\sqrt[3]{-\frac{27}{8}}\right)^2 && \text{Take the cube root.} \\ &= \left(-\frac{3}{2}\right)^2 && \text{Square the result.} \\ &= \frac{9}{4} \end{aligned}$$

e) $\left(\frac{81}{16}\right)^{-\frac{3}{4}} = \left(\frac{16}{81}\right)^{\frac{3}{4}}$ Write with a positive exponent.

$= \left(\sqrt[4]{\frac{16}{81}}\right)^3$ Take the fourth root.

$= \left(\frac{2}{3}\right)^3$ Cube the result.

$= \frac{8}{27}$

f) $\left(\frac{9}{4}\right)^{-\frac{5}{2}} = \left(\frac{4}{9}\right)^{\frac{5}{2}}$ Write with a positive exponent.

$= \left(\sqrt{\frac{4}{9}}\right)^5$ Take the square root.

$= \left(\frac{2}{3}\right)^5$ Raise the result to the fifth power.

$= \frac{32}{243}$

14. Use a calculator to evaluate:

$P = \frac{150\left[1 - 1.032^{-10}\right]}{0.032}$

$= 1266.5690\ldots$

Michelle must invest $1266.57 on January 1st.

15. Use the formula: $I = 100d^{-2}$

Substitute: $d = 23$

$I = 100(23)^{-2}$ Use a calculator.

$= 0.1890\ldots$

The intensity of the light 23 cm from the source is approximately 0.19%.

16. Write each power with a positive exponent.

$2^{-5} = \frac{1}{2^5}$

$= \frac{1}{32}$

$5^{-2} = \frac{1}{5^2}$

$= \frac{1}{25}$

Since $\frac{1}{25} > \frac{1}{32}$, 5^{-2} is greater.

17. a) The numbers at the left are divided by 2 each time.
The exponents in the powers at the right decrease by 1 each time.

b)

$16 = 2^4$

$8 = 2^3$

$4 = 2^2$

$2 = 2^1$

$1 = 2^0$

$\frac{1}{2} = 2^{-1}$

$\frac{1}{4} = 2^{-2}$

$\frac{1}{8} = 2^{-3}$

c) Look at the last 3 rows.

$$2^{-1} = \frac{1}{2} = \frac{1}{2^1} \qquad 2^{-2} = \frac{1}{4} = \frac{1}{2^2} \qquad 2^{-3} = \frac{1}{8} = \frac{1}{2^3}$$

So, this pattern shows that $a^{-n} = \frac{1}{a^n}$.

18. $3^3 = 27$ $\qquad$ $3^{-5} = \frac{1}{3^5} = \frac{1}{243}$

Divide the greater number by the smaller number:

$$\frac{3^3}{3^{-5}} = \frac{27}{\frac{1}{243}} = 27\left(\frac{243}{1}\right) = 6561$$

So, 3^3 is 6561 times as great as 3^{-5}.
Or, divide the powers:

$$\frac{3^3}{3^{-5}} = 3^3 \cdot \frac{1}{3^{-5}} = 3^3 \cdot 3^5 = 3^8$$

So, 3^3 is 3^8 times as great as 3^{-5}.

19. Make an organized list to test powers of 3.

Power of 3	Value
3^3	27
3^2	9
3^1	3
3^0	1
3^{-1}	$\frac{1}{3}$
3^{-2}	$\frac{1}{3^2}$, or $\frac{1}{9}$
3^{-3}	$\frac{1}{3^3}$, or $\frac{1}{27}$

a) From the table, when the exponent is greater than 0, the value is greater than 1. So, when the sign of the exponent is positive, $3^x > 1$.

b) From the table, when the exponent is less than 0, the value is less than 1.
So, when the sign of the exponent is negative, $3^x < 1$.

c) From the table, when the exponent is 0, the value is 1.
So, when the exponent is 0, $3^x = 1$.

C

20. Try different bases and different negative exponents.

$$8^{-\frac{1}{3}} = \left(\frac{1}{8}\right)^{\frac{1}{3}} = \frac{1}{2}$$

$$(-4)^{-2} = \frac{1}{(-4)^2} = \frac{1}{16}$$

$$\left(\frac{1}{4}\right)^{-\frac{1}{2}} = 4^{\frac{1}{2}} = 2$$

$$\left(-\frac{1}{2}\right)^{-2} = (-2)^2 = 4$$

The values of $\left(\frac{1}{4}\right)^{-\frac{1}{2}}$ and $\left(-\frac{1}{2}\right)^{-2}$ are greater than 1.

So, when a number is raised to a negative exponent,
the value of the power is not always less than 1.

21. a) Use the formula: $F = \left(6.67 \times 10^{-11}\right)Mmr^{-2}$

Write the distance between Earth and the moon in metres:
382 260 km = 382 260 000 m
Substitute: $M = 5.9736 \times 10^{24}$, $m = 7.349 \times 10^{22}$, and $r = 382\ 260\ 000$

$$F = \left(6.67 \times 10^{-11}\right)\left(5.9736 \times 10^{24}\right)\left(7.349 \times 10^{22}\right)\left(382\ 260\ 000\right)^{-2}$$

Use a calculator.
$F = 2.0038\ldots \times 10^{20}$
The gravitational force between Earth and the moon is about 2.0×10^{20} N.

b) Answers may vary. They depend on the diameters of Earth and the moon.
The diameter of Earth is about 12 756 km.
The diameter of the moon is about 3475 km.
So, the distance between the centres of Earth and the moon is:

$382\ 260 \text{ km} + \frac{1}{2}$ diameter of Earth $+ \frac{1}{2}$ diameter of the moon

$= 382\ 260 \text{ km} + \frac{1}{2}(12\ 756 \text{ km}) + \frac{1}{2}(3475 \text{ km})$

$= 382\ 260 \text{ km} + 6378 \text{ km} + 1737.5 \text{ km}$

$= 390\ 375.5 \text{ km}$

Write this distance in metres: 390 375.5 km = 390 375 500 m

Use the formula: $F = \left(6.67 \times 10^{-11}\right)Mmr^{-2}$

Substitute: $M = 5.9736 \times 10^{24}$, $m = 7.349 \times 10^{22}$, and $r = 390\ 375\ 500$

$F = \left(6.67 \times 10^{-11}\right)\left(5.9736 \times 10^{24}\right)\left(7.349 \times 10^{22}\right)\left(390\ 375\ 500\right)^{-2}$

Use a calculator.

$F = 1.9214\ldots \times 10^{20}$

The gravitational force between Earth and the moon is about 1.9×10^{20} N.

Checkpoint 2 — Assess Your Understanding (page 236)

4.4

1. a) $16^{\frac{1}{4}} = \sqrt[4]{16}$
$= 2$

b) $0.5 = \frac{1}{2}$

So, $49^{0.5} = 49^{\frac{1}{2}}$
$= \sqrt{49}$
$= 7$

c) $(-64)^{\frac{2}{3}} = \left(\sqrt[3]{-64}\right)^2$
$= (-4)^2$
$= 16$

d) $1.5 = \frac{3}{2}$

So, $\left(\frac{49}{9}\right)^{1.5} = \left(\frac{49}{9}\right)^{\frac{3}{2}}$
$= \left(\sqrt{\frac{49}{9}}\right)^3$
$= \left(\frac{7}{3}\right)^3$
$= \frac{343}{27}$

e) $(-8)^{\frac{5}{3}} = \left(\sqrt[3]{-8}\right)^5$
$= (-2)^5$
$= -32$

2. a) Use $a^{\frac{m}{n}} = \left(\sqrt[n]{a}\right)^m$ or $\sqrt[n]{a^m}$.

i) $35^{\frac{2}{3}} = \left(\sqrt[3]{35}\right)^2$ or $\sqrt[3]{35^2}$

ii) $32^{\frac{3}{2}} = \left(\sqrt{32}\right)^3$ or $\sqrt{32^3}$

iii) $(-32)^{\frac{2}{5}} = \left(\sqrt[5]{-32}\right)^2$ or $\sqrt[5]{(-32)^2}$

iv) $1.5 = \frac{3}{2}$

So, $400^{1.5} = 400^{\frac{3}{2}}$

$= \left(\sqrt{400}\right)^3$ or $\sqrt{400^3}$

v) $(-125)^{\frac{1}{3}} = \sqrt[3]{-125}$

vi) $\left(\frac{8}{125}\right)^{\frac{2}{3}} = \left(\sqrt[3]{\frac{8}{125}}\right)^2$ or $\sqrt[3]{\left(\frac{8}{125}\right)^2}$

b) i) $35^{\frac{2}{3}} = \left(\sqrt[3]{35}\right)^2$

I cannot evaluate this radical without using a calculator because 35 is not a perfect cube.

ii) $32^{\frac{3}{2}} = \left(\sqrt{32}\right)^3$

I cannot evaluate this radical without using a calculator because 32 is not a perfect square.

iii) $(-32)^{\frac{2}{5}} = \left(\sqrt[5]{-32}\right)^2$

$= (-2)^2$

$= 4$

iv) $400^{1.5} = \left(\sqrt{400}\right)^3$
$= (20)^3$
$= 8000$

v) $(-125)^{\frac{1}{3}} = \sqrt[3]{-125}$
$= -5$

vi) $\left(\frac{8}{125}\right)^{\frac{2}{3}} = \left(\sqrt[3]{\frac{8}{125}}\right)^2$
$= \left(\frac{2}{5}\right)^2$
$= \frac{4}{25}$

3. Use $\sqrt[n]{a^m} = a^{\frac{m}{n}}$ or $\left(\sqrt[n]{a}\right)^m = a^{\frac{m}{n}}$.

a) $\sqrt[3]{4} = 4^{\frac{1}{3}}$

b) $\sqrt{9} = 9^{\frac{1}{2}}$

Or, since $\frac{1}{2} = 0.5$, $\sqrt{9} = 9^{0.5}$

c) $\sqrt[4]{18} = 18^{\frac{1}{4}}$

Or, since $\frac{1}{4} = 0.25$, $\sqrt[4]{18} = 18^{0.25}$

d) $\left(\sqrt{10}\right)^3 = 10^{\frac{3}{2}}$

Or, since $\frac{3}{2} = 1.5$, $\left(\sqrt{10}\right)^3 = 10^{1.5}$

e) $\left(\sqrt[3]{-10}\right)^2 = (-10)^{\frac{2}{3}}$

4. Use the formula: $T \doteq 17.4m^{\frac{1}{4}}$ Substitute: $m = 85$

$T \doteq 17.4(85)^{\frac{1}{4}}$ Since 85 is not a perfect fourth power, use a calculator.
$= 52.8328...$

The circulation time for a mammal with mass 85 kg is about 53 s.

5. Since every base is 3, write each number with a fractional exponent.

$3^{\frac{3}{2}}$

$\sqrt[3]{3} = 3^{\frac{1}{3}}$

$\left(\sqrt{3}\right)^5 = 3^{\frac{5}{2}}$

$3^{\frac{2}{3}}$

$\left(\sqrt[3]{3}\right)^4 = 3^{\frac{4}{3}}$

Order the exponents from least to greatest: $\frac{1}{3}, \frac{2}{3}, \frac{4}{3}, \frac{3}{2}, \frac{5}{2}$

So, from least to greatest, the numbers are: $\sqrt[3]{3}$, $3^{\frac{2}{3}}$, $\left(\sqrt[3]{3}\right)^4$, $3^{\frac{3}{2}}$, $\left(\sqrt{3}\right)^5$

6. The formula for the volume, V, of a cube with edge length e units is:
$V = e^3$
To determine the value of e, take the cube root of each side.

$\sqrt[3]{V} = \sqrt[3]{e^3}$

$\sqrt[3]{V} = e$ Substitute: $V = 421\ 875$

$e = \sqrt[3]{421\,875}$ or $e = 421\,875^{\frac{1}{3}}$

The cube has edge length $\sqrt[3]{421\,875}$ mm or $421\,875^{\frac{1}{3}}$ mm.

Use a calculator to determine the edge length: $\sqrt[3]{421\,875} = 75$

The cube has edge length 75 mm.

4.5

7. a) $\left(\frac{2}{3}\right)^{-4} = \left(\frac{3}{2}\right)^{4}$ Write with a positive exponent.

$= \frac{81}{16}$

b) Write 0.5 as a fraction: $\frac{1}{2}$

Then,

$0.5^{-2} = \left(\frac{1}{2}\right)^{-2}$

$= \left(\frac{2}{1}\right)^{2}$

$= 4$

c) $(-1000)^{-\frac{2}{3}} = \left(\frac{1}{-1000}\right)^{\frac{2}{3}}$ Write with a positive exponent.

$= \left(\sqrt[3]{\frac{1}{-1000}}\right)^{2}$ Take the cube root.

$= \left(-\frac{1}{10}\right)^{2}$ Square the result.

$= \frac{1}{100}$

d) $\left(\frac{1}{4}\right)^{-\frac{1}{2}} = 4^{\frac{1}{2}}$ Write with a positive exponent.

$= \sqrt{4}$ Take the square root.

$= 2$

e) $\left(\frac{1}{10}\right)^{-2} = 10^{2}$ Write with a positive exponent.

$= 100$

f) Write –0.008 as a fraction: $-\frac{8}{1000}$

Then,

$$(-0.008)^{-\frac{4}{3}} = \left(-\frac{8}{1000}\right)^{-\frac{4}{3}}$$

$$= \left(-\frac{1000}{8}\right)^{\frac{4}{3}}$$ Write with a positive exponent.

$$= \left(\sqrt[3]{-\frac{1000}{8}}\right)^{4}$$ Take the cube root.

$$= \left(-\frac{10}{2}\right)^{4}$$

$$= (-5)^{4}$$ Raise the result to the fourth power.

$$= 625$$

8. Use a calculator to evaluate:

$$P = 5000(1.029)^{-3}$$
$$= 4589.0615\ldots$$

To have \$5000 in 3 years, \$4589.06 must be invested now.

Lesson 4.6 Applying the Exponent Laws Exercises (pages 241–243)

A

3. Use the product of powers law:
When the bases are the same, add the exponents.

a) $x^3 \cdot x^4 = x^{3+4}$
$= x^7$

b) $a^2 \cdot a^{-5} = a^{2+(-5)}$
$= a^{-3}$ Write with a positive exponent.
$= \dfrac{1}{a^3}$

c) $b^{-3} \cdot b^5 = b^{-3+5}$
$= b^2$

d) $m^2 \cdot m^{-3} = m^{2+(-3)}$
$= m^{-1}$ Write with a positive exponent.
$= \dfrac{1}{m}$

4. a) $0.5^2 \cdot 0.5^3$
Use the product of powers law:
When the bases are the same, add the exponents.
$0.5^2 \cdot 0.5^3 = 0.5^{2+3}$
$= 0.5^5$

b) $0.5^2 \cdot 0.5^{-3}$
Use the product of powers law:
When the bases are the same, add the exponents.
$0.5^2 \cdot 0.5^{-3} = 0.5^{2+(-3)}$
$= 0.5^{-1}$ Write with a positive exponent.
$= \dfrac{1}{0.5}$

c) $\dfrac{0.5^2}{0.5^3}$

Use the quotient of powers law:
When the bases are the same, subtract the exponents.

$$\frac{0.5^2}{0.5^3} = 0.5^{2-3}$$

$$= 0.5^{-1} \quad \text{Write with a positive exponent.}$$

$$= \frac{1}{0.5}$$

d) $\dfrac{0.5^2}{0.5^{-3}}$

Use the quotient of powers law:
When the bases are the same, subtract the exponents.

$$\frac{0.5^2}{0.5^{-3}} = 0.5^{2-(-3)}$$

$$= 0.5^5$$

5. Use the quotient of powers law:
When the bases are the same, subtract the exponents.

a) $\dfrac{x^4}{x^2} = x^{4-2}$

$$= x^2$$

b) $\dfrac{x^2}{x^5} = x^{2-5}$

$$= x^{-3} \quad \text{Write with a positive exponent.}$$

$$= \frac{1}{x^3}$$

c) $n^6 \div n^5 = n^{6-5}$

$$= n^1$$

$$= n$$

d) $\dfrac{a^2}{a^6} = a^{2-6}$

$$= a^{-4} \quad \text{Write with a positive exponent.}$$

$$= \frac{1}{a^4}$$

6. Use the power of a power law: multiply the exponents

a) $\left(n^2\right)^3 = n^{(2)(3)}$

$= n^6$

b) $\left(z^2\right)^{-3} = z^{(2)(-3)}$

$= z^{-6}$ Write with a positive exponent.

$= \dfrac{1}{z^6}$

c) $\left(n^{-4}\right)^{-3} = n^{(-4)(-3)}$

$= n^{12}$

d) $\left(c^{-2}\right)^2 = c^{(-2)(2)}$

$= c^{-4}$ Write with a positive exponent.

$= \dfrac{1}{c^4}$

7. Use the power of a power law: multiply the exponents

a) $\left[\left(\dfrac{3}{5}\right)^3\right]^4 = \left(\dfrac{3}{5}\right)^{(3)(4)}$

$= \left(\dfrac{3}{5}\right)^{12}$

b) $\left[\left(\dfrac{3}{5}\right)^3\right]^{-4} = \left(\dfrac{3}{5}\right)^{(3)(-4)}$

$= \left(\dfrac{3}{5}\right)^{-12}$ Write with a positive exponent.

$= \left(\dfrac{5}{3}\right)^{12}$

c) $\left[\left(\dfrac{3}{5}\right)^{-3}\right]^{-4} = \left(\dfrac{3}{5}\right)^{(-3)(-4)}$

$= \left(\dfrac{3}{5}\right)^{12}$

d) $\left[\left(-\frac{3}{5}\right)^{-3}\right]^{-4} = \left(-\frac{3}{5}\right)^{(-3)(-4)}$

$= \left(-\frac{3}{5}\right)^{12}$

8. **a)** Use the power of a quotient law.

$\left(\frac{a}{b}\right)^2 = \frac{a^2}{b^2}$

b) Use the power of a quotient law.

$\left(\frac{n^2}{m}\right)^3 = \frac{\left(n^2\right)^3}{m^3}$ Use the power of a power law.

$= \frac{n^{(2)(3)}}{m^3}$

$= \frac{n^6}{m^3}$

c) $\left(\frac{c^2}{d^2}\right)^{-4} = \left(\frac{d^2}{c^2}\right)^4$ Writing with a positive exponent

$= \frac{\left(d^2\right)^4}{\left(c^2\right)^4}$ Using the power of a quotient law

$= \frac{d^{(2)(4)}}{c^{(2)(4)}}$ Using the power of a power law

$= \frac{d^8}{c^8}$

d) Use the power of a quotient law.

$\left(\frac{2b}{5c}\right)^2 = \frac{(2b)^2}{(5c)^2}$ Use the power of a product law.

$= \frac{2^2 b^2}{5^2 c^2}$

$= \frac{4b^2}{25c^2}$

e) $(ab)^2 = a^2b^2$ Using the power of a product law

f) $(n^2m)^3 = (n^2)^3 \cdot m^3$ Using the power of a product law

$= n^{(2)(3)}m^3$ Using the power of a power law

$= n^6m^3$

g) $(c^3d^2)^{-4} = (c^3)^{-4} \cdot (d^2)^{-4}$ Using the power of a product law

$= c^{(3)(-4)}d^{(2)(-4)}$ Using the power of a power law

$= c^{-12}d^{-8}$ Write with positive exponents.

$= \frac{1}{c^{12}d^8}$

h) $(xy^{-1})^3 = (x)^3 \cdot (y^{-1})^3$ Using the power of a product law

$= x^3y^{(-1)(3)}$ Using the power of a power law

$= x^3y^{-3}$ Write with a positive exponent.

$= \frac{x^3}{y^3}$

B

9. a) $x^{-3} \cdot x^4 = x^{-3+4}$ Using the product of powers law

$= x^1$

$= x$

b) $a^{-4} \cdot a^{-1} = a^{-4+(-1)}$ Using the product of powers law

$= a^{-5}$ Write with a positive exponent.

$= \frac{1}{a^5}$

c) $b^4 \cdot b^{-3} \cdot b^2 = b^{4+(-3)+2}$ Using the product of powers law

$= b^3$

d) $m^8 \cdot m^{-2} \cdot m^{-6} = m^{8+(-2)+(-6)}$ Using the product of powers law

$= m^0$

$= 1$

e) $\frac{x^{-5}}{x^{2}} = x^{-5-2}$ Using the quotient of powers law

$= x^{-7}$ Write with a positive exponent.

$= \frac{1}{x^{7}}$

f) $\frac{s^{5}}{s^{-5}} = s^{5-(-5)}$ Using the quotient of powers law

$= s^{10}$

g) $\frac{b^{-8}}{b^{-3}} = b^{-8-(-3)}$ Using the quotient of powers law

$= b^{-5}$ Write with a positive exponent.

$= \frac{1}{b^{5}}$

h) $\frac{t^{-4}}{t^{-4}} = t^{-4-(-4)}$ Using the quotient of powers law

$= t^{0}$

$= 1$

10. a) $1.5^{\frac{3}{2}} \cdot 1.5^{\frac{1}{2}} = 1.5^{\frac{3}{2}+\frac{1}{2}}$ Using the product of powers law

$= 1.5^{\frac{4}{2}}$

$= 1.5^{2}$

$= 2.25$

b) $\left(\frac{3}{4}\right)^{\frac{3}{4}} \cdot \left(\frac{3}{4}\right)^{\frac{5}{4}} = \left(\frac{3}{4}\right)^{\frac{3}{4}+\frac{5}{4}}$ Using the product of powers law

$= \left(\frac{3}{4}\right)^{\frac{8}{4}}$

$= \left(\frac{3}{4}\right)^{2}$

$= \frac{9}{16}$

c) $(-0.6)^{\frac{1}{3}} \cdot (-0.6)^{\frac{5}{3}} = (-0.6)^{\frac{1}{3}+\frac{5}{3}}$ Using the product of powers law

$= (-0.6)^{\frac{6}{3}}$

$= (-0.6)^{2}$

$= 0.36$

d) $\left(\frac{4}{5}\right)^{\frac{4}{3}} \cdot \left(\frac{4}{5}\right)^{-\frac{4}{3}} = \left(\frac{4}{5}\right)^{\frac{4}{3}+\left(-\frac{4}{3}\right)}$ Using the product of powers law

$= \left(\frac{4}{5}\right)^{0}$

$= 1$

e) $\frac{0.6^{\frac{1}{2}}}{0.6^{\frac{3}{2}}} = 0.6^{\frac{1}{2}-\frac{3}{2}}$ Using the quotient of powers law

$= 0.6^{-\frac{2}{2}}$

$= 0.6^{-1}$ Write with a positive exponent.

$= \frac{1}{0.6}$

$= \frac{1}{\frac{6}{10}}$

$= \frac{10}{6}$

$= \frac{5}{3}$

f)

$\frac{\left(-\frac{3}{8}\right)^{\frac{2}{3}}}{\left(-\frac{3}{8}\right)^{-\frac{1}{3}}} = \left(-\frac{3}{8}\right)^{\frac{2}{3}-\left(-\frac{1}{3}\right)}$ Using the quotient of powers law.

$= \left(-\frac{3}{8}\right)^{\frac{3}{3}}$

$= \left(-\frac{3}{8}\right)^{1}$

$= -\frac{3}{8}$

g)

$$\frac{0.49^{\frac{5}{2}}}{0.49^{4}} = 0.49^{\frac{5}{2}-4}$$ Using the quotient of powers law

$$= 0.49^{\frac{5}{2}-\frac{8}{2}}$$

$$= 0.49^{-\frac{3}{2}}$$ Write with a positive exponent.

$$= \frac{1}{0.49^{\frac{3}{2}}}$$

$$= \frac{1}{\left(\sqrt{0.49}\right)^{3}}$$

$$= \frac{1}{0.7^{3}}$$

$$= \frac{1}{0.343}$$

$$= \frac{1}{\frac{343}{1000}}$$

$$= \frac{1000}{343}$$

h) $$\frac{0.027^{\frac{5}{3}}}{0.027^{\frac{4}{3}}} = 0.027^{\frac{5}{3}-\frac{4}{3}}$$ Using the quotient of powers law

$$= 0.027^{\frac{1}{3}}$$

$$= \sqrt[3]{0.027}$$

$$= 0.3, \text{ or } \frac{3}{10}$$

11. a) $$\left(x^{-1}y^{-2}\right)^{-3} = \left(x^{-1}\right)^{-3} \cdot \left(y^{-2}\right)^{-3}$$ Using the power of a product law

$$= x^{(-1)(-3)} \cdot y^{(-2)(-3)}$$ Using the power of a power law

$$= x^{3} \cdot y^{6}$$

$$= x^{3}y^{6}$$

b) $\left(2a^{-2}b^{2}\right)^{-2} = \left(2^{1}\right)^{-2} \cdot \left(a^{-2}\right)^{-2} \cdot \left(b^{2}\right)^{-2}$ Using the power of a product law

$= 2^{(1)(-2)} \cdot a^{(-2)(-2)} \cdot b^{(2)(-2)}$ Using the power of a power law

$= 2^{-2} \cdot a^{4} \cdot b^{-4}$ Write with positive exponents.

$= \dfrac{a^{4}}{2^{2} \cdot b^{4}}$

$= \dfrac{a^{4}}{4b^{4}}$

c) $\left(4m^{2}n^{3}\right)^{-3} = \left(4^{1}\right)^{-3} \cdot \left(m^{2}\right)^{-3} \cdot \left(n^{3}\right)^{-3}$ Using the power of a product law

$= 4^{(1)(-3)} \cdot m^{(2)(-3)} \cdot n^{(3)(-3)}$ Using the power of a power law

$= 4^{-3} \cdot m^{-6} \cdot n^{-9}$ Write with positive exponents.

$= \dfrac{1}{4^{3} \cdot m^{6} \cdot n^{9}}$

$= \dfrac{1}{64m^{6}n^{9}}$

d)

$\left(\dfrac{3}{2}m^{-2}n^{-3}\right)^{-4} = \left(\dfrac{3^{1}}{2}\right)^{-4} \cdot \left(m^{-2}\right)^{-4} \cdot \left(n^{-3}\right)^{-4}$ Using the power of a product law

$= \left(\dfrac{3}{2}\right)^{(1)(-4)} \cdot m^{(-2)(-4)} \cdot n^{(-3)(-4)}$ Using the power of a power law

$= \left(\dfrac{3}{2}\right)^{-4} \cdot m^{8} \cdot n^{12}$ Write with a positive exponent.

$= \left(\dfrac{2}{3}\right)^{4} \cdot m^{8} \cdot n^{12}$

$= \dfrac{16}{81} \cdot m^{8} \cdot n^{12}$

$= \dfrac{16m^{8}n^{12}}{81}$

12. The volume of a cone with base radius r and height h is given by the formula:

$$V = \frac{1}{3}\pi r^2 h$$

The cone has equal height and radius.
So, substitute: $r = h$

$$V = \frac{1}{3}\pi h^2 h$$

$$= \frac{1}{3}\pi h^2 h^1 \quad \text{Use the product of powers law.}$$

$$= \frac{1}{3}\pi h^{2+1}$$

$$= \frac{1}{3}\pi h^3$$

Substitute $V = 1234$, then solve for h.

$$1234 = \frac{1}{3}\pi h^3 \quad \text{Multiply each side by 3.}$$

$$3(1234) = 3\left(\frac{1}{3}\pi h^3\right)$$

$$3702 = \pi h^3 \quad \text{Divide each side by } \pi.$$

$$\frac{3702}{\pi} = \frac{\pi h^3}{\pi}$$

$$\frac{3702}{\pi} = h^3$$

To solve for h, take the cube root of each side by raising each side to the one-third power.

$$\left(\frac{3702}{\pi}\right)^{\frac{1}{3}} = \left(h^3\right)^{\frac{1}{3}} \quad \text{Use the power of a power law.}$$

$$\left(\frac{3702}{\pi}\right)^{\frac{1}{3}} = h \quad \text{Use a calculator.}$$

$$h = 10.5623...$$

The height of the cone is approximately 10.6 cm.

13. The volume, V, of a sphere with radius r is given by the formula: $V = \frac{4}{3}\pi r^3$

Substitute $V = 375$, then solve for r.

$375 = \frac{4}{3}\pi r^3$ Multiply each side by 3.

$3(375) = 3\left(\frac{4}{3}\pi r^3\right)$

$1125 = 4\pi r^3$ Divide each side by 4π.

$\frac{1125}{4\pi} = \frac{4\pi r^3}{4\pi}$

$\frac{1125}{4\pi} = r^3$ To solve for r, take the cube root of each side by raising each side to the one-third power.

$\left(\frac{1125}{4\pi}\right)^{\frac{1}{3}} = \left(r^3\right)^{\frac{1}{3}}$ Use the power of a power law.

$\left(\frac{1125}{4\pi}\right)^{\frac{1}{3}} = r$ Use a calculator.

$r = 4.4735...$

The surface area, SA, of a sphere with radius r is given by the formula:

$SA = 4\pi r^2$ Substitute: $r = 4.4735...$

$= 4\pi(4.4735...)^2$

$= 251.4808...$

The surface area of the sphere is approximately 251 square feet.

14. a) $\frac{\left(a^2b^{-1}\right)^{-2}}{\left(a^{-3}b\right)^3} = \frac{a^{2(-2)} \cdot b^{(-1)(-2)}}{a^{(-3)(3)} \cdot b^{(1)(3)}}$ Using the power of a power law

$= \frac{a^{-4} \cdot b^2}{a^{-9} \cdot b^3}$ Use the quotient of powers law.

$= a^{-4-(-9)} \cdot b^{2-3}$

$= a^5 \cdot b^{-1}$ Write with a positive exponent.

$= \frac{a^5}{b}$

b)

$$\left(\frac{\left(c^{-3}d\right)^{-1}}{c^2d}\right)^{-2} = \left(\frac{c^{(-3)(-1)} \cdot d^{(1)(-1)}}{c^2d}\right)^{-2}$$ Using the power of a power law inside the large brackets

$$= \left(\frac{c^3 \cdot d^{-1}}{c^2d}\right)^{-2}$$ Use the quotient of powers law.

$$= \left(c^{3-2} \cdot d^{-1-1}\right)^{-2}$$

$$= \left(c^1 \cdot d^{-2}\right)^{-2}$$ Use the power of a power law.

$$= c^{1(-2)} \cdot d^{(-2)(-2)}$$

$$= c^{-2}d^4$$ Write with a positive exponent.

$$= \frac{d^4}{c^2}$$

15. a) Simplify first.

$$\left(a^3b^2\right)\left(a^2b^3\right) = a^3 \cdot b^2 \cdot a^2 \cdot b^3$$

$$= a^3 \cdot a^2 \cdot b^2 \cdot b^3$$ Use the product of powers law.

$$= a^{3+2} \cdot b^{2+3}$$

$$= a^5b^5$$

Substitute: $a = -2$, $b = 1$

$$a^5b^5 = (-2)^5(1)^5$$

$$= (-32)(1)$$

$$= -32$$

b) Simplify first.

$$\left(a^{-1}b^{-2}\right)\left(a^{-2}b^{-3}\right) = a^{-1} \cdot b^{-2} \cdot a^{-2} \cdot b^{-3}$$

$$= a^{-1} \cdot a^{-2} \cdot b^{-2} \cdot b^{-3}$$ Use the product of powers law.

$$= a^{-1+(-2)} \cdot b^{-2+(-3)}$$

$$= a^{-3}b^{-5}$$ Write with positive exponents.

$$= \frac{1}{a^3b^5}$$

Substitute: $a = -2$, $b = 1$

$$\frac{1}{a^3b^5} = \frac{1}{(-2)^3(1)^5}$$

$$= \frac{1}{(-8)(1)}$$

$$= -\frac{1}{8}$$

c) Simplify first.

$$\frac{a^{-4}b^5}{ab^3} = a^{-4-1} \cdot b^{5-3}$$ Using the quotient of powers law

$$= a^{-5} \cdot b^2$$ Write with a positive exponent.

$$= \frac{b^2}{a^5}$$

Substitute: $a = -2$, $b = 1$

$$\frac{b^2}{a^5} = \frac{1^2}{(-2)^5}$$

$$= -\frac{1}{32}$$

d) Simplify first.

$$\left(\frac{a^{-7}b^7}{a^{-9}b^{10}}\right)^{-5} = \left(a^{-7-(-9)} \cdot b^{7-10}\right)^{-5}$$ Using the quotient of powers law inside the brackets

$$= \left(a^2 \cdot b^{-3}\right)^{-5}$$ Write the expression inside the brackets with a positive exponent.

$$= \left(\frac{a^2}{b^3}\right)^{-5}$$ Write with a positive exponent.

$$= \left(\frac{b^3}{a^2}\right)^5$$ Use the power of a power law.

$$= \frac{b^{3(5)}}{a^{2(5)}}$$

$$= \frac{b^{15}}{a^{10}}$$

Substitute: $a = -2$, $b = 1$

$$\frac{b^{15}}{a^{10}} = \frac{1^{15}}{(-2)^{10}}$$

$$= \frac{1}{1024}$$

16. a) $m^{\frac{2}{3}} \cdot m^{\frac{4}{3}} = m^{\frac{2}{3}+\frac{4}{3}}$ Using the product of powers law

$$= m^{\frac{6}{3}}$$

$$= m^2$$

b) $x^{-\frac{3}{2}} \div x^{-\frac{1}{4}} = x^{-\frac{3}{2}-\left(-\frac{1}{4}\right)}$ Using the quotient of powers law

$= x^{-\frac{6}{4}-\left(-\frac{1}{4}\right)}$

$= x^{-\frac{5}{4}}$ Write with a positive exponent.

$= \dfrac{1}{x^{\frac{5}{4}}}$

c)

$\dfrac{-9a^{-4}b^{\frac{3}{4}}}{3a^{2}b^{\frac{1}{4}}} = \dfrac{-9}{3} \cdot \dfrac{a^{-4}}{a^{2}} \cdot \dfrac{b^{\frac{3}{4}}}{b^{\frac{1}{4}}}$ Use the quotient of powers law.

$= -3 \cdot a^{-4-2} \cdot b^{\frac{3}{4}-\frac{1}{4}}$

$= -3 \cdot a^{-6} \cdot b^{\frac{2}{4}}$

$= -3 \cdot a^{-6} \cdot b^{\frac{1}{2}}$ Write with a positive exponent.

$= -\dfrac{3b^{\frac{1}{2}}}{a^{6}}$

d)

$\left(\dfrac{-64c^{6}}{a^{9}b^{-\frac{1}{2}}}\right)^{\frac{1}{3}} = \left(-64 \cdot \dfrac{1}{a^{9}} \cdot \dfrac{1}{b^{-\frac{1}{2}}} \cdot c^{6}\right)^{\frac{1}{3}}$ Simplify inside the brackets first.

$= \left(-64 \cdot a^{-9} \cdot b^{\frac{1}{2}} \cdot c^{6}\right)^{\frac{1}{3}}$ Use the power of a power law.

$= (-64)^{\frac{1}{3}} \cdot a^{-9\left(\frac{1}{3}\right)} \cdot b^{\left(\frac{1}{2}\right)\left(\frac{1}{3}\right)} \cdot c^{6\left(\frac{1}{3}\right)}$

$= (-64)^{\frac{1}{3}} \cdot a^{-3} \cdot b^{\frac{1}{6}} \cdot c^{2}$

$= -4 \cdot a^{-3} \cdot b^{\frac{1}{6}} \cdot c^{2}$ Write with a positive exponent.

$= \dfrac{-4b^{\frac{1}{6}}c^{2}}{a^{3}}$

17. a) In the second line, the exponents were multiplied instead of added.
A correct solution is:

$$\left(x^2y^{-3}\right)\left(x^{\frac{1}{2}}y^{-1}\right) = x^2 \cdot x^{\frac{1}{2}} \cdot y^{-3} \cdot y^{-1}$$ Use the product of powers law.

$$= x^{2+\frac{1}{2}} \cdot y^{-3+(-1)}$$

$$= x^{\frac{4}{2}+\frac{1}{2}} \cdot y^{-3+(-1)}$$

$$= x^{\frac{5}{2}}y^{-4}$$ Write with a positive exponent.

$$= \frac{x^{\frac{5}{2}}}{y^4}$$

b) In the first line, –5 was multiplied by –2 instead of being raised to the power –2.
A correct solution is:

$$\left(\frac{-5a^2}{b^{\frac{1}{2}}}\right)^{-2} = \frac{(-5)^{-2} \cdot a^{2(-2)}}{b^{\left(\frac{1}{2}\right)(-2)}}$$ Using the power of a power law

$$= \frac{(-5)^{-2} \cdot a^{-4}}{b^{-1}}$$ Write with positive exponents.

$$= \frac{b^1}{(-5)^2 \cdot a^4}$$

$$= \frac{b}{25a^4}$$

18. I record the volume of water in the measuring cylinder, in millilitres. Then I carefully place the marble in the cylinder. The water level rises; this is the total volume of the water and the marble. I record this volume. I subtract the volume of the water alone from the total volume of the water and the marble to determine the volume of the marble in millilitres. Since 1 mL = 1 cm^3, I can write the volume of the marble in cubic centimetres.

The volume, V, of a sphere with radius r is given by the formula: $V = \frac{4}{3}\pi r^3$

I substitute the volume of the marble in cubic centimetres for V.
I then multiply both sides of the equation by 3, then divide both sides by 4π. To solve for r, I raise each side to the one-third power. I then use the power of a power law to write $\left(r^3\right)^{\frac{1}{3}}$ as r. Once I have determined r, I multiply the radius by 2 to get the diameter of the marble.

19. a) There are two errors in the first line: the quotient of powers law was used before the power of a power law, and when using the quotient of powers law, the powers outside the brackets were subtracted. There is an error in the second line: when using the power of a power law, the product of 5 and –6 should have been –30.
A correct solution is:

$$\frac{\left(m^{-3}\cdot n^{2}\right)^{-4}}{\left(m^{2}\cdot n^{-3}\right)^{2}}=\frac{m^{(-3)(-4)}\cdot n^{2(-4)}}{m^{2(2)}\cdot n^{-3(2)}}$$ Using the power of a power law

$$=\frac{m^{12}\cdot n^{-8}}{m^{4}\cdot n^{-6}}$$ Use the quotient of powers law.

$$=m^{12-4}\cdot n^{-8-(-6)}$$

$$=m^{8}n^{-2}$$ Write with a positive exponent.

$$=\frac{m^{8}}{n^{2}}$$

b) There is an error in the first line: when using the power of a power law, the exponents were added when they should have been multiplied.
A correct solution is:

$$\left(r^{\frac{1}{2}}\cdot s^{-\frac{3}{2}}\right)^{\frac{1}{2}}\cdot\left(r^{-\frac{1}{4}}\cdot s^{\frac{1}{2}}\right)^{-1}=r^{\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)}\cdot s^{\left(-\frac{3}{2}\right)\left(\frac{1}{2}\right)}\cdot r^{\left(-\frac{1}{4}\right)(-1)}\cdot s^{\left(\frac{1}{2}\right)(-1)}$$ Using the power of a power law

$$=r^{\frac{1}{4}}\cdot s^{-\frac{3}{4}}\cdot r^{\frac{1}{4}}\cdot s^{-\frac{1}{2}}$$ Use the product of powers law.

$$=r^{\frac{1}{4}+\frac{1}{4}}\cdot s^{-\frac{3}{4}-\frac{1}{2}}$$

$$=r^{\frac{2}{4}}\cdot s^{-\frac{3}{4}-\frac{2}{4}}$$

$$=r^{\frac{1}{2}}\cdot s^{-\frac{5}{4}}$$ Write with a positive exponent.

$$=\frac{r^{\frac{1}{2}}}{s^{\frac{5}{4}}}$$

20. Width, in metres, of a piece of An paper: $2^{-\frac{2n+1}{4}}$

Length, in metres, of a piece of An paper: $2^{-\frac{2n-1}{4}}$

a) **i)** To determine the dimensions of a piece of A3 paper, substitute: $n = 3$

Width: $2^{-\frac{2n+1}{4}} = 2^{-\frac{2(3)+1}{4}}$

$= 2^{-\frac{7}{4}}$ Write with a positive exponent.

$= \dfrac{1}{2^{\frac{7}{4}}}$

This is the width in metres. Since 1 m = 1000 mm, to write the width in millimetres, multiply by 1000.

$$\frac{1}{2^{\frac{7}{4}}} \cdot 1000 = \frac{1000}{2^{\frac{7}{4}}}$$

So, the width of the paper is $\dfrac{1000}{2^{\frac{7}{4}}}$ mm.

Use a calculator to evaluate: $\dfrac{1000}{2^{\frac{7}{4}}} = 297.3017\ldots$

The width of a piece of A3 paper is about 297 mm.

Length: $2^{-\frac{2n-1}{4}} = 2^{-\frac{2(3)-1}{4}}$

$= 2^{-\frac{5}{4}}$ Write with a positive exponent.

$= \dfrac{1}{2^{\frac{5}{4}}}$

This is the length in metres. Since 1 m = 1000 mm, to write the length in millimetres, multiply by 1000.

$$\frac{1}{2^{\frac{5}{4}}} \cdot 1000 = \frac{1000}{2^{\frac{5}{4}}}$$

So, the length of the paper is $\dfrac{1000}{2^{\frac{5}{4}}}$ mm.

Use a calculator to evaluate: $\dfrac{1000}{2^{\frac{5}{4}}} = 420.4482\ldots$

The length of a piece of A3 paper is about 420 mm.

A piece of A3 paper has dimensions $\dfrac{1000}{2^{\frac{5}{4}}}$ mm by $\dfrac{1000}{2^{\frac{7}{4}}}$ mm, or 297 mm by 420 mm.

ii) To determine the dimensions of a piece of A4 paper, substitute: $n = 4$

Width: $2^{-\frac{2n+1}{4}} = 2^{-\frac{2(4)+1}{4}}$

$= 2^{-\frac{9}{4}}$ Write with a positive exponent.

$= \dfrac{1}{2^{\frac{9}{4}}}$

This is the width in metres. To write the width in millimetres, multiply by 1000.

$\dfrac{1}{2^{\frac{9}{4}}} \cdot 1000 = \dfrac{1000}{2^{\frac{9}{4}}}$

So, the width of the paper is $\dfrac{1000}{2^{\frac{9}{4}}}$ mm.

Use a calculator to evaluate: $\dfrac{1000}{2^{\frac{9}{4}}} = 210.2241\ldots$

The width of a piece of A4 paper is about 210 mm.

Length: $2^{-\frac{2n-1}{4}} = 2^{-\frac{2(4)-1}{4}}$

$= 2^{-\frac{7}{4}}$ Write with a positive exponent.

$= \dfrac{1}{2^{\frac{7}{4}}}$

From part i, $\dfrac{1000}{2^{\frac{7}{4}}} = 297.3017\ldots$

The length of a piece of A4 paper is about 297 mm.

A piece of A4 paper has dimensions $\dfrac{1000}{2^{\frac{7}{4}}}$ mm by $\dfrac{1000}{2^{\frac{9}{4}}}$ mm, or 210 mm by 297 mm.

iii) To determine the dimensions of a piece of A5 paper, substitute: $n = 5$

Width: $2^{-\frac{2n+1}{4}} = 2^{-\frac{2(5)+1}{4}}$

$= 2^{-\frac{11}{4}}$ Write with a positive exponent.

$= \dfrac{1}{2^{\frac{11}{4}}}$

This is the width in metres. To write the width in millimetres, multiply by 1000.

$$\frac{1}{2^{\frac{11}{4}}} \cdot 1000 = \frac{1000}{2^{\frac{11}{4}}}$$

So, the width of the paper is $\frac{1000}{2^{\frac{11}{4}}}$ mm.

Use a calculator to evaluate: $\frac{1000}{2^{\frac{11}{4}}} = 148.6508\ldots$

The width of a piece of A5 paper is about 149 mm.

Length: $2^{-\frac{2n-1}{4}} = 2^{-\frac{2(5)-1}{4}}$

$= 2^{-\frac{9}{4}}$ Write with a positive exponent.

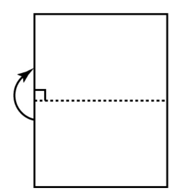
$= \frac{1}{2^{\frac{9}{4}}}$

From part ii, $\frac{1000}{2^{\frac{9}{4}}} = 210.2241\ldots$

The length of a piece of A5 paper is about 210 mm.

A piece of A5 paper has dimensions $\frac{1000}{2^{\frac{9}{4}}}$ mm by $\frac{1000}{2^{\frac{11}{4}}}$ mm, or 149 mm by 210 mm.

b) **i)** The paper is folded along a line perpendicular to its length.
So, one dimension of a folded piece of A3 paper, in millimetres, is one-half the original length:



$$\frac{1000}{2^{\frac{5}{4}}} \cdot \frac{1}{2} = \frac{1000}{2^{\frac{5}{4}} \cdot 2^1}$$

$$= \frac{1000}{2^{\frac{5}{4}+1}}$$

$$= \frac{1000}{2^{\frac{5}{4}+\frac{4}{4}}}$$

$$= \frac{1000}{2^{\frac{9}{4}}}$$

The other dimension is the original width: $\dfrac{1000}{2^{\frac{7}{4}}}$ mm

So, a folded piece of A3 paper has dimensions $\dfrac{1000}{2^{\frac{7}{4}}}$ mm by $\dfrac{1000}{2^{\frac{9}{4}}}$ mm.

ii) The paper is folded along a line perpendicular to its length.
So, one dimension of a folded piece of A4 paper, in millimetres, is one-half the original length:

$$\begin{aligned}\frac{1000}{2^{\frac{7}{4}}}\cdot\frac{1}{2} &= \frac{1000}{2^{\frac{7}{4}}\cdot 2^{1}}\\ &= \frac{1000}{2^{\frac{7}{4}+1}}\\ &= \frac{1000}{2^{\frac{7}{4}+\frac{4}{4}}}\\ &= \frac{1000}{2^{\frac{11}{4}}}\end{aligned}$$

The other dimension is the original width: $\dfrac{1000}{2^{\frac{9}{4}}}$ mm

So, a folded piece of A4 paper has dimensions $\dfrac{1000}{2^{\frac{9}{4}}}$ mm by $\dfrac{1000}{2^{\frac{11}{4}}}$ mm.

iii) The paper is folded along a line perpendicular to its length.
So, one dimension of a folded piece of A5 paper, in millimetres, is one-half the original length:

$$\begin{aligned}\frac{1000}{2^{\frac{9}{4}}}\cdot\frac{1}{2} &= \frac{1000}{2^{\frac{9}{4}}\cdot 2^{1}}\\ &= \frac{1000}{2^{\frac{9}{4}+1}}\\ &= \frac{1000}{2^{\frac{9}{4}+\frac{4}{4}}}\\ &= \frac{1000}{2^{\frac{13}{4}}}\end{aligned}$$

The other dimension is the original width: $\dfrac{1000}{2^{\frac{11}{4}}}$ mm

So, a folded piece of A5 paper has dimensions $\dfrac{1000}{2^{\frac{11}{4}}}$ mm by $\dfrac{1000}{2^{\frac{13}{4}}}$ mm.

c) I noticed that a piece of A4 paper has the same dimensions as a folded piece of A3 paper, and a piece of A5 paper has the same dimensions as a folded piece of A4 paper.

C

21. a)

$$\left(\frac{a^{-3}b}{c^2}\right)^{-4}\cdot\left(\frac{c^5}{a^4b^{-3}}\right)^{-1}=\left(\frac{a^{(-3)(-4)}\cdot b^{1(-4)}}{c^{2(-4)}}\right)\cdot\left(\frac{c^{5(-1)}}{a^{4(-1)}\cdot b^{(-3)(-1)}}\right)$$ Using the power of a power law

$$=\left(\frac{a^{12}\cdot b^{-4}}{c^{-8}}\right)\cdot\left(\frac{c^{-5}}{a^{-4}\cdot b^{3}}\right)$$ Rewrite without using fractions.

$$=a^{12}b^{-4}c^{8}\cdot a^{4}b^{-3}c^{-5}$$ Use the product of powers law.

$$=a^{12+4}b^{-4-3}c^{8-5}$$

$$=a^{16}b^{-7}c^{3}$$ Write with a positive exponent.

$$=\frac{a^{16}c^{3}}{b^{7}}$$

b)

$$\frac{\left(2a^{-1}b^{4}c^{-3}\right)^{-2}}{\left(4a^{2}bc^{-4}\right)^{2}}=\frac{2^{-2}\cdot a^{(-1)(-2)}\cdot b^{4(-2)}\cdot c^{(-3)(-2)}}{4^{2}\cdot a^{2(2)}\cdot b^{2}\cdot c^{(-4)(2)}}$$ Using the power of a power law

$$=\frac{2^{-2}\cdot a^{2}\cdot b^{-8}\cdot c^{6}}{16\cdot a^{4}\cdot b^{2}\cdot c^{-8}}$$ Rewrite 16 as a power of 2.

$$=\frac{2^{-2}\cdot a^{2}\cdot b^{-8}\cdot c^{6}}{2^{4}\cdot a^{4}\cdot b^{2}\cdot c^{-8}}$$ Use the quotient of powers law.

$$=2^{-2-4}\cdot a^{2-4}\cdot b^{-8-2}\cdot c^{6-(-8)}$$

$$=2^{-6}\cdot a^{-2}\cdot b^{-10}\cdot c^{14}$$ Write with positive exponents.

$$=\frac{c^{14}}{2^{6}a^{2}b^{10}}$$

$$=\frac{c^{14}}{64a^{2}b^{10}}$$

22. a)

$$\left(x^{\frac{1}{2}}y^{\frac{2}{3}}\right)^2 = x^{\frac{1}{2}(2)} \cdot y^{\frac{2}{3}(2)}$$ Using the power of a power law

$$= x \cdot y^{\frac{4}{3}}$$ Substitute: $x = a^{-2}$, $y = a^{\frac{2}{3}}$

$$= a^{-2} \cdot \left(a^{\frac{2}{3}}\right)^{\frac{4}{3}}$$ Use the power of a power law.

$$= a^{-2} \cdot a^{\frac{2}{3} \cdot \frac{4}{3}}$$

$$= a^{-2} \cdot a^{\frac{8}{9}}$$ Use the product of powers law.

$$= a^{-2 + \frac{8}{9}}$$

$$= a^{-\frac{18}{9} + \frac{8}{9}}$$

$$= a^{-\frac{10}{9}}$$ Write with a positive exponent.

$$= \frac{1}{a^{\frac{10}{9}}}$$

b)

$$\left(x^{\frac{3}{4}} \div y^{-\frac{1}{2}}\right)^3 = \left(\frac{x^{\frac{3}{4}}}{y^{-\frac{1}{2}}}\right)^3$$ Rewriting as a fraction

$$= \frac{x^{\frac{3}{4}(3)}}{y^{\left(-\frac{1}{2}\right)3}}$$ Using the power of a power law

$$= \frac{x^{\frac{9}{4}}}{y^{-\frac{3}{2}}}$$ Substitute: $x = a^{-2}$, $y = a^{\frac{2}{3}}$

$$= \frac{\left(a^{-2}\right)^{\frac{9}{4}}}{\left(a^{\frac{2}{3}}\right)^{-\frac{3}{2}}}$$ Use the power of a power law.

$$= \frac{a^{(-2)\frac{9}{4}}}{a^{\frac{2}{3}\cdot\left(-\frac{3}{2}\right)}}$$

$$= \frac{a^{-\frac{9}{2}}}{a^{-1}}$$ Use the quotient of powers law.

$$= a^{-\frac{9}{2}-(-1)}$$

$$= a^{-\frac{9}{2}+\frac{2}{2}}$$

$$= a^{-\frac{7}{2}}$$ Write with a positive exponent.

$$= \frac{1}{a^{\frac{7}{2}}}$$

23. a) The product of powers law is: $x^m \cdot x^n = x^{m+n}$

So, for the product to be $x^{\frac{3}{2}}$, I need to find two exponents whose sum is $\frac{3}{2}$.

For example:

$$x^{\frac{1}{2}} \cdot x^1 = x^{\frac{1}{2}+\frac{2}{2}} = x^{\frac{3}{2}}$$

$$x^{\frac{1}{4}} \cdot x^{\frac{5}{4}} = x^{\frac{1}{4}+\frac{5}{4}} = x^{\frac{6}{4}} = x^{\frac{3}{2}}$$

$$x^{\frac{11}{8}} \cdot x^{\frac{1}{8}} = x^{\frac{11}{8}+\frac{1}{8}} = x^{\frac{12}{8}} = x^{\frac{3}{2}}$$

b) The quotient of powers law is: $x^m \div x^n = x^{m-n}$

So, for the quotient to be $x^{\frac{3}{2}}$, I need to find two exponents whose difference is $\frac{3}{2}$.

For example:

$$x^2 \div x^{\frac{1}{2}} = x^{2-\frac{1}{2}} = x^{\frac{3}{2}}$$

$$x^{\frac{5}{2}} \div x^1 = x^{\frac{5}{2}-1} = x^{\frac{5}{2}-\frac{2}{2}} = x^{\frac{3}{2}}$$

$$x^{\frac{7}{2}} \div x^2 = x^{\frac{7}{2}-2} = x^{\frac{7}{2}-\frac{4}{2}} = x^{\frac{3}{2}}$$

c) The power of a power law is: $\left(x^m\right)^n = x^{mn}$

So, for the result to be $x^{\frac{3}{2}}$, I need to find two exponents whose product is $\frac{3}{2}$.

For example:

$$\left(x^3\right)^{\frac{1}{2}} = x^{3\left(\frac{1}{2}\right)} = x^{\frac{3}{2}}$$

$$\left(x^{\frac{1}{3}}\right)^{\frac{9}{2}} = x^{\left(\frac{1}{3}\right)\left(\frac{9}{2}\right)} = x^{\frac{9}{6}} = x^{\frac{3}{2}}$$

$$\left(x^9\right)^{\frac{1}{6}} = x^{9\left(\frac{1}{6}\right)} = x^{\frac{9}{6}} = x^{\frac{3}{2}}$$

24. In $\frac{AC}{AB}=\frac{AD}{AE}$, substitute $AB=\left(\frac{2}{3}\right)^{\frac{1}{2}}$, $AC=\frac{3}{2}\left(\frac{1}{3}\right)^{\frac{1}{2}}$, $AE=\left(\frac{1}{3}\right)^{\frac{1}{2}}$, then solve for AD.

$$\frac{\frac{3}{2}\left(\frac{1}{3}\right)^{\frac{1}{2}}}{\left(\frac{2}{3}\right)^{\frac{1}{2}}}=\frac{AD}{\left(\frac{1}{3}\right)^{\frac{1}{2}}}$$

Multiply both sides by $\left(\frac{1}{3}\right)^{\frac{1}{2}}$.

$$\left(\frac{1}{3}\right)^{\frac{1}{2}}\left(\frac{\frac{3}{2}\left(\frac{1}{3}\right)^{\frac{1}{2}}}{\left(\frac{2}{3}\right)^{\frac{1}{2}}}\right)=\left(\frac{1}{3}\right)^{\frac{1}{2}}\left(\frac{AD}{\left(\frac{1}{3}\right)^{\frac{1}{2}}}\right)$$

$$\frac{\frac{3}{2}\left(\frac{1}{3}\right)^{\frac{1}{2}}\left(\frac{1}{3}\right)^{\frac{1}{2}}}{\left(\frac{2}{3}\right)^{\frac{1}{2}}}=AD$$

Use the product of powers law.

$$\frac{\frac{3}{2}\left(\frac{1}{3}\right)^{\frac{1}{2}+\frac{1}{2}}}{\left(\frac{2}{3}\right)^{\frac{1}{2}}}=AD$$

$$\frac{\frac{3}{2}\left(\frac{1}{3}\right)}{\left(\frac{2}{3}\right)^{\frac{1}{2}}}=AD$$

$$\frac{\frac{1}{2}}{\left(\frac{2}{3}\right)^{\frac{1}{2}}}=AD$$

Use a negative exponent to move the denominator to the numerator.

$$\left(\frac{1}{2}\right)\left(\frac{2}{3}\right)^{-\frac{1}{2}}=AD$$

Write with a positive exponent.

$$\left(\frac{1}{2}\right)\left(\frac{3}{2}\right)^{\frac{1}{2}}=AD$$

Use a calculator to evaluate:

$$AD=\left(\frac{1}{2}\right)\left(\frac{3}{2}\right)^{\frac{1}{2}}$$
$$=0.6123...$$

The length of AD is $\left(\frac{1}{2}\right)\left(\frac{3}{2}\right)^{\frac{1}{2}}$ cm, or approximately 0.6 cm.

Review (pages 246–248)

4.1

1. a) $\sqrt[3]{1000} = \sqrt[3]{10 \cdot 10 \cdot 10}$
$= 10$

b) $\sqrt{0.81} = \sqrt{0.9 \cdot 0.9}$
$= 0.9$

c) $\sqrt[6]{64} = \sqrt[6]{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2}$
$= 2$

d) $\sqrt[4]{\dfrac{81}{625}} = \sqrt[3]{\dfrac{3}{5} \cdot \dfrac{3}{5} \cdot \dfrac{3}{5} \cdot \dfrac{3}{5}}$
$= \dfrac{3}{5}$

I did not need a calculator because 1000 is a perfect cube, 0.81 is a perfect square, 64 is a perfect sixth power, and $\dfrac{81}{625}$ is a perfect fourth power.

2. The index of a radical tells me which root I have to take.
For example, the index of $\sqrt[3]{27}$ is 3, so I take the cube root.
The index of $\sqrt[4]{16}$ is 4, so I take the fourth root.
The index of $\sqrt[5]{243}$ is 5, so I take the fifth root.
When there is no index written, it is assumed to be 2, and I take the square root; for example, $\sqrt{25}$.

3. Use benchmarks with guess and check.

a) 11 is between the perfect squares 9 and 16, but closer to 9.
So, $\sqrt{11}$ is between 3 and 4, but closer to 3.
Estimate to 1 decimal place: $\sqrt{11} \doteq 3.4$
Square the estimate: $3.4^2 = 11.56$ (too large, but close)
Revise the estimate: $\sqrt{11} \doteq 3.3$
Square the estimate: $3.3^2 = 10.89$ (very close)
10.89 is closer to 11, so $\sqrt{11}$ is approximately 3.3.

b) –12 is between the perfect cubes –8 and –27, but closer to –8.
So, $\sqrt[3]{-12}$ is between –2 and –3, but closer to –2.
Estimate to 1 decimal place: $\sqrt[3]{-12} \doteq -2.2$
Cube the estimate: $(-2.2)^3 = -10.648$ (too large)
Revise the estimate: $\sqrt[3]{-12} \doteq -2.3$

Cube the estimate: $(-2.3)^3 = -12.167$ (very close)

-12.167 is closer to -12, so $\sqrt[3]{-12}$ is approximately -2.3.

c) 15 is between the perfect fourth powers 1 and 16, but closer to 16.
So, $\sqrt[4]{15}$ is between 1 and 2, but closer to 2.
Estimate to 1 decimal place: $\sqrt[4]{15} \doteq 1.9$
Raise the estimate to the fourth power: $1.9^4 = 13.0321$ (too small)
Revise the estimate: $\sqrt[4]{15} \doteq 2.0$
Raise the estimate to the fourth power: $2.0^4 = 16.0$ (closer)
16 is closer to 15, so $\sqrt[4]{15}$ is approximately 2.0.

4. a) $5^2 = 25$, so $5 = \sqrt{25}$
The number is 25.

b) $6^3 = 216$, so $6 = \sqrt[3]{216}$
The number is 216.

c) $7^4 = 2401$, so $7 = \sqrt[4]{2401}$
The number is 2401.

5. $\sqrt[3]{35}$ is irrational because 35 is not a perfect cube.
The decimal form of $\sqrt[3]{35}$ neither terminates nor repeats.

4.2

6. a) -2 is rational because it can be written as a quotient of integers, $\frac{-2}{1}$.

b) 17 is rational because it can be written as a quotient of integers, $\frac{17}{1}$.

c) $\sqrt{16}$ is rational because 16 is a perfect square.
Its decimal form is 4.0, which terminates.

d) $\sqrt{32}$ is irrational because 32 is not a perfect square.
The decimal form of $\sqrt{32}$ neither terminates nor repeats.

e) 0.756 is rational because it is a terminating decimal.

f) $12.\overline{3}$ is rational because it is a repeating decimal.

g) 0 is rational because it can be written as a quotient of integers, $\frac{0}{1}$.

h) $\sqrt[3]{81}$ is irrational because 81 is not a perfect cube.
The decimal form of $\sqrt[3]{81}$ neither terminates nor repeats.

i) π is irrational because the decimal form of π neither terminates nor repeats.

7. The formula for the area, A, of a square with side length s units is:
$A = s^2$
To determine the value of s, take the square root of each side.
$\sqrt{A} = \sqrt{s^2}$
$\sqrt{A} = s$ Substitute: $A = 23$
$s = \sqrt{23}$
Use a calculator.
$\sqrt{23} = 4.7958\ldots$
The side length of the square is approximately 4.8 cm.
I could check my answer by squaring the side length.
If the result is close to 23 cm^2, my answer is correct.

8. a) 3.141 592 654 is a terminating decimal, so 3.141 592 654 is a rational number.

b) The number π is irrational because it cannot be written as a quotient of integers.
The calculator screen shows only the first 10 digits for the value of π.

9. 30 is between the perfect cubes 27 and 64, and is closer to 27.

$\sqrt[3]{27}$ $\sqrt[3]{30}$ $\sqrt[3]{64}$
↓ ↓ ↓
3 ? 4

Use a calculator.
$\sqrt[3]{30} = 3.1072\ldots$

20 is between the perfect squares 16 and 25, and is closer to 16.

$\sqrt{16}$ $\sqrt{20}$ $\sqrt{25}$
↓ ↓ ↓
4 ? 5

Use a calculator.
$\sqrt{20} = 4.4721\ldots$

18 is between the perfect fourth powers 16 and 81, and is closer to 16.

$\sqrt[4]{16}$ $\sqrt[4]{18}$ $\sqrt[4]{81}$

↓ ↓ ↓

2 ? 3

Use a calculator.

$\sqrt[4]{18} = 2.0597\ldots$

–30 is between the perfect cubes –27 and –64, and is closer to –27.

$\sqrt[3]{-27}$ $\sqrt[3]{-30}$ $\sqrt[3]{-64}$

↓ ↓ ↓

–3 ? –4

Use a calculator.

$\sqrt[3]{-30} = -3.1072\ldots$

30 is between the perfect squares 25 and 36, and is closer to 25.

$\sqrt{25}$ $\sqrt{30}$ $\sqrt{36}$

↓ ↓ ↓

5 ? 6

Use a calculator.

$\sqrt{30} = 5.4772\ldots$

10 is between the perfect fourth powers 1 and 16, and is closer to 16.

$\sqrt[4]{1}$ $\sqrt[4]{10}$ $\sqrt[4]{16}$

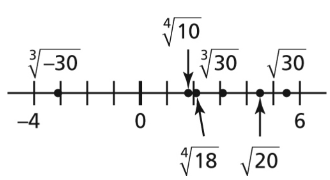
↓ ↓ ↓

1 ? 2

Use a calculator.

$\sqrt[4]{10} = 1.7782\ldots$

Mark each number on a number line.



From least to greatest: $\sqrt[3]{-30}$, $\sqrt[4]{10}$, $\sqrt[4]{18}$, $\sqrt[3]{30}$, $\sqrt{20}$, $\sqrt{30}$

10. Use the formula: $T = 2\pi\sqrt{\frac{L}{9.8}}$

Substitute: $L = 0.25$

$T = 2\pi\sqrt{\frac{0.25}{9.8}}$ Use a calculator.

$= 1.0035...$

It takes the pendulum approximately 1 s to complete one swing.

4.3

11. a)

$$\begin{aligned}\sqrt{150} &= \sqrt{25 \cdot 6}\\ &= \sqrt{5 \cdot 5 \cdot 6}\\ &= \sqrt{(5 \cdot 5) \cdot 6}\\ &= \sqrt{5 \cdot 5} \cdot \sqrt{6}\\ &= 5 \cdot \sqrt{6}\\ &= 5\sqrt{6}\end{aligned}$$

b)

$$\begin{aligned}\sqrt[3]{135} &= \sqrt[3]{27 \cdot 5}\\ &= \sqrt[3]{3 \cdot 3 \cdot 3 \cdot 5}\\ &= \sqrt[3]{(3 \cdot 3 \cdot 3) \cdot 5}\\ &= \sqrt[3]{(3 \cdot 3 \cdot 3)} \cdot \sqrt[3]{5}\\ &= 3 \cdot \sqrt[3]{5}\\ &= 3\sqrt[3]{5}\end{aligned}$$

c)

$$\begin{aligned}\sqrt{112} &= \sqrt{16 \cdot 7}\\ &= \sqrt{4 \cdot 4 \cdot 7}\\ &= \sqrt{(4 \cdot 4) \cdot 7}\\ &= \sqrt{4 \cdot 4} \cdot \sqrt{7}\\ &= 4 \cdot \sqrt{7}\\ &= 4\sqrt{7}\end{aligned}$$

d)

$$\begin{aligned}\sqrt[4]{162} &= \sqrt[4]{81 \cdot 2}\\ &= \sqrt[4]{3 \cdot 3 \cdot 3 \cdot 3 \cdot 2}\\ &= \sqrt[4]{(3 \cdot 3 \cdot 3 \cdot 3) \cdot 2}\\ &= \sqrt[4]{(3 \cdot 3 \cdot 3 \cdot 3)} \cdot \sqrt[4]{2}\\ &= 3 \cdot \sqrt[4]{2}\\ &= 3\sqrt[4]{2}\end{aligned}$$

12. a) Write 6 as: $\sqrt{6 \cdot 6} = \sqrt{36}$

$$\begin{aligned} 6\sqrt{5} &= \sqrt{36} \cdot \sqrt{5} \\ &= \sqrt{36 \cdot 5} \\ &= \sqrt{180} \end{aligned}$$

b) Write 3 as: $\sqrt{3 \cdot 3} = \sqrt{9}$

$$\begin{aligned} 3\sqrt{14} &= \sqrt{9} \cdot \sqrt{14} \\ &= \sqrt{9 \cdot 14} \\ &= \sqrt{126} \end{aligned}$$

c) Write 4 as: $\sqrt[3]{4 \cdot 4 \cdot 4} = \sqrt[3]{64}$

$$\begin{aligned} 4\sqrt[3]{3} &= \sqrt[3]{64} \cdot \sqrt[3]{3} \\ &= \sqrt[3]{64 \cdot 3} \\ &= \sqrt[3]{192} \end{aligned}$$

d) Write 2 as: $\sqrt[4]{2 \cdot 2 \cdot 2 \cdot 2} = \sqrt[4]{16}$

$$\begin{aligned} 2\sqrt[4]{2} &= \sqrt[4]{16} \cdot \sqrt[4]{2} \\ &= \sqrt[4]{16 \cdot 2} \\ &= \sqrt[4]{32} \end{aligned}$$

13. The formula for the volume, V, of a cube with edge length e units is:
$V = e^3$
To determine the value of e, take the cube root of each side.

$$\sqrt[3]{V} = \sqrt[3]{e^3}$$
$$\sqrt[3]{V} = e$$

To determine the edge length of a cube with volume 32 cm^3, substitute: $V = 32$
$e = \sqrt[3]{32}$
Write 32 as $8 \cdot 4$.

$$\begin{aligned} e &= \sqrt[3]{8 \cdot 4} \\ &= \sqrt[3]{8} \cdot \sqrt[3]{4} \\ &= 2 \cdot \sqrt[3]{4} \\ &= 2\sqrt[3]{4} \end{aligned}$$

The cube has edge length $2\sqrt[3]{4}$ cm.
Use a calculator to evaluate: $2\sqrt[3]{4} = 3.1748\ldots$
The edge length of the cube is approximately 3.2 cm.

To determine the edge length of a cube with volume 11 cm^3,
in $e = \sqrt[3]{V}$ substitute: $V = 11$
$e = \sqrt[3]{11}$
$\sqrt[3]{11}$ cannot be simplified.
Use a calculator to evaluate: $\sqrt[3]{11} = 2.2239\ldots$
The edge length of the cube is approximately 2.2 cm.

The difference in the edge lengths is approximately 3.2 cm – 2.2 cm = 1.0 cm.

14. In the second line, the student wrote $\sqrt{100}$ as $\sqrt{50} \cdot \sqrt{50}$; the student should have written $\sqrt{100}$ as 10 because 100 is a perfect square. From the third line to the fourth line, the student wrote $\sqrt{3} \cdot \sqrt{2}$ as 3, which is not correct.
A correct solution is:

$$\begin{aligned}\sqrt{300} &= \sqrt{100 \cdot 3}\\ &= \sqrt{100} \cdot \sqrt{3}\\ &= 10 \cdot \sqrt{3}\\ &= 10\sqrt{3}\end{aligned}$$

15. Since all the radicals are square roots, I will rewrite each mixed radical as an entire radical then order the entire radicals from the greatest radicand to the least radicand.
Write 5 as: $\sqrt{5 \cdot 5} = \sqrt{25}$

$$\begin{aligned}5\sqrt{2} &= \sqrt{25} \cdot \sqrt{2}\\ &= \sqrt{25 \cdot 2}\\ &= \sqrt{50}\end{aligned}$$

Write 4 as: $\sqrt{4 \cdot 4} = \sqrt{16}$

$$\begin{aligned}4\sqrt{3} &= \sqrt{16} \cdot \sqrt{3}\\ &= \sqrt{16 \cdot 3}\\ &= \sqrt{48}\end{aligned}$$

Write 3 as: $\sqrt{3 \cdot 3} = \sqrt{9}$

$$\begin{aligned}3\sqrt{6} &= \sqrt{9} \cdot \sqrt{6}\\ &= \sqrt{9 \cdot 6}\\ &= \sqrt{54}\end{aligned}$$

Write 2 as: $\sqrt{2 \cdot 2} = \sqrt{4}$

$$\begin{aligned} 2\sqrt{7} &= \sqrt{4} \cdot \sqrt{7} \\ &= \sqrt{4 \cdot 7} \\ &= \sqrt{28} \end{aligned}$$

Write 6 as: $\sqrt{6 \cdot 6} = \sqrt{36}$

$$\begin{aligned} 6\sqrt{2} &= \sqrt{36} \cdot \sqrt{2} \\ &= \sqrt{36 \cdot 2} \\ &= \sqrt{72} \end{aligned}$$

From greatest to least, the entire radicals are: $\sqrt{72}$, $\sqrt{54}$, $\sqrt{50}$, $\sqrt{48}$, $\sqrt{28}$

So, from greatest to least, the mixed radicals are: $6\sqrt{2}$, $3\sqrt{6}$, $5\sqrt{2}$, $4\sqrt{3}$, $2\sqrt{7}$

4.4

16. The product of powers law is: $a^m \cdot a^n = a^{m+n}$

A multiplication property of radicals is: $\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{a \cdot b}$

So, $5^{\frac{1}{2}} \cdot 5^{\frac{1}{2}} = 5^{\frac{1}{2}+\frac{1}{2}} = 5^1 = 5$ and $\sqrt{5} \cdot \sqrt{5} = \sqrt{25} = 5$

So, $5^{\frac{1}{2}}$ and $\sqrt{5}$ are equivalent expressions; that is, $5^{\frac{1}{2}} = \sqrt{5}$.

Similarly, $5^{\frac{1}{3}} \cdot 5^{\frac{1}{3}} \cdot 5^{\frac{1}{3}} = 5^{\frac{1}{3}+\frac{1}{3}+\frac{1}{3}} = 5^1 = 5$ and $\sqrt[3]{5} \cdot \sqrt[3]{5} \cdot \sqrt[3]{5} = \sqrt[3]{125} = 5$

So, $5^{\frac{1}{3}}$ and $\sqrt[3]{5}$ are equivalent expressions; that is, $5^{\frac{1}{3}} = \sqrt[3]{5}$.

So, when *n* is a natural number and *a* is a rational number, $a^{\frac{1}{n}} = \sqrt[n]{a}$.

17. a) Use the rule: $x^{\frac{1}{n}} = \sqrt[n]{x}$

$$12^{\frac{1}{4}} = \sqrt[4]{12}$$

b) Use $a^{\frac{m}{n}} = \left(\sqrt[n]{a}\right)^m$ or $\sqrt[n]{a^m}$.

$$(-50)^{\frac{5}{3}} = \left(\sqrt[3]{-50}\right)^5, \text{ or } \sqrt[3]{(-50)^5}$$

c) Use the rule: $x^{\frac{1}{n}} = \sqrt[n]{x}$

$$1.2^{0.5} = 1.2^{\frac{1}{2}}$$
$$= \sqrt{1.2}$$

d) Use the rule: $x^{\frac{1}{n}} = \sqrt[n]{x}$

$$\left(\frac{3}{8}\right)^{\frac{1}{3}} = \sqrt[3]{\frac{3}{8}}$$

18. a) Use: $\sqrt[n]{x} = x^{\frac{1}{n}}$

$$\sqrt{1.4} = 1.4^{\frac{1}{2}}$$

b) Use: $\sqrt[n]{a^m} = a^{\frac{m}{n}}$

$$\sqrt[3]{13^2} = 13^{\frac{2}{3}}$$

c) Use $\left(\sqrt[n]{a}\right)^m = a^{\frac{m}{n}}$.

$$\left(\sqrt[5]{2.5}\right)^4 = 2.5^{\frac{4}{5}}$$

d) Use: $\left(\sqrt[n]{a}\right)^m = a^{\frac{m}{n}}$

$$\left(\sqrt[4]{\frac{2}{5}}\right)^3 = \left(\frac{2}{5}\right)^{\frac{3}{4}}$$

19. a) The exponent $0.25 = \frac{1}{4}$

So, $16^{0.25} = 16^{\frac{1}{4}}$
$$= \sqrt[4]{16}$$
$$= \sqrt[4]{2 \cdot 2 \cdot 2 \cdot 2}$$
$$= 2$$

b) $$1.44^{\frac{1}{2}} = \sqrt{1.44}$$
$$= \sqrt{1.2 \cdot 1.2}$$
$$= 1.2$$

c) $$(-8)^{\frac{5}{3}} = \left[(-8)^{\frac{1}{3}}\right]^5$$
$$= \left(\sqrt[3]{-8}\right)^5$$
$$= (-2)^5$$
$$= -32$$

d)
$$\left(\frac{9}{16}\right)^{\frac{3}{2}} = \left[\left(\frac{9}{16}\right)^{\frac{1}{2}}\right]^3$$
$$= \left(\sqrt{\frac{9}{16}}\right)^3$$
$$= \left(\frac{3}{4}\right)^3$$
$$= \frac{27}{64}$$

20. In the formula: $P = 100(0.5)^{\frac{t}{20}}$, substitute: $t = 30$

$$P = 100(0.5)^{\frac{30}{20}}$$ Use a calculator.
$$= 35.3553...$$

Approximately 35% of polonium remains after 30 weeks.

21. The radicand of each number is 5, so write each number with a fractional exponent.
To order the numbers from greatest to least, order the exponents from greatest to least.
The number with the greatest exponent is the greatest number.
The number with the least exponent is the least number.

$\sqrt[4]{5} = 5^{\frac{1}{4}}$ $\quad 5^{\frac{2}{3}}$ $\quad \sqrt[3]{5} = 5^{\frac{1}{3}}$ $\quad 5^{\frac{3}{4}}$ $\quad \left(\sqrt{5}\right)^3 = 5^{\frac{3}{2}}$

From greatest to least, the exponents are: $\frac{3}{2}, \frac{3}{4}, \frac{2}{3}, \frac{1}{3}, \frac{1}{4}$

So, from greatest to least, the numbers are: $\left(\sqrt{5}\right)^3$, $5^{\frac{3}{4}}$, $5^{\frac{2}{3}}$, $\sqrt[3]{5}$, $\sqrt[4]{5}$

22. a) In the formula $q = 70M^{\frac{3}{4}}$, substitute: $M = 475$

$$q = 70(475)^{\frac{3}{4}}$$
$$= 7122.2669...$$

Use a calculator.

The metabolic rate of a cow with mass 475 kg is approximately 7122 Calories per day.

b) In the formula $q = 70M^{\frac{3}{4}}$, the mass must be in kilograms.
To write 25 g in kilograms, divide by 1000: 25 g = 0.025 kg
Substitute: $M = 0.025$

$$q = 70(0.025)^{\frac{3}{4}}$$
$$= 4.4010...$$

Use a calculator.

The metabolic rate of a mouse with mass 25 g is approximately 4 Calories per day.

4.5

23. a) The numbers at the left are divided by 3 each time.
The exponents in the powers at the right decrease by 1 each time.

b)

$$81 = 3^4$$
$$27 = 3^3$$
$$9 = 3^2$$
$$3 = 3^1$$
$$1 = 3^0$$
$$\frac{1}{3} = 3^{-1}$$
$$\frac{1}{9} = 3^{-2}$$
$$\frac{1}{27} = 3^{-3}$$

c) Look at the last 3 rows.

$$3^{-1} = \frac{1}{3} = \frac{1}{3^1}$$

$$3^{-2} = \frac{1}{9} = \frac{1}{3^2}$$

$$3^{-3} = \frac{1}{27} = \frac{1}{3^3}$$

So, this pattern shows that $a^{-n} = \frac{1}{a^n}$.

24. a) $2^{-2} = \frac{1}{2^2}$

$= \frac{1}{4}$

b) $\left(\frac{2}{3}\right)^{-3} = \left(\frac{3}{2}\right)^{3}$ Writing with a positive exponent

$= \frac{3^3}{2^3}$ Cubing the result

$= \frac{27}{8}$

c) $\left(\frac{4}{25}\right)^{-\frac{3}{2}} = \left(\frac{25}{4}\right)^{\frac{3}{2}}$ Writing with a positive exponent

$= \left(\sqrt{\frac{25}{4}}\right)^{3}$ Taking the square root

$= \left(\frac{5}{2}\right)^{3}$ Cubing the result

$= \frac{125}{8}$

25. Use a calculator to evaluate:

$P = 1000(1.0325)^{-3}$

$= 908.5102\ldots$

To have \$1000 in 3 years, Kyle should invest \$908.51 today.

26. In the formula: $h = 2000(3)^{-\frac{1}{2}}s^{-2}$, substitute: $s = 8.0$

$h = 2000(3)^{-\frac{1}{2}}(8.0)^{-2}$ Use a calculator.

$= 18.0421...$

The height of a container with base side length 8.0 cm is approximately 18.0 cm.

27. Use the formula: $F = 440\left(\sqrt[12]{2}\right)^{n}$

Middle C is 9 semitones below the concert pitch, so substitute $n = -9$.

$F = 440\left(\sqrt[12]{2}\right)^{-9}$ Use a calculator.

$= 261.6255...$

The frequency of middle C is approximately 262 Hz.

4.6

28. a) $(3m^4n)^2 = (3^1)^2 \cdot (m^4)^2 \cdot (n^1)^2$ Using the power of a product law

$= 3^{(1)(2)} \cdot m^{(4)(2)} \cdot n^{(1)(2)}$

$= 3^2 \cdot m^8 \cdot n^2$

$= 9m^8n^2$

b) $\left(\frac{x^2y}{y^{-2}}\right)^{-2} = \left(x^2 \cdot y^{1-(-2)}\right)^{-2}$ Using the quotient of powers law inside the brackets

$= \left(x^2 \cdot y^3\right)^{-2}$ Use the power of a product law.

$= x^{2(-2)} \cdot y^{3(-2)}$

$= x^{-4} \cdot y^{-6}$

$= x^{-4}y^{-6}$ Write with positive exponents.

$= \frac{1}{x^4y^6}$

c)

$(16a^2b^6)^{-\frac{1}{2}} = \frac{1}{(16a^2b^6)^{\frac{1}{2}}}$ Writing with a positive exponent

$= \frac{1}{16^{\frac{1}{2}} \cdot a^{2\left(\frac{1}{2}\right)} \cdot b^{6\left(\frac{1}{2}\right)}}$ Using the power of a product law

$= \frac{1}{16^{\frac{1}{2}} \cdot a^1 \cdot b^3}$

$= \frac{1}{4ab^3}$

d)

$\left(\frac{r^3s^{-1}}{s^{-2}r^{-2}}\right)^{-\frac{2}{3}} = \left(\frac{r^3}{r^{-2}} \cdot \frac{s^{-1}}{s^{-2}}\right)^{-\frac{2}{3}}$ Simplify inside the brackets.

$= \left(r^{3-(-2)} \cdot s^{-1-(-2)}\right)^{-\frac{2}{3}}$ Using the quotient of powers law

$= \left(r^5 \cdot s^1\right)^{-\frac{2}{3}}$ Use the power of a product law.

$= r^{5\left(-\frac{2}{3}\right)} \cdot s^{1\left(-\frac{2}{3}\right)}$

$= r^{-\frac{10}{3}} \cdot s^{-\frac{2}{3}}$ Write with positive exponents.

$= \frac{1}{r^{\frac{10}{3}}s^{\frac{2}{3}}}$

29. a)

$$\begin{aligned}\left(a^3b\right)\left(a^{-1}b^4\right) &= a^3 \cdot a^{-1} \cdot b^1 \cdot b^4 && \text{Use the product of powers law.}\\ &= a^{3-1} \cdot b^{1+4}\\ &= a^2 \cdot b^5\\ &= a^2b^5\end{aligned}$$

b)

$$\begin{aligned}\left(x^{\frac{1}{2}}y\right)\left(x^{\frac{3}{2}}y^{-2}\right) &= x^{\frac{1}{2}} \cdot x^{\frac{3}{2}} \cdot y^1 \cdot y^{-2} && \text{Use the product of powers law.}\\ &= x^{\frac{1}{2}+\frac{3}{2}} \cdot y^{1+(-2)}\\ &= x^{\frac{4}{2}} \cdot y^{-1}\\ &= x^2y^{-1} && \text{Write with a positive exponent.}\\ &= \frac{x^2}{y}\end{aligned}$$

c)

$$\begin{aligned}\frac{a^3}{a^5} \cdot a^{-3} &= a^{3-5} \cdot a^{-3} && \text{Using the quotient of powers law}\\ &= a^{-2} \cdot a^{-3} && \text{Use the product of powers law.}\\ &= a^{-2+(-3)}\\ &= a^{-5} && \text{Write with a positive exponent.}\\ &= \frac{1}{a^5}\end{aligned}$$

d)

$$\begin{aligned}\frac{x^2y}{x^{\frac{1}{2}}y^{-2}} &= \frac{x^2}{x^{\frac{1}{2}}} \cdot \frac{y}{y^{-2}} && \text{Use the quotient of powers law.}\\ &= x^{2-\frac{1}{2}} \cdot y^{1-(-2)}\\ &= x^{\frac{3}{2}} \cdot y^3\\ &= x^{\frac{3}{2}}y^3\end{aligned}$$

30. a) $\left(\frac{3}{2}\right)^{\frac{3}{2}} \cdot \left(\frac{3}{2}\right)^{\frac{1}{2}} = \left(\frac{3}{2}\right)^{\frac{3}{2}+\frac{1}{2}}$ Using the product of powers law

$= \left(\frac{3}{2}\right)^{\frac{4}{2}}$

$= \left(\frac{3}{2}\right)^{2}$

$= \frac{9}{4}$

b) $\frac{(-5.5)^{\frac{2}{3}}}{(-5.5)^{-\frac{4}{3}}} = (-5.5)^{\frac{2}{3}-\left(-\frac{4}{3}\right)}$ Using the quotient of powers law

$= (-5.5)^{\frac{6}{3}}$

$= (-5.5)^{2}$

$= 30.25$

c) $\left[\left(-\frac{12}{5}\right)^{\frac{1}{3}}\right]^{6} = \left(-\frac{12}{5}\right)^{\frac{1}{3}(6)}$ Using the power of a power law

$= \left(-\frac{12}{5}\right)^{2}$

$= \frac{144}{25}$

d) $\frac{0.16^{\frac{3}{4}}}{0.16^{\frac{1}{4}}} = 0.16^{\frac{3}{4}-\frac{1}{4}}$ Using the quotient of powers law

$= 0.16^{\frac{2}{4}}$

$= 0.16^{\frac{1}{2}}$

$= \sqrt{0.16}$

$= 0.4$

31. The volume, V, of a sphere with radius r is given by the formula: $V = \frac{4}{3}\pi r^3$

Substitute $V = 1100$, then solve for r.

$1100 = \frac{4}{3}\pi r^3$ Multiply each side by 3.

$3(1100) = 3\left(\frac{4}{3}\pi r^3\right)$

$3300 = 4\pi r^3$ Divide each side by 4π.

$\frac{3300}{4\pi} = \frac{4\pi r^3}{4\pi}$

$\frac{3300}{4\pi} = r^3$ To solve for r, take the cube root of each side by raising each side to the one-third power.

$\left(\frac{3300}{4\pi}\right)^{\frac{1}{3}} = \left(r^3\right)^{\frac{1}{3}}$ Use the power of a power law.

$\left(\frac{3300}{4\pi}\right)^{\frac{1}{3}} = r$ Use a calculator.

$r = 6.4037...$

The radius of the sphere is approximately 6.4 cm.

32. a) In the second line, the exponents were multiplied instead of added.
A correct solution is:

$\left(s^{-1}t^{\frac{1}{3}}\right)\left(s^4t^3\right) = s^{-1} \cdot s^4 \cdot t^{\frac{1}{3}} \cdot t^3$ Use the product of powers law.

$= s^{-1+4} \cdot t^{\frac{1}{3}+3}$

$= s^3 \cdot t^{\frac{10}{3}}$

$= s^3t^{\frac{10}{3}}$

b) In the first line, 4 was multiplied by –3, instead of being raised to the power of –3; and, to raise d^3 to the power –3, the exponents were added instead of multiplied.

In the third line, –12 was incorrectly written as $\frac{1}{12}$.

A correct solution is:

$$\left(\frac{4c^{\frac{1}{3}}}{d^3}\right)^{-3} = \frac{4^{-3} \cdot c^{\left(\frac{1}{3}\right)(-3)}}{d^{3(-3)}}$$ Using the power of a power law

$$= \frac{4^{-3} \cdot c^{-1}}{d^{-9}}$$ Write with positive exponents.

$$= \frac{d^9}{4^3 \cdot c}$$

$$= \frac{d^9}{64c}$$

Practice Test (page 249)

1. The formula for the volume, V, of a cube with edge length s units is:
$V = s^3$
To determine the value of s, take the cube root of each side.
$\sqrt[3]{V} = \sqrt[3]{s^3}$
$\sqrt[3]{V} = s$
For s to be irrational, V must be a positive number that is not a perfect cube.
Try to write each given volume as a perfect cube.
$5^3 = 125$, $4^3 = 64$, $6^3 = 216$
75 is not a perfect cube.
This is the volume of the cube in choice B.
So, choice B is correct.

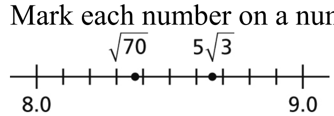
2. $\sqrt{0.09}$ is rational because 0.09 is a perfect square; $0.3^2 = 0.09$
$\sqrt{50}$ is irrational because 50 is not a perfect square.
$\sqrt[3]{-\frac{64}{121}}$ is irrational because 121 is not a perfect cube.
π is irrational because it cannot be written as a quotient of integers.
So, choice A is correct.

3. a) Since both radicals have an index of 2, the greater radical has the greater radicand.
Rewrite $5\sqrt{3}$ as an entire radical.
Write 5 as: $\sqrt{5 \cdot 5} = \sqrt{25}$

$$5\sqrt{3} = \sqrt{25} \cdot \sqrt{3}$$
$$= \sqrt{25 \cdot 3}$$
$$= \sqrt{75}$$

Compare $\sqrt{75}$ and $\sqrt{70}$.
Since $75 > 70$, $\sqrt{75} > \sqrt{70}$
So, $\sqrt{75}$ or $5\sqrt{3}$ is the greater radical.

b) Use a calculator.
$5\sqrt{3} = 8.6602\ldots$ $\qquad$ $\sqrt{70} = 8.3666\ldots$
Mark each number on a number line.



4. a)
$$\sqrt[4]{\frac{256}{81}} = \sqrt[4]{\frac{4}{3} \cdot \frac{4}{3} \cdot \frac{4}{3} \cdot \frac{4}{3}}$$
$$= \frac{4}{3}$$

b) $$(-4)^{-2} = \frac{1}{(-4)^2} = \frac{1}{16}$$

c) $$0.81^{\frac{3}{2}} = \left(\sqrt{0.81}\right)^3 = 0.9^3 = 0.729$$

d) $$16^{-\frac{1}{2}} = \frac{1}{16^{\frac{1}{2}}} = \frac{1}{\sqrt{16}} = \frac{1}{4}$$

5. $44^{\frac{1}{2}} = \sqrt{44}$

The factors of 44 are: 1, 2, 4, 11, 22, 44

The greatest perfect square is $4 = 2 \cdot 2$, so write 44 as $4 \cdot 11$.

$$\sqrt{44} = \sqrt{4 \cdot 11} = \sqrt{4} \cdot \sqrt{11} = 2 \cdot \sqrt{11} = 2\sqrt{11}$$

6. The student is incorrect. When using the quotient of powers law, the exponents should be subtracted, not added.

A correct solution is:

$$\frac{x^{-1}y^3}{xy^{-2}} = x^{-1-1} \cdot y^{3-(-2)}$$ Using the quotient of powers law

$$= x^{-2} \cdot y^5$$

$$= x^{-2}y^5$$ Write with a positive exponent.

$$= \frac{y^5}{x^2}$$

7. a)

$$\left(p^{-2}q^{-1}\right)^{2}\left(pq^{\frac{1}{2}}\right)^{2} = \left(p^{(-2)(2)}q^{-1(2)}\right)\left(p^{2}q^{\frac{1}{2}(2)}\right)$$ Using the power of a product law

$$= \left(p^{-4}q^{-2}\right)\left(p^{2}q\right)$$

$$= p^{-4}\cdot q^{-2}\cdot p^{2}\cdot q$$ Use the product of powers law.

$$= p^{-4+2}\cdot q^{-2+1}$$

$$= p^{-2}q^{-1}$$ Write with positive exponents.

$$= \frac{1}{p^{2}q}$$

b)

$$\left(\frac{c^{6}d^{5}}{c^{3}d^{4}}\right)^{-\frac{1}{3}} = \left(\frac{c^{3}d^{4}}{c^{6}d^{5}}\right)^{\frac{1}{3}}$$ Writing with a positive exponent

$$= \frac{c^{3\left(\frac{1}{3}\right)}\cdot d^{4\left(\frac{1}{3}\right)}}{c^{6\left(\frac{1}{3}\right)}\cdot d^{5\left(\frac{1}{3}\right)}}$$ Using the power of a product law

$$= \frac{c^{1}\cdot d^{\frac{4}{3}}}{c^{2}\cdot d^{\frac{5}{3}}}$$ Use the quotient of powers law.

$$= c^{1-2}\cdot d^{\frac{4}{3}-\frac{5}{3}}$$

$$= c^{-1}d^{-\frac{1}{3}}$$ Write with positive exponents.

$$= \frac{1}{cd^{\frac{1}{3}}}$$

8. In the formula: $d = 0.099m^{\frac{9}{10}}$, substitute: $m = 550$

$$d = 0.099(550)^{\frac{9}{10}}$$ Use a calculator.

$$= 28.9708...$$

A 550-kg moose should drink approximately 29 L of water in one day.