Lesson 3.1 **Factors and Multiples of Whole Numbers** **Exercises (pages 140–141)**

A

3. Multiply each number by 1, 2, 3, 4, 5, and 6.

a) $6 \times 1 = 6$
$6 \times 2 = 12$
$6 \times 3 = 18$
$6 \times 4 = 24$
$6 \times 5 = 30$
$6 \times 6 = 36$
So, the first 6 multiples of 6 are: 6, 12, 18, 24, 30, 36

b) $13 \times 1 = 13$
$13 \times 2 = 26$
$13 \times 3 = 39$
$13 \times 4 = 52$
$13 \times 5 = 65$
$13 \times 6 = 78$
So, the first 6 multiples of 13 are: 13, 26, 39, 52, 65, 78

c) $22 \times 1 = 22$
$22 \times 2 = 44$
$22 \times 3 = 66$
$22 \times 4 = 88$
$22 \times 5 = 110$
$22 \times 6 = 132$
So, the first 6 multiples of 22 are: 22, 44, 66, 88, 110, 132

d) $31 \times 1 = 31$
$31 \times 2 = 62$
$31 \times 3 = 93$
$31 \times 4 = 124$
$31 \times 5 = 155$
$31 \times 6 = 186$
So, the first 6 multiples of 31 are: 31, 62, 93, 124, 155, 186

e) $45 \times 1 = 45$
$45 \times 2 = 90$
$45 \times 3 = 135$
$45 \times 4 = 180$
$45 \times 5 = 225$
$45 \times 6 = 270$
So, the first 6 multiples of 45 are: 45, 90, 135, 180, 225, 270

f) $27 \times 1 = 27$
$27 \times 2 = 54$
$27 \times 3 = 81$
$27 \times 4 = 108$
$27 \times 5 = 135$
$27 \times 6 = 162$
So, the first 6 multiples of 27 are: 27, 54, 81, 108, 135, 162

4. The first 10 prime numbers are: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29
Divide each number by these primes.

a) $40 \div 2 = 20$
$40 \div 3 = 13.3333\ldots$
$40 \div 5 = 8$
$40 \div 7 = 5.7142\ldots$
There are no more primes between 5.7142… and 7.
40 is divisible by the prime numbers 2 and 5.
So, the prime factors of 40 are 2 and 5.

b) $75 \div 2 = 37.5$
$75 \div 3 = 25$
$75 \div 5 = 15$
$75 \div 7 = 10.7142\ldots$
There are no more primes between 7 and 10.7142….
75 is divisible by the prime numbers 3 and 5.
So, the prime factors of 75 are 3 and 5.

c) $81 \div 2 = 40.5$
$81 \div 3 = 27$
$81 \div 5 = 16.2$
$81 \div 7 = 11.5714\ldots$
$81 \div 11 = 7.3636\ldots$
There are no more primes between 7.3636… and 11.
81 is divisible by the prime number 3.
So, the prime factor of 81 is 3.

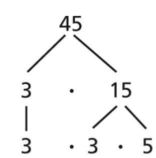
d) $120 \div 2 = 60$
$120 \div 3 = 40$
$120 \div 5 = 24$
$120 \div 7 = 17.1428\ldots$
$120 \div 11 = 10.9090\ldots$
There are no more primes between 10.9090… and 11.
120 is divisible by the prime numbers 2, 3, and 5.
So, the prime factors of 120 are 2, 3, and 5.

e) $140 \div 2 = 70$
$140 \div 3 = 46.6666\ldots$
$140 \div 5 = 28$
$140 \div 7 = 20$
$140 \div 11 = 12.7272\ldots$
There are no more primes between 11 and 12.7272….
140 is divisible by the prime numbers 2, 5, and 7.
So, the prime factors of 140 are 2, 5, and 7.

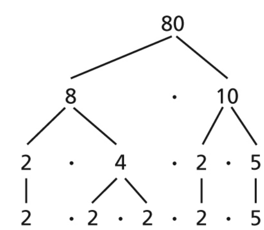
f) $192 \div 2 = 96$
$192 \div 3 = 64$
$192 \div 5 = 38.4$
$192 \div 7 = 27.4285\ldots$
$192 \div 11 = 17.4545\ldots$
$192 \div 13 = 14.7692\ldots$
There are no more primes between 13 and 14.7692….
192 is divisible by the prime numbers 2 and 3.
So, the prime factors of 192 are 2 and 3.

5. Draw a factor tree for each number.

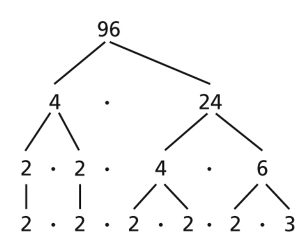
a) Write 45 as a product of 2 factors: $45 = 3 \cdot 15$
3 is a prime factor, but 15 can be factored further.
$15 = 3 \cdot 5$
3 and 5 are prime factors, so there are no more factors.
$45 = 3 \cdot 3 \cdot 5$, or $3^2 \cdot 5$



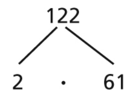
b) Write 80 as a product of 2 factors: $80 = 8 \cdot 10$
Both 8 and 10 are composite numbers, so factor again.
$8 = 2 \cdot 4$ and $10 = 2 \cdot 5$
2 and 5 are prime factors, but 4 can be factored further.
$4 = 2 \cdot 2$
2 is a prime factor, so there are no more factors.
$80 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 5$, or $2^4 \cdot 5$



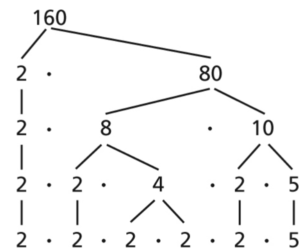
c) Write 96 as a product of 2 factors: $96 = 4 \cdot 24$
Both 4 and 24 are composite numbers, so factor again.
$4 = 2 \cdot 2$ and $24 = 4 \cdot 6$
2 is a prime factor, but both 4 and 6 can be factored further.
$4 = 2 \cdot 2$ and $6 = 2 \cdot 3$
2 and 3 are prime factors, so there are no more factors.
$96 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3$, or $2^5 \cdot 3$



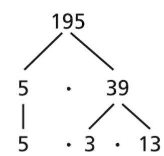
d) Write 122 as a product of 2 factors: 122 = 2 · 61
2 and 61 are prime factors, so there are no more factors.
122 = 2 · 61



e) Write 160 as a product of 2 factors:
160 = 2 · 80
From part b, 80 = 2 · 2 · 2 · 2 · 5, or $2^4 \cdot 5$
So, 160 = 2 · 2 · 2 · 2 · 2 · 5, or $2^5 \cdot 5$



f) Write 195 as a product of 2 factors: 195 = 5 · 39
5 is a prime factor, but 39 can be factored further.
39 = 3 · 13
3 and 13 are prime factors, so there are no more factors.
195 = 3 · 5 · 13



B

6. Use a calculator and repeated division by prime factors.

a) 600 ÷ 2 = 300
300 ÷ 2 = 150
150 ÷ 2 = 75
75 ÷ 3 = 25
25 ÷ 5 = 5
5 ÷ 5 = 1
So, 600 = 2 · 2 · 2 · 3 · 5 · 5
Using powers: $600 = 2^3 \cdot 3 \cdot 5^2$

b) 1150 ÷ 2 = 575
575 ÷ 5 = 115
115 ÷ 5 = 23
23 ÷ 23 = 1
So, 1150 = 2 · 5 · 5 · 23
Using powers: $1150 = 2 \cdot 5^2 \cdot 23$

c) 1022 ÷ 2 = 511
511 ÷ 7 = 73
73 ÷ 73 = 1
So, 1150 = 2 · 7 · 73

d) $2250 \div 2 = 1125$
$1125 \div 3 = 375$
$375 \div 3 = 125$
$125 \div 5 = 25$
$25 \div 5 = 5$
$5 \div 5 = 1$
So, $2250 = 2 \cdot 3 \cdot 3 \cdot 5 \cdot 5 \cdot 5$
Using powers: $2250 = 2 \cdot 3^2 \cdot 5^3$

e) $4500 \div 2 = 2250$
From part b, $2250 = 2 \cdot 3^2 \cdot 5^3$
So, $4500 = 2 \cdot 2 \cdot 3^2 \cdot 5^3$
Using powers: $4500 = 2^2 \cdot 3^2 \cdot 5^3$

f) $6125 \div 5 = 1225$
$1225 \div 5 = 245$
$245 \div 5 = 49$
$49 \div 7 = 7$
$7 \div 7 = 1$
So, $6125 = 5 \cdot 5 \cdot 5 \cdot 7 \cdot 7$
Using powers: $6125 = 5^3 \cdot 7^2$

7. I cannot write 0 as a product of prime numbers, so 0 has no prime factors.
1 is the only whole number that divides into 1, so 1 has no prime factors.

8. Write the prime factorization of each number.
Highlight the factors that appear in each prime factorization.

a) $46 = \mathbf{2} \cdot 23$
$84 = \mathbf{2} \cdot 2 \cdot 3 \cdot 7$
The greatest common factor is 2.

b) $64 = \mathbf{2} \cdot \mathbf{2} \cdot \mathbf{2} \cdot 2 \cdot 2 \cdot 2$
$120 = \mathbf{2} \cdot \mathbf{2} \cdot \mathbf{2} \cdot 3 \cdot 5$
The greatest common factor is $2 \cdot 2 \cdot 2$, which is 8.

c) $81 = \mathbf{3} \cdot \mathbf{3} \cdot \mathbf{3} \cdot 3$
$216 = 2 \cdot 2 \cdot 2 \cdot \mathbf{3} \cdot \mathbf{3} \cdot \mathbf{3}$
The greatest common factor is $3 \cdot 3 \cdot 3$, which is 27.

d) $180 = \mathbf{2} \cdot \mathbf{2} \cdot 3 \cdot 3 \cdot 5$
$224 = \mathbf{2} \cdot \mathbf{2} \cdot 2 \cdot 2 \cdot 2 \cdot 7$
The greatest common factor is $2 \cdot 2$, which is 4.

e) $160 = \mathbf{2} \cdot \mathbf{2} \cdot \mathbf{2} \cdot \mathbf{2} \cdot \mathbf{2} \cdot 5$
$672 = \mathbf{2} \cdot \mathbf{2} \cdot \mathbf{2} \cdot \mathbf{2} \cdot \mathbf{2} \cdot 3 \cdot 7$
The greatest common factor is $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$, which is 32.

f) $220 = \mathbf{2} \cdot \mathbf{2} \cdot \mathbf{5} \cdot 11$
$860 = \mathbf{2} \cdot \mathbf{2} \cdot \mathbf{5} \cdot 43$
The greatest common factor is $2 \cdot 2 \cdot 5$, which is 20.

9. Write the prime factorization of each number.
Highlight the factors that appear in each prime factorization.

a) $150 = 2 \cdot 3 \cdot \mathbf{5} \cdot 5$
$275 = \mathbf{5} \cdot 5 \cdot 11$
$420 = 2 \cdot 2 \cdot 3 \cdot \mathbf{5} \cdot 7$
The greatest common factor is 5.

b) $120 = \mathbf{2} \cdot \mathbf{2} \cdot \mathbf{2} \cdot 3 \cdot \mathbf{5}$
$960 = \mathbf{2} \cdot \mathbf{2} \cdot \mathbf{2} \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot \mathbf{5}$
$1400 = \mathbf{2} \cdot \mathbf{2} \cdot \mathbf{2} \cdot \mathbf{5} \cdot 5 \cdot 7$
The greatest common factor is $2 \cdot 2 \cdot 2 \cdot 5$, which is 40.

c) $126 = \mathbf{2} \cdot \mathbf{3} \cdot 3 \cdot \mathbf{7}$
$210 = \mathbf{2} \cdot \mathbf{3} \cdot 5 \cdot \mathbf{7}$
$546 = \mathbf{2} \cdot \mathbf{3} \cdot \mathbf{7} \cdot 13$
$714 = \mathbf{2} \cdot \mathbf{3} \cdot \mathbf{7} \cdot 17$
The greatest common factor is $2 \cdot 3 \cdot 7$, which is 42.

d) $220 = \mathbf{2} \cdot \mathbf{2} \cdot 5 \cdot 11$
$308 = \mathbf{2} \cdot \mathbf{2} \cdot 7 \cdot 11$
$484 = \mathbf{2} \cdot \mathbf{2} \cdot 11 \cdot 11$
$988 = \mathbf{2} \cdot \mathbf{2} \cdot 13 \cdot 19$
The greatest common factor is $2 \cdot 2$, which is 4.

10. Write the prime factorization of each number. Highlight the greater power of each prime factor in any list. The least common multiple is the product of the greater power of each prime factor.

a) $12 = 2 \cdot 2 \cdot 3 = \mathbf{2^2} \cdot \mathbf{3}$
$14 = 2 \cdot \mathbf{7}$
The greater power of 2 in either list is 2^2.
The greater power of 3 in either list is 3.
The greater power of 7 in either list is 7.
The least common multiple is:
$$\begin{aligned} 2^2 \cdot 3 \cdot 7 &= 4 \cdot 3 \cdot 7 \\ &= 84 \end{aligned}$$
So, the least common multiple of 12 and 14 is 84.

b) $21 = 3 \cdot \mathbf{7}$
$45 = 3 \cdot 3 \cdot 5 = \mathbf{3^2} \cdot \mathbf{5}$
The greater power of 3 in either list is 3^2.
The greater power of 5 in either list is 5.
The greater power of 7 in either list is 7.
The least common multiple is:
$$\begin{aligned} 3^2 \cdot 5 \cdot 7 &= 9 \cdot 5 \cdot 7 \\ &= 315 \end{aligned}$$
So, the least common multiple of 21 and 45 is 315.

c) $45 = 3 \cdot 3 \cdot 5 = \mathbf{3^2 \cdot 5}$
$60 = 2 \cdot 2 \cdot 3 \cdot 5 = \mathbf{2^2} \cdot 3 \cdot 5$
The greater power of 2 in either list is 2^2.
The greater power of 3 in either list is 3^2.
The greater power of 5 in either list is 5.
The least common multiple is:
$$\begin{aligned}2^2 \cdot 3^2 \cdot 5 &= 4 \cdot 9 \cdot 5 \\ &= 180\end{aligned}$$
So, the least common multiple of 45 and 60 is 180.

d) $38 = \mathbf{2 \cdot 19}$
$42 = 2 \cdot \mathbf{3 \cdot 7}$
The greater power of 2 in either list is 2.
The greater power of 3 in either list is 3.
The greater power of 7 in either list is 7.
The greater power of 19 in either list is 19.
The least common multiple is:
$2 \cdot 3 \cdot 7 \cdot 19 = 798$
So, the least common multiple of 38 and 42 is 798.

e) $32 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = \mathbf{2^5}$
$45 = 3 \cdot 3 \cdot 5 = \mathbf{3^2 \cdot 5}$
The greater power of 2 in either list is 2^5.
The greater power of 3 in either list is 3^2.
The greater power of 5 in either list is 5.
The least common multiple is:
$$\begin{aligned}2^5 \cdot 3^2 \cdot 5 &= 32 \cdot 9 \cdot 5 \\ &= 1440\end{aligned}$$
So, the least common multiple of 32 and 45 is 1440.

f) $28 = 2 \cdot 2 \cdot 7 = \mathbf{2^2 \cdot 7}$
$52 = 2 \cdot 2 \cdot 13 = 2^2 \cdot \mathbf{13}$
The greater power of 2 in either list is 2^2.
The greater power of 7 in either list is 7.
The greater power of 13 in either list is 13.
The least common multiple is:
$$\begin{aligned}2^2 \cdot 7 \cdot 13 &= 4 \cdot 7 \cdot 13 \\ &= 364\end{aligned}$$
So, the least common multiple of 28 and 52 is 364.

11. Write the prime factorization of each number. Highlight the greatest power of each prime factor in any list. The least common multiple is the product of the greatest power of each prime factor.

a) $20 = 2 \cdot 2 \cdot 5 = \mathbf{2^2} \cdot \mathbf{5}$
$36 = 2 \cdot 2 \cdot 3 \cdot 3 = 2^2 \cdot \mathbf{3^2}$
$38 = 2 \cdot \mathbf{19}$
The greatest power of 2 in any list is 2^2.
The greatest power of 3 in any list is 3^2.
The greatest power of 5 in any list is 5.
The greatest power of 19 in any list is 19.
The least common multiple is:
$$\begin{aligned} 2^2 \cdot 3^2 \cdot 5 \cdot 19 &= 4 \cdot 9 \cdot 5 \cdot 19 \\ &= 3420 \end{aligned}$$
So, the least common multiple of 20, 36, and 38 is 3420.

b) $15 = \mathbf{3} \cdot \mathbf{5}$
$32 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = \mathbf{2^5}$
$44 = 2 \cdot 2 \cdot 11 = 2^2 \cdot \mathbf{11}$
The greatest power of 2 in any list is 2^5.
The greatest power of 3 in any list is 3.
The greatest power of 5 in any list is 5.
The greatest power of 11 in any list is 11.
The least common multiple is:
$$\begin{aligned} 2^5 \cdot 3 \cdot 5 \cdot 11 &= 32 \cdot 3 \cdot 5 \cdot 11 \\ &= 5280 \end{aligned}$$
So, the least common multiple of 15, 32, and 44 is 5280.

c) $12 = 2 \cdot 2 \cdot 3 = \mathbf{2^2} \cdot 3$
$18 = 2 \cdot 3 \cdot 3 = 2 \cdot \mathbf{3^2}$
$25 = 5 \cdot 5 = \mathbf{5^2}$
$30 = 2 \cdot 3 \cdot 5$
The greatest power of 2 in any list is 2^2.
The greatest power of 3 in any list is 3^2.
The greatest power of 5 in any list is 5^2.
The least common multiple is:
$$\begin{aligned} 2^2 \cdot 3^2 \cdot 5^2 &= 4 \cdot 9 \cdot 25 \\ &= 900 \end{aligned}$$
So, the least common multiple of 12, 18, 25, and 30 is 900.

d) $15 = 3 \cdot \mathbf{5}$
$20 = 2 \cdot 2 \cdot 5 = 2^2 \cdot 5$
$24 = 2 \cdot 2 \cdot 2 \cdot 3 = \mathbf{2^3} \cdot 3$
$27 = 3 \cdot 3 \cdot 3 = \mathbf{3^3}$
The greatest power of 2 in any list is 2^3.
The greatest power of 3 in any list is 3^3.
The greatest power of 5 in any list is 5.
The least common multiple is:
$$\begin{aligned} 2^3 \cdot 3^3 \cdot 5 &= 8 \cdot 27 \cdot 5 \\ &= 1080 \end{aligned}$$
So, the least common multiple of 15, 20, 24, and 27 is 1080.

12. The greatest common factor of 12 and 14 is the greatest number that divides into 12 and 14 with no remainder. This number is less than both 12 and 14.
Since $12 = 2 \cdot 6$, and $14 = 2 \cdot 7$, then the greatest common factor is 2.
The least common multiple of 12 and 14 is the least number that both 12 and 14 divide into with no remainder. This number is greater than both 12 and 14.
Since $12 = 2 \cdot 6$ and $14 = 2 \cdot 7$, then the least common multiple is $2 \cdot 6 \cdot 7 = 84$.

13. The first band has 42 members. So, the number of columns in the array is a factor of 42.
The second band has 36 members. So, the number of columns in the array is a factor of 36.
The arrays have the same number of columns.
So, the number of columns in each array is a common factor of 42 and 36.
The greatest number of columns is the greatest common factor of 42 and 36.
Write the prime factorization of each number.
Highlight the prime factors that appear in both lists.
$42 = \mathbf{2} \cdot \mathbf{3} \cdot 7$
$36 = 2 \cdot \mathbf{2} \cdot \mathbf{3} \cdot 3$
The greatest common factor is: $2 \cdot 3 = 6$
The array has no more than 6 columns.

14. The product of two numbers is equal to their least common multiple when both numbers have no common factors, except 1. For example, two prime numbers such as 5 and 11 have a product of 55 and this is also their least common multiple. And, two numbers such as 6 and 35 have a product of 210 and this is also their least common multiple.

15. A fraction is simplified when its numerator and denominator have no common factors.
So, divide the numerator and denominator of each fraction by their greatest common factor.

a) $\frac{185}{325}$
Write the prime factorization of the numerator and of the denominator.
Highlight the prime factors that appear in both lists.
$185 = \mathbf{5} \cdot 37$
$325 = \mathbf{5} \cdot 5 \cdot 13$
The greatest common factor is 5.
$$\begin{aligned} \frac{185}{325} &= \frac{185 \div 5}{325 \div 5} \\ &= \frac{37}{65} \end{aligned}$$

b) $\frac{340}{380}$

Write the prime factorization of the numerator and of the denominator.
Highlight the prime factors that appear in both lists.
$340 = \mathbf{2} \cdot \mathbf{2} \cdot \mathbf{5} \cdot 17$
$380 = \mathbf{2} \cdot \mathbf{2} \cdot \mathbf{5} \cdot 19$
The greatest common factor is $2 \cdot 2 \cdot 5$, or 20.

$$\frac{340}{380} = \frac{340 \div 20}{380 \div 20}$$
$$= \frac{17}{19}$$

c) $\frac{650}{900}$

Write the prime factorization of the numerator and of the denominator.
Highlight the prime factors that appear in both lists.
$650 = \mathbf{2} \cdot \mathbf{5} \cdot \mathbf{5} \cdot 13$
$900 = \mathbf{2} \cdot 2 \cdot 3 \cdot 3 \cdot \mathbf{5} \cdot \mathbf{5}$
The greatest common factor is $2 \cdot 5 \cdot 5$, or 50.

$$\frac{650}{900} = \frac{650 \div 50}{900 \div 50}$$
$$= \frac{13}{18}$$

d) $\frac{840}{1220}$

Write the prime factorization of the numerator and of the denominator.
Highlight the prime factors that appear in both lists.
$840 = \mathbf{2} \cdot \mathbf{2} \cdot 2 \cdot 3 \cdot \mathbf{5} \cdot 7$
$1220 = \mathbf{2} \cdot \mathbf{2} \cdot \mathbf{5} \cdot 61$
The greatest common factor is $2 \cdot 2 \cdot 5$, or 20.

$$\frac{840}{1220} = \frac{840 \div 20}{1220 \div 20}$$
$$= \frac{42}{61}$$

e) $\frac{1225}{2750}$

Write the prime factorization of the numerator and of the denominator.
Highlight the prime factors that appear in both lists.
$1225 = \mathbf{5} \cdot \mathbf{5} \cdot 7 \cdot 7$
$2750 = 2 \cdot \mathbf{5} \cdot \mathbf{5} \cdot 5 \cdot 11$
The greatest common factor is $5 \cdot 5$, or 25.

$$\begin{aligned}\frac{1225}{2750} &= \frac{1225 \div 25}{2750 \div 25}\\ &= \frac{49}{110}\end{aligned}$$

f) $\frac{2145}{1105}$

Write the prime factorization of the numerator and of the denominator.
Highlight the prime factors that appear in both lists.
$2145 = 3 \cdot \mathbf{5} \cdot 11 \cdot \mathbf{13}$
$1105 = \mathbf{5} \cdot \mathbf{13} \cdot 17$
The greatest common factor is $5 \cdot 13$, or 65.

$$\begin{aligned}\frac{2145}{1105} &= \frac{2145 \div 65}{1105 \div 65}\\ &= \frac{33}{17}\end{aligned}$$

16. Add, subtract, or divide fractions with a common denominator by adding, subtracting, or dividing their numerators.
So, write the fractions in each part with denominator equal to the least common multiple of the original denominators.

a) $\frac{9}{14} + \frac{11}{16}$

Write the prime factorization of the denominators.
Highlight the greatest power of each prime factor in either list.
$14 = 2 \cdot \mathbf{7}$
$16 = \mathbf{2^4}$
The least common multiple is $2^4 \cdot 7$, or 112.

$$\begin{aligned}\frac{9}{14} + \frac{11}{16} &= \frac{72}{112} + \frac{77}{112}\\ &= \frac{72 + 77}{112}\\ &= \frac{149}{112}\end{aligned}$$

b) $\frac{8}{15}+\frac{11}{20}$

Write the prime factorization of the denominators.
Highlight the greatest power of each prime factor in either list.
$15 = \mathbf{3} \cdot \mathbf{5}$
$20 = \mathbf{2^2} \cdot 5$
The least common multiple is $2^2 \cdot 3 \cdot 5$, or 60.

$$\begin{aligned}\frac{8}{15}+\frac{11}{20} &= \frac{32}{60}+\frac{33}{60}\\ &= \frac{32+33}{60}\\ &= \frac{65}{60}, \text{ or } \frac{13}{12}\end{aligned}$$

c) $\frac{5}{24}-\frac{1}{22}$

Write the prime factorization of the denominators.
Highlight the greatest power of each prime factor in either list.
$24 = \mathbf{2^3} \cdot \mathbf{3}$
$22 = 2 \cdot \mathbf{11}$
The least common multiple is $2^3 \cdot 3 \cdot 11$, or 264.

$$\begin{aligned}\frac{5}{24}-\frac{1}{22} &= \frac{55}{264}-\frac{12}{264}\\ &= \frac{55-12}{264}\\ &= \frac{43}{264}\end{aligned}$$

d) $\frac{9}{10}+\frac{5}{14}+\frac{4}{21}$

Write the prime factorization of the denominators.
Highlight the greatest power of each prime factor in any list.
$10 = \mathbf{2} \cdot \mathbf{5}$
$14 = 2 \cdot \mathbf{7}$
$21 = \mathbf{3} \cdot 7$
The least common multiple is $2 \cdot 3 \cdot 5 \cdot 7$, or 210.

$$\begin{aligned}\frac{9}{10}+\frac{5}{14}+\frac{4}{21} &= \frac{189}{210}+\frac{75}{210}+\frac{40}{210}\\ &= \frac{189+75+40}{210}\\ &= \frac{304}{210}, \text{ or } \frac{152}{105}\end{aligned}$$

e) $\frac{9}{25}+\frac{7}{15}-\frac{5}{8}$

Write the prime factorization of the denominators.
Highlight the greatest power of each prime factor in any list.
$25 = \mathbf{5^2}$
$15 = \mathbf{3} \cdot 5$
$8 = \mathbf{2^3}$
The least common multiple is $2^3 \cdot 3 \cdot 5^2$, or 600.

$$\begin{aligned}\frac{9}{25}+\frac{7}{15}-\frac{5}{8} &= \frac{216}{600}+\frac{280}{600}-\frac{375}{600}\\ &= \frac{216+280-375}{600}\\ &= \frac{121}{600}\end{aligned}$$

f) $\frac{3}{5}-\frac{5}{18}+\frac{7}{3}$

Write the prime factorization of the denominators.
Highlight the greatest power of each prime factor in any list.
$5 = \mathbf{5}$
$18 = \mathbf{2} \cdot \mathbf{3^2}$
$3 = 3$
The least common multiple is $2 \cdot 3^2 \cdot 5$, or 90.

$$\begin{aligned}\frac{3}{5}-\frac{5}{18}+\frac{7}{3} &= \frac{54}{90}-\frac{25}{90}+\frac{210}{90}\\ &= \frac{54-25+210}{90}\\ &= \frac{239}{90}\end{aligned}$$

g) $\frac{3}{5}\div\frac{4}{9}$

Write the prime factorization of the denominators.
Highlight the greatest power of each prime factor in either list.
$5 = \mathbf{5}$
$9 = \mathbf{3^2}$
The least common multiple is $3^2 \cdot 5$, or 45.

$$\begin{aligned}\frac{3}{5}\div\frac{4}{9} &= \frac{27}{45}\div\frac{20}{45}\\ &= 27\div 20\\ &= \frac{27}{20}\end{aligned}$$

h) $\frac{11}{6} \div \frac{2}{7}$

Write the prime factorization of the denominators.
Highlight the greatest power of each prime factor in either list.
6 = **2** · **3**
7 = **7**
The least common multiple is 2 · 3 · 7, or 42.

$$\frac{11}{6} \div \frac{2}{7} = \frac{77}{42} \div \frac{12}{42}$$
$$= 77 \div 12$$
$$= \frac{77}{12}$$

17. The shorter side of the rectangular plot of land measures 2400 m. So, the side length of the square plot must be a factor of 2400.
The longer side of the rectangular plot of land measures 3200 m. So, the side length of the square plot must be a factor of 3200.
So, the side length of the square must be a common factor of 2400 and 3200.
Write the prime factorization of each number.
Highlight the prime factors that appear in both lists.
2400 = **2** · **2** · **2** · **2** · **2** · 3 · **5** · **5**
3200 = **2** · **2** · **2** · **2** · **2** · 2 · 2 · **5** · **5**
The greatest common factor is:
2 · 2 · 2 · 2 · 2 · 5 · 5 = 800
The largest square has side length 800 m.

18. No. The number 1 is a whole number, and its only factor is 1.
A prime number has exactly 2 factors: 1 and itself
So, 1 is a whole number, but not a prime number, and it has no prime factors.

19. a) The side length of the square is a common multiple of 18 and 24. The least common multiple will produce the smallest square.
The multiples of 18 are: 18, 36, 54, 72, …
The multiples of 24 are: 24, 48, 72, …
The least common multiple of 18 and 24 is 72.
So, the dimensions of the square are 72 cm by 72 cm.

b) The dimensions of the floor in centimetres are 648 cm by 1512 cm.
If 18 or 24 is a factor of each dimension, then the tiles will cover the floor.

648 ÷ 18 = 36	648 ÷ 24 = 27
1512 ÷ 18 = 84	1512 ÷ 24 = 63

Since both dimensions of the tile are factors of the dimensions of the floor, the tiles could be used to cover the floor.

20. a) A square with side length 1 mi. measures 5280 ft. by 5280 ft.
Determine whether 660 is a factor of 5280.
$5280 \div 660 = 8$
Since 66 is a factor of 660, then 66 is a factor of 5280, and $5280 \div 66 = 80$
So, the rectangles for 1 acre fit exactly into a section.

b) From part a, 8 rectangles fit along the one side of a section and 80 rectangles fit along the side at right angles.
For a quarter section, which is a square with side length $\frac{1}{2}$ mi., 4 rectangles will fit along one side and 40 rectangles will fit along the side at right angles. So, the rectangles for 1 acre do fit exactly into a quarter section.

c) The side length of the square is a common multiple of 660 and 66. The least common multiple will produce the smallest square.
Since $660 = 66 \cdot 10$, then the least common multiple of 660 and 66 is 660.
So, the side length of the smallest square is 660 ft.

C

21. Yes, $61 \div 7 = 8.7142\ldots$
And, 61 is not divisible by 8 because 61 is an odd number.
There are no more natural numbers between 7 and 8.7142….
So, 61 is not divisible by any natural number.

22. The edge length, in centimetres, of the smallest cube that could be filled with these bars is the least common multiple of 10, 6, and 3.
The multiples of 10 are: 10, 20, 30, …
The multiples of 6 are: 6, 12, 18, 24, 30, …
The multiples of 3 are: 3, 6, 9, 12, 15, 18, 21, 24, 27, 30, …
The least common multiple is 30.
So, the edge length of the cube is 30 cm.

Lesson 3.2 Perfect Squares, Perfect Cubes, and Their Roots Exercises (pages 146–147)

A

4. Use a calculator to write each number as a product of its prime factors, then arrange the factors in 2 equal groups. The product of the factors in one group is the square root.

a) $196 = 2 \cdot 2 \cdot 7 \cdot 7$
$= (2 \cdot 7)(2 \cdot 7)$
$= 14 \cdot 14$
$\sqrt{196} = 14$

b) $256 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$
$= (2 \cdot 2 \cdot 2 \cdot 2)(2 \cdot 2 \cdot 2 \cdot 2)$
$= 16 \cdot 16$
$\sqrt{256} = 16$

c) $361 = 19 \cdot 19$
$\sqrt{361} = 19$

d) $289 = 17 \cdot 17$
$\sqrt{289} = 17$

e) $441 = 3 \cdot 3 \cdot 7 \cdot 7$
$= (3 \cdot 7)(3 \cdot 7)$
$= 21 \cdot 21$
$\sqrt{441} = 21$

5. Use a calculator to write each number as a product of its prime factors, then arrange the factors in 3 equal groups. The product of the factors in one group is the cube root.

a) $343 = 7 \cdot 7 \cdot 7$
$\sqrt[3]{343} = 7$

b) $512 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$
$= (2 \cdot 2 \cdot 2)(2 \cdot 2 \cdot 2)(2 \cdot 2 \cdot 2)$
$= 8 \cdot 8 \cdot 8$
$\sqrt[3]{512} = 8$

c) $1000 = 2 \cdot 2 \cdot 2 \cdot 5 \cdot 5 \cdot 5$
$= (2 \cdot 5)(2 \cdot 5)(2 \cdot 5)$
$= 10 \cdot 10 \cdot 10$
$\sqrt[3]{1000} = 10$

d) $1331 = 11 \cdot 11 \cdot 11$
$\sqrt[3]{1331} = 11$

e) $3375 = 3 \cdot 3 \cdot 3 \cdot 5 \cdot 5 \cdot 5$
$= (3 \cdot 5)(3 \cdot 5)(3 \cdot 5)$
$= 15 \cdot 15 \cdot 15$
$\sqrt[3]{3375} = 15$

B

6. Use a calculator to write each number as a product of its prime factors. If the factors can be arranged in 2 equal groups, then the number is a perfect square. If the factors can be arranged in 3 equal groups, then the number is a perfect cube.

a) $225 = 3 \cdot 3 \cdot 5 \cdot 5$
$= (3 \cdot 5)(3 \cdot 5)$
The factors can be arranged in 2 equal groups, so 225 is a perfect square.

b) $729 = 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3$
$= (3 \cdot 3)(3 \cdot 3)(3 \cdot 3)$
$= (3 \cdot 3 \cdot 3)(3 \cdot 3 \cdot 3)$
The factors can be arranged in 2 equal groups and in 3 equal groups, so 729 is both a perfect square and a perfect cube.

c) $1944 = 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3$
The factors cannot be arranged in 2 or 3 equal groups, so 1944 is neither a perfect square nor a perfect cube.

d) $1444 = 2 \cdot 2 \cdot 19 \cdot 19$
$= (2 \cdot 19)(2 \cdot 19)$
The factors can be arranged in 2 equal groups, so 1444 is a perfect square.

e) $4096 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$
$= (2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2)(2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2)$
$= (2 \cdot 2 \cdot 2 \cdot 2)(2 \cdot 2 \cdot 2 \cdot 2)(2 \cdot 2 \cdot 2 \cdot 2)$
The factors can be arranged in 2 equal groups and 3 equal groups, so 4096 is both a perfect square and a perfect cube.

f) $13\,824 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot 3$
$= (2 \cdot 2 \cdot 2 \cdot 3)(2 \cdot 2 \cdot 2 \cdot 3)(2 \cdot 2 \cdot 2 \cdot 3)$
The factors can be arranged 3 equal groups, so 13 824 is a perfect cube.

7. The side length of each square is the square root of its area. Use a calculator to write each number as a product of its prime factors, then arrange the factors in 2 equal groups. The product of the factors in one group is the square root.

a) $484 = 2 \cdot 2 \cdot 11 \cdot 11$
$= (2 \cdot 11)(2 \cdot 11)$
$= 22 \cdot 22$
$\sqrt{484} = 22$
The side length of the square is 22 mm.

b) $1764 = 2 \cdot 2 \cdot 3 \cdot 3 \cdot 7 \cdot 7$
$= (2 \cdot 3 \cdot 7)(2 \cdot 3 \cdot 7)$
$= 42 \cdot 42$

$\sqrt{1764} = 42$

The side length of the square is 42 yd.

8. The edge length of each cube is the cube root of its volume. Use a calculator to write each number as a product of its prime factors, then arrange the factors in 3 equal groups. The product of the factors in one group is the cube root.

a) $5832 = 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3$
$= (2 \cdot 3 \cdot 3)(2 \cdot 3 \cdot 3)(2 \cdot 3 \cdot 3)$
$= 18 \cdot 18 \cdot 18$

$\sqrt[3]{5832} = 18$

The edge length of the cube is 18 in.

b) $15\,625 = 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5$
$= (5 \cdot 5)(5 \cdot 5)(5 \cdot 5)$
$= 25 \cdot 25 \cdot 25$

$\sqrt[3]{15\,625} = 25$

The edge length of the cube is 25 ft.

9. The volume of the cube is 64 cubic feet.

The edge length of the cube, in feet, is: $\sqrt[3]{64} = 4$

The surface area of the cube, in square feet, is 6 times the area of one face: $6(4^2) = 96$

The surface area of the cube was 96 square feet.

10. The surface area of the cube is 6534 square feet.

The area of one face, in square feet, is: $\dfrac{6534}{6} = 1089$

The side length of a face, in feet, is $\sqrt{1089}$.

$1089 = 3 \cdot 3 \cdot 11 \cdot 11$
$= (3 \cdot 11)(3 \cdot 11)$
$= 33 \cdot 33$

$\sqrt{1089} = 33$

The volume of the cube, in cubic feet, is: $33^3 = 35\,937$

The volume of the cube is 35 937 cubic feet.

11. If 2000 is a perfect cube, then a cube could be constructed with 2000 interlocking cubes. Determine the factors of 2000.

$2000 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 5 \cdot 5 \cdot 5$

Since these factors cannot be arranged in 3 equal groups, 2000 is not a perfect cube, and a cube could not be constructed with 2000 interlocking cubes.

12. Use estimation or guess and check to determine the perfect square and perfect cube closest to the first number in each given pair, then calculate the squares and cubes of all whole numbers until the second number in each pair is reached or exceeded.

a) 315 – 390

$17^2 = 289$ $18^2 = 324$ $19^2 = 361$ $20^2 = 400$

$6^3 = 216$ $7^3 = 343$ $8^3 = 512$

Between 315 and 390, the perfect squares are 324 and 361; and the perfect cube is 343.

b) 650 – 750
$25^2 = 625$ $26^2 = 676$ $27^2 = 729$ $28^2 = 784$
$8^3 = 512$ $9^3 = 729$ $10^3 = 1000$
Between 650 and 750, the perfect squares are 676 and 729; and the perfect cube is 729.

c) 800 – 925
$28^2 = 784$ $29^2 = 841$ $30^2 = 900$ $31^2 = 961$
$9^3 = 729$ $10^3 = 1000$
Between 800 and 925, the perfect squares are 841 and 900; there are no perfect cubes.

d) 1200 – 1350
$34^2 = 1156$ $35^2 = 1225$ $36^2 = 1296$ $37^2 = 1369$
$10^3 = 1000$ $11^3 = 1331$ $12^3 = 1728$
Between 1200 and 1350, the perfect squares are 1225 and 1296; and the perfect cube is 1331.

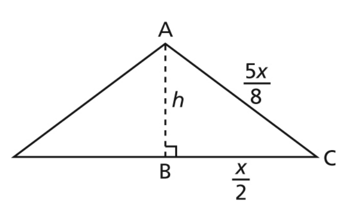
13. For a number to be a perfect square and a perfect cube, its prime factors must be arranged in 2 equal groups and 3 equal groups; that is, each factor occurs 2(3), or 6 times, or a multiple of 6 times.
The first number, after 0 and 1, that is a perfect square and perfect cube is:
$2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 64$
Another number that is a perfect square and a perfect cube is: $3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 = 729$
Another number that is a perfect square and a perfect cube is: $5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 = 15\,625$

14. Since the rectangular prism has a square cross-section, the prism is a square prism. Its volume is 1440 cubic feet.
Its height is 10 ft., so its base area, in square feet, is: $\frac{1440}{10} = 144$
The side length of the square base, in feet, is: $\sqrt{144} = 12$
So, the length and width of the base are 12 ft.

C

15. a) The tent has 4 congruent square faces; 2 congruent rectangular faces, and 2 congruent triangular faces.
The area, in square feet, of each square face is: $(x)(x) = x^2$
The area, in square feet, of each rectangular face is: $(x)\left(\frac{5x}{8}\right) = \frac{5x^2}{8}$
To determine the area of each triangle, first determine its height, h.

Use the Pythagorean Theorem in ΔABC.



$$h^2 = \left(\frac{5x}{8}\right)^2 - \left(\frac{x}{2}\right)^2$$

$$h^2 = \frac{25x^2}{64} - \frac{x^2}{4}$$

$$h^2 = \frac{25x^2}{64} - \frac{16x^2}{64}$$
$$h^2 = \frac{9x^2}{64}$$
$$h = \sqrt{\frac{9x^2}{64}}$$
$$h = \frac{3x}{8}$$

The area, in square feet, of each triangular face is: $\frac{1}{2}(x)\left(\frac{3x}{8}\right) = \frac{3x^2}{16}$

So, the surface area of the tent is:

$$4(x^2) + 2\left(\frac{5x^2}{8}\right) + 2\left(\frac{3x^2}{16}\right) = 4x^2 + \frac{10x^2}{8} + \frac{3x^2}{8}$$
$$= \frac{32x^2}{8} + \frac{13x^2}{8}$$
$$= \frac{45x^2}{8}$$

The surface area of the tent is $\frac{45x^2}{8}$ square feet.

b) Write an equation.

$\frac{45x^2}{8} = 90$ Multiply each side by 8.

$45x^2 = 720$ Divide each side by 45.

$x^2 = 16$

$x = \sqrt{16}$

$x = 4$

16. Let the edge length of the cube be x.
Then the area of one face is x^2.
And its surface area is $6x^2$.
The volume of the cube is x^3.
The volume and surface area are equal, so:
$x^3 = 6x^2$
Since x is not equal to 0, divide each side by x^2.
$x = 6$
The dimensions of a cube that has its surface area numerically the same as its volume are 6 units by 6 units by 6 units.

17. a) The side length of a square with area $121x^4y^2$ is: $\sqrt{121x^4y^2}$

Factor $121x^4y^2$: $11 \cdot 11 \cdot x \cdot x \cdot x \cdot x \cdot y \cdot y$

Rearrange the factors in 2 equal groups: $(11 \cdot x \cdot x \cdot y)(11 \cdot x \cdot x \cdot y)$

So, $\sqrt{121x^4y^2} = 11x^2y$

The side length of the square is $11x^2y$.

b) The edge length of a cube with volume $64x^6y^3$ is: $\sqrt[3]{64x^6y^3}$

Factor $64x^6y^3$: $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot y \cdot y \cdot y$

Rearrange the factors in 3 equal groups:

$(2 \cdot 2 \cdot x \cdot x \cdot y)(2 \cdot 2 \cdot x \cdot x \cdot y)(2 \cdot 2 \cdot x \cdot x \cdot y)$

So, $\sqrt[3]{64x^6y^3} = 4x^2y$

The edge length of the cube is $4x^2y$.

18. List all the perfect cubes up to a number close to 1729:
1, 8, 27, 64, 125, 216, 343, 512, 729, 1000, 1331, 1728
Use guess and check.
The 1st and 12th perfect cubes have the sum: 1 + 1728 = 1729
The 9th and 10th perfect cubes have the sum: 729 + 1000 = 1729
No other perfect cubes have the sum 1729.

Checkpoint 1 **Assess Your Understanding (page 149)**

3.1

1. Use a calculator to divide each number by its prime factors.

a) $1260 = 2 \cdot 2 \cdot 3 \cdot 3 \cdot 5 \cdot 7$
$= 2^2 \cdot 3^2 \cdot 5 \cdot 7$

b) $4224 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot 11$
$= 2^7 \cdot 3 \cdot 11$

c) $6120 = 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot 5 \cdot 17$
$= 2^3 \cdot 3^2 \cdot 5 \cdot 17$

d) $1045 = 5 \cdot 11 \cdot 19$

e) $3024 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot 3 \cdot 7$
$= 2^4 \cdot 3^3 \cdot 7$

f) $3675 = 3 \cdot 5 \cdot 5 \cdot 7 \cdot 7$
$= 3 \cdot 5^2 \cdot 7^2$

2. List the factors of each number, then highlight the common factors.

a) $40 = \mathbf{2} \cdot \mathbf{2} \cdot \mathbf{2} \cdot 5$
$48 = \mathbf{2} \cdot \mathbf{2} \cdot \mathbf{2} \cdot 6$
$56 = \mathbf{2} \cdot \mathbf{2} \cdot \mathbf{2} \cdot 7$
The greatest common factor is: $2 \cdot 2 \cdot 2 = 8$

b) $84 = \mathbf{2} \cdot \mathbf{2} \cdot \mathbf{3} \cdot 7$
$120 = \mathbf{2} \cdot \mathbf{2} \cdot 2 \cdot \mathbf{3} \cdot 5$
$144 = \mathbf{2} \cdot \mathbf{2} \cdot 2 \cdot 2 \cdot \mathbf{3} \cdot 3$
The greatest common factor is: $2 \cdot 2 \cdot 3 = 12$

c) $145 = \mathbf{5} \cdot 29$
$205 = \mathbf{5} \cdot 41$
$320 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot \mathbf{5}$
The greatest common factor is: 5

d) $208 = \mathbf{2} \cdot \mathbf{2} \cdot \mathbf{2} \cdot \mathbf{2} \cdot 13$
$368 = \mathbf{2} \cdot \mathbf{2} \cdot \mathbf{2} \cdot \mathbf{2} \cdot 23$
$528 = \mathbf{2} \cdot \mathbf{2} \cdot \mathbf{2} \cdot \mathbf{2} \cdot 3 \cdot 11$
The greatest common factor is: $2 \cdot 2 \cdot 2 \cdot 2 = 16$

e) $856 = \mathbf{2} \cdot \mathbf{2} \cdot \mathbf{2} \cdot 107$
$1200 = \mathbf{2} \cdot \mathbf{2} \cdot \mathbf{2} \cdot 2 \cdot 3 \cdot 5 \cdot 5$
$1368 = \mathbf{2} \cdot \mathbf{2} \cdot \mathbf{2} \cdot 3 \cdot 3 \cdot 19$
The greatest common factor is: $2 \cdot 2 \cdot 2 = 8$

f) $950 = 2 \cdot \mathbf{5} \cdot \mathbf{5} \cdot 19$
$1225 = \mathbf{5} \cdot \mathbf{5} \cdot 7 \cdot 7$
$1550 = 2 \cdot \mathbf{5} \cdot \mathbf{5} \cdot 31$
The greatest common factor is: $5 \cdot 5 = 25$

3. Factor each number.
Highlight the greatest power of each prime factor in any list.
The least common multiple is the product of the greatest power of each prime factor.

a) $12 = 2 \cdot 2 \cdot 3 = \mathbf{2^2} \cdot \mathbf{3}$
$15 = 3 \cdot \mathbf{5}$
$21 = 3 \cdot \mathbf{7}$
The least common multiple is: $2^2 \cdot 3 \cdot 5 \cdot 7 = 420$

b) $12 = 2 \cdot 2 \cdot 3 = 2^2 \cdot \mathbf{3}$
$20 = 2 \cdot 2 \cdot 5 = 2^2 \cdot \mathbf{5}$
$32 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = \mathbf{2^5}$
The least common multiple is: $2^5 \cdot 3 \cdot 5 = 480$

c) $18 = 2 \cdot 3 \cdot 3 = 2 \cdot \mathbf{3^2}$
$24 = 2 \cdot 2 \cdot 2 \cdot 3 = \mathbf{2^3} \cdot 3$
$30 = 2 \cdot 3 \cdot \mathbf{5}$
The least common multiple is: $2^3 \cdot 3^2 \cdot 5 = 360$

d) $30 = 2 \cdot \mathbf{3} \cdot \mathbf{5}$
$32 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = \mathbf{2^5}$
$40 = 2 \cdot 2 \cdot 2 \cdot 5 = 2^3 \cdot 5$
The least common multiple is: $2^5 \cdot 3 \cdot 5 = 480$

e) $49 = 7 \cdot 7 = \mathbf{7^2}$
$56 = 2 \cdot 2 \cdot 2 \cdot 7 = 2^3 \cdot 7$
$64 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = \mathbf{2^6}$
The least common multiple is: $2^6 \cdot 7^2 = 3136$

f) $50 = 2 \cdot 5 \cdot 5 = \mathbf{2} \cdot \mathbf{5^2}$
$55 = 5 \cdot 11$
$66 = 2 \cdot \mathbf{3} \cdot \mathbf{11}$
The least common multiple is: $2 \cdot 3 \cdot 5^2 \cdot 11 = 1650$

4. a) $$\frac{8}{3} + \frac{5}{11} = \frac{88}{33} + \frac{15}{33} = \frac{103}{33}$$

b) $$\frac{13}{5} - \frac{4}{7} = \frac{91}{35} - \frac{20}{35} = \frac{71}{35}$$

c) $$\frac{9}{10} \div \frac{7}{3} = \frac{27}{30} \div \frac{70}{30} = \frac{27}{70}$$

5. Find the least common multiple of 365 and 260.
$365 = 5 \cdot \mathbf{73}$
$260 = \mathbf{2} \cdot \mathbf{2} \cdot \mathbf{5} \cdot \mathbf{13}$
The least common multiple is: $2 \cdot 2 \cdot 5 \cdot 13 \cdot 73 = 18\,980$
The first day on both calendars would occur again after 18 980 days, which is $\frac{18\,980}{365}$ years, or 52 years.

3.2

6. I could list the prime factors, then arrange them in two equal groups, or I could estimate then use guess and check. I will use prime factors.

a) $400 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 5 \cdot 5$
$= (2 \cdot 2 \cdot 5)(2 \cdot 2 \cdot 5)$
$= 20 \cdot 20$
$\sqrt{400} = 20$

b) $784 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 7 \cdot 7$
$= (2 \cdot 2 \cdot 7)(2 \cdot 2 \cdot 7)$
$= 28 \cdot 28$
$\sqrt{784} = 28$

c) $576 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3$
$= (2 \cdot 2 \cdot 2 \cdot 3)(2 \cdot 2 \cdot 2 \cdot 3)$
$= 24 \cdot 24$
$\sqrt{576} = 24$

d) $1089 = 3 \cdot 3 \cdot 11 \cdot 11$
$= (3 \cdot 11)(3 \cdot 11)$
$= 33 \cdot 33$
$\sqrt{1089} = 33$

e) $1521 = 3 \cdot 3 \cdot 13 \cdot 13$
$= (3 \cdot 13)(3 \cdot 13)$
$= 39 \cdot 39$
$\sqrt{1521} = 39$

f) $3025 = 5 \cdot 5 \cdot 11 \cdot 11$
$= (5 \cdot 11)(5 \cdot 11)$
$= 55 \cdot 55$
$\sqrt{3025} = 55$

7. I could list the prime factors, then arrange them in three equal groups, or I could estimate then use guess and check. I will use prime factors.

a) $1728 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot 3$
$= (2 \cdot 2 \cdot 3)(2 \cdot 2 \cdot 3)(2 \cdot 2 \cdot 3)$
$= 12 \cdot 12 \cdot 12$
$\sqrt[3]{1728} = 12$

b) $3375 = 3 \cdot 3 \cdot 3 \cdot 5 \cdot 5 \cdot 5$
$= (3 \cdot 5)(3 \cdot 5)(3 \cdot 5)$
$= 15 \cdot 15 \cdot 15$
$\sqrt[3]{3375} = 15$

c) $8000 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 5 \cdot 5 \cdot 5$
$= (2 \cdot 2 \cdot 5)(2 \cdot 2 \cdot 5)(2 \cdot 2 \cdot 5)$
$= 20 \cdot 20 \cdot 20$
$\sqrt[3]{8000} = 20$

d) $5832 = 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3$
$= (2 \cdot 3 \cdot 3)(2 \cdot 3 \cdot 3)(2 \cdot 3 \cdot 3)$
$= 18 \cdot 18 \cdot 18$
$\sqrt[3]{5832} = 18$

e) $10\,648 = 2 \cdot 2 \cdot 2 \cdot 11 \cdot 11 \cdot 11$
$= (2 \cdot 11)(2 \cdot 11)(2 \cdot 11)$
$= 22 \cdot 22 \cdot 22$
$\sqrt[3]{10\,648} = 22$

f) $9261 = 3 \cdot 3 \cdot 3 \cdot 7 \cdot 7 \cdot 7$
$= (3 \cdot 7)(3 \cdot 7)(3 \cdot 7)$
$= 21 \cdot 21 \cdot 21$
$\sqrt[3]{9261} = 21$

8. Use a calculator to write each number as a product of its prime factors. If the factors can be arranged in 2 equal groups, then the number is a perfect square. If the factors can be arranged in 3 equal groups, then the number is a perfect cube.

a) $2808 = 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot 3 \cdot 13$
The factors cannot be arranged in 2 or 3 equal groups, so 2808 is neither a perfect square nor a perfect cube.

b) $3136 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 7 \cdot 7$
$= (2 \cdot 2 \cdot 2 \cdot 7)(2 \cdot 2 \cdot 2 \cdot 7)$
The factors can be arranged in 2 equal groups, so 3136 is a perfect square.

c) $4096 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$
$= (2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2)(2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2)$
$= (2 \cdot 2 \cdot 2 \cdot 2)(2 \cdot 2 \cdot 2 \cdot 2)(2 \cdot 2 \cdot 2 \cdot 2)$
The factors can be arranged in 2 equal groups and 3 equal groups, so 4096 is both a perfect square and a perfect cube.

d) $4624 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 17 \cdot 17$
$= (2 \cdot 2 \cdot 17)(2 \cdot 2 \cdot 17)$
The factors can be arranged in 2 equal groups, so 4624 is a perfect square.

e) $5832 = 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3$
$= (2 \cdot 3 \cdot 3)(2 \cdot 3 \cdot 3)(2 \cdot 3 \cdot 3)$
The factors can be arranged in 3 equal groups, so 5832 is a perfect cube.

f) $9270 = 2 \cdot 3 \cdot 3 \cdot 5 \cdot 103$
The factors cannot be arranged in 2 or 3 equal groups, so 9270 is neither a perfect square nor a perfect cube.

9. Use estimation or guess and check to determine the perfect square and perfect cube closest to the first number in each given pair, then calculate the squares and cubes of all whole numbers until the second number in each pair is reached or exceeded.

a) 400 – 500
$20^2 = 400$ $21^2 = 441$ $22^2 = 484$ $23^2 = 529$
$7^3 = 343$ $8^3 = 512$
Between 400 and 500, the perfect squares are 441 and 484; there are no perfect cubes.

b) 900 – 1000
$30^2 = 900$ $31^2 = 961$ $32^2 = 1024$
$9^3 = 729$ $10^3 = 1000$
Between 900 and 1000, the perfect square is 961; there are no perfect cubes.

c) 1100 – 1175
$33^2 = 1089$ $34^2 = 1156$ $35^2 = 1225$
$10^3 = 1000$ $11^3 = 1331$
Between 1100 and 1175, the perfect square is 1156; there are no perfect cubes.

10. The edge length of the cube, in metres, is $\sqrt[3]{2197}$.
$2197 = 13 \cdot 13 \cdot 13$
So, $\sqrt[3]{2197} = 13$
The surface area of one face, in square metres, is: $13^2 = 169$
The surface area of the cube, in square metres, is: $6(169) = 1014$
One can of paint covers about 40 m^2.
So, the number of cans of paint needed is: $\frac{1014}{40} = 25.35$
Twenty-six cans of paint are needed.

Lesson 3.3 Common Factors of a Polynomial Exercises (pages 155–156)

A

4. a) There are 3 x-tiles and twelve 1-tiles.
So, the polynomial is: $3x + 12$
The length of the rectangle is $x + 4$ and the width is 3; these are the factors.
So, $3x + 12 = 3(x + 4)$

b) There are 4 x^2-tiles and 10 x-tiles.
So, the polynomial is: $4x^2 + 10x$
The length of the rectangle is $2x + 5$ and the width is $2x$; these are the factors.
So, $4x^2 + 10x = 2x(2x + 5)$

c) There are 12 x^2-tiles, 8 negative x-tiles, and sixteen 1-tiles.
So, the polynomial is: $12x^2 – 8x + 16$
There are 4 equal groups of tiles; 4 is one factor.
Each group of tiles contains 3 x^2-tiles, 2 negative x-tiles, and four 1-tiles; $3x^2 – 2x + 4$ is the other factor.
So, $12x^2 – 8x + 16 = 4(3x^2 – 2x + 4)$

5. a) $6 = 2 \cdot \mathbf{3}$
$15n = \mathbf{3} \cdot 5 \cdot n$
The greatest common factor of 6 and $15n$ is 3.

b) $4m = 2 \cdot 2 \cdot \boldsymbol{m}$
$m^2 = \boldsymbol{m} \cdot m$
The greatest common factor of $4m$ and m^2 is m.

6. For each polynomial, write each term as the product of the greatest common factor and another monomial. Then use the distributive property to write the expression as a product.

a) The greatest common factor of 6 and $15n$ is 3.

i) $6 + 15n = 3(2) + 3(5n)$
$= 3(2 + 5n)$

ii) $6 – 15n = 3(2) – 3(5n)$
$= 3(2 – 5n)$

iii) $15n – 6 = 3(5n) – 3(2)$
$= 3(5n – 2)$

iv) $–15n + 6 = 3(–5n) + 3(2)$
$= 3(–5n + 2)$

b) The greatest common factor of $4m$ and m^2 is m.

i) $4m + m^2 = m(4) + m(m)$
$= m(4 + m)$

ii) $m^2 + 4m = m(m) + m(4)$
$= m(m + 4)$

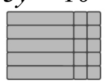
iii) $4m – m^2 = m(4) – m(m)$
$= m(4 – m)$

iv) $m^2 – 4m = m(m) – m(4)$
$= m(m – 4)$

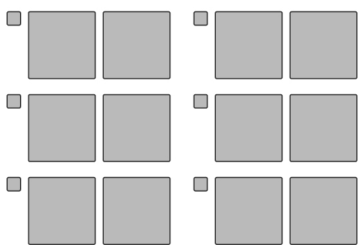
B

7. Arrange the algebra tiles in a rectangle; if more than one rectangle is possible, make the rectangle that is closest to a square. Or, make as many equal groups of tiles as possible.

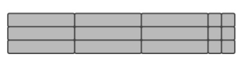
a) Arrange 5 y-tiles and ten 1-tiles to form a rectangle.
The length of the rectangle is $y + 2$ and the width is 5.
$5y + 10 = 5(y + 2)$



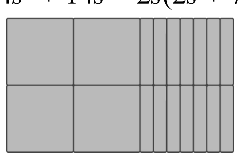
b) Arrange six 1-tiles and 12 x^2-tiles into equal groups.
There are 6 groups; each contains one 1-tile and 2 x^2-tiles.
$6 + 12x^2 = 6(1 + 2x^2)$



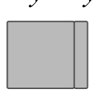
c) Arrange 9 k-tiles and six 1-tiles in a rectangle.
The length of the rectangle is $3k + 2$ and the width is 3.
$9k + 6 = 3(3k + 2)$



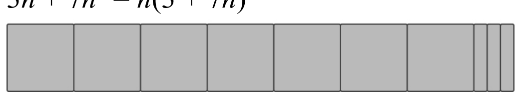
d) Arrange 4 s^2-tiles and 14 s-tiles in a rectangle.
The length of the rectangle is $2s + 7$ and the width is $2s$.
$4s^2 + 14s = 2s(2s + 7)$



e) Arrange 1 y-tile and 1 y^2-tile in a rectangle.
The length of the rectangle is $1 + y$ and the width is y.
$y + y^2 = y(1 + y)$



f) Arrange 3 h-tiles and 7 h^2-tiles in a rectangle.
The length of the rectangle is $3 + 7h$ and the width is h.
$3h + 7h^2 = h(3 + 7h)$



8. For each binomial, factor each term then identify the greatest common factor. Write each term as the product of the greatest common factor and another monomial. Then use the distributive property to write the expression as a product.
I cannot use algebra tiles because I do not have x^3-tiles, or tiles for greater powers of x.

a) $9b^2 - 12b^3$
$9b^2 = \mathbf{3} \cdot 3 \cdot \boldsymbol{b} \cdot \boldsymbol{b}$
$12b^3 = 2 \cdot 2 \cdot \mathbf{3} \cdot \boldsymbol{b} \cdot \boldsymbol{b} \cdot b$
The greatest common factor is: $3 \cdot b \cdot b = 3b^2$
$9b^2 - 12b^3 = 3b^2(3) - 3b^2(4b)$
$= 3b^2(3 - 4b)$
Check: $3b^2(3 - 4b) = 3b^2(3) - 3b^2(4b)$
$= 9b^2 - 12b^3$

b) $48s^3 - 12$
$48s^3 = \mathbf{2} \cdot \mathbf{2} \cdot 2 \cdot 2 \cdot \mathbf{3} \cdot b \cdot b \cdot b$
$12 = \mathbf{2} \cdot \mathbf{2} \cdot \mathbf{3}$
The greatest common factor is: $2 \cdot 2 \cdot 3 = 12$
$48s^3 - 12 = 12(4s^3) - 12(1)$
$= 12(4s^3 - 1)$
Check: $12(4s^3 - 1) = 12(4s^3) - 12(1)$
$= 48s^3 - 12$

c) $-a^2 - a^3$
$a^2 = \boldsymbol{a} \cdot \boldsymbol{a}$
$a^3 = \boldsymbol{a} \cdot \boldsymbol{a} \cdot a$
The greatest common factor is: $a \cdot a = a^2$
$-a^2 - a^3 = a^2(-1) - a^2(a)$
$= a^2(-1 - a)$
$= a^2(-1)(1 + a)$
$= -a^2(1 + a)$
Check: $-a^2(1 + a) = -a^2(1) + (-a^2)(a)$
$= -a^2 - a^3$

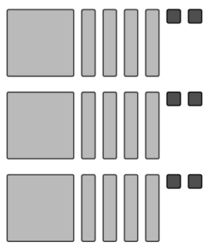
d) $3x^2 + 6x^4$
$3x^2 = \mathbf{3} \cdot \boldsymbol{x} \cdot \boldsymbol{x}$
$6x^4 = 2 \cdot \mathbf{3} \cdot \boldsymbol{x} \cdot \boldsymbol{x} \cdot x \cdot x$
The greatest common factor is: $3 \cdot x \cdot x = 3x^2$
$3x^2 + 6x^4 = 3x^2(1) + 3x^2(2x^2)$
$= 3x^2(1 + 2x^2)$
Check: $3x^2(1 + 2x^2) = 3x^2(1) + 3x^2(2x^2)$
$= 3x^2 + 6x^4$

e) $8y^3 - 12y$
$8y^3 = \mathbf{2} \cdot \mathbf{2} \cdot 2 \cdot \boldsymbol{y} \cdot y \cdot y$
$12y = \mathbf{2} \cdot \mathbf{2} \cdot 3 \cdot \boldsymbol{y}$
The greatest common factor is: $2 \cdot 2 \cdot y = 4y$
$8y^3 - 12y = 4y(2y^2) - 4y(3)$
$= 4y(2y^2 - 3)$
Check: $4y(2y^2 - 3) = 4y(2y^2) - 4y(3)$
$= 8y^3 - 12y$

f) $-7d - 14d^4$
$7d = \mathbf{7} \cdot \boldsymbol{d}$
$14d^4 = 2 \cdot \mathbf{7} \cdot \boldsymbol{d} \cdot d \cdot d \cdot d$
The greatest common factor is: $7 \cdot d = 7d$
$-7d - 14d^4 = 7d(-1) - 7d(2d^3)$
$= 7d(-1 - 2d^3)$
$= 7d(-1)(1 + 2d^3)$
$= -7d(1 + 2d^3)$
Check: $-7d(1 + 2d^3) = -7d(1) + (-7d)(2d^3)$
$= -7d - 14d^4$

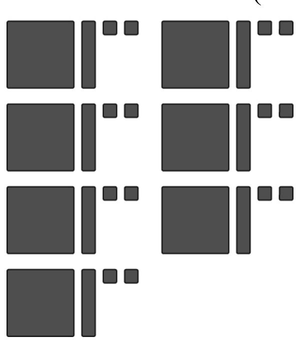
9. Arrange the algebra tiles in as many equal groups as possible.

a) Arrange 3 x^2-tiles, 12 x-tiles, and 6 negative 1-tiles into equal groups. There are 3 groups; each contains 1 x^2-tile, 4 x-tiles, and 2 negative 1-tiles.
$3x^2 + 12x - 6 = 3(x^2 + 4x - 2)$



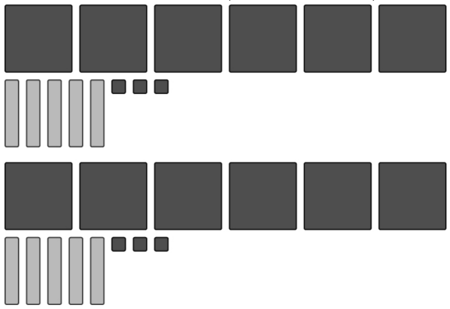
b) Arrange four 1-tiles, 6 negative y-tiles, and 8 negative y^2-tiles into equal groups. There are 2 groups; each contains two 1-tiles, 3 negative y-tiles, and 4 negative y^2-tiles.
$4 - 6y - 8y^2 = 2(2 - 3y - 4y^2)$

c) Arrange 7 negative m-tiles, 7 negative m^2-tiles, and 14 negative 1-tiles into equal groups. There are 7 groups; each contains 1 negative m-tile, 1 negative m^2-tile, and 2 negative 1-tiles.
$-7m - 7m^2 - 14 = 7(-m - m^2 - 2)$
$= 7(-1)(m + m^2 + 2)$
$= -7(m + m^2 + 2)$

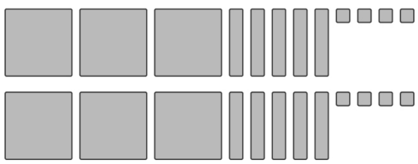


d) Arrange 10 n-tiles, 6 negative 1-tiles, and 12 negative n^2-tiles into equal groups. There are 2 groups; each contains 5 n-tiles, 3 negative 1-tiles, and 6 negative n^2-tiles.

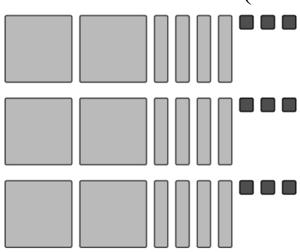
$10n - 6 - 12n^2 = 2(5n - 3 - 6n^2)$



e) Arrange eight 1-tiles, 10 x-tiles, and 6 x^2-tiles into equal groups. There are 2 groups; each contains four 1-tiles, 5 x-tiles, and 3 x^2-tiles.
$8 + 10x + 6x^2 = 2(4 + 5x + 3x^2)$



f) Arrange 9 negative 1-tiles, 12 b-tiles, and 6 b^2-tiles into equal groups. There are 3 groups; each contains 3 negative 1-tiles, 4 b-tiles, and 2 b^2-tiles.
$–9 + 12b + 6b^2 = 3(–3 + 4b + 2b^2)$
$= 3(–1)(3 – 4b – 2b^2)$
$= –3(3 – 4b – 2b^2)$



10. For each trinomial, factor each term then identify the greatest common factor. Write each term as the product of the greatest common factor and another monomial. Then use the distributive property to write the expression as a product.
I cannot use algebra tiles because I do not have x^3-tiles, or tiles for greater powers of x.

a) $5 + 15m^2 – 10m^3$
$5 = \mathbf{5}$
$15m^2 = 3 \cdot \mathbf{5} \cdot m \cdot m$
$10m^3 = 2 \cdot \mathbf{5} \cdot m \cdot m \cdot m$
The greatest common factor is: 5
$5 + 15m^2 – 10m^3 = 5(1) + 5(3m^2) – 5(2m^3)$
$= 5(1 + 3m^2 – 2m^3)$
Check: $5(1 + 3m^2 – 2m^3) = 5(1) + 5(3m^2) – 5(2m^3)$
$= 5 + 15m^2 – 10m^3$

b) $27n + 36 – 18n^3$
$27n = \mathbf{3} \cdot \mathbf{3} \cdot 3 \cdot n$
$36 = 2 \cdot 2 \cdot \mathbf{3} \cdot \mathbf{3}$
$18n^3 = 2 \cdot \mathbf{3} \cdot \mathbf{3} \cdot n \cdot n \cdot n$

The greatest common factor is: $3 \cdot 3 = 9$

$27n + 36 - 18n^3 = 9(3n) + 9(4) - 9(2n^3)$
$= 9(3n + 4 - 2n^3)$

Check: $9(3n + 4 - 2n^3) = 9(3n) + 9(4) - 9(2n^3)$
$= 27n + 36 - 18n^3$

c) $6v^4 + 7v - 8v^3$

$6v^4 = 2 \cdot 3 \cdot \boldsymbol{v} \cdot v \cdot v \cdot v$
$7v = 7 \cdot \boldsymbol{v}$
$8v^3 = 2 \cdot 2 \cdot 2 \cdot \boldsymbol{v} \cdot v \cdot v$

The greatest common factor is: v

$6v^4 + 7v - 8v^3 = v(6v^3) + v(7) - v(8v^2)$
$= v(6v^3 + 7 - 8v^2)$

Check: $v(6v^3 + 7 - 8v^2) = v(6v^3) + v(7) - v(8v^2)$
$= 6v^4 + 7v - 8v^3$

d) $-3c^2 - 13c^4 - 12c^3$

$3c^2 = 3 \cdot \boldsymbol{c} \cdot \boldsymbol{c}$
$13c^4 = 13 \cdot \boldsymbol{c} \cdot \boldsymbol{c} \cdot c \cdot c$
$12c^3 = 2 \cdot 2 \cdot 3 \cdot \boldsymbol{c} \cdot \boldsymbol{c} \cdot c$

The greatest common factor is: $c \cdot c = c^2$

$-3c^2 - 13c^4 - 12c^3 = c^2(-3) - c^2(13c^2) - c^2(12c)$
$= c^2(-3 - 13c^2 - 12c)$
$= c^2(-1)(3 + 13c^2 + 12c)$
$= -c^2(3 + 13c^2 + 12c)$

Check: $-c^2(3 + 13c^2 + 12c) = -c^2(3) + (-c^2)(13c^2) + (-c^2)(12c)$
$= -3c^2 - 13c^4 - 12c^3$

e) $24x + 30x^2 - 12x^4$

$24x = \mathbf{2} \cdot 2 \cdot 2 \cdot \mathbf{3} \cdot \boldsymbol{x}$
$30x^2 = \mathbf{2} \cdot \mathbf{3} \cdot 5 \cdot \boldsymbol{x} \cdot x$
$12x^4 = \mathbf{2} \cdot 2 \cdot \mathbf{3} \cdot \boldsymbol{x} \cdot x \cdot x \cdot x$

The greatest common factor is: $2 \cdot 3 \cdot x = 6x$

$24x + 30x^2 - 12x^4 = 6x(4) + 6x(5x) - 6x(2x^3)$
$= 6x(4 + 5x - 2x^3)$

Check: $6x(4 + 5x - 2x^3) = 6x(4) + 6x(5x) - 6x(2x^3)$
$= 24x + 30x^2 - 12x^4$

f) $s^4 + s^2 - 4s$

$s^4 = \boldsymbol{s} \cdot s \cdot s \cdot s$
$s^2 = \boldsymbol{s} \cdot s$
$4s = 2 \cdot 2 \cdot \boldsymbol{s}$

The greatest common factor is: s

$s^4 + s^2 - 4s = s(s^3) + s(s) - s(4)$
$= s(s^3 + s - 4)$

Check: $s(s^3 + s - 4) = s(s^3) + s(s) - s(4)$
$= s^4 + s^2 - 4s$

11. a) There are 12 negative x^2-tiles and 20 x-tiles.
So, the polynomial is: $-12x^2 + 20x$

b) Factor each term, then identify the greatest common factor.
$12x^2 = \mathbf{2} \cdot \mathbf{2} \cdot 3 \cdot \boldsymbol{x} \cdot x$
$20x = \mathbf{2} \cdot \mathbf{2} \cdot 5 \cdot \boldsymbol{x}$
The greatest common factor is: $2 \cdot 2 \cdot x = 4x$
Write each term as the product of the greatest common factor and another monomial.
$-12x^2 + 20x = 4x(-3x) + 4x(5)$
Use the distributive property to write the expression as a product.
$$\begin{aligned}-12x^2 + 20x &= 4x(-3x + 5)\\ &= 4x(-1)(3x - 5)\\ &= -4x(3x - 5)\end{aligned}$$

c) The factors can be written in two ways: $4x$ and $-3x + 5$; or $-4x$ and $3x - 5$
The width of the rectangle is $-3x + 5$ and its length is $-4x$.
So, for each way the factors are written, only one of the factors matches the dimensions of the rectangle.

12. a) i) When the student wrote each term as a product of the common factor and another monomial, he probably wrote the 3rd term, $3m$, as $3m(0)$, so when the student used the distributive property, this term disappeared.
The correct solution is:
$$\begin{aligned}3m^2 + 9m^3 - 3m &= 3m(m) + 3m(3m^2) - 3m(1)\\ &= 3m(m + 3m^2 - 1)\end{aligned}$$

ii) When the student removed –4 as a common factor, he or she forgot to consider the negative sign when writing the 2nd term as the product of the common factor and another monomial. Also, the student wrote the other monomial for the 3rd term incorrectly.
The correct solution is:
$$\begin{aligned}-16 + 8n - 4n^3 &= -4(4) + (-4)(-2n) + (-4)(n^3)\\ &= -4(4 - 2n + n^3)\end{aligned}$$

b) The student should have expanded to check that the factored expression is equal to the given expression.

13. The monomial is 1 when the factor is equal to the term; for example, if the factor is $2x$ and the term is $2x$, I write: $2x = 2x(1)$
The monomial is –1 when the factor and the term are opposites; for example, if the factor is $2x$ and the term is $-2x$, I write: $-2x = 2x(-1)$

14. a) $$\begin{aligned}x^2 + 6x - 7 - x^2 - 2x + 3 &= x^2 - x^2 + 6x - 2x - 7 + 3\\ &= 4x - 4\end{aligned}$$
Factor. The common factor is 4.
$$\begin{aligned}4x - 4 &= 4(x) - 4(1)\\ &= 4(x - 1)\end{aligned}$$

b) $$\begin{aligned}12m^2 - 24m - 3 + 4m^2 - 13 &= 12m^2 + 4m^2 - 24m - 3 - 13\\ &= 16m^2 - 24m - 16\end{aligned}$$
Factor. The common factor is 8.
$$\begin{aligned}16m^2 - 24m - 16 &= 8(2m^2) - 8(3m) - 8(2)\\ &= 8(2m^2 - 3m - 2)\end{aligned}$$

c) $-7n^3 - 5n^2 + 2n - n^2 - n^3 - 12n = -7n^3 - n^3 - 5n^2 - n^2 + 2n - 12n$
$= -8n^3 - 6n^2 - 10n$

Factor. The common factor is $2n$.

$-8n^3 - 6n^2 - 10n = 2n(-4n^2) - 2n(3n) - 2n(5)$
$= 2n(-4n^2 - 3n - 5)$
$= 2n(-1)(4n^2 + 3n + 5)$
$= -2n(4n^2 + 3n + 5)$

15. a) i) $4s^2t^2 = \mathbf{2} \cdot \mathbf{2} \cdot \boldsymbol{s} \cdot s \cdot \boldsymbol{t} \cdot \boldsymbol{t}$
$12s^2t^3 = \mathbf{2} \cdot \mathbf{2} \cdot 3 \cdot \boldsymbol{s} \cdot s \cdot \boldsymbol{t} \cdot \boldsymbol{t} \cdot t$
$36st^2 = \mathbf{2} \cdot \mathbf{2} \cdot 3 \cdot 3 \cdot \boldsymbol{s} \cdot \boldsymbol{t} \cdot \boldsymbol{t}$
The greatest common factor is: $2 \cdot 2 \cdot s \cdot t \cdot t = 4st^2$

ii) $3a^3b = 3 \cdot \boldsymbol{a} \cdot \boldsymbol{a} \cdot a \cdot \boldsymbol{b}$
$8a^2b = 2 \cdot 2 \cdot 2 \cdot \boldsymbol{a} \cdot \boldsymbol{a} \cdot \boldsymbol{b}$
$9a^4b = 3 \cdot 3 \cdot \boldsymbol{a} \cdot \boldsymbol{a} \cdot a \cdot a \cdot \boldsymbol{b}$
The greatest common factor is: $a \cdot a \cdot b = a^2b$

iii) $12x^3y^2 = \mathbf{2} \cdot \mathbf{2} \cdot \mathbf{3} \cdot \boldsymbol{x} \cdot \boldsymbol{x} \cdot x \cdot \boldsymbol{y} \cdot \boldsymbol{y}$
$12x^4y^3 = \mathbf{2} \cdot \mathbf{2} \cdot \mathbf{3} \cdot \boldsymbol{x} \cdot \boldsymbol{x} \cdot x \cdot x \cdot \boldsymbol{y} \cdot \boldsymbol{y} \cdot y$
$36x^2y^4 = \mathbf{2} \cdot \mathbf{2} \cdot \mathbf{3} \cdot 3 \cdot \boldsymbol{x} \cdot \boldsymbol{x} \cdot \boldsymbol{y} \cdot \boldsymbol{y} \cdot y \cdot y$
The greatest common factor is: $2 \cdot 2 \cdot 3 \cdot x \cdot x \cdot y \cdot y = 12x^2y^2$

b) i) $4s^2t^2 + 12s^2t^3 + 36st^2 = 4st^2(s) + 4st^2(3st) + 4st^2(9)$
$= 4st^2(s + 3st + 9)$

ii) $12s^2t^3 - 4s^2t^2 - 36st^2 = 4st^2(3st) - 4st^2(s) - 4st^2(9)$
$= 4st^2(3st - s - 9)$

iii) $-3a^3b - 9a^4b + 8a^2b = a^2b(-3a) - a^2b(9a^2) + a^2b(8)$
$= a^2b(-3a - 9a^2 + 8)$
$= a^2b(-1)(3a + 9a^2 - 8)$
$= -a^2b(3a + 9a^2 - 8)$

iv) $9a^4b + 3a^3b - 8a^2b = a^2b(9a^2) + a^2b(3a) - a^2b(8)$
$= a^2b(9a^2 + 3a - 8)$

v) $36x^2y^4 + 12x^3y^2 + 12x^4y^3 = 12x^2y^2(3y^2) + 12x^2y^2(x) + 12x^2y^2(x^2y)$
$= 12x^2y^2(3y^2 + x + x^2y)$

vi) $-36x^2y^4 - 12x^4y^3 - 12x^3y^2 = 12x^2y^2(-3y^2) + 12x^2y^2(-x^2y) + 12x^2y^2(-x)$
$= 12x^2y^2(-3y^2 - x^2y - x)$
$= 12x^2y^2(-1)(3y^2 + x^2y + x)$
$= -12x^2y^2(3y^2 + x^2y + x)$

16. For each trinomial, factor each term then identify the greatest common factor. Write each term as the product of the greatest common factor and another monomial. Then use the distributive property to write the expression as a product.

a) $25xy + 15x^2 - 30x^2y^2$
$25xy = \mathbf{5} \cdot 5 \cdot \boldsymbol{x} \cdot y$
$15x^2 = 3 \cdot \mathbf{5} \cdot \boldsymbol{x} \cdot x$

$30x^2y^2 = 2 \cdot 3 \cdot \mathbf{5} \cdot \boldsymbol{x} \cdot x \cdot y \cdot y$
The greatest common factor is: $5 \cdot x = 5x$
$25xy + 15x^2 - 30x^2y^2 = 5x(5y) + 5x(3x) - 5x(6xy^2)$
$= 5x(5y + 3x - 6xy^2)$
Check: $5x(5y + 3x - 6xy^2) = 5x(5y) + 5x(3x) - 5x(6xy^2)$
$= 25xy + 15x^2 - 30x^2y^2$

b) $51m^2n + 39mn^2 - 72mn$
$51m^2n = \mathbf{3} \cdot 17 \cdot \boldsymbol{m} \cdot m \cdot \boldsymbol{n}$
$39mn^2 = \mathbf{3} \cdot 13 \cdot \boldsymbol{m} \cdot \boldsymbol{n} \cdot n$
$72mn = 2 \cdot 2 \cdot 2 \cdot \mathbf{3} \cdot 3 \cdot \boldsymbol{m} \cdot \boldsymbol{n}$
The greatest common factor is: $3 \cdot m \cdot n = 3mn$
$51m^2n + 39mn^2 - 72mn = 3mn(17m) + 3mn(13n) - 3mn(24)$
$= 3mn(17m + 13n - 24)$
Check: $3mn(17m + 13n - 24) = 3mn(17m) + 3mn(13n) - 3mn(24)$
$= 51m^2n + 39mn^2 - 72mn$

c) $9p^4q^2 - 6p^3q^3 + 12p^2q^4$
$9p^4q^2 = \mathbf{3} \cdot 3 \cdot \boldsymbol{p} \cdot \boldsymbol{p} \cdot p \cdot p \cdot \boldsymbol{q} \cdot \boldsymbol{q}$
$6p^3q^3 = 2 \cdot \mathbf{3} \cdot \boldsymbol{p} \cdot \boldsymbol{p} \cdot p \cdot \boldsymbol{q} \cdot \boldsymbol{q} \cdot q$
$12p^2q^4 = 2 \cdot 2 \cdot \mathbf{3} \cdot \boldsymbol{p} \cdot \boldsymbol{p} \cdot \boldsymbol{q} \cdot \boldsymbol{q} \cdot q \cdot q$
The greatest common factor is: $3 \cdot p \cdot p \cdot q \cdot q = 3p^2q^2$
$9p^4q^2 - 6p^3q^3 + 12p^2q^4 = 3p^2q^2(3p^2) - 3p^2q^2(2pq) + 3p^2q^2(4q^2)$
$= 3p^2q^2(3p^2 - 2pq + 4q^2)$
Check: $3p^2q^2(3p^2 - 2pq + 4q^2) = 3p^2q^2(3p^2) - 3p^2q^2(2pq) + 3p^2q^2(4q^2)$
$= 9p^4q^2 - 6p^3q^3 + 12p^2q^4$

d) $10a^3b^2 + 12a^2b^4 - 5a^2b^2$
$10a^3b^2 = 2 \cdot 5 \cdot \boldsymbol{a} \cdot \boldsymbol{a} \cdot a \cdot \boldsymbol{b} \cdot \boldsymbol{b}$
$12a^2b^4 = 2 \cdot 2 \cdot 3 \cdot \boldsymbol{a} \cdot \boldsymbol{a} \cdot \boldsymbol{b} \cdot \boldsymbol{b} \cdot b \cdot b$
$5a^2b^2 = 5 \cdot \boldsymbol{a} \cdot \boldsymbol{a} \cdot \boldsymbol{b} \cdot \boldsymbol{b}$
The greatest common factor is: $a \cdot a \cdot b \cdot b = a^2b^2$
$10a^3b^2 + 12a^2b^4 - 5a^2b^2 = a^2b^2(10a) + a^2b^2(12b^2) - a^2b^2(5)$
$= a^2b^2(10a + 12b^2 - 5)$
Check: $a^2b^2(10a + 12b^2 - 5) = a^2b^2(10a) + a^2b^2(12b^2) - a^2b^2(5)$
$= 10a^3b^2 + 12a^2b^4 - 5a^2b^2$

e) $12cd^2 - 8cd - 20c^2d$
$12cd^2 = \mathbf{2} \cdot \mathbf{2} \cdot 3 \cdot \boldsymbol{c} \cdot \boldsymbol{d} \cdot d$
$8cd = \mathbf{2} \cdot \mathbf{2} \cdot 2 \cdot \boldsymbol{c} \cdot \boldsymbol{d}$
$20c^2d = \mathbf{2} \cdot \mathbf{2} \cdot 5 \cdot \boldsymbol{c} \cdot c \cdot \boldsymbol{d}$
The greatest common factor is: $2 \cdot 2 \cdot c \cdot d = 4cd$
$12cd^2 - 8cd - 20c^2d = 4cd(3d) - 4cd(2) - 4cd(5c)$
$= 4cd(3d - 2 - 5c)$
Check: $4cd(3d - 2 - 5c) = 4cd(3d) - 4cd(2) - 4cd(5c)$
$= 12cd^2 - 8cd - 20c^2d$

f) $7r^3s^3 + 14r^2s^2 - 21rs^2$
$7r^3s^3 = \mathbf{7} \cdot \boldsymbol{r} \cdot r \cdot r \cdot \boldsymbol{s} \cdot \boldsymbol{s} \cdot s$
$14r^2s^2 = 2 \cdot \mathbf{7} \cdot \boldsymbol{r} \cdot r \cdot \boldsymbol{s} \cdot \boldsymbol{s}$
$21rs^2 = 3 \cdot \mathbf{7} \cdot \boldsymbol{r} \cdot \boldsymbol{s} \cdot \boldsymbol{s}$

The greatest common factor is: $7 \cdot r \cdot s \cdot s = 7rs^2$
$7r^3s^3 + 14r^2s^2 - 21rs^2 = 7rs^2(r^2s) + 7rs^2(2r) - 7rs^2(3)$
$= 7rs^2(r^2s + 2r - 3)$
Check: $7rs^2(r^2s + 2r - 3) = 7rs^2(r^2s) + 7rs^2(2r) - 7rs^2(3)$
$= 7r^3s^3 + 14r^2s^2 - 21rs^2$

17. a) Factor each term then identify the greatest common factor. Write each term as the product of the greatest common factor and another monomial. Then use the distributive property to write the expression as a product.
$SA = 2\pi r^2 + 2\pi rh$
$2\pi r^2 = \mathbf{2} \cdot \boldsymbol{\pi} \cdot \boldsymbol{r} \cdot r$
$2\pi rh = \mathbf{2} \cdot \boldsymbol{\pi} \cdot \boldsymbol{r} \cdot h$
The greatest common factor is: $2 \cdot \pi \cdot r = 2\pi r$
$2\pi r^2 + 2\pi rh = 2\pi r(r) + 2\pi r(h)$
$= 2\pi r(r + h)$
So, in factored form, $SA = 2\pi r(r + h)$

b) Use: $SA = 2\pi r^2 + 2\pi rh$ Substitute: $r = 12$ and $h = 23$
$SA = 2\pi(12)^2 + 2\pi(12)(23)$
$= 288\pi + 552\pi$
$= 840\pi$
$= 2638.9378\ldots$

Use: $SA = 2\pi r(r + h)$ Substitute: $r = 12$ and $h = 23$
$SA = 2\pi(12)(12 + 23)$
$= 24\pi(35)$
$= 840\pi$
$= 2638.9378\ldots$
The surface area of the cylinder is approximately 2639 cm^2.
The factored form of the formula is more efficient to use because fewer calculations are required, and I can use mental math for some calculations.

18. a) Factor each term then identify the greatest common factor. Write each term as the product of the greatest common factor and another monomial. Then use the distributive property to write the expression as a product.
$SA = \pi r^2 + \pi rs$
$\pi r^2 = \boldsymbol{\pi} \cdot \boldsymbol{r} \cdot r$
$\pi rs = \boldsymbol{\pi} \cdot \boldsymbol{r} \cdot s$
The greatest common factor is: $\pi \cdot r = \pi r$
$\pi r^2 + \pi rs = \pi r(r) + \pi r(s)$
$= \pi r(r + s)$
So, in factored form, $SA = \pi r(r + s)$

b) Use: $SA = \pi r^2 + \pi rs$ Substitute: $r = 9$ and $s = 15$
$SA = \pi(9)^2 + \pi(9)(15)$
$= 81\pi + 135\pi$
$= 216\pi$
$= 678.5840\ldots$

Use: $SA = \pi r(r + s)$ Substitute: $r = 9$ and $s = 15$
$SA = \pi(9)(9 + 15)$

$= 9\pi(24)$
$= 216\pi$
$= 678.5840\ldots$

The surface area of the cone is approximately 679 cm^2.
The factored form of the formula is more efficient to use because fewer calculations are required, and I can use mental math for some calculations.

19. a) Assume the base of the silo is not included in the surface area.
The surface area of the silo is:
the curved surface area of a cylinder + the curved surface area of a hemisphere
The curved surface area of the cylinder is: $2\pi rh$
The curved surface area of the hemisphere is: $2\pi r^2$
The surface area of the silo is:
$SA = 2\pi rh + 2\pi r^2$
Factor each term then identify the greatest common factor. Write each term as the product of the greatest common factor and another monomial. Then use the distributive property to write the expression as a product.
$2\pi rh = \mathbf{2} \cdot \boldsymbol{\pi} \cdot \boldsymbol{r} \cdot h$
$2\pi r^2 = \mathbf{2} \cdot \boldsymbol{\pi} \cdot \boldsymbol{r} \cdot r$
The greatest common factor is: $2 \cdot \pi \cdot r = 2\pi r$
$2\pi rh + 2\pi r^2 = 2\pi r(h) + 2\pi r(r)$
$= 2\pi r(h + r)$
I will use the factored form because I have fewer calculations and I can use mental math for some of them.
$SA = 2\pi r(h + r)$ Substitute: $r = 6$ and $h = 10$
$= 2\pi(6)(10 + 6)$
$= 12\pi(16)$
$= 192\pi$
$= 603.1857$
The surface area of the silo is approximately 603 m^2.

b) The volume of the silo is:
the volume of a cylinder + the volume of a hemisphere
The volume of the cylinder is: $\pi r^2 h$
The volume of the hemisphere is: $\frac{2}{3}\pi r^3$
The volume of the silo is:
$V = \pi r^2 h + \frac{2}{3}\pi r^3$
Factor each term then identify the greatest common factor. Write each term as the product of the greatest common factor and another monomial. Then use the distributive property to write the expression as a product.
$\pi r^2 h = \boldsymbol{\pi} \cdot \boldsymbol{r} \cdot \boldsymbol{r} \cdot h$
$\frac{2}{3}\pi r^3 = \frac{2}{3} \cdot \boldsymbol{\pi} \cdot \boldsymbol{r} \cdot \boldsymbol{r} \cdot r$
The greatest common factor is: $\pi \cdot r \cdot r = \pi r^2$
$\pi r^2 h + \frac{2}{3}\pi r^3 = \pi r^2(h) + \pi r^2\left(\frac{2}{3}r\right)$

$= \pi r^2(h + \frac{2}{3}r)$

So, in factored form, $V = \pi r^2(h + \frac{2}{3}r)$

I will use the factored form because I have fewer calculations and I can use mental math for some of them.

$V = \pi r^2(h + \frac{2}{3}r)$ Substitute: $r = 6$ and $h = 10$

$= \pi(6)^2(10 + \frac{2}{3}(6))$

$= 36\pi(14)$

$= 504\pi$

$= 1583.3626\ldots$

The volume of the silo is approximately 1583 m^3.

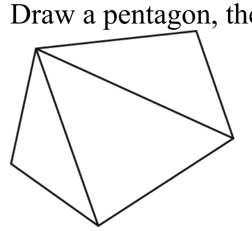
20. Factor: $n^2 - n = n(n - 1)$
When n is an integer, $n - 1$ is also an integer, and the product of two integers is an integer.
So, when n is an integer, then $n^2 - n$ is always an integer.

C

21. a) The surface area of a cylinder with base radius r and height h is: $2\pi r^2 + 2\pi rh$
From question 17a, this expression factors as: $2\pi r(r + h)$
The curved surface area of the cylinder is: $2\pi rh$
The fraction of the surface area that will be painted is: $\frac{2\pi rh}{2\pi r(r + h)}$

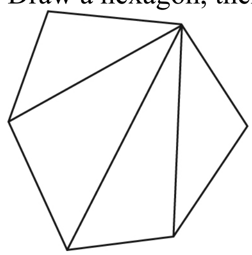
b) To simplify the fraction $\frac{2\pi rh}{2\pi r(r + h)}$, divide numerator and denominator by their common factor, $2\pi r$.
The fraction simplifies to: $\frac{h}{r + h}$

22. a) Draw a pentagon, then draw the diagonals from one vertex.



From the diagram, 2 diagonals can be drawn from one vertex of a pentagon.

Draw a hexagon, then draw the diagonals from one vertex.



From the diagram, 3 diagonals can be drawn from one vertex of a hexagon.

b) For a polygon with n sides and n vertices, $(n - 1)$ line segments can be drawn from one vertex to all the other vertices. But two of these line segments are sides of the polygon, so $(n - 3)$ of the line segments are diagonals.
For a polygon with n sides, $(n - 3)$ diagonals can be drawn from one vertex.

c) Factor: $\frac{n^2}{2} - \frac{3n}{2}$

The common factor of the two terms is $\frac{n}{2}$.

So, $\frac{n^2}{2} - \frac{3n}{2} = \frac{n}{2}(n) - \frac{n}{2}(3)$

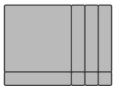
$= \frac{n}{2}(n - 3)$

The total number of diagonals in a polygon can be written as $\frac{n}{2}(n - 3)$.

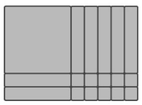
To explain why this formula is reasonable:
There are n vertices; so from all the vertices, $n(n - 3)$ diagonals can be drawn. But each diagonal has been counted twice, so divide by 2 to get the total number of diagonals:
$\frac{n}{2}(n - 3)$

Lesson 3.4 **Math Lab: Modelling Trinomials as Binomial Products** **Assess Your Understanding (page 158)**

1. a) Try to arrange 1 y^2-tile, 4 y-tiles, and three 1-tiles in a rectangle. A rectangle is possible.



b) Try to arrange 1 d^2-tile, 7 d-tiles, and ten 1-tiles in a rectangle. A rectangle is possible.



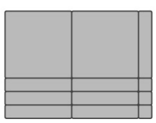
c) Try to arrange 1 m^2-tile, 7 m-tiles, and seven 1-tiles in a rectangle. A rectangle is not possible.

d) Try to arrange 1 r^2-tile, 14 r-tiles, and fourteen 1-tiles in a rectangle. A rectangle is not possible.

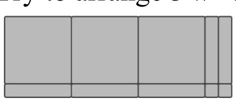
e) Try to arrange 1 t^2-tile, 6 t-tiles, and six 1-tiles in a rectangle. A rectangle is not possible.

f) Try to arrange 1 p^2-tile, 9 p-tiles, and two 1-tiles in a rectangle. A rectangle is not possible.

2. a) Try to arrange 2 s^2-tiles, 7 s-tiles, and three 1-tiles in a rectangle. A rectangle is possible.



b) Try to arrange 3 w^2-tiles, 5 w-tiles, and two 1-tiles in a rectangle. A rectangle is possible.



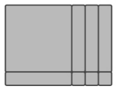
c) Try to arrange 2 f^2-tiles, 3 f-tiles, and two 1-tiles in a rectangle. A rectangle is not possible.

d) Try to arrange 2 h^2-tiles, 10 h-tiles, and six 1-tiles in a rectangle. A rectangle is not possible.

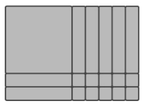
e) Try to arrange 4 n^2-tiles, 2 n-tiles, and one 1-tile in a rectangle. A rectangle is not possible.

Lesson 3.4 Math Lab: Modelling Trinomials as Binomial Products — Assess Your Understanding (page 158)

1. a) Try to arrange 1 y^2-tile, 4 y-tiles, and three 1-tiles in a rectangle. A rectangle is possible.



b) Try to arrange 1 d^2-tile, 7 d-tiles, and ten 1-tiles in a rectangle. A rectangle is possible.



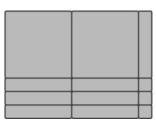
c) Try to arrange 1 m^2-tile, 7 m-tiles, and seven 1-tiles in a rectangle. A rectangle is not possible.

d) Try to arrange 1 r^2-tile, 14 r-tiles, and fourteen 1-tiles in a rectangle. A rectangle is not possible.

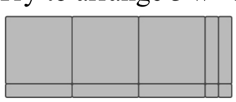
e) Try to arrange 1 t^2-tile, 6 t-tiles, and six 1-tiles in a rectangle. A rectangle is not possible.

f) Try to arrange 1 p^2-tile, 9 p-tiles, and two 1-tiles in a rectangle. A rectangle is not possible.

2. a) Try to arrange 2 s^2-tiles, 7 s-tiles, and three 1-tiles in a rectangle. A rectangle is possible.



b) Try to arrange 3 w^2-tiles, 5 w-tiles, and two 1-tiles in a rectangle. A rectangle is possible.

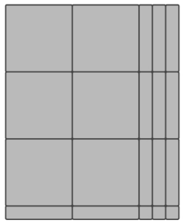


c) Try to arrange 2 f^2-tiles, 3 f-tiles, and two 1-tiles in a rectangle. A rectangle is not possible.

d) Try to arrange 2 h^2-tiles, 10 h-tiles, and six 1-tiles in a rectangle. A rectangle is not possible.

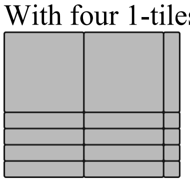
e) Try to arrange 4 n^2-tiles, 2 n-tiles, and one 1-tile in a rectangle. A rectangle is not possible.

f) Try to arrange 6 k^2-tiles, 11 k-tiles, and three 1-tiles in a rectangle. A rectangle is possible.

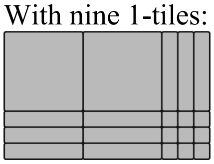


3. Use the patterns you found in *Construct Understanding*. There are twelve 1-tiles. Write all the pairs of numbers that have a product of 12. The numbers of x-tiles that you could use to make rectangles are equal to the sums of the numbers in these pairs.
$12 = 1 \cdot 12$ $1 + 12 = 13$
$12 = 2 \cdot 6$ $2 + 6 = 8$
$12 = 3 \cdot 4$ $3 + 4 = 7$
These sets of tiles make rectangles:
1 x^2-tile, 13 x-tiles, and twelve 1-tiles
1 x^2-tile, 8 x-tiles, and twelve 1-tiles
1 x^2-tile, 7 x-tiles, and twelve 1-tiles

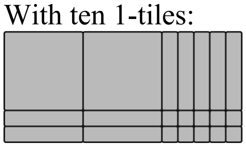
4. Use algebra tiles. Arrange 9 x-tiles on two sides of 2 x^2-tiles in as many different ways as possible, so that rectangles can be completed by adding 1-tiles. For each arrangement, add 1-tiles to complete a rectangle. Here are the possible arrangements.
With four 1-tiles:



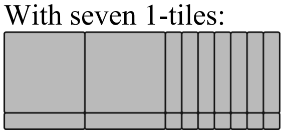
With nine 1-tiles:



With ten 1-tiles:



With seven 1-tiles:



Lesson 3.5 Polynomials of the Form $x^2 + bx + c$ Exercises (pages 166–167)

A

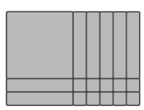
4. a) There are 1 x^2-tile, 4 x-tiles, and three 1-tiles.
So, the trinomial is: $x^2 + 4x + 3$
The length of the rectangle is $x + 3$ and the width is $x + 1$.
So, the multiplication sentence is: $(x + 3)(x + 1) = x^2 + 4x + 3$

b) There are 1 x^2-tile, 6 x-tiles, and eight 1-tiles.
So, the trinomial is: $x^2 + 6x + 8$
The length of the rectangle is $x + 4$ and the width is $x + 2$.
So, the multiplication sentence is: $(x + 4)(x + 2) = x^2 + 6x + 8$

c) There are 1 x^2-tile, 10 x-tiles, and twenty-five 1-tiles.
So, the trinomial is: $x^2 + 10x + 25$
The length of the rectangle is $x + 5$ and the width is also $x + 5$.
So, the multiplication sentence is: $(x + 5)(x + 5) = x^2 + 10x + 25$

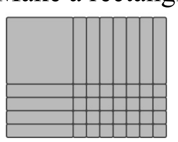
d) There are 1 x^2-tile, 9 x-tiles, and eighteen 1-tiles.
So, the trinomial is: $x^2 + 9x + 18$
The length of the rectangle is $x + 6$ and the width is $x + 3$.
So, the multiplication sentence is: $(x + 6)(x + 3) = x^2 + 9x + 18$

5. a) Make a rectangle with length $b + 5$ and width $b + 2$.



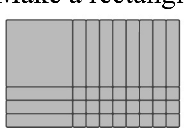
The tiles that form the product are: 1 b^2-tile, 7 b-tiles, and ten 1-tiles.
So, $(b + 2)(b + 5) = b^2 + 7b + 10$

b) Make a rectangle with length $n + 7$ and width $n + 4$.



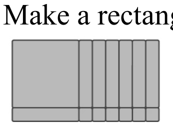
The tiles that form the product are: 1 n^2-tile, 11 n-tiles, and twenty-eight 1-tiles.
So, $(n + 4)(n + 7) = n^2 + 11n + 28$

c) Make a rectangle with length $h + 8$ and width $h + 3$.



The tiles that form the product are: 1 h^2-tile, 11 h-tiles, and twenty-four 1-tiles.
So, $(h + 8)(h + 3) = h^2 + 11h + 24$

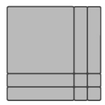
d) Make a rectangle with length $k + 6$ and width $k + 1$.



The tiles that form the product are: 1 k^2-tile, 7 k-tiles, and six 1-tiles.
So, $(k + 1)(k + 6) = k^2 + 7k + 6$

6. a) i) There are 1 x^2-tile, 4 x-tiles, and four 1-tiles.
So, the trinomial is: $x^2 + 4x + 4$

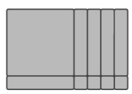
ii)



iii) The length of the rectangle is $x + 2$ and the width is also $x + 2$.
So, the factors of $x^2 + 4x + 4$ are: $(x + 2)(x + 2)$

b) i) There are 1 x^2-tile, 5 x-tiles, and four 1-tiles.
So, the trinomial is: $x^2 + 5x + 4$

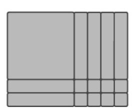
ii)



iii) The length of the rectangle is $x + 4$ and the width is $x + 1$.
So, the factors of $x^2 + 5x + 4$ are: $(x + 4)(x + 1)$

c) i) There are 1 x^2-tile, 6 x-tiles, and eight 1-tiles.
So, the trinomial is: $x^2 + 6x + 8$

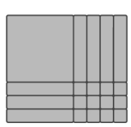
ii)



iii) The length of the rectangle is $x + 4$ and the width is $x + 2$.
So, the factors of $x^2 + 6x + 8$ are: $(x + 4)(x + 2)$

d) i) There are 1 x^2-tile, 7 x-tiles, and twelve 1-tiles.
So, the trinomial is: $x^2 + 7x + 12$

ii)



iii) The length of the rectangle is $x + 4$ and the width is $x + 3$.
So, the factors of $x^2 + 7x + 12$ are: $(x + 4)(x + 3)$

7. a) i) Only two numbers have a product of 2; the numbers are 1 and 2.
The sum is: $1 + 2 = 3$
So, $a = 1$ and $b = 2$

ii) The numbers with a product of 6 are: 1 and 6; 2 and 3
The numbers with a sum of 5 are 2 and 3.
So, $a = 2$ and $b = 3$

iii) The numbers with a product of 9 are: 1 and 9; 3 and 3
The numbers with a sum of 10 are 1 and 9.
So, $a = 1$ and $b = 9$

iv) The numbers with a product of 10 are: 1 and 10; 2 and 5
The numbers with a sum of 7 are 2 and 5.
So, $a = 2$ and $b = 5$

v) The numbers with a product of 12 are: 1 and 12; 2 and 6; 3 and 4
The numbers with a sum of 7 are 3 and 4.
So, $a = 3$ and $b = 4$

vi) The numbers with a product of 15 are: 1 and 15; 3 and 5
The numbers with a sum of 8 are 3 and 5.
So, $a = 3$ and $b = 5$

b) i) To factor $v^2 + 3v + 2$, find two numbers with a sum of 3 and a product of 2.
From part a, i, these numbers are 1 and 2.
So, $v^2 + 3v + 2 = (v + 1)(v + 2)$

ii) To factor $w^2 + 5w + 6$, find two numbers with a sum of 5 and a product of 6.
From part a, ii, these numbers are 2 and 3.
So, $w^2 + 5w + 6 = (w + 2)(w + 3)$

iii) To factor $s^2 + 10s + 9$, find two numbers with a sum of 10 and a product of 9.
From part a, iii, these numbers are 1 and 9.
So, $s^2 + 10s + 9 = (s + 1)(s + 9)$

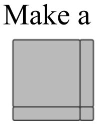
iv) To factor $t^2 + 7t + 10$, find two numbers with a sum of 7 and a product of 10.
From part a, iv, these numbers are 2 and 5.
So, $t^2 + 7t + 10 = (t + 2)(t + 5)$

v) To factor $y^2 + 7y + 12$, find two numbers with a sum of 7 and a product of 12.
From part a, v, these numbers are 3 and 4.
So, $y^2 + 7y + 12 = (y + 3)(y + 4)$

vi) To factor $h^2 + 8h + 15$, find two numbers with a sum of 8 and a product of 15.
From part a, vi, these numbers are 3 and 5.
So, $h^2 + 8h + 15 = (h + 3)(h + 5)$

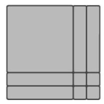
B

8. a) i) Make a rectangle with 1 v^2-tile, 2 v-tiles, and one 1-tile.



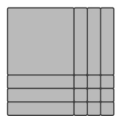
The length of the rectangle is $v + 1$ and the width is also $v + 1$.
So, $v^2 + 2v + 1 = (v + 1)(v + 1)$

ii) Make a rectangle with 1 v^2-tile, 4 v-tiles, and four 1-tiles.



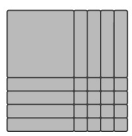
The length of the rectangle is $v + 2$ and the width is also $v + 2$.
So, $v^2 + 4v + 4 = (v + 2)(v + 2)$

iii) Make a rectangle with 1 v^2-tile, 6 v-tiles, and nine 1-tiles.



The length of the rectangle is $v + 3$ and the width is also $v + 3$.
So, $v^2 + 6v + 9 = (v + 3)(v + 3)$

iv) Make a rectangle with 1 v^2-tile, 8 v-tiles, and sixteen 1-tiles.



The length of the rectangle is $v + 4$ and the width is also $v + 4$.
So, $v^2 + 8v + 16 = (v + 4)(v + 4)$

b) All the rectangles are squares. For each trinomial, its binomial factors are equal because the sides of a square are equal.

c) In the trinomials in part a, the coefficients of the v-terms are consecutive even numbers and the constant terms are the squares of one-half of these coefficients.
So, the next 3 trinomials and their factors are:
$v^2 + 10v + 25 = (v + 5)(v + 5)$
$v^2 + 12v + 36 = (v + 6)(v + 6)$
$v^2 + 14v + 49 = (v + 7)(v + 7)$

9. Use the distributive property to expand. Draw a rectangle and label its sides with the binomial factors. Divide the rectangle into 4 smaller rectangles, and label each with a term in the expansion.

a) $(m + 5)(m + 8) = m(m + 8) + 5(m + 8)$
$= m(m) + m(8) + 5(m) + 5(8)$
$= m^2 + 8m + 5m + 40$

$= m^2 + 13m + 40$

	m	8
m	$(m)(m) = m^2$	$(m)(8) = 8m$
5	$(5)(m) = 5m$	$(5)(8) = 40$

b) $(y + 9)(y + 3) = y(y + 3) + 9(y + 3)$
$= y(y) + y(3) + 9(y) + 9(3)$
$= y^2 + 3y + 9y + 27$
$= y^2 + 12y + 27$

	y	3
y	$(y)(y) = y^2$	$(y)(3) = 3y$
9	$(9)(y) = 9y$	$(9)(3) = 27$

c) $(w + 2)(w + 16) = w(w + 16) + 2(w + 16)$
$= w(w) + w(16) + 2(w) + 2(16)$
$= w^2 + 16w + 2w + 32$
$= w^2 + 18w + 32$

	w	16
w	$(w)(w) = w^2$	$(w)(16) = 16w$
2	$(2)(w) = 2w$	$(2)(16) = 32$

d) $(k + 13)(k + 1) = k(k + 1) + 13(k + 1)$
$= k(k) + k(1) + 13(k) + 13(1)$
$= k^2 + 1k + 13k + 13$

$= k^2 + 14k + 13$

	k	1
k	$(k)(k) = k^2$	$(k)(1) = k$
13	$(13)(k) = 13k$	$(13)(1) = 13$

10. a) $(w + 3)(w + 2) = w^2 + \square w + 6$
The coefficient of w in the trinomial is the sum of the constant terms in the binomial factors.
So, $\square = 3 + 2$, or 5
Then, $(w + 3)(w + 2) = w^2 + 5w + 6$

b) $(x + 5)(x + \square) = x^2 + \bigcirc x + 10$
The product of the constant terms in the binomial factors is equal to the constant term in the trinomial.
This means that 5 times the $\square$ in the binomial factor is 10.
So, $\square = 2$
The coefficient of x in the trinomial is the sum of the constant terms in the binomial factors.
So, $\bigcirc = 5 + 2$, or 7
Then, $(x + 5)(x + 2) = x^2 + 7x + 10$

c) $(y + \bigcirc)(y + \square) = y^2 + 12y + 20$
The constant terms in the binomial factors have a sum of 12 and a product of 20.
Two numbers with a product of 20 are: 1 and 20; 2 and 10; 4 and 5
The two numbers with a sum of 12 are 2 and 10.
So, $\bigcirc = 2$ and $\square = 10$
Then, $(y + 2)(y + 10) = y^2 + 12y + 20$

11. For each trinomial, find two numbers whose sum is equal to the coefficient of the middle term and whose product is equal to the constant term. These numbers are the constants in the binomial factors.

a) $x^2 + 10x + 24$
Since all the terms are positive, consider only positive factors of 24.
The factors of 24 are: 1 and 24; 2 and 12; 3 and 8; 4 and 6
The two factors with a sum of 10 are 4 and 6.
So, $x^2 + 10x + 24 = (x + 4)(x + 6)$
Check by expanding.

$$\begin{aligned}(x + 4)(x + 6) &= x(x + 6) + 4(x + 6)\\ &= x(x) + x(6) + 4(x) + 4(6)\\ &= x^2 + 6x + 4x + 24\\ &= x^2 + 10x + 24\end{aligned}$$

b) $m^2 + 10m + 16$
Since all the terms are positive, consider only positive factors of 16.
The factors of 16 are: 1 and 16; 2 and 8; 4 and 4
The two factors with a sum of 10 are 2 and 8.
So, $m^2 + 10m + 16 = (m + 2)(m + 8)$
Check by expanding.
$$\begin{aligned}(m + 2)(m + 8) &= m(m + 8) + 2(m + 8)\\ &= m(m) + m(8) + 2(m) + 2(8)\\ &= m^2 + 8m + 2m + 16\\ &= m^2 + 10m + 16\end{aligned}$$

c) $p^2 + 13p + 12$
Since all the terms are positive, consider only positive factors of 12.
The factors of 12 are: 1 and 12; 2 and 6; 3 and 4
The two factors with a sum of 13 are 1 and 12.
So, $p^2 + 13p + 12 = (p + 1)(p + 12)$
Check by expanding.
$$\begin{aligned}(p + 1)(p + 12) &= p(p + 12) + 1(p + 12)\\ &= p(p) + p(12) + 1(p) + 1(12)\\ &= p^2 + 12p + 1p + 12\\ &= p^2 + 13p + 12\end{aligned}$$

d) $s^2 + 12s + 20$
Since all the terms are positive, consider only positive factors of 20.
The factors of 20 are: 1 and 20; 2 and 10; 4 and 5
The two factors with a sum of 12 are 2 and 10.
So, $s^2 + 12s + 20 = (s + 2)(s + 10)$
Check by expanding.
$$\begin{aligned}(s + 2)(s + 10) &= s(s + 10) + 2(s + 10)\\ &= s(s) + s(10) + 2(s) + 2(10)\\ &= s^2 + 10s + 2s + 20\\ &= s^2 + 12s + 20\end{aligned}$$

e) $n^2 + 12n + 11$
Since all the terms are positive, consider only positive factors of 11.
The factors of 11 are: 1 and 11; these numbers have a sum of 12.
So, $n^2 + 12n + 11 = (n + 1)(n + 11)$
Check by expanding.
$$\begin{aligned}(n + 1)(n + 11) &= n(n + 11) + 1(n + 11)\\ &= n(n) + n(11) + 1(n) + 1(11)\\ &= n^2 + 11n + 1n + 11\\ &= n^2 + 12n + 11\end{aligned}$$

f) $h^2 + 8h + 12$
Since all the terms are positive, consider only positive factors of 12.
The factors of 12 are: 1 and 12; 2 and 6; 3 and 4
The two factors with a sum of 8 are 2 and 6.
So, $h^2 + 8h + 12 = (h + 2)(h + 6)$
Check by expanding.
$$\begin{aligned}(h + 2)(h + 6) &= h(h + 6) + 2(h + 6)\\ &= h(h) + h(6) + 2(h) + 2(6)\end{aligned}$$

$= h^2 + 6h + 2h + 12$
$= h^2 + 8h + 12$

g) $q^2 + 7q + 6$
Since all the terms are positive, consider only positive factors of 6.
The factors of 6 are: 1 and 6; 2 and 3
The two factors with a sum of 7 are 1 and 6.
So, $q^2 + 7q + 6 = (q + 1)(q + 6)$
Check by expanding.
$(q + 1)(q + 6) = q(q + 6) + 1(q + 6)$
$= q(q) + q(6) + 1(q) + 1(6)$
$= q^2 + 6q + 1q + 6$
$= q^2 + 7q + 6$

h) $b^2 + 11b + 18$
Since all the terms are positive, consider only positive factors of 18.
The factors of 18 are: 1 and 18; 2 and 9; 3 and 6
The two factors with a sum of 11 are 2 and 9.
So, $b^2 + 11b + 18 = (b + 2)(b + 9)$
Check by expanding.
$(b + 2)(b + 9) = b(b + 9) + 2(b + 9)$
$= b(b) + b(9) + 2(b) + 2(9)$
$= b^2 + 9b + 2b + 18$
$= b^2 + 11b + 18$

12. Use the distributive property to expand. Draw a rectangle and label its sides with the binomial factors. Divide the rectangle into 4 smaller rectangles, and label each with a term in the expansion.

a) $(g – 3)(g + 7) = g(g + 7) – 3(g + 7)$
$= g(g) + g(7) – 3(g) – 3(7)$
$= g^2 + 7g – 3g – 21$
$= g^2 + 4g – 21$

	g	7
g	$(g)(g) = g^2$	$(g)(7) = 7g$
$–3$	$(–3)(g) = –3g$	$(–3)(7) = –21$

b) $(h + 2)(h – 7) = h(h – 7) + 2(h – 7)$
$= h(h) + h(–7) + 2(h) + 2(–7)$
$= h^2 – 7h + 2h – 14$
$= h^2 – 5h – 14$

	h	$–7$
h	$(h)(h) = h^2$	$(h)(–7) = –7h$
2	$(2)(h) = 2h$	$(2)(–7) = –14$

c) $(11 – j)(2 – j) = 11(2 – j) – j(2 – j)$
$= 11(2) + 11(–j) – j(2) – j(–j)$

$= 22 - 11j - 2j + j^2$
$= 22 - 13j + j^2$

	2	$-j$
11	$(11)(2) = 22$	$(11)(-j) = -11j$
$-j$	$(-j)(2) = -2j$	$(-j)(-j) = j^2$

d) $(k - 3)(k + 11) = k(k + 11) - 3(k + 11)$
$= k(k) + k(11) - 3(k) - 3(11)$
$= k^2 + 11k - 3k - 33$
$= k^2 + 8k - 33$

	k	11
k	$(k)(k) = k^2$	$(k)(11) = 11k$
-3	$(-3)(k) = -3k$	$(-3)(11) = -33$

e) $(12 + h)(7 - h) = 12(7 - h) + h(7 - h)$
$= 12(7) + 12(-h) + h(7) + h(-h)$
$= 84 - 12h + 7h - h^2$
$= 84 - 5h - h^2$

	7	$-h$
12	$(12)(7) = 84$	$(12)(-h) = -12h$
h	$(h)(7) = 7h$	$(h)(-h) = -h^2$

f) $(m - 9)(m + 9) = m(m + 9) - 9(m + 9)$
$= m(m) + m(9) - 9(m) - 9(9)$
$= m^2 + 9m - 9m - 81$
$= m^2 - 81$

	m	9
m	$(m)(m) = m^2$	$(m)(9) = 9m$
-9	$(-9)(m) = -9m$	$(-9)(9) = -81$

g) $(n - 14)(n - 4) = n(n - 4) - 14(n - 4)$
$= n(n) + n(-4) - 14(n) - 14(-4)$
$= n^2 - 4n - 14n + 56$
$= n^2 - 18n + 56$

	n	-4
n	$(n)(n) = n^2$	$(n)(-4) = -4n$
-14	$(-14)(n) = -14n$	$(-14)(-4) = 56$

h) $(p + 6)(p - 17) = p(p - 17) + 6(p - 17)$
$= p(p) + p(-17) + 6(p) + 6(-17)$
$= p^2 - 17p + 6p - 102$
$= p^2 - 11p - 102$

	p	-17
p	$(p)(p) = p^2$	$(p)(-17) = -17p$
6	$(6)(p) = 6p$	$(6)(-17) = -102$

13. a) In the second line of the solution, the product of –13 and 4 is –52; and the sum + 4r – 13r is –9r. The correct solution is:
$(r - 13)(r + 4) = r(r + 4) - 13(r + 4)$
$= r^2 + 4r - 13r - 52$
$= r^2 - 9r - 52$

b) In the first line of the solution, on the right side of the equals sign, both binomial factors should be (s – 5); and the sign between the products of a monomial and a binomial should be negative. The correct solution is:
$(s - 15)(s - 5) = s(s - 5) - 15(s - 5)$
$= s^2 - 5s - 15s + 75$
$= s^2 - 20s + 75$

14. For each trinomial, find two numbers whose sum is equal to the coefficient of the middle term and whose product is equal to the constant term. These numbers are the constants in the binomial factors.

a) $b^2 + 19b - 20$
The factors of –20 are: 1 and –20; –1 and 20; 2 and –10; –2 and 10; 4 and –5; –4 and 5
Use mental math to calculate each sum.
The two factors with a sum of 19 are –1 and 20.
So, $b^2 + 19b - 20 = (b - 1)(b + 20)$
Check by expanding.
$(b - 1)(b + 20) = b(b + 20) - 1(b + 20)$
$= b(b) + b(20) - 1(b) - 1(20)$
$= b^2 + 20b - 1b - 20$
$= b^2 + 19b - 20$

b) $t^2 + 15t - 54$
The factors of –54 are: 1 and –54; –1 and 54; 2 and –27; –2 and 27; 3 and –18; –3 and 18; 6 and –9; –6 and 9
Use mental math to calculate each sum.
The two factors with a sum of 15 are –3 and 18.
So, $t^2 + 15t - 54 = (t - 3)(t + 18)$
Check by expanding.
$(t - 3)(t + 18) = t(t + 18) - 3(t + 18)$
$= t(t) + t(18) - 3(t) - 3(18)$
$= t^2 + 18t - 3t - 54$
$= t^2 + 15t - 54$

c) $x^2 + 12x - 28$
The factors of –28 are: 1 and –28; –1 and 28; 2 and –14; –2 and 14; 4 and –7; –4 and 7
Use mental math to calculate each sum.
The two factors with a sum of 12 are –2 and 14.
So, $x^2 + 12x - 28 = (x - 2)(x + 14)$
Check by expanding.
$$\begin{aligned}(x - 2)(x + 14) &= x(x + 14) - 2(x + 14)\\ &= x(x) + x(14) - 2(x) - 2(14)\\ &= x^2 + 14x - 2x - 28\\ &= x^2 + 12x - 28\end{aligned}$$

d) $n^2 - 5n - 24$
The factors of –24 are: 1 and –24; –1 and 24; 2 and –12; –2 and 12; 3 and –8; –3 and 8; 4 and –6; –4 and 6
Use mental math to calculate each sum.
The two factors with a sum of –5 are 3 and –8.
So, $n^2 - 5n - 24 = (n + 3)(n - 8)$
Check by expanding.
$$\begin{aligned}(n + 3)(n - 8) &= n(n - 8) + 3(n - 8)\\ &= n(n) + n(-8) + 3(n) + 3(-8)\\ &= n^2 - 8n + 3n - 24\\ &= n^2 - 5n - 24\end{aligned}$$

e) $a^2 - a - 20$
The factors of –20 are: 1 and –20; –1 and 20; 2 and –10; –2 and 10; 4 and –5; –4 and 5
Use mental math to calculate each sum.
The two factors with a sum of –1 are 4 and –5.
So, $a^2 - a - 20 = (a + 4)(a - 5)$
Check by expanding.
$$\begin{aligned}(a + 4)(a - 5) &= a(a - 5) + 4(a - 5)\\ &= a(a) + a(-5) + 4(a) + 4(-5)\\ &= a^2 - 5a + 4a - 20\\ &= a^2 - a - 20\end{aligned}$$

f) $y^2 - 2y - 48$
The factors of –48 are: 1 and –48; –1 and 48; 2 and –24; –2 and 24; 3 and –16; –3 and 16; 4 and –12; –4 and 12; 6 and –8; –6 and 8
Use mental math to calculate each sum.
The two factors with a sum of –2 are 6 and –8.
So, $y^2 - 2y - 48 = (y + 6)(y - 8)$
Check by expanding.
$$\begin{aligned}(y + 6)(y - 8) &= y(y - 8) + 6(y - 8)\\ &= y(y) + y(-8) + 6(y) + 6(-8)\\ &= y^2 - 8y + 6y - 48\\ &= y^2 - 2y - 48\end{aligned}$$

g) $m^2 - 15m + 50$
Since the constant term is positive, the two numbers in the binomials have the same sign.
Since the coefficient of m is negative, both numbers are negative. So, list only the negative factors of 50.
The negative factors of 50 are: –1 and –50; –2 and –25; –5 and –10
Use mental math to calculate each sum.
The two factors with a sum of –15 are –5 and –10.
So, $m^2 - 15m + 50 = (m - 5)(m - 10)$
Check by expanding.

$$\begin{aligned}(m-5)(m-10) &= m(m-10) - 5(m-10)\\ &= m(m) + m(-10) - 5(m) - 5(-10)\\ &= m^2 - 10m - 5m + 50\\ &= m^2 - 15m + 50\end{aligned}$$

h) $a^2 - 12a + 36$
Since the constant term is positive, the two numbers in the binomials have the same sign.
Since the coefficient of a is negative, both numbers are negative. So, list only the negative factors of 36.
The negative factors of 36 are: –1 and –36; –2 and –18; –3 and –12; –4 and –9; –6 and –6
Use mental math to calculate each sum.
The two factors with a sum of –12 are –6 and –6.
So, $a^2 - 12a + 36 = (a - 6)(a - 6)$
Check by expanding.

$$\begin{aligned}(a-6)(a-6) &= a(a-6) - 6(a-6)\\ &= a(a) + a(-6) - 6(a) - 6(-6)\\ &= a^2 - 6a - 6a + 36\\ &= a^2 - 12a + 36\end{aligned}$$

15. For each trinomial, find two numbers whose product is equal to the constant term and whose sum is equal to the coefficient of the middle term. These numbers are the constants in the binomial factors.

a) $12 + 13k + k^2$
Since all the terms are positive, consider only positive factors of 12.
The factors of 12 are: 1 and 12; 2 and 6; 3 and 4
The two factors with a sum of 13 are 1 and 12.
So, $12 + 13k + k^2 = (1 + k)(12 + k)$
Check by expanding.

$$\begin{aligned}(1+k)(12+k) &= 1(12+k) + k(12+k)\\ &= 1(12) + 1(k) + k(12) + k(k)\\ &= 12 + 1k + 12k + k^2\\ &= 12 + 13k + k^2\end{aligned}$$

b) $-16 - 6g + g^2$

The factors of –16 are: 1 and –16; –1 and 16; 2 and –8; –2 and 8; 4 and –4

The two factors with a sum of –6 are 2 and –8.

So, $-16 - 6g + g^2 = (2 + g)(-8 + g)$

Check by expanding.

$$\begin{aligned}(2 + g)(-8 + g) &= 2(-8 + g) + g(-8 + g)\\ &= 2(-8) + 2(g) + g(-8) + g(g)\\ &= -16 + 2g - 8g + g^2\\ &= -16 - 6g + g^2\end{aligned}$$

c) $60 + 17y + y^2$

Since all the terms are positive, consider only positive factors of 60.

The factors of 60 are: 1 and 60; 2 and 30; 3 and 20; 4 and 15; 5 and 12; 6 and 10

The two factors with a sum of 17 are 5 and 12.

So, $60 + 17y + y^2 = (5 + y)(12 + y)$

Check by expanding.

$$\begin{aligned}(5 + y)(12 + y) &= 5(12 + y) + y(12 + y)\\ &= 5(12) + 5(y) + y(12) + y(y)\\ &= 60 + 5y + 12y + y^2\\ &= 60 + 17y + y^2\end{aligned}$$

d) $72 - z - z^2$

Remove –1 as a common factor to make the z^2-term positive.

$72 - z - z^2 = -1(-72 + z + z^2)$

The factors of –72 are: 1 and –72; –1 and 72; 2 and –36; –2 and 36; 3 and –24; –3 and 24; 4 and –18; –4 and 18; 6 and –12; –6 and 12; 8 and –9; –8 and 9

The two factors with a sum of 1 are –8 and 9.

So, $72 - z - z^2 = -(-8 + z)(9 + z)$
$= (8 - z)(9 + z)$

Check by expanding.

$$\begin{aligned}(8 - z)(9 + z) &= 8(9 + z) - z(9 + z)\\ &= 8(9) + 8(z) - z(9) - z(z)\\ &= 72 + 8z - 9z - z^2\\ &= 72 - z - z^2\end{aligned}$$

16. a) i)

$$\begin{aligned}(x + 1)(x + 2) &= x(x + 2) + 1(x + 2)\\ &= x(x) + x(2) + 1(x) + 1(2)\\ &= x^2 + 2x + 1x + 2\\ &= x^2 + 3x + 2\end{aligned}$$

$$\begin{aligned}11 \cdot 12 &= (10 + 1)(10 + 2)\\ &= 10(10 + 2) + 1(10 + 2)\\ &= 10(10) + 10(2) + 1(10) + 1(2)\\ &= 100 + 20 + 10 + 2\\ &= 132\end{aligned}$$

ii)

$$\begin{aligned}(x + 1)(x + 3) &= x(x + 3) + 1(x + 3)\\ &= x(x) + x(3) + 1(x) + 1(3)\\ &= x^2 + 3x + 1x + 3\\ &= x^2 + 4x + 3\end{aligned}$$

$$\begin{aligned}11 \cdot 13 &= (10 + 1)(10 + 3)\\ &= 10(10 + 3) + 1(10 + 3)\\ &= 10(10) + 10(3) + 1(10) + 1(3)\\ &= 100 + 30 + 10 + 3\\ &= 143\end{aligned}$$

b) For each pair of products, if x is substituted for 10, then the products are equal. The coefficients in the terms of the trinomial are the same as the digits in the product of the whole numbers.

17. a) The constant terms in the binomials have a sum of –17 and a product of 60. Their sum should be –7 and their product should be –60. So, these constant terms should be 5 and –12. The correct solution is:
$m^2 - 7m - 60 = (m + 5)(m - 12)$

b) The constant terms in the binomials have a sum of –12 and a product of –45. Their sum should be –14 and their product should be 45. So, these constant terms should be –5 and –9. The correct solution is:
$w^2 - 14w + 45 = (w - 5)(w - 9)$

c) The constant terms in the binomials have a sum of –9 and a product of –36. Their sum should be 9. So, these constant terms should be –3 and 12. The correct solution is:
$b^2 + 9b - 36 = (b - 3)(b + 12)$

18. a) i)

$$\begin{aligned}(t + 4)(t + 7) &= t(t + 7) + 4(t + 7)\\ &= t^2 + 7t + 4t + 28\\ &= t^2 + 11t + 28\end{aligned}$$

ii)

$$\begin{aligned}(t - 4)(t - 7) &= t(t - 7) - 4(t - 7)\\ &= t^2 - 7t - 4t + 28\\ &= t^2 - 11t + 28\end{aligned}$$

iii)

$$\begin{aligned}(t - 4)(t + 7) &= t(t + 7) - 4(t + 7)\\ &= t^2 + 7t - 4t - 28\\ &= t^2 + 3t - 28\end{aligned}$$

iv) $(t + 4)(t - 7) = t(t - 7) + 4(t - 7)$
$= t^2 - 7t + 4t - 28$
$= t^2 - 3t - 28$

b) i) The constant terms in the trinomials in parts i and ii above are positive because the constant terms in the binomials have the same sign.

ii) The constant terms in the trinomials in parts iii and iv above are negative because the constant terms in the binomials have opposite signs.

iii) The coefficient of the t-term in the trinomial is the sum of the constant terms in the binomials.

19. To be able to factor each trinomial, the coefficient of the middle term must be the sum of the factors of the constant term.

a) $x^2 + \square x + 10$
The factors of 10 are: 1 and 10; –1 and –10; 2 and 5; –2 and –5
The sums of the factors are: $1 + 10 = 11$; $-1 - 10 = -11$; $2 + 5 = 7$; $-2 - 5 = -7$
So, $\square = 11, -11, 7$, or -7

b) $a^2 + \square a - 9$
The factors of –9 are: 1 and –9; –1 and 9; 3 and –3
The sums of the factors are: $1 - 9 = -8$; $-1 + 9 = 8$; $3 - 3 = 0$
So, $\square = -8, 8$, or 0

c) $t^2 + \square t + 8$
The factors of 8 are: 1 and 8; –1 and –8; 2 and 4; –2 and –4
The sums of the factors are: $1 + 8 = 9$; $-1 - 8 = -9$; $2 + 4 = 6$; $-2 - 4 = -6$
So, $\square = 9, -9, 6$, or -6

d) $y^2 + \square y - 12$
The factors of –12 are: 1 and –12; –1 and 12; 2 and –6; –2 and 6; 3 and –4; –3 and 4
The sums of the factors are: $1 - 12 = -11$; $-1 + 12 = 11$; $2 - 6 = -4$; $-2 + 6 = 4$;
$3 - 4 = -1$; $-3 + 4 = 1$
So, $\square = -11, 11, -4, 4, -1$, or 1

e) $h^2 + \square h + 18$
The factors of 18 are: 1 and 18; –1 and –18; 2 and 9; –2 and –9; 3 and 6; –3 and –6
The sums of the factors are: $1 + 18 = 19$; $-1 - 18 = -19$; $2 + 9 = 11$; $-2 - 9 = -11$;
$3 + 6 = 9$; $-3 - 6 = -9$
So, $\square = 19, -19, 11, -11, 9$, or -9

f) $p^2 + \square p - 16$
The factors of –16 are: 1 and –16; –1 and 16; 2 and –8; –2 and 8; 4 and –4
The sums of the factors are: $1 - 16 = -15$; $-1 + 16 = 15$; $2 - 8 = -6$; $-2 + 8 = 6$;
$4 - 4 = 0$
So, $\square = -15, 15, -6, 6$, or 0

20. To be able to factor each trinomial, the constant term must be the product of two numbers whose sum is the coefficient of the middle term.

a) $r^2 + r + \square$
Two numbers whose sum is 1 are: 0 + 1; 2 – 1; 3 – 2; 4 – 3; 5 – 4; and so on
The products of these numbers are: 0(1) = 0; 2(–1) = –2; 3(–2) = –6; 4(–3) = –12; 5(–4) = –20; and so on
So, there are infinitely many values of □.
Some of these values are: 0, –2, –6, –12, and –20

b) $h^2 - h + \square$
Two numbers whose sum is –1 are: 0 – 1; 1 – 2; 2 – 3; 3 – 4; 4 – 5; and so on
The products of these numbers are: 0(–1) = 0; 1(–2) = –2; 2(–3) = –6; 3(–4) = –12; 4(–5) = –20; and so on
So, there are infinitely many values of □.
Some of these values are: 0, –2, –6, –12, and –20

c) $b^2 + 2b + \square$
Two numbers whose sum is 2 are: 0 + 2; 1 + 1; 3 – 1; 4 – 2; 5 – 3; and so on
The products of these numbers are: 0(2) = 0; 1(1) = 1; 3(–1) = –3; 4(–2) = –8; 5(–3) = –15; and so on
So, there are infinitely many values of □.
Some of these values are: 0, 1, –3, –8, and –15

d) $z^2 - 2z + \square$
Two numbers whose sum is –2 are: 0 – 2; –1 – 1; 1 – 3; 2 – 4; 3 – 5; and so on
The products of these numbers are: 0(–2) = 0; (–1)(–1) = 1; 1(–3) = –3; 2(–4) = –8; 3(–5) = –15; and so on
So, there are infinitely many values of □.
Some of these values are: 0, 1, –3, –8, and –15

e) $q^2 + 3q + \square$
Two numbers whose sum is 3 are: 0 + 3; 1 + 2; 4 – 1; 5 – 2; 6 – 3; and so on
The products of these numbers are: 0(3) = 0; 1(2) = 2; 4(–1) = –4; 5(–2) = –10; 6(–3) = –18; and so on
So, there are infinitely many values of □.
Some of these values are: 0, 2, –4, –10, and –18

f) $g^2 - 3g + \square$
Two numbers whose sum is –3 are: 0 – 3; –1 – 2; 1 – 4; 2 – 5; 3 – 6; and so on
The products of these numbers are: 0(–3) = 0; (–1)(–2) = 2; 1(–4) = –4; 2(–5) = –10; 3(–6) = –18; and so on
So, there are infinitely many values of □.
Some of these values are: 0, 2, –4, –10, and –18

21. Remove common factors first.

a) $4y^2 - 20y - 56$
The common factor is 4.
$4y^2 - 20y - 56 = 4(y^2) - 4(5y) - 4(14)$
$= 4(y^2 - 5y - 14)$
Find two numbers with a product of –14 and a sum of –5.
The numbers are –7 and 2.

So, $4y^2 – 20y – 56 = 4(y – 7)(y + 2)$

b) $–3m^2 – 18m – 24$
The common factor is –3.
$–3m^2 – 18m – 24 = –3(m^2) + (–3)(6m) + (–3)(8)$
$= –3(m^2 + 6m + 8)$
Find two numbers with a product of 8 and a sum of 6.
The numbers are 2 and 4.
So, $–3m^2 – 18m – 24 = –3(m + 2)(m + 4)$

c) $4x^2 + 4x – 48$
The common factor is 4.
$4x^2 + 4x – 48 = 4(x^2) + 4(x) – 4(12)$
$= 4(x^2 + x – 12)$
Find two numbers with a product of –12 and a sum of 1.
The numbers are 4 and –3.
So, $4x^2 + 4x – 48 = 4(x + 4)(x – 3)$

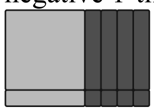
d) $10x^2 + 80x + 120$
The common factor is 10.
$10x^2 + 80x + 120 = 10(x^2) + 10(8x) + 10(12)$
$= 10(x^2 + 8x + 12)$
Find two numbers with a product of 12 and a sum of 8.
The numbers are 2 and 6.
So, $10x^2 + 80x + 120 = 10(x + 2)(x + 6)$

e) $–5n^2 + 40n – 35$
The common factor is –5.
$–5n^2 + 40n – 35 = –5(n^2) + (–5)(–8n) + (–5)(7)$
$= –5(n^2 – 8n + 7)$
Find two numbers with a product of 7 and a sum of –8.
The numbers are –1 and –7.
So, $–5n^2 + 40n – 35 = –5(n – 1)(n – 7)$

f) $7c^2 – 35c + 42$
The common factor is 7.
$7c^2 – 35c + 42 = 7(c^2) – 7(5c) + 7(6)$
$= 7(c^2 – 5c + 6)$
Find two numbers with a product of 6 and a sum of –5.
The numbers are –2 and –3.
So, $7c^2 – 35c + 42 = 7(c – 2)(c – 3)$

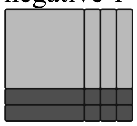
C

22. a) To expand $(r – 4)(r + 1)$ with algebra tiles:
Make a rectangle with length $r – 4$ and width $r + 1$; that is, use 1 r^2-tile, then 4 negative r-tiles for the length and 1 r-tile for the width. Complete the rectangle with 4 negative 1-tiles.



From the diagram, the polynomial is: $r^2 - 4r + 1r - 4 = r^2 - 3r - 4$
So, $(r - 4)(r + 1) = r^2 - 3r - 4$

b) To factor $t^2 + t - 6$ with algebra tiles:
Use 1 t^2-tile. Arrange positive and negative t-tiles on two sides of the t^2-tile, so there is 1 more positive t-tile than negative t-tile, and the rectangle is completed with 6 negative 1-tiles.



From the diagram, the length of the rectangle is $t + 3$, and the width is $t - 2$.
So, $t^2 + t - 6 = (t + 3)(t - 2)$

23. a) For each trinomial, find two numbers whose sum is equal to the coefficient of the middle term and whose product is equal to the constant term. These numbers are the constants in the binomial factors.

i) $h^2 - 10h - 24$
The factors of –24 are: 1 and –24; –1 and 24; 2 and –12; –2 and 12; 3 and –8; –3 and 8; 4 and –6; –4 and 6
Use mental math to calculate each sum.
The two factors with a sum of –10 are 2 and –12.
So, $h^2 - 10h - 24 = (h + 2)(h - 12)$

ii) $h^2 + 10h - 24$
Use the factors from part i.
The two factors with a sum of 10 are –2 and 12.
So, $h^2 + 10h - 24 = (h - 2)(h + 12)$

iii) $h^2 - 10h + 24$
The factors of 24 are: 1 and 24; –1 and –24; 2 and 12; –2 and –12; 3 and 8; –3 and –8; 4 and 6; –4 and –6
Use mental math to calculate each sum.
The two factors with a sum of –10 are –4 and –6.
So, $h^2 - 10h + 24 = (h - 4)(h - 6)$

iv) $h^2 + 10h + 24$
Use the factors from part iii.
The two factors with a sum of 10 are 4 and 6.
So, $h^2 + 10h + 24 = (h + 4)(h + 6)$

b) In part a, there are 4 pairs of factors. The two numbers in two pairs are opposites, and the 4 sums are two opposite numbers. All these numbers must be factors of the same number.

$2 - 12 = -10$	$4 + 6 = 10$
$-4 - 6 = -10$	$-2 + 12 = 10$

Use guess and check to find other numbers.
The factors of 6 are: 1, 2, 3, 6
$2 + 3 = 5$ and $6 - 1 = 5$
So, four trinomials and their factors are:

$(x + 2)(x + 3) = x^2 + 5x + 6$
$(x - 2)(x - 3) = x^2 - 5x + 6$
$(x - 6)(x + 1) = x^2 - 5x - 6$
$(x + 6)(x - 1) = x^2 + 5x - 6$

The factors of 30 are: 1, 2, 3, 5, 6, 10, 15, 30
3 + 10 = 13 and 15 – 2 = 13
So, four trinomials and their factors are:
$(x + 3)(x + 10) = x^2 + 13x + 30$
$(x - 3)(x - 10) = x^2 - 13x + 30$
$(x - 15)(x + 2) = x^2 - 13x - 30$
$(x + 15)(x - 2) = x^2 + 13x - 30$

The factors of 54 are: 1, 2, 3, 6, 9, 18, 27, 54
6 + 9 = 15 and 18 – 3 = 15
So, four trinomials and their factors are:
$(x + 6)(x + 9) = x^2 + 15x + 54$
$(x - 6)(x - 9) = x^2 - 15x + 54$
$(x - 18)(x + 3) = x^2 - 15x - 54$
$(x + 18)(x - 3) = x^2 + 15x - 54$

The factors of 60 are: 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60
5 + 12 = 17 and 20 – 3 = 17
So, four trinomials and their factors are:
$(x + 5)(x + 12) = x^2 + 17x + 60$
$(x - 5)(x - 12) = x^2 - 17x + 60$
$(x - 20)(x + 3) = x^2 - 17x - 60$
$(x + 20)(x - 3) = x^2 + 17x - 60$

There are many other possible trinomials.

Lesson 3.6 Polynomials of the Form $ax^2 + bx + c$ Exercises (pages 177–178)

A

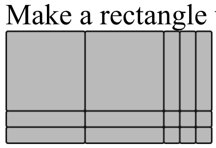
5. a) There are 2 x^2-tiles, 7 x-tiles, and three 1-tiles.
So, the trinomial is: $2x^2 + 7x + 3$
The length of the rectangle is $2x + 1$ and the width is $x + 3$.
So, the multiplication sentence is: $(2x + 1)(x + 3) = 2x^2 + 7x + 3$

b) There are 3 x^2-tiles, 14 x-tiles, and eight 1-tiles.
So, the trinomial is: $3x^2 + 14x + 8$
The length of the rectangle is $3x + 2$ and the width is $x + 4$.
So, the multiplication sentence is: $(3x + 2)(x + 4) = 3x^2 + 14x + 8$

c) There are 6 x^2-tiles, 5 x-tiles, and one 1-tile.
So, the trinomial is: $6x^2 + 5x + 1$
The length of the rectangle is $3x + 1$ and the width is $2x + 1$.
So, the multiplication sentence is: $(3x + 1)(2x + 1) = 6x^2 + 5x + 1$

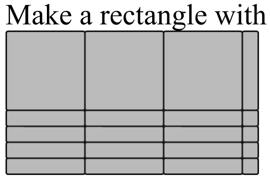
d) There are 12 x^2-tiles, 17 x-tiles, and six 1-tiles.
So, the trinomial is: $12x^2 + 17x + 6$
The length of the rectangle is $4x + 3$ and the width is $3x + 2$.
So, the multiplication sentence is: $(4x + 3)(3x + 2) = 12x^2 + 17x + 6$

6. a) Make a rectangle with length $2v + 3$ and width $v + 2$.



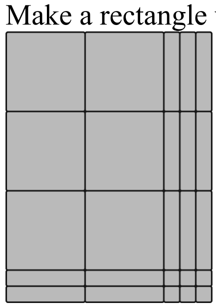
The tiles that form the product are: 2 v^2-tiles, 7 v-tiles, and six 1-tiles
So, $(2v + 3)(v + 2) = 2v^2 + 7v + 6$

b) Make a rectangle with length $3r + 1$ and width $r + 4$.



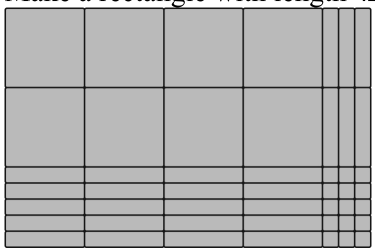
The tiles that form the product are: 3 r^2-tiles, 13 r-tiles, and four 1-tiles
So, $(3r + 1)(r + 4) = 3r^2 + 13r + 4$

c) Make a rectangle with length $2g + 3$ and width $3g + 2$.



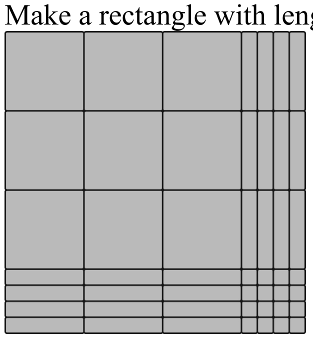
The tiles that form the product are: 6 g^2-tiles, 13 g-tiles, and six 1-tiles
So, $(2g + 3)(3g + 2) = 6g^2 + 13g + 6$

d) Make a rectangle with length $4z + 3$ and width $2z + 5$.



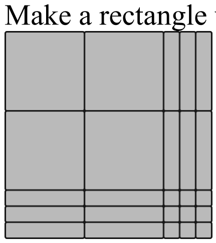
The tiles that form the product are: 8 z^2-tiles, 26 z-tiles, and fifteen 1-tiles
So, $(4z + 3)(2z + 5) = 8z^2 + 26z + 15$

e) Make a rectangle with length $3t + 4$ and width $3t + 4$.



The tiles that form the product are: 9 t^2-tiles, 24 t-tiles, and sixteen 1-tiles
So, $(3t + 4)(3t + 4) = 9t^2 + 24t + 16$

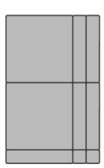
f) Make a rectangle with length $2r + 3$ and width $2r + 3$.



The tiles that form the product are: 4 r^2-tiles, 12 r-tiles, and nine 1-tiles
So, $(2r + 3)(2r + 3) = 4r^2 + 12r + 9$

7. a) i) There are 2 x^2-tiles, 5 x-tiles, and two 1-tiles.
So, the trinomial is: $2x^2 + 5x + 2$

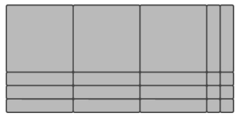
ii)



iii) The length of the rectangle is $2x + 1$ and the width is $x + 2$.
So, the factors of $2x^2 + 5x + 2$ are: $(2x + 1)(x + 2)$

b) i) There are 3 x^2-tiles, 11 x-tiles, and six 1-tiles.
So, the trinomial is: $3x^2 + 11x + 6$

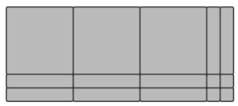
ii)



iii) The length of the rectangle is $3x + 2$ and the width is $x + 3$.
So, the factors of $3x^2 + 11x + 6$ are: $(3x + 2)(x + 3)$

c) i) There are 3 x^2-tiles, 8 x-tiles, and four 1-tiles.
So, the trinomial is: $3x^2 + 8x + 4$

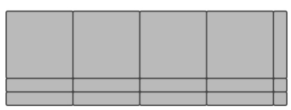
ii)



iii) The length of the rectangle is $3x + 2$ and the width is $x + 2$.
So, the factors of $3x^2 + 8x + 4$ are: $(3x + 2)(x + 2)$

d) i) There are 4 x^2-tiles, 9 x-tiles, and two 1-tiles.
So, the trinomial is: $4x^2 + 9x + 2$

ii)



iii) The length of the rectangle is $4x + 1$ and the width is $x + 2$.
So, the factors of $4x^2 + 9x + 2$ are: $(4x + 1)(x + 2)$

B

8. a) $(2w + 1)(w + 6) = 2w^2 + \square w + 6$
The coefficient of w in the trinomial is: (the coefficient of the 1st term in the 1st binomial)(the constant term in the 2nd binomial) + (the constant term in the 1st binomial)(the coefficient of the 1st term in the 2nd binomial)
That is: $2(6) + 1(1) = 13$
So, $\square = 13$
Then, $(2w + 1)(w + 6) = 2w^2 + 13w + 6$

b) $(2g – 5)(3g – 3) = 6g^2 + \square + \bigcirc$
$\square$ is: (the 1st term in the 1st binomial)(the constant term in the 2nd binomial) + (the constant term in the 1st binomial)(the 1st term in the 2nd binomial)
That is: $2g(–3) + (–5)(3g) = –6g – 15g$
$= –21g$
$\bigcirc$ is the product of the constant terms in the binomials.
That is: $(–5)(–3) = 15$
Then, $(2g – 5)(3g – 3) = 6g^2 – 21g + 15$

c) $(-4v-3)(-2v-7) = \square + \bigcirc + 21$
$\square$ is the product of the 1st terms in the binomials.
That is: $(-4v)(-2v) = 8v^2$
$\bigcirc$ is: (the 1st term in the 1st binomial)(the constant term in the 2nd binomial) + (the constant term in the 1st binomial)(the 1st term in the 2nd binomial)
That is: $(-4v)(-7) + (-3)(-2v) = 28v + 6v$
$= 34v$
Then, $(-4v-3)(-2v-7) = 8v^2 + 34v + 21$

9. Use the distributive property, then collect and combine like terms.

a)
$$\begin{aligned}(5+f)(3+4f) &= 5(3+4f) + f(3+4f)\\ &= 5(3) + 5(4f) + f(3) + f(4f)\\ &= 15 + 20f + 3f + 4f^2\\ &= 15 + 23f + 4f^2\end{aligned}$$

b)
$$\begin{aligned}(3-4t)(5-3t) &= 3(5-3t) - 4t(5-3t)\\ &= 3(5) + 3(-3t) - 4t(5) - 4t(-3t)\\ &= 15 - 9t - 20t + 12t^2\\ &= 15 - 29t + 12t^2\end{aligned}$$

c)
$$\begin{aligned}(10-r)(9+2r) &= 10(9+2r) - r(9+2r)\\ &= 10(9) + 10(2r) - r(9) - r(2r)\\ &= 90 + 20r - 9r - 2r^2\\ &= 90 + 11r - 2r^2\end{aligned}$$

d)
$$\begin{aligned}(-6+2m)(-6+2m) &= (-6)(-6+2m) + 2m(-6+2m)\\ &= (-6)(-6) + (-6)(2m) + 2m(-6) + 2m(2m)\\ &= 36 - 12m - 12m + 4m^2\\ &= 36 - 24m + 4m^2\end{aligned}$$

e)
$$\begin{aligned}(-8-2x)(3-7x) &= (-8)(3-7x) - 2x(3-7x)\\ &= (-8)(3) - 8(-7x) - 2x(3) - 2x(-7x)\\ &= -24 + 56x - 6x + 14x^2\\ &= -24 + 50x + 14x^2\end{aligned}$$

f)
$$\begin{aligned}(6-5n)(-6+5n) &= 6(-6+5n) - 5n(-6+5n)\\ &= -36 + 30n + 30n - 25n^2\\ &= -36 + 60n - 25n^2\end{aligned}$$

10. Use the distributive property, then collect and combine like terms.

a)
$$\begin{aligned}(3c+4)(5+2c) &= 3c(5+2c) + 4(5+2c)\\ &= 3c(5) + 3c(2c) + 4(5) + 4(2c)\\ &= 15c + 6c^2 + 20 + 8c\\ &= 6c^2 + 15c + 8c + 20\\ &= 6c^2 + 23c + 20\end{aligned}$$
Arrange the terms in descending order.

b)
$$\begin{aligned}(1-7t)(3t+5) &= 1(3t+5) - 7t(3t+5)\\ &= 3t + 5 - 21t^2 - 35t\\ &= -21t^2 - 35t + 3t + 5\\ &= -21t^2 - 32t + 5\end{aligned}$$
Arrange the terms in descending order.

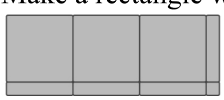
c) $(-4r - 7)(2 - 8r) = (-4r)(2 - 8r) - 7(2 - 8r)$
$= (-4r)(2) - 4r(-8r) - 7(2) - 7(-8r)$
$= -8r + 32r^2 - 14 + 56r$ Arrange the terms in descending order.
$= 32r^2 - 8r + 56r - 14$
$= 32r^2 + 48r - 14$

d) $(-9 - t)(-5t - 1) = (-9)(-5t - 1) - t(-5t - 1)$
$= (-9)(-5t) - 9(-1) - t(-5t) - t(-1)$
$= 45t + 9 + 5t^2 + t$ Arrange the terms in descending order.
$= 5t^2 + 45t + t + 9$
$= 5t^2 + 46t + 9$

e) $(7h + 10)(-3 + 5h) = 7h(-3 + 5h) + 10(-3 + 5h)$
$= 7h(-3) + 7h(5h) + 10(-3) + 10(5h)$
$= -21h + 35h^2 - 30 + 50h$ Arrange the terms in descending order.
$= 35h^2 - 21h + 50h - 30$
$= 35h^2 + 29h - 30$

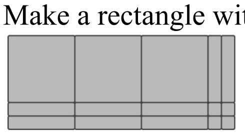
f) $(7 - 6y)(6y - 7) = 7(6y - 7) - 6y(6y - 7)$
$= 7(6y) + 7(-7) - 6y(6y) - 6y(-7)$
$= 42y - 49 - 36y^2 + 42y$ Arrange the terms in descending order.
$= -36y^2 + 42y + 42y - 49$
$= -36y^2 + 84y - 49$

11. a) i) Make a rectangle with 3 t^2-tiles, 4 t-tiles, and one 1-tile.



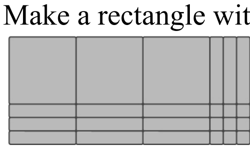
The length of the rectangle is $3t + 1$ and the width is $t + 1$.
So, $3t^2 + 4t + 1 = (3t + 1)(t + 1)$

ii) Make a rectangle with 3 t^2-tiles, 8 t-tiles, and four 1-tiles.



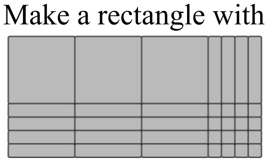
The length of the rectangle is $3t + 2$ and the width is $t + 2$.
So, $3t^2 + 8t + 4 = (3t + 2)(t + 2)$

iii) Make a rectangle with 3 t^2-tiles, 12 t-tiles, and nine 1-tiles.



The length of the rectangle is $3t + 3$ and the width is $t + 3$.
So, $3t^2 + 12t + 9 = (3t + 3)(t + 3)$

iv) Make a rectangle with 3 t^2-tiles, 16 t-tiles, and sixteen 1-tiles.



The length of the rectangle is $3t + 4$ and the width is $t + 4$.
So, $3t^2 + 16t + 16 = (3t + 4)(t + 4)$

b) In the rectangles, each side length increases by 1 each time. In the binomial factors, the constant terms increase by 1 each time.

c) In the trinomials in part a:
- The coefficients of the t^2-terms are 3, so these do not change.
- The coefficients of the t-terms are consecutive multiples of 4.
- The constant terms are consecutive squares.

So, the next 3 trinomials and their factors are:
$3t^2 + 20t + 25 = (3t + 5)(t + 5)$
$3t^2 + 24t + 36 = (3t + 6)(t + 6)$
$3t^2 + 28t + 49 = (3t + 7)(t + 7)$

12. To factor each trinomial, use number sense and reasoning, with mental math.

a) i) $2n^2 + 13n + 6$
The factors of $2n^2$ are: $2n$ and n
The factors of 6 are: 1 and 6; and 2 and 3
Do not consider negative factors of 6 because there are no negative terms in the trinomial.
Arrange the factor combinations vertically. Then form the products and add.

$2n$ 1	$2n$ 6	$2n$ 2	$2n$ 3
n 6	n 1	n 3	n 2
$12n + 1n = 13n$	$2n + 6n = 8n$	$6n + 2n = 8n$	$4n + 3n = 7n$

The first set of factors is correct because the coefficient of n is 13.
So, $2n^2 + 13n + 6 = (2n + 1)(n + 6)$

ii) $2n^2 - 13n + 6$
The factors of 6 in the binomials are numerically the same as those in part i; but both factors are negative because the coefficient of n in the trinomial is negative, and the constant term is positive.
So, $2n^2 - 13n + 6 = (2n - 1)(n - 6)$

b) i) $2n^2 + 11n - 6$
The factors of 6 in the binomials have opposite signs because the constant term in the trinomial is negative.
Use the factors from part a, but amend the signs so they are opposites.
Stop when the sum of the n-terms is $11n$.

$2n$ 1	$2n$ –1
n –6	n 6
$-12n + 1n = -11n$	$12n - 1n = 11n$

So, $2n^2 + 11n - 6 = (2n - 1)(n + 6)$

ii) $2n^2 - 11n - 6$
The factors of 6 in the binomials have opposite signs because the constant term in the trinomial is negative.
The factors that produce the sum of n-terms as $-11n$ are shown in part i.
So, $2n^2 - 11n - 6 = (2n + 1)(n - 6)$

c) i) $2n^2 + 7n + 6$
The factors of 6 in the binomials have the same sign because the constant term in the trinomial is positive. Both factors are positive because the n-term in the trinomial is positive.
The factors that produce the sum of n-terms as $7n$ are shown in part a, i.
So, $2n^2 + 7n + 6 = (2n + 3)(n + 2)$

ii) $2n^2 - 7n + 6$
The factors of 6 in the binomials have the same sign because the constant term in the trinomial is positive. Both factors are negative because the n-term in the trinomial is negative.
The factors that produce the sum of n-terms as $-7n$ are the opposites of those in part i.
So, $2n^2 - 7n + 6 = (2n - 3)(n - 2)$

All 6 trinomials have the same coefficient of n^2, and either 6 or –6 as their constant term. All 6 pairs of binomial factors have $2n$ as the first term in each binomial, and a factor of 6 as the constant term in each binomial.
For each pair of trinomials, their constant terms are equal and the coefficients of their n-terms are opposites. For each related pair of binomial factors, the constant terms in one pair of factors are the opposites of the constant terms in the other pair.

13. To factor each trinomial, use number sense and reasoning, with mental math.

a) $2y^2 + 5y + 2$
The factors of $2y^2$ are: $2y$ and y
The factors of 2 are: 1 and 2
The possible binomial factors are:
$(2y + 2)(y + 1)$ The middle term is: $2y(1) + 2(y) = 4y$
$(2y + 1)(y + 2)$ The middle term is: $2y(2) + 1(y) = 5y$
The second set of factors is correct because the middle term in the trinomial is $5y$.
$2y^2 + 5y + 2 = (2y + 1)(y + 2)$
Check: $(2y + 1)(y + 2) = 2y(y + 2) + 1(y + 2)$
$= 2y^2 + 4y + y + 2$
$= 2y^2 + 5y + 2$

b) $2a^2 + 11a + 12$
The factors of $2a^2$ are: $2a$ and a
The factors of 12 are: 1 and 12; 2 and 6; 3 and 4
Do not consider negative factors of 6 because there are no negative terms in the trinomial.
Arrange the factor combinations vertically. Then form the products and add.
Stop when the sum of the a-terms is $11a$.

$2a$ 1	$2a$ 12	$2a$ 2	$2a$ 6
a 12	a 1	a 6	a 2
$24a + 1a = 25a$	$2a + 12a = 14a$	$12a + 2a = 14a$	$4a + 6a = 10a$

$2a$ 3
a 4
$8a + 3a = 11a$

So, $2a^2 + 11a + 12 = (2a + 3)(a + 4)$
Check: $(2a + 3)(a + 4) = 2a(a + 4) + 3(a + 4)$
$= 2a^2 + 8a + 3a + 12$
$= 2a^2 + 11a + 12$

c) $2k^2 + 13k + 15$
The factors of $2k^2$ are: $2k$ and k
The factors of 15 are: 1 and 15; 3 and 5
Do not consider negative factors of 15 because there are no negative terms in the trinomial.
Arrange the factor combinations vertically. Then form the products and add.
Stop when the sum of the k-terms is $13k$.

$2k$ ╲╱ 1	$2k$ ╲╱ 15	$2k$ ╲╱ 3
k ╱╲ 15	k ╱╲ 1	k ╱╲ 5
$30k + 1k = 31k$	$2k + 15k = 17k$	$10k + 3k = 13k$

So, $2k^2 + 13k + 15 = (2k + 3)(k + 5)$
Check: $(2k + 3)(k + 5) = 2k(k + 5) + 3(k + 5)$
$= 2k^2 + 10k + 3k + 15$
$= 2k^2 + 13k + 15$

d) $2m^2 - 11m + 12$
The factors of $2m^2$ are: $2m$ and m
The factors of 12 are: –1 and –12; –2 and –6; –3 and –4
Consider only negative factors of 12 because the constant term in the trinomial is positive, and the m-term is negative.
Arrange the factor combinations vertically. Then form the products and add.
Stop when the sum of the m-terms is $-11m$.

$2m$ ╲╱ -1	$2m$ ╲╱ -12	$2m$ ╲╱ -2
m ╱╲ -12	m ╱╲ -1	m ╱╲ -6
$-24m - 1m = -25m$	$-2m - 12m = -14m$	$-12m - 2m = -14m$
$2m$ ╲╱ -6	$2m$ ╲╱ -3	
m ╱╲ -2	m ╱╲ -4	
$-4m - 6m = -10m$	$-8m - 3m = -11m$	

So, $2m^2 - 11m + 12 = (2m - 3)(m - 4)$
Check: $(2m - 3)(m - 4) = 2m(m - 4) - 3(m - 4)$
$= 2m^2 - 8m - 3m + 12$
$= 2m^2 - 11m + 12$

e) $2k^2 - 11k + 15$
The factors of $2k^2$ are: $2k$ and k
The factors of 15 are: –1 and –15; –3 and –5
Consider only negative factors of 15 because the constant term in the trinomial is positive, and the k-term is negative.

Arrange the factor combinations vertically. Then form the products and add.
Stop when the sum of the k-terms is $-11k$.

$2k$ -1 k -15 $-30k - 1k = -31k$	$2k$ -15 k -1 $-2k - 15k = -17k$	$2k$ -3 k -5 $-10k - 3k = -13k$

$2k$ -5
k -3
$-6k - 5k = -11k$

So, $2k^2 - 11k + 15 = (2k - 5)(k - 3)$
Check: $(2k - 5)(k - 3) = 2k(k - 3) - 5(k - 3)$
$= 2k^2 - 6k - 5k + 15$
$= 2k^2 - 11k + 15$

f) $2m^2 + 15m + 7$
The factors of $2m^2$ are: $2m$ and m
The factors of 7 are: 1 and 7
The possible binomial factors are:
$(2m + 7)(m + 1)$ The middle term is: $2m(1) + 7(m) = 9m$
$(2m + 1)(m + 7)$ The middle term is: $2m(7) + 1(m) = 15m$
The second set of factors is correct because the middle term in the trinomial is $15m$.
$2m^2 + 15m + 7 = (2m + 1)(m + 7)$
Check: $(2m + 1)(m + 7) = 2m(m + 7) + 1(m + 7)$
$= 2m^2 + 14m + m + 7$
$= 2m^2 + 15m + 7$

g) $2g^2 + 15g + 18$
The factors of $2g^2$ are: $2g$ and g
The factors of 18 are: 1 and 18; 2 and 9; 3 and 6
Do not consider negative factors of 18 because there are no negative terms in the trinomial.
Arrange the factor combinations vertically. Then form the products and add.
Stop when the sum of the g-terms is $15g$.

$2g$ 1 g 18 $36g + 1g = 37g$	$2g$ 18 g 1 $2g + 18g = 20g$	$2g$ 2 g 9 $18g + 2g = 20g$	$2g$ 9 g 2 $4g + 9g = 13g$

$2g$ 3
g 6
$12g + 3g = 15g$

So, $2g^2 + 15g + 18 = (2g + 3)(g + 6)$
Check: $(2g + 3)(g + 6) = 2g(g + 6) + 3(g + 6)$
$= 2g^2 + 12g + 3g + 18$
$= 2g^2 + 15g + 18$

h) $2n^2 + 9n - 18$
The factors of $2n^2$ are: $2n$ and n
The factors of –18 are: 1 and –18; –1 and 18; 2 and –9; –2 and 9; 3 and –6; –3 and 6
Arrange the factor combinations vertically. Then form the products and add.
Stop when the sum of the n-terms is $9n$.

$2n$ 1 / n –18	$2n$ –18 / n 1	$2n$ –1 / n 18
$-36n + 1n = -35n$	$2n - 18n = -16n$	$36n - 1n = 35n$
$2n$ 18 / n –1	$2n$ 2 / n –9	$2n$ –9 / n 2
$-2n + 18n = 16n$	$-18n + 2n = -16n$	$4n - 9n = -5n$

$2n$ –2 / n 9	$2n$ 9 / n –2	$2n$ 3 / n –6	$2n$ –6 / n 3
$18n - 2n = 16n$	$-4n + 9n = 5n$	$-12n + 3n = -9n$	$6n - 6n = 0$

$2n$ –3
n 6
$12n - 3n = 9n$

So, $2n^2 + 9n - 18 = (2n - 3)(n + 6)$
Check: $(2n - 3)(n + 6) = 2n(n + 6) - 3(n + 6)$
$= 2n^2 + 12n - 3n - 18$
$= 2n^2 + 9n - 18$

14. a) i) The factors of 15 are: 1 and 15; 3 and 5
The sums of the factors are: 16 and 8
So, the two integers with a sum of 16 are 1 and 15.

ii) The factors of 24 are: 1 and 24; 2 and 12; 3 and 8; 4 and 6
The sums of the factors are: 25; 14; 11; and 10
So, the two integers with a sum of 14 are 2 and 12.

iii) The factors of 15 are: 1 and 15; 3 and 5
The sums of the factors are: 16 and 8
So, the two integers with a sum of 8 are 3 and 5.

iv) The factors of 12 are: 1 and 12; 2 and 6; 3 and 4
The sums of the factors are: 13; 8; and 7
So, the two integers with a sum of 7 are 3 and 4.

v) The factors of 12 are: 1 and 12; 2 and 6; 3 and 4
The sums of the factors are: 13; 8; and 7
So, the two integers with a sum of 13 are 1 and 12.

vi) The factors of 24 are: 1 and 24; 2 and 12; 3 and 8; 4 and 6

The sums of the factors are: 25; 14; 11; and 10
So, the two integers with a sum of 11 are 3 and 8.

b) For each trinomial, write the middle term as the sum of two terms whose coefficients have a product equal to the product of the coefficient of the 1st term and the constant term. Remove common factors from the 1st two terms and from the last two terms, then remove a binomial as a common factor.

i) $3v^2 + 16v + 5$
The product is: 3(5) = 15
From part a) i), the factors of 15 with a sum of 16 are: 1 and 15
So, $3v^2 + 16v + 5 = 3v^2 + 1v + 15v + 5$
$= v(3v + 1) + 5(3v + 1)$
$= (3v + 1)(v + 5)$

ii) $3m^2 + 14m + 8$
The product is: 3(8) = 24
From part a) ii), the factors of 24 with a sum of 14 are: 2 and 12
So, $3m^2 + 14m + 8 = 3m^2 + 2m + 12m + 8$
$= m(3m + 2) + 4(3m + 2)$
$= (3m + 2)(m + 4)$

iii) $3b^2 + 8b + 5$
The product is: 3(5) = 15
From part a) iii), the factors of 15 with a sum of 8 are: 3 and 5
So, $3b^2 + 8b + 5 = 3b^2 + 3b + 5b + 5$
$= 3b(b + 1) + 5(b + 1)$
$= (b + 1)(3b + 5)$

iv) $4a^2 + 7a + 3$
The product is: 4(3) = 12
From part a) iv), the factors of 12 with a sum of 7 are: 3 and 4
So, $4a^2 + 7a + 3 = 4a^2 + 3a + 4a + 3$
$= a(4a + 3) + 1(4a + 3)$
$= (4a + 3)(a + 1)$

v) $4d^2 + 13d + 3$
The product is: 4(3) = 12
From part a) v), the factors of 12 with a sum of 13 are: 1 and 12
So, $4d^2 + 13d + 3 = 4d^2 + 1d + 12d + 3$
$= d(4d + 1) + 3(4d + 1)$
$= (4d + 1)(d + 3)$

vi) $4v^2 + 11v + 6$
The product is: 4(6) = 24
From part a) vi), the factors of 24 with a sum of 11 are: 3 and 8
So, $4v^2 + 11v + 6 = 4v^2 + 3v + 8v + 6$
$= v(4v + 3) + 2(4v + 3)$
$= (4v + 3)(v + 2)$

15. Factor by decomposition. For each trinomial, write the middle term as the sum of two terms whose coefficients have a product equal to the product of the coefficient of the 1st term and the constant term. Remove common factors from the 1st two terms and from the last two terms, then remove a binomial as a common factor.

a) $5a^2 - 7a - 6$

The product is: 5(–6) = –30

The factors of –30 are: 1 and –30; –1 and 30; 2 and –15; –2 and 15; 3 and –10; –3 and 10; 5 and –6; –5 and 6

The two factors with a sum of –7 are: 3 and –10

So, $5a^2 - 7a - 6 = 5a^2 + 3a - 10a - 6$
$= a(5a + 3) - 2(5a + 3)$
$= (5a + 3)(a - 2)$

Check: $(5a + 3)(a - 2) = 5a(a - 2) + 3(a - 2)$
$= 5a^2 - 10a + 3a - 6$
$= 5a^2 - 7a - 6$

b) $3y^2 - 13y - 10$

The product is: 3(–10) = –30

The factors of –30 are: 1 and –30; –1 and 30; 2 and –15; –2 and 15; 3 and –10; –3 and 10; 5 and –6; –5 and 6

The two factors with a sum of –13 are: 2 and –15

So, $3y^2 - 13y - 10 = 3y^2 + 2y - 15y - 10$
$= y(3y + 2) - 5(3y + 2)$
$= (3y + 2)(y - 5)$

Check: $(3y + 2)(y - 5) = 3y(y - 5) + 2(y - 5)$
$= 3y^2 - 15y + 2y - 10$
$= 3y^2 - 13y - 10$

c) $5s^2 + 19s - 4$

The product is: 5(–4) = –20

The factors of –20 are: 1 and –20; –1 and 20; 2 and –10; –2 and 10; 4 and –5; –4 and 5

The two factors with a sum of 19 are: –1 and 20

So, $5s^2 + 19s - 4 = 5s^2 - 1s + 20s - 4$
$= s(5s - 1) + 4(5s - 1)$
$= (5s - 1)(s + 4)$

Check: $(5s - 1)(s + 4) = 5s(s + 4) - 1(s + 4)$
$= 5s^2 + 20s - 1s - 4$
$= 5s^2 + 19s - 4$

d) $14c^2 - 19c - 3$

The product is: 14(–3) = –42

The factors of –42 are: 1 and –42; –1 and 42; 2 and –21; –2 and 21; 3 and –14; –3 and 14; 6 and –7; –6 and 7

The two factors with a sum of –19 are: 2 and –21

So, $14c^2 - 19c - 3 = 14c^2 + 2c - 21c - 3$
$= 2c(7c + 1) - 3(7c + 1)$
$= (7c + 1)(2c - 3)$

Check: $(7c + 1)(2c - 3) = 7c(2c - 3) + 1(2c - 3)$
$= 14c^2 - 21c + 2c - 3$
$= 14c^2 - 19c - 3$

e) $8a^2 + 18a - 5$

The product is: 8(–5) = –40

The factors of –40 are: 1 and –40; –1 and 40; 2 and –20; –2 and 20; 4 and –10; –4 and 10; 8 and –5; –8 and 5
The two factors with a sum of 18 are: –2 and 20
So, $8a^2 + 18a - 5 = 8a^2 - 2a + 20a - 5$
$= 2a(4a - 1) + 5(4a - 1)$
$= (4a - 1)(2a + 5)$
Check: $(4a - 1)(2a + 5) = 4a(2a + 5) - 1(2a + 5)$
$= 8a^2 + 20a - 2a - 5$
$= 8a^2 + 18a - 5$

f) $8r^2 - 14r + 3$
The product is: 8(3) = 24
Consider only negative factors of 24 because the constant term in the trinomial is positive, and the *r*-term is negative.
The factors of 24 are: –1 and –24; –2 and –12; –3 and –8; –4 and –6
The two factors with a sum of –14 are: –2 and –12
So, $8r^2 - 14r + 3 = 8r^2 - 2r - 12r + 3$
$= 2r(4r - 1) - 3(4r - 1)$
$= (4r - 1)(2r - 3)$
Check: $(4r - 1)(2r - 3) = 4r(2r - 3) - 1(2r - 3)$
$= 8r^2 - 12r - 2r + 3$
$= 8r^2 - 14r + 3$

g) $6d^2 + d - 5$
The product is: 6(–5) = –30
The factors of –30 are: 1 and –30; –1 and 30; 2 and –15; –2 and 15; 3 and –10; –3 and 10; 5 and –6; –5 and 6
The two factors with a sum of 1 are: –5 and 6
So, $6d^2 + d - 5 = 6d^2 - 5d + 6d - 5$
$= d(6d - 5) + 1(6d - 5)$
$= (6d - 5)(d + 1)$
Check: $(6d - 5)(d + 1) = 6d(d + 1) - 5(d + 1)$
$= 6d^2 + 6d - 5d - 5$
$= 6d^2 + d - 5$

h) $15e^2 - 7e - 2$
The product is: 15(–2) = –30
The factors of –30 are: 1 and –30; –1 and 30; 2 and –15; –2 and 15; 3 and –10; –3 and 10; 5 and –6; –5 and 6
The two factors with a sum of –7 are: 3 and –10
So, $15e^2 - 7e - 2 = 15e^2 + 3e - 10e - 2$
$= 3e(5e + 1) - 2(5e + 1)$
$= (5e + 1)(3e - 2)$
Check: $(5e + 1)(3e - 2) = 5e(3e - 2) + 1(3e - 2)$
$= 15e^2 - 10e + 3e - 2$
$= 15e^2 - 7e - 2$

16. a) $6u^2 + 17u - 14 = (2u - 7)(3u + 2)$
Consider how the mistake might have been made.
The incorrect signs could have been used in the binomial factors.
Check to see if the factors are: $(2u + 7)(3u - 2)$

Expand: $(2u + 7)(3u - 2) = 2u(3u - 2) + 7(3u - 2)$
$= 6u^2 - 4u + 21u - 14$
$= 6u^2 + 17u - 14$
This trinomial is the same as the given trinomial.
So, the correct factorization is: $6u^2 + 17u - 14 = (2u + 7)(3u - 2)$

Alternative solution:
Expand the binomial factors.
$(2u - 7)(3u + 2) = 2u(3u + 2) - 7(3u + 2)$
$= 6u^2 + 4u - 21u - 14$
$= 6u^2 - 17u - 14$
Compare this trinomial with the given trinomial.
The sign of $17u$ should be positive.
So, to correct the error, transpose the signs in the binomial factors.
The correct factorization is:
$6u^2 + 17u - 14 = (2u + 7)(3u - 2)$

b) $3k^2 - k - 30 = (3k - 3)(k + 10)$
Consider how the mistake might have been made.
The incorrect signs could have been used in the binomial factors.
Check to see if the factors are: $(3k + 3)(k - 10)$
Expand: $(3k + 3)(k - 10) = 3k(k - 10) + 3(k - 10)$
$= 3k^2 - 30k + 3k - 30$
$= 3k^2 - 27k - 30$
This is not the correct trinomial.
The constant terms in the binomial factors could have been transposed.
Check to see if the factors are: $(3k - 10)(k + 3)$
Expand: $(3k - 10)(k + 3) = 3k(k + 3) - 10(k + 3)$
$= 3k^2 + 9k - 10k - 30$
$= 3k^2 - k - 30$
This is the correct trinomial.
The correct factorization is:
$3k^2 - k - 30 = (3k - 10)(k + 3)$

Alternative solution:
Expand the binomial factors.
$(3k - 3)(k + 10) = 3k(k + 10) - 3(k + 10)$
$= 3k^2 + 30k - 3k - 30$
$= 3k^2 + 27k - 30$
Compare this trinomial with the given trinomial.
The middle term, $27k$, should be $-k$.
Factor $3k^2 - k - 30$ by decomposition.
The product is: $3(-30) = -90$
Since the difference of the factors is -1, find factors that are close in numerical value:
-10 and 9
So, $3k^2 - k - 30 = 3k^2 - 10k + 9k - 30$
$= k(3k - 10) + 3(3k - 10)$
$= (3k - 10)(k + 3)$
The correct factorization is:
$3k^2 - k - 30 = (3k - 10)(k + 3)$

c) $4v^2 - 21v + 20 = (4v - 4)(v + 5)$
Consider how the mistake might have been made.
The constant term in the trinomial is positive and the v-term is negative, so both the constant terms in the binomials must be negative.
Check to see if the factors are: $(4v - 4)(v - 5)$
Expand: $(4v - 4)(v - 5) = 4v(v - 5) - 4(v - 5)$
$= 4v^2 - 20v - 4v + 20$
$= 4v^2 - 24v + 20$
This is not the correct trinomial.
The constant terms in the binomial factors could have been transposed.
Check to see if the factors are: $(4v - 5)(v - 4)$
Expand: $(4v - 5)(v - 4) = 4v(v - 4) - 5(v - 4)$
$= 4v^2 - 16v - 5v + 20$
$= 4v^2 - 21v + 20$
This is the correct trinomial.
The correct factorization is:
$4v^2 - 21v + 20 = (4v - 5)(v - 4)$

Alternative solution:
$4v^2 - 21v + 20 = (4v - 4)(v + 5)$
Expand the binomial factors.
$(4v - 4)(v + 5) = 4v(v + 5) - 4(v + 5)$
$= 4v^2 + 20v - 4v - 20$
$= 4v^2 + 16v - 20$
Compare this trinomial with the given trinomial.
The middle term, $16v$, should be $-21v$ and the constant term, -20, should be 20.
Factor $4v^2 - 21v + 20$ by decomposition.
The product is: $4(20) = 80$
The factors of 80 are: –1 and –80; –2 and –40; –4 and –20; –5 and –16, –8 and –10
Consider only negative factors of 80 because the constant term in the trinomial is positive, and the v-term is negative.
The two factors with a sum of –21 are: –5 and –16
So, $4v^2 - 21v + 20 = 4v^2 - 5v - 16v + 20$
$= v(4v - 5) - 4(4v - 5)$
$= (4v - 5)(v - 4)$
The correct factorization is:
$4v^2 - 21v + 20 = (4v - 5)(v - 4)$

17. In the second line of the solution, when 7 is removed as the common factor of the last 2 terms, the sign in the binomial should be negative because $(-42) \div 7 = -6$
The correct solution is:
$15g^2 + 17g - 42 = 15g^2 - 18g + 35g - 42$
$= 3g(5g - 6) + 7(5g - 6)$
$= (5g - 6)(3g + 7)$

18. Remove any common factors before factoring each trinomial by decomposition. For each trinomial, write the middle term as the sum of two terms whose coefficients have a product equal to the product of the coefficient of the 1st term and the constant term. Remove common factors from the 1st two terms and from the last two terms, then remove a binomial as a common factor.
a) $20r^2 + 70r + 60 = 10(2r^2 + 7r + 6)$

Factor: $2r^2 + 7r + 6$
The product is: 2(6) = 12
The factors of 12 are: 1 and 12; 2 and 6; 3 and 4
The factors with a sum of 7 are 3 and 4.
$2r^2 + 7r + 6 = 2r^2 + 3r + 4r + 6$
$= r(2r + 3) + 2(2r + 3)$
$= (2r + 3)(r + 2)$
So, $20r^2 + 70r + 60 = 10(2r + 3)(r + 2)$

b) $15a^2 - 65a + 20 = 5(3a^2 - 13a + 4)$
Factor: $3a^2 - 13a + 4$
The product is: 3(4) = 12
Consider only the negative factors of 12 because the *a*-term is negative and the constant term is positive.
The negative factors of 12 are: –1 and –12; –2 and –6; –3 and –4
The factors with a sum of –13 are –1 and –12.
$3a^2 - 13a + 4 = 3a^2 - 1a - 12a + 4$
$= a(3a - 1) - 4(3a - 1)$
$= (3a - 1)(a - 4)$
So, $15a^2 - 65a + 20 = 5(3a - 1)(a - 4)$

c) $18h^2 + 15h - 18 = 3(6h^2 + 5h - 6)$
Factor: $6h^2 + 5h - 6$
The product is: 6(–6) = –36
The factors of –36 are: 1 and –36; –1 and 36; 2 and –18; –2 and 18; 3 and –12; –3 and 12; 4 and –9; –4 and 9; 6 and –6
The factors with a sum of 5 are –4 and 9.
$6h^2 + 5h - 6 = 6h^2 - 4h + 9h - 6$
$= 2h(3h - 2) + 3(3h - 2)$
$= (3h - 2)(2h + 3)$
So, $18h^2 + 15h - 18 = 3(3h - 2)(2h + 3)$

d) $24u^2 - 72u + 54 = 6(4u^2 - 12u + 9)$
Factor: $4u^2 - 12u + 9$
The product is: 4(9) = 36
Consider only the negative factors of 36 because the *u*-term is negative and the constant term is positive.
The negative factors of 36 are: –1 and –36; –2 and –18; –3 and –12; –4 and –9; –6 and –6
The factors with a sum of –12 are –6 and –6.
$4u^2 - 12u + 9 = 4u^2 - 6u - 6u + 9$
$= 2u(2u - 3) - 3(2u - 3)$
$= (2u - 3)(2u - 3)$
So, $24u^2 - 72u + 54 = 6(2u - 3)(2u - 3)$

e) $12m^2 - 52m - 40 = 4(3m^2 - 13m - 10)$
Factor: $3m^2 - 13m - 10$
The product is: 3(–10) = –30
The factors of –30 are: 1 and –30; –1 and 30; 2 and –15; –2 and 15; 3 and –10; –3 and 10; 5 and –6; –5 and 6
The factors with a sum of –13 are 2 and –15.
$3m^2 - 13m - 10 = 3m^2 + 2m - 15m - 10$

$= m(3m + 2) - 5(3m + 2)$
$= (3m + 2)(m - 5)$
So, $12m^2 - 52m - 40 = 4(3m + 2)(m - 5)$

f) $24g^2 - 2g - 70 = 2(12g^2 - g - 35)$
Factor: $12g^2 - g - 35$
The product is: $12(-35) = -420$
Since the coefficient of g is –1, start with factors of –420 that are close in numerical value: 20 and –21
$12g^2 - g - 35 = 12g^2 + 20g - 21g - 35$
$= 4g(3g + 5) - 7(3g + 5)$
$= (3g + 5)(4g - 7)$
So, $24g^2 - 2g - 70 = 2(3g + 5)(4g - 7)$

19. Factor by decomposition. For each trinomial, write the middle term as the sum of two terms whose coefficients have a product equal to the product of the coefficient of the 1st term and the constant term. Remove common factors from the 1st two terms and from the last two terms, then remove a binomial as a common factor.

a) $14y^2 - 13y + 3$
The product is: $14(3) = 42$
Consider only the negative factors of 42 because the constant term in the trinomial is positive, and the y-term is negative.
The negative factors of 42 are: –1 and –42; –2 and –21; –3 and –14; –6 and –7
The two factors with a sum of –13 are: –6 and –7
So, $14y^2 - 13y + 3 = 14y^2 - 6y - 7y + 3$
$= 2y(7y - 3) - 1(7y - 3)$
$= (7y - 3)(2y - 1)$

b) $10p^2 - 17p - 6$
The product is: $10(-6) = -60$
The factors of –60 are: 1 and –60; –1 and 60; 2 and –30; –2 and 30; 3 and –20; –3 and 20; 4 and –15; –4 and 15; 5 and –12; –5 and 12; 6 and –10; –6 and 10
The factors with a sum of –17 are 3 and –20.
$10p^2 - 17p - 6 = 10p^2 - 20p + 3p - 6$
$= 10p(p - 2) + 3(p - 2)$
$= (p - 2)(10p + 3)$

c) $10r^2 - 33r - 7$
The product is: $10(-7) = -70$
The factors of –70 are: 1 and –70; –1 and 70; 2 and –35; –2 and 35; 5 and –14; –5 and 14; 7 and –10; –7 and 10
The factors with a sum of –33 are 2 and –35.
$10r^2 - 33r - 7 = 10r^2 + 2r - 35r - 7$
$= 2r(5r + 1) - 7(5r + 1)$
$= (5r + 1)(2r - 7)$

d) $15g^2 - g - 2$
The product is: $15(-2) = -30$
The factors of –30 are: 1 and –30; –1 and 30; 2 and –15; –2 and 15; 3 and –10; –3 and 10; 5 and –6; –5 and 6
The factors with a sum of –1 are 5 and –6.

$15g^2 - g - 2 = 15g^2 + 5g - 6g - 2$
$= 5g(3g + 1) - 2(3g + 1)$
$= (3g + 1)(5g - 2)$

e) $4x^2 + 4x - 15$
The product is: $4(-15) = -60$
The factors of –60 are: 1 and –60; –1 and 60; 2 and –30; –2 and 30; 3 and –20; –3 and 20; 4 and –15; –4 and 15; 5 and –12; –5 and 12; 6 and –10; –6 and 10
The factors with a sum of 4 are –6 and 10.
$4x^2 + 4x - 15 = 4x^2 - 6x + 10x - 15$
$= 2x(2x - 3) + 5(2x - 3)$
$= (2x - 3)(2x + 5)$

f) $9d^2 - 24d + 16$
The product is: $9(16) = 144$
Consider only the negative factors of 144 because the constant term in the trinomial is positive, and the *d*-term is negative.
The negative factors of 144 are: –1 and –144; –2 and –72; –3 and –48; –4 and –36; –6 and –24; –8 and –18; –9 and –16; –12 and –12
The two factors with a sum of –24 are –12 and –12.
So, $9d^2 - 24d + 16 = 9d^2 - 12d - 12d + 16$
$= 3d(3d - 4) - 4(3d - 4)$
$= (3d - 4)(3d - 4)$

g) $9t^2 + 12t + 4$
The product is: $9(4) = 36$
Consider only the positive factors of 36 because both the constant term and the *t*-term in the trinomial are positive.
The positive factors of 36 are: 1 and 36; 2 and 18; 3 and 12; 4 and 9; 6 and 6
The two factors with a sum of 12 are: 6 and 6
So, $9t^2 + 12t + 4 = 9t^2 + 6t + 6t + 4$
$= 3t(3t + 2) + 2(3t + 2)$
$= (3t + 2)(3t + 2)$

h) $40y^2 + y - 6$
The product is: $40(-6) = -240$
Since the coefficient of *y* is 1, start with factors of –240 that are close in numerical value: –15 and 16
$40y^2 + y - 6 = 40y^2 - 15y + 16y - 6$
$= 5y(8y - 3) + 2(8y - 3)$
$= (8y - 3)(5y + 2)$

i) $24c^2 + 26c - 15$
The product is: $24(-15) = -360$
Since the coefficient of *c* is 26, start with factors of –360 that have a difference close to 26: –10 and 36
$24c^2 + 26c - 15 = 24c^2 - 10c + 36c - 15$
$= 2c(12c - 5) + 3(12c - 5)$
$= (12c - 5)(2c + 3)$

j) $8x^2 + 14x - 15$
The product is: $8(-15) = -120$
Since the coefficient of x is 14, start with factors of –120 that have a difference close to 14: –6 and 20
$8x^2 + 14x - 15 = 8x^2 - 6x + 20x - 15$
$= 2x(4x - 3) + 5(4x - 3)$
$= (4x - 3)(2x + 5)$

20. a) $4s^2 + \square s + 3$
The possible values of □ are the sums of the factors of 4(3) = 12
The factors of 12 are: 1 and 12; –1 and –12; 2 and 6; –2 and –6; 3 and 4; –3 and –4
The sums of these factors are: 13; –13; 8; –8; 7; –7
So, □ = 13; –13; 8; –8; 7; –7

b) $4h^2 + \square h + 25$
The possible values of □ are the sums of the factors of 4(25) = 100
The factors of 100 are: 1 and 100; –1 and –100; 2 and 50; –2 and –50; 4 and 25; –4 and –25; 5 and 20; –5 and –20; 10 and 10; –10 and –10
The sums of these factors are: 101; –101; 52; –52; 29; –29; 25; –25; 20; –20
So, □ = 101; –101; 52; –52; 29; –29; 25; –25; 20; –20

c) $6y^2 + \square y - 9$
The possible values of □ are the sums of the factors of 6(–9) = –54
The factors of –54 are: 1 and –54; –1 and 54; 2 and –27; –2 and 27; 3 and –18; –3 and 18; 6 and –9; –6 and 9
The sums of these factors are: –53; 53; –25; 25; –15; 15; –3; 3
So, □ = –53; 53; –25; 25; –15; 15; –3; 3

d) $12t^2 + \square t + 10$
The possible values of □ are the sums of the factors of 12(10) = 120
The factors of 120 are: 1 and 120; –1 and –120; 2 and 60; –2 and –60; 3 and 40; –3 and –40; 4 and 30; –4 and –30; 5 and 24; –5 and –24; 6 and 20; –6 and –20; 8 and 15; –8 and –15; 10 and 12; –10 and –12
The sums of these factors are: 121; –121; 62; –62; 43; –43; 34; –34; 29; –29; 26; –26; 23; –23; 22; –22
So, □ = 121; –121; 62; –62; 43; –43; 34; –34; 29; –29; 26; –26; 23; –23; 22; –22

e) $9z^2 + \square z + 1$
The possible values of □ are the sums of the factors of 9(1) = 9
The factors of 9 are: 1 and 9; –1 and –9; 3 and 3; –3 and –3
The sums of these factors are: 10; –10; 6; –6
So, □ = 10; –10; 6; –6

f) $\square f^2 + 2f + \square$
The possible values of the two □ are numbers whose products have factors with a sum or difference of 2.
Factors with a sum of 2 are: 1 + 1; 3 – 1; 4 – 2; 5 – 3; 6 – 4; and so on
If both □ have the same value; then, □ = 1
If both □ have different values; then these values are: 3 and –1; or 4 and –2; or 5 and –3; or 6 and –4; and so on; there are infinite possible values.

C

21. a) i) $4r^2 - r - 5$

Attempt to factor by decomposition.
The product is: $4(-5) = -20$
Find two factors of –20 that have a sum of –1; these factors are 4 and –5.
So, $4r^2 - r - 5 = 4r^2 + 4r - 5r - 5$
$= 4r(r + 1) - 5(r + 1)$
$= (r + 1)(4r - 5)$

ii) $2t^2 + 10t + 3$

Attempt to factor by decomposition.
The product is: $2(3) = 6$
There are no two factors of 6 that have a sum of 10.
So, $2t^2 + 10t + 3$ does not factor.

iii) $5y^2 + 4y - 2$

Attempt to factor by decomposition.
The product is: $5(-2) = -10$
The factors of –10 are: 1 and –10; –1 and 10; 2 and –5; –2 and 5
There are no two factors of –10 that have a sum of 4.
So, $5y^2 + 4y - 2$ does not factor.

iv) $2w^2 - 5w + 2$

Attempt to factor by decomposition.
The product is: $2(2) = 4$
Find two factors of 4 that have a sum of –5; these factors are –1 and –4.
So, $2w^2 - 5w + 2 = 2w^2 - 1w - 4w + 2$
$= w(2w - 1) - 2(2w - 1)$
$= (2w - 1)(w - 2)$

v) $3h^2 - 8h - 3$

Attempt to factor by decomposition.
The product is: $3(-3) = -9$
Find two factors of –9 that have a sum of –8; these factors are 1 and –9.
So, $3h^2 - 8h - 3 = 3h^2 + 1h - 9h - 3$
$= h(3h + 1) - 3(3h + 1)$
$= (3h + 1)(h - 3)$

vi) $2f^2 - f + 1$

Attempt to factor by decomposition.
The product is: $2(1) = 2$
There are no two factors of 2 that have a sum of –1.
So, $2f^2 - f + 1$ does not factor.

b) From part a) v), $3h^2 - 8h - 3$ can be factored because there are two numbers with a product of –9 and a sum of –8. From part a) vi), $2f^2 - f + 1$ cannot be factored because there are no two numbers with a product of 2 and a sum of –1.

22. Factor using decomposition.

a) i) $3n^2 + 11n + 10$

The product is: $3(10) = 30$

The positive factors of 30 are: 1 and 30; 2 and 15; 3 and 10; 5 and 6

The factors of 30 with a sum of 11 are 5 and 6.

So, $3n^2 + 11n + 10 = 3n^2 + 5n + 6n + 10$
$= n(3n + 5) + 2(3n + 5)$
$= (3n + 5)(n + 2)$

ii) $3n^2 - 11n + 10$

The product is: $3(10) = 30$

Two factors of 30 with a sum of –11 are –5 and –6.

So, $3n^2 - 11n + 10 = 3n^2 - 5n - 6n + 10$
$= n(3n - 5) - 2(3n - 5)$
$= (3n - 5)(n - 2)$

iii) $3n^2 + 13n + 10$

The product is: $3(10) = 30$

From part i, two factors of 30 with a sum of 13 are 3 and 10.

So, $3n^2 + 13n + 10 = 3n^2 + 3n + 10n + 10$
$= 3n(n + 1) + 10(n + 1)$
$= (n + 1)(3n + 10)$

iv) $3n^2 - 13n + 10$

The product is: $3(10) = 30$

Two factors of 30 with a sum of –13 are –3 and –10.

So, $3n^2 - 13n + 10 = 3n^2 - 3n - 10n + 10$
$= 3n(n - 1) - 10(n - 1)$
$= (n - 1)(3n - 10)$

v) $3n^2 + 17n + 10$

The product is: $3(10) = 30$

From part i, two factors of 30 with a sum of 17 are 2 and 15.

So, $3n^2 + 17n + 10 = 3n^2 + 2n + 15n + 10$
$= n(3n + 2) + 5(3n + 2)$
$= (3n + 2)(n + 5)$

vi) $3n^2 - 17n + 10$

The product is: $3(10) = 30$

Two factors of 30 with a sum of –17 are –2 and –15.

So, $3n^2 - 17n + 10 = 3n^2 - 2n - 15n + 10$
$= n(3n - 2) - 5(3n - 2)$
$= (3n - 2)(n - 5)$

b) There are 2 more trinomials that begin with $3n^2$ and end in +10. From the list of factors of 30 in part a) i), these are the products of the binomial factors with constant terms: 1 and 30; and the negative constant terms –1 and –30

The sums of these factors are: 31 and –31

So, the trinomials are: $3n^2 + 31n + 10$ and $3n^2 - 31n + 10$

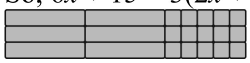
23. For all the trinomials that begin with $9m^2$ and end with +16, find the sums of the pairs of factors of 9(16) = 144. These sums are the coefficients of the *m*-terms in the trinomials.
The factors of 144 are: 1 and 144; –1 and –144; 2 and 72; –2 and –72; 3 and 48; –3 and –48; 4 and 36; –4 and –36; 6 and 24; –6 and –24; 8 and 18; –8 and –18; 9 and 16; –9 and –16; 12 and 12; –12 and –12
The sums of these factors are: 145; –145; 74; –74; 51; –51; 40; –40; 30; –30; 26; –26; 25; 25; 24; –24
So, the trinomials are: $9m^2 + 145m + 16$; $9m^2 - 145m + 16$; $9m^2 + 74m + 16$; $9m^2 - 74m + 16$; $9m^2 + 51m + 16$; $9m^2 - 51m + 16$; $9m^2 + 40m + 16$; $9m^2 - 40m + 16$; $9m^2 + 30m + 16$; $9m^2 - 30m + 16$; $9m^2 + 26m + 16$; $9m^2 - 26m + 16$; $9m^2 + 25m + 16$; $9m^2 - 25m + 16$; $9m^2 + 24m + 16$; $9m^2 - 24m + 16$

Checkpoint 2 **Assess Your Understanding (pages 180–181)**

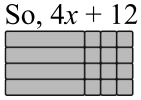
3.3

1. Arrange each set of tiles to form a rectangle.

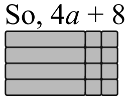
a) There are 6 x-tiles and fifteen 1-tiles.
So, the polynomial is: $6x + 15$
The length of the rectangle is $2x + 5$ and the width is 3; these are the factors.
So, $6x + 15 = 3(2x + 5)$



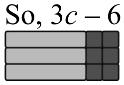
b) There are 4 x-tiles and twelve 1-tiles.
So, the polynomial is: $4x + 12$
The length of the rectangle is $x + 3$ and the width is 4; these are the factors.
So, $4x + 12 = 4(x + 3)$



2. a) i) $4a + 8$
Arrange 4 a-tiles and eight 1-tiles to form a rectangle.
The length of the rectangle is $a + 2$ and the width is 4.
So, $4a + 8 = 4(a + 2)$



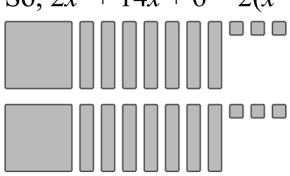
ii) $3c – 6$
Arrange 3 c-tiles and 6 negative 1-tiles to form a rectangle.
The length of the rectangle is $c – 2$ and the width is 3.
So, $3c – 6 = 3(c – 2)$



iii) $–2v^2 – 5v$
The greatest common factor is v.
Write each term as the product of the greatest common factor and another monomial. Then use the distributive property to write the expression as a product.

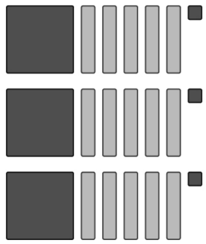
$$
\begin{aligned}
–2v^2 – 5v &= v(–2v) – v(5) \\
&= v(–2v – 5) \qquad \text{Remove } –1 \text{ as a common factor.} \\
&= –v(2v + 5)
\end{aligned}
$$

iv) $2x^2 + 14x + 6$
Arrange 2 x^2-tiles, 14 x-tiles, and six 1-tiles in equal groups.
There are 2 equal groups.
Each group has 1 x^2-tile, 7 x-tiles, and three 1-tiles.
So, $2x^2 + 14x + 6 = 2(x^2 + 7x + 3)$



v) $-3r^2 + 15r - 3$

Arrange 3 negative r^2-tiles, 15 r-tiles, and 3 negative 1-tiles in equal groups.
There are 3 equal groups.
Each group has 1 negative r^2-tile, 5 r-tiles, and 1 negative 1-tile.
So, $-3r^2 + 15r - 3 = 3(-r^2 + 5r - 1)$ Remove –1 as a common factor.
$= -3(r^2 - 5r + 1)$



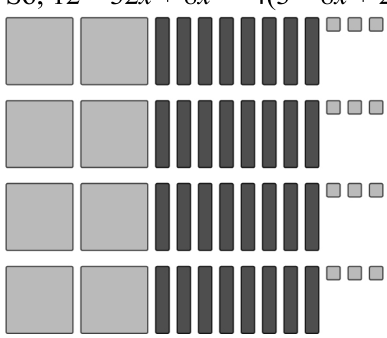
vi) $15a^3 - 3a^2b - 6ab^2$

Factor each term then identify the greatest common factor. Write each term as the product of the greatest common factor and another monomial. Then use the distributive property to write the expression as a product.

$15a^3 = \mathbf{3} \cdot 5 \cdot \boldsymbol{a} \cdot a \cdot a$
$3a^2b = \mathbf{3} \cdot \boldsymbol{a} \cdot a \cdot b$
$6ab^2 = 2 \cdot \mathbf{3} \cdot \boldsymbol{a} \cdot b \cdot b$
The greatest common factor is: $3 \cdot a = 3a$
$15a^3 - 3a^2b - 6ab^2 = 3a(5a^2) - 3a(ab) - 3a(2b^2)$
$= 3a(5a^2 - ab - 2b^2)$

vii) $12 - 32x + 8x^2$

Arrange twelve 1-tiles, 32 negative x-tiles, and 8 x^2-tiles in equal groups.
There are 4 equal groups.
Each group has three 1-tiles, 8 negative x-tiles, and 2 x^2-tiles.
So, $12 - 32x + 8x^2 = 4(3 - 8x + 2x^2)$



viii) $12x^2y - 8xy - 16y$

Factor each term then identify the greatest common factor. Write each term as the product of the greatest common factor and another monomial. Then use the distributive property to write the expression as a product.

$12x^2y = \mathbf{2} \cdot \mathbf{2} \cdot 3 \cdot x \cdot x \cdot \boldsymbol{y}$
$8xy = \mathbf{2} \cdot \mathbf{2} \cdot 2 \cdot x \cdot \boldsymbol{y}$
$16y = \mathbf{2} \cdot \mathbf{2} \cdot 2 \cdot 2 \cdot \boldsymbol{y}$
The greatest common factor is: $2 \cdot 2 \cdot y = 4y$
$12x^2y - 8xy - 16y = 4y(3x^2) + 4y(-2x) + 4y(-4)$

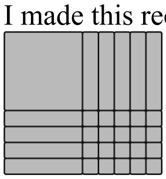
$= 4y(3x^2 - 2x - 4)$

b) iii) For $-2v^2 - 5v$, I could have used algebra tiles to make a rectangle, but both sides of the rectangle appear to be negative, which would suggest both factors are negative. But, from part a) iii), the factors have opposite signs. So, the algebra tiles show the terms in the factors, but I have to check the signs of the factors.

vi), viii) I cannot use algebra tiles when terms have more than one variable.

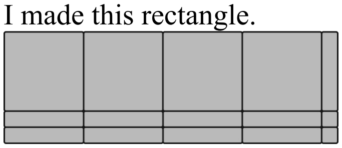
3.4

3. I used 1 x^2-tile, 9 x-tiles, and twenty 1-tiles.
I made this rectangle.



This rectangle represents the multiplication sentence:
$(x + 4)(x + 5) = x^2 + 9x + 20$

4. I used 4 x^2-tiles, 9 x-tiles, and two 1-tiles.
I made this rectangle.

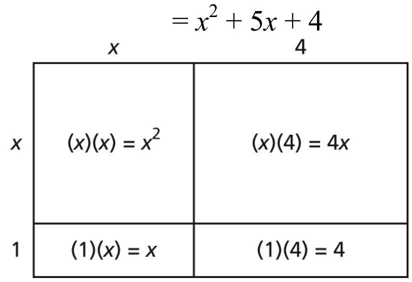


This rectangle represents the multiplication sentence:
$(4x + 1)(x + 2) = 4x^2 + 9x + 2$

3.5

5. Use the distributive property to expand. Draw a rectangle and label its sides with the binomial factors. Divide the rectangle into 4 smaller rectangles, and label each with a term in the expansion.

a) $(x + 1)(x + 4) = x(x + 4) + 1(x + 4)$
$= x^2 + 4x + x + 4$
$= x^2 + 5x + 4$



b) $(d - 2)(d + 3) = d(d + 3) - 2(d + 3)$
$= d^2 + 3d - 2d - 6$

$= d^2 + d - 6$

	d	3
d	$(d)(d) = d^2$	$(d)(3) = 3d$
–2	$(-2)(d) = -2d$	$(-2)(3) = -6$

c) $(x - 4)(x - 2) = x(x - 2) - 4(x - 2)$
$= x^2 - 2x - 4x + 8$
$= x^2 - 6x + 8$

	x	–2
x	$(x)(x) = x^2$	$(x)(-2) = -2x$
–4	$(-4)(x) = -4x$	$(-4)(-2) = 8$

d) $(5 - r)(6 + r) = 5(6 + r) - r(6 + r)$
$= 30 + 5r - 6r - r^2$
$= 30 - r - r^2$

	6	r
5	$(5)(6) = 30$	$(5)(r) = 5r$
$-r$	$(-r)(6) = -6r$	$(-r)(r) = -r^2$

e) $(g + 5)(g - 1) = g(g - 1) + 5(g - 1)$
$= g^2 - g + 5g - 5$
$= g^2 + 4g - 5$

	g	–1
g	$(g)(g) = g^2$	$(g)(-1) = -g$
5	$(5)(g) = 5g$	$(5)(-1) = -5$

f) $(2 - t)(10 - t) = 2(10 - t) - t(10 - t)$
$= 20 - 2t - 10t + t^2$
$= 20 - 12t + t^2$

	10	$-t$
2	$(2)(10) = 20$	$(2)(-t) = -2t$
$-t$	$(-t)(10) = -10t$	$(-t)(-t) = t^2$

6. For each trinomial, find two integers whose sum is equal to the coefficient of the middle term and whose product is equal to the constant term.

a) $s^2 + 11s + 30$
Two numbers whose sum is 11 and whose product is 30 are 5 and 6.
So, $s^2 + 11s + 30 = (s + 5)(s + 6)$

Check: $(s+5)(s+6) = s(s+6) + 5(s+6)$
$= s^2 + 6s + 5s + 30$
$= s^2 + 11s + 30$

b) $n^2 - n - 30$
Two numbers whose sum is –1 and whose product is –30 are 5 and –6.
So, $n^2 - n - 30 = (n+5)(n-6)$
Check: $(n+5)(n-6) = n(n-6) + 5(n-6)$
$= n^2 - 6n + 5n - 30$
$= n^2 - n - 30$

c) $20 - 9b + b^2$
Two numbers whose sum is –9 and whose product is 20 are –4 and –5.
So, $20 - 9b + b^2 = (-4+b)(-5+b)$
This can be written as $(4-b)(5-b)$, or $(b-4)(b-5)$
Check: $(b-4)(b-5) = b(b-5) - 4(b-5)$
$= b^2 - 5b - 4b + 20$
$= b^2 - 9b + 20$

d) $-11 - 10t + t^2$
Rearrange the trinomial and write it as: $t^2 - 10t - 11$
Two numbers whose sum is –10 and whose product is –11 are 1 and –11.
So, $t^2 - 10t - 11 = (t+1)(t-11)$
Check: $(t+1)(t-11) = t(t-11) + 1(t-11)$
$= t^2 - 11t + 1t - 11$
$= t^2 - 10t - 11$

e) $z^2 + 13z + 30$
Two numbers whose sum is 13 and whose product is 30 are 3 and 10.
So, $z^2 + 13z + 30 = (z+3)(z+10)$
Check: $(z+3)(z+10) = z(z+10) + 3(z+10)$
$= z^2 + 10z + 3z + 30$
$= z^2 + 13z + 30$

f) $-k^2 + 9k - 18$
Remove –1 as a common factor.
$-k^2 + 9k - 18 = -(k^2 - 9k + 18)$
Two numbers whose sum is –9 and whose product is 18 are –3 and –6.
So, $-k^2 + 9k - 18 = -(k-3)(k-6)$
Check: $-(k-3)(k-6) = -[k(k-6) - 3(k-6)]$
$= -[k^2 - 6k - 3k + 18]$
$= -[k^2 - 9k + 18]$
$= -k^2 + 9k - 18$

7. Remove a common factor first.

a) $3x^2 + 15x - 42 = 3(x^2 + 5x - 14)$
Two numbers whose sum is 5 and whose product is –14 are –2 and 7.
So, $3x^2 + 15x - 42 = 3(x - 2)(x + 7)$

b) $-2y^2 + 22y - 48 = -2(y^2 - 11y + 24)$
Two numbers whose sum is –11 and whose product is 24 are –3 and –8.
So, $-2y^2 + 22y - 48 = -2(y - 3)(y - 8)$

c) $-24 - 11m - m^2 = -(24 + 11m + m^2)$
Two numbers whose sum is 11 and whose product is 24 are 3 and 8.
So, $-24 - 11m - m^2 = -(3 + m)(8 + m)$

d) $50 - 23y - y^2 = -(-50 + 23y + y^2)$
Two numbers whose sum is 23 and whose product is –50 are –2 and 25.
So, $50 - 23y - y^2 = -(-2 + y)(25 + y)$
This can be written as: $(2 - y)(25 + y)$

3.6

8. Use the distributive property.

a)
$$\begin{aligned}(2c + 1)(c + 3) &= 2c(c + 3) + 1(c + 3)\\ &= 2c^2 + 6c + c + 3\\ &= 2c^2 + 7c + 3\end{aligned}$$

b)
$$\begin{aligned}(-m + 5)(4m - 1) &= (-m)(4m - 1) + 5(4m - 1)\\ &= -4m^2 + m + 20m - 5\\ &= -4m^2 + 21m - 5\end{aligned}$$

c)
$$\begin{aligned}(3f - 4)(3f + 1) &= 3f(3f + 1) - 4(3f + 1)\\ &= 9f^2 + 3f - 12f - 4\\ &= 9f^2 - 9f - 4\end{aligned}$$

d)
$$\begin{aligned}(6z - 1)(2z - 3) &= 6z(2z - 3) - 1(2z - 3)\\ &= 12z^2 - 18z - 2z + 3\\ &= 12z^2 - 20z + 3\end{aligned}$$

e)
$$\begin{aligned}(5 - 3r)(6 + 2r) &= 5(6 + 2r) - 3r(6 + 2r)\\ &= 30 + 10r - 18r - 6r^2\\ &= 30 - 8r - 6r^2\end{aligned}$$

f)
$$\begin{aligned}(-4 - 2h)(-2 - 4h) &= -4(-2 - 4h) - 2h(-2 - 4h)\\ &= 8 + 16h + 4h + 8h^2\\ &= 8 + 20h + 8h^2\end{aligned}$$

9. Factor using decomposition.

a) $2j^2 + 13j + 20$

The product is: 2(20) = 40

Consider only positive factors of 40 because all the terms in the trinomial are positive.

The factors of 40 are: 1 and 40; 2 and 20; 4 and 10; 5 and 8

The factors of 40 with a sum of 13 are 5 and 8.

$$\begin{aligned} \text{So, } 2j^2 + 13j + 20 &= 2j^2 + 5j + 8j + 20 \\ &= j(2j + 5) + 4(2j + 5) \\ &= (2j + 5)(j + 4) \end{aligned}$$

$$\begin{aligned} \text{Check: } (2j + 5)(j + 4) &= 2j(j + 4) + 5(j + 4) \\ &= 2j^2 + 8j + 5j + 20 \\ &= 2j^2 + 13j + 20 \end{aligned}$$

b) $3v^2 + v - 10$

The product is: 3(–10) = –30

The factors of –30 are: 1 and –30; –1 and 30; 2 and –15; –2 and 15; 3 and –10; –3 and 10; 5 and –6; –5 and 6

The factors of –30 with a sum of 1 are –5 and 6.

$$\begin{aligned} \text{So, } 3v^2 + v - 10 &= 3v^2 - 5v + 6v - 10 \\ &= v(3v - 5) + 2(3v - 5) \\ &= (3v - 5)(v + 2) \end{aligned}$$

$$\begin{aligned} \text{Check: } (3v - 5)(v + 2) &= 3v(v + 2) - 5(v + 2) \\ &= 3v^2 + 6v - 5v - 10 \\ &= 3v^2 + v - 10 \end{aligned}$$

c) $5k^2 - 23k + 12$

The product is: 5(12) = 60

Consider only negative factors of 60 because the constant term in the trinomial is positive, and the *k*-term is negative.

The factors of 60 are: –1 and –60; –2 and –30; –3 and –20; –4 and –15; –5 and –12; –6 and –10

The factors of 60 with a sum of –23 are –3 and –20.

$$\begin{aligned} \text{So, } 5k^2 - 23k + 12 &= 5k^2 - 3k - 20k + 12 \\ &= k(5k - 3) - 4(5k - 3) \\ &= (5k - 3)(k - 4) \end{aligned}$$

$$\begin{aligned} \text{Check: } (5k - 3)(k - 4) &= 5k(k - 4) - 3(k - 4) \\ &= 5k^2 - 20k - 3k + 12 \\ &= 5k^2 - 23k + 12 \end{aligned}$$

d) $9h^2 + 18h + 8$

The product is: 9(8) = 72

Consider only positive factors of 72 because all the terms in the trinomial are positive.

The factors of 72 are: 1 and 72; 2 and 36; 3 and 24; 4 and 18; 6 and 12; 8 and 9

The factors of 72 with a sum of 18 are 6 and 12.

$$\begin{aligned} \text{So, } 9h^2 + 18h + 8 &= 9h^2 + 6h + 12h + 8 \\ &= 3h(3h + 2) + 4(3h + 2) \\ &= (3h + 2)(3h + 4) \end{aligned}$$

$$\begin{aligned} \text{Check: } (3h + 2)(3h + 4) &= 3h(3h + 4) + 2(3h + 4) \\ &= 9h^2 + 12h + 6h + 8 \\ &= 9h^2 + 18h + 8 \end{aligned}$$

e) $8y^2 - 2y - 1$
The product is: $8(-1) = -8$
The factors of –8 are: 1 and –8; –1 and 8; 2 and –4; –2 and 4
The factors of –8 with a sum of –2 are 2 and –4.
So, $8y^2 - 2y - 1 = 8y^2 + 2y - 4y - 1$
$= 2y(4y + 1) - 1(4y + 1)$
$= (4y + 1)(2y - 1)$
Check: $(4y + 1)(2y - 1) = 4y(2y - 1) + 1(2y - 1)$
$= 8y^2 - 4y + 2y - 1$
$= 8y^2 - 2y - 1$

f) $6 - 23u + 20u^2$
The product is: $6(20) = 120$
Consider only negative factors of 120 because the constant term in the trinomial is positive, and the *u*-term is negative.
The negative factors of 120 are: –1 and –120; –2 and –60; –3 and –40; –4 and –30; –5 and –24; –6 and –20; –8 and –15; –10 and –12
The factors of 120 with a sum of –23 are –8 and –15.
So, $6 - 23u + 20u^2 = 6 - 8u - 15u + 20u^2$
$= 2(3 - 4u) - 5u(3 - 4u)$
$= (3 - 4u)(2 - 5u)$
Check: $(3 - 4u)(2 - 5u) = 3(2 - 5u) - 4u(2 - 5u)$
$= 6 - 15u - 8u + 20u^2$
$= 6 - 23u + 20u^2$

Lesson 3.7 **Multiplying Polynomials** **Exercises (pages 186–187)**

A

4. a) $(g+1)(g^2+2g+3)$

Use the distributive property. Multiply each term in the trinomial by each term in the binomial. Write the terms in a list.

$(g+1)(g^2+2g+3)$

$= g(g^2+2g+3)+1(g^2+2g+3)$

$= g(g^2)+g(2g)+g(3)+1(g^2)+1(2g)+1(3)$

$= g^3+2g^2+3g+g^2+2g+3$ Collect like terms.

$= g^3+2g^2+g^2+3g+2g+3$ Combine like terms.

$= g^3+3g^2+5g+3$

b) $(2+t+t^2)(1+3t+t^2)$

Use the distributive property. Multiply each term in the 1st trinomial by each term in the 2nd trinomial. Write the terms in a list.

$(2+t+t^2)(1+3t+t^2)$

$= 2(1+3t+t^2)+t(1+3t+t^2)+t^2(1+3t+t^2)$

$= 2(1)+2(3t)+2(t^2)+t(1)+t(3t)+t(t^2)+t^2(1)+t^2(3t)+t^2(t^2)$

$= 2+6t+2t^2+t+3t^2+t^3+t^2+3t^3+t^4$ Collect like terms.

$= 2+6t+t+2t^2+3t^2+t^2+t^3+3t^3+t^4$ Combine like terms.

$= 2+7t+6t^2+4t^3+t^4$

c) $(2w+3)(w^2+4w+7)$

Use the distributive property. Multiply each term in the trinomial by each term in the binomial. Write the terms in a list.

$(2w+3)(w^2+4w+7)$

$= 2w(w^2+4w+7)+3(w^2+4w+7)$

$= 2w(w^2)+2w(4w)+2w(7)+3(w^2)+3(4w)+3(7)$

$= 2w^3+8w^2+14w+3w^2+12w+21$ Collect like terms.

$= 2w^3+8w^2+3w^2+14w+12w+21$ Combine like terms.

$= 2w^3+11w^2+26w+21$

d) $(4+3n+n^2)(3+5n+n^2)$

Use the distributive property. Multiply each term in the 1st trinomial by each term in the 2nd trinomial. Write the terms in a list.

$(4+3n+n^2)(3+5n+n^2)$

$=4(3+5n+n^2)+3n(3+5n+n^2)+n^2(3+5n+n^2)$

$=4(3)+4(5n)+4(n^2)+3n(3)+3n(5n)+3n(n^2)+n^2(3)+n^2(5n)+n^2(n^2)$

$=12+20n+4n^2+9n+15n^2+3n^3+3n^2+5n^3+n^4$ Collect like terms.

$=12+20n+9n+4n^2+15n^2+3n^2+3n^3+5n^3+n^4$ Combine like terms.

$=12+29n+22n^2+8n^3+n^4$

5. a) $(2z+y)(3z+y)$

Use the distributive property. Multiply each term in the 1st binomial by each term in the 2nd binomial. Write the terms in a list.

$(2z+y)(3z+y)$

$=2z(3z+y)+y(3z+y)$

$=2z(3z)+2z(y)+y(3z)+y(y)$

$=6z^2+2yz+3yz+y^2$ Combine like terms.

$=6z^2+5yz+y^2$

b) $(4f-3g)(3f-4g+1)$

Use the distributive property. Multiply each term in the trinomial by each term in the binomial. Write the terms in a list.

$(4f-3g)(3f-4g+1)$

$=4f(3f-4g+1)-3g(3f-4g+1)$

$=4f(3f)+4f(-4g)+4f(1)-3g(3f)-3g(-4g)-3g(1)$

$=12f^2-16fg+4f-9fg+12g^2-3g$ Collect like terms.

$=12f^2+4f-16fg-9fg-3g+12g^2$ Combine like terms.

$=12f^2+4f-25fg-3g+12g^2$

c) $(2a+3b)(4a+5b)$

Use the distributive property. Multiply each term in the 1st binomial by each term in the 2nd binomial. Write the terms in a list.

$(2a+3b)(4a+5b)$

$=2a(4a+5b)+3b(4a+5b)$

$=2a(4a)+2a(5b)+3b(4a)+3b(5b)$

$=8a^2+10ab+12ab+15b^2$ Combine like terms.

$=8a^2+22ab+15b^2$

d) $(3a - 4b + 1)(4a - 5b)$

Use the distributive property. Multiply each term in the binomial by each term in the trinomial. Write the terms in a list.

$(3a - 4b + 1)(4a - 5b)$
$= 3a(4a - 5b) - 4b(4a - 5b) + 1(4a - 5b)$
$= 3a(4a) + 3a(-5b) - 4b(4a) - 4b(-5b) + 1(4a) + 1(-5b)$
$= 12a^2 - 15ab - 16ab + 20b^2 + 4a - 5b$ Collect like terms.
$= 12a^2 + 4a - 15ab - 16ab - 5b + 20b^2$ Combine like terms.
$= 12a^2 + 4a - 31ab - 5b + 20b^2$

e) $(2r + s)^2 = (2r + s)(2r + s)$

Use the distributive property. Multiply each term in the 1st binomial by each term in the 2nd binomial. Write the terms in a list.

$(2r + s)(2r + s)$
$= 2r(2r + s) + s(2r + s)$
$= 2r(2r) + 2r(s) + s(2r) + s(s)$
$= 4r^2 + 2rs + 2rs + s^2$ Combine like terms.
$= 4r^2 + 4rs + s^2$

f) $(3t - 2u)^2 = (3t - 2u)(3t - 2u)$

Use the distributive property. Multiply each term in the 1st binomial by each term in the 2nd binomial. Write the terms in a list.

$(3t - 2u)(3t - 2u)$
$= 3t(3t - 2u) - 2u(3t - 2u)$
$= 3t(3t) + 3t(-2u) - 2u(3t) - 2u(-2u)$
$= 9t^2 - 6tu - 6tu + 4u^2$ Combine like terms.
$= 9t^2 - 12tu + 4u^2$

B

6. a) i) $(2x + y)(2x + y)$
$= 2x(2x + y) + y(2x + y)$
$= 2x(2x) + 2x(y) + y(2x) + y(y)$
$= 4x^2 + 2xy + 2xy + y^2$ Combine like terms.
$= 4x^2 + 4xy + y^2$

ii) $(5r + 2s)(5r + 2s)$

$= 5r(5r + 2s) + 2s(5r + 2s)$

$= 5r(5r) + 5r(2s) + 2s(5r) + 2s(2s)$

$= 25r^2 + 10rs + 10rs + 4s^2$ Combine like terms.

$= 25r^2 + 20rs + 4s^2$

iii) $(6c + 5d)(6c + 5d)$

$= 6c(6c + 5d) + 5d(6c + 5d)$

$= 6c(6c) + 6c(5d) + 5d(6c) + 5d(5d)$

$= 36c^2 + 30cd + 30cd + 25d^2$ Combine like terms.

$= 36c^2 + 60cd + 25d^2$

iv) $(5v + 7w)(5v + 7w)$

$= 5v(5v + 7w) + 7w(5v + 7w)$

$= 5v(5v) + 5v(7w) + 7w(5v) + 7w(7w)$

$= 25v^2 + 35vw + 35vw + 49w^2$ Combine like terms.

$= 25v^2 + 70vw + 49w^2$

v) $(2x - y)(2x - y)$

$= 2x(2x - y) - y(2x - y)$

$= 2x(2x) + 2x(-y) - y(2x) - y(-y)$

$= 4x^2 - 2xy - 2xy + y^2$ Combine like terms.

$= 4x^2 - 4xy + y^2$

vi) $(5r - 2s)(5r - 2s)$

$= 5r(5r - 2s) - 2s(5r - 2s)$

$= 5r(5r) + 5r(-2s) - 2s(5r) - 2s(-2s)$

$= 25r^2 - 10rs - 10rs + 4s^2$ Combine like terms.

$= 25r^2 - 20rs + 4s^2$

vii) $(6c - 5d)(6c - 5d)$

$= 6c(6c - 5d) - 5d(6c - 5d)$

$= 6c(6c) + 6c(-5d) - 5d(6c) - 5d(-5d)$

$= 36c^2 - 30cd - 30cd + 25d^2$ Combine like terms.

$= 36c^2 - 60cd + 25d^2$

viii) $(5v-7w)(5v-7w)$

$= 5v(5v-7w) - 7w(5v-7w)$

$= 5v(5v) + 5v(-7w) - 7w(5v) - 7w(-7w)$

$= 25v^2 - 35vw - 35vw + 49w^2$ Combine like terms.

$= 25v^2 - 70vw + 49w^2$

b) The factors being multiplied are the same, so each time a binomial is being squared.
The first term in the trinomial is the square of the first term in the binomial.
The second term in the trinomial is twice the product of the terms in the binomial.
The third term in the trinomial is the square of the second term in the binomial.

i) $(p+3q)(p+3q)$

$= p(p) + 2(p)(3q) + 3q(3q)$

$= p^2 + 6pq + 9q^2$

ii) $(2s-7t)(2s-7t)$

$= 2s(2s) + 2(2s)(-7t) + (-7t)(-7t)$

$= 4s^2 - 28st + 49t^2$

iii) $(5g+4h)(5g+4h)$

$= 5g(5g) + 2(5g)(4h) + 4h(4h)$

$= 25g^2 + 40gh + 16h^2$

iv) $(10h-7k)(10h-7k)$

$= 10h(10h) + 2(10h)(-7k) + (-7k)(-7k)$

$= 100h^2 - 140hk + 49k^2$

7. a) i) $(x+2y)(x-2y)$

$= x(x-2y) + 2y(x-2y)$

$= x(x) + x(-2y) + 2y(x) + 2y(-2y)$

$= x^2 - 2xy + 2xy - 4y^2$ Combine like terms.

$= x^2 - 4y^2$

ii) $(3r-4s)(3r+4s)$

$=3r(3r+4s)-4s(3r+4s)$

$=3r(3r)+3r(4s)-4s(3r)-4s(4s)$

$=9r^2+12rs-12rs-16s^2$ Combine like terms.

$=9r^2-16s^2$

iii) $(5c+3d)(5c-3d)$

$=5c(5c-3d)+3d(5c-3d)$

$=5c(5c)+5c(-3d)+3d(5c)+3d(-3d)$

$=25c^2-15cd+15cd-9d^2$ Combine like terms.

$=25c^2-9d^2$

iv) $(2v-7w)(2v+7w)$

$=2v(2v+7w)-7w(2v+7w)$

$=2v(2v)+2v(7w)-7w(2v)-7w(7w)$

$=4v^2+14vw-14vw-49w^2$ Combine like terms.

$=4v^2-49w^2$

b) The factors in each pair are the same except the second terms have opposite signs.
The product is the square of the first term in the binomial minus the square of the second term in the binomial.

i) $(11g+5h)(11g-5h)$

$=11g(11g)-5h(5h)$

$=121g^2-25h^2$

ii) $(25m-7n)(25m+7n)$

$=25m(25m)-7n(7n)$

$=625m^2-49n^2$

8. a) $(3y-2)(y^2+y-8)$

$=3y(y^2+y-8)-2(y^2+y-8)$

$=3y(y^2)+3y(y)+3y(-8)-2(y^2)-2(y)-2(-8)$

$=3y^3+3y^2-24y-2y^2-2y+16$ Collect like terms.

$=3y^3+3y^2-2y^2-24y-2y+16$ Combine like terms.

$=3y^3+y^2-26y+16$

b) $(4r+1)(r^2-2r-3)$

$= 4r(r^2-2r-3)+1(r^2-2r-3)$

$= 4r(r^2)+4r(-2r)+4r(-3)+1(r^2)+1(-2r)+1(-3)$

$= 4r^3-8r^2-12r+r^2-2r-3$ Collect like terms.

$= 4r^3-8r^2+r^2-12r-2r-3$ Combine like terms.

$= 4r^3-7r^2-14r-3$

c) $(b^2+9b-2)(2b-1)$

$= b^2(2b-1)+9b(2b-1)-2(2b-1)$

$= b^2(2b)+b^2(-1)+9b(2b)+9b(-1)-2(2b)-2(-1)$

$= 2b^3-b^2+18b^2-9b-4b+2$ Combine like terms.

$= 2b^3+17b^2-13b+2$

d) $(x^2+6x+1)(3x-7)$

$= x^2(3x-7)+6x(3x-7)+1(3x-7)$

$= x^2(3x)+x^2(-7)+6x(3x)+6x(-7)+1(3x)+1(-7)$

$= 3x^3-7x^2+18x^2-42x+3x-7$ Combine like terms.

$= 3x^3+11x^2-39x-7$

9. a) $(x+y)(x+y+3)$

$= x(x+y+3)+y(x+y+3)$

$= x(x)+x(y)+x(3)+y(x)+y(y)+y(3)$

$= x^2+xy+3x+xy+y^2+3y$ Collect like terms.

$= x^2+3x+xy+xy+3y+y^2$ Combine like terms.

$= x^2+3x+2xy+3y+y^2$

b) $(x+2)(x+y+1)$

$= x(x+y+1)+2(x+y+1)$

$= x(x)+x(y)+x(1)+2(x)+2(y)+2(1)$

$= x^2+xy+x+2x+2y+2$ Collect like terms.

$= x^2+2x+x+xy+2y+2$ Combine like terms.

$= x^2+3x+xy+2y+2$

c) $(a+b)(a+b+c)$
$= a(a+b+c)+b(a+b+c)$
$= a(a)+a(b)+a(c)+b(a)+b(b)+b(c)$
$= a^2+ab+ac+ab+b^2+bc$ Collect like terms.
$= a^2+ab+ab+b^2+ac+bc$ Combine like terms.
$= a^2+2ab+b^2+ac+bc$

d) $(3+t)(2+t+s)$
$= 3(2+t+s)+t(2+t+s)$
$= 3(2)+3(t)+3(s)+t(2)+t(t)+t(s)$
$= 6+3t+3s+2t+t^2+st$ Collect like terms.
$= 3s+st+3t+2t+t^2+6$ Combine like terms.
$= 3s+st+5t+t^2+6$

10. a) $(x+2y)(x-2y-1)$
$= x(x-2y-1)+2y(x-2y-1)$
$= x(x)+x(-2y)+x(-1)+2y(x)+2y(-2y)+2y(-1)$
$= x^2-2xy-x+2xy-4y^2-2y$ Collect like terms.
$= x^2-x-2y-2xy+2xy-4y^2$ Combine like terms.
$= x^2-x-2y-4y^2$

b) $(2c-3d)(c+d+1)$
$= 2c(c+d+1)-3d(c+d+1)$
$= 2c(c)+2c(d)+2c(1)-3d(c)-3d(d)-3d(1)$
$= 2c^2+2cd+2c-3cd-3d^2-3d$ Collect like terms.
$= 2c^2+2c+2cd-3cd-3d-3d^2$ Combine like terms.
$= 2c^2+2c-cd-3d-3d^2$

c) $(a-5b)(a+2b-4)$
$= a(a+2b-4)-5b(a+2b-4)$
$= a(a)+a(2b)+a(-4)-5b(a)-5b(2b)-5b(-4)$
$= a^2+2ab-4a-5ab-10b^2+20b$ Collect like terms.
$= a^2-4a+2ab-5ab+20b-10b^2$ Combine like terms.
$= a^2-4a-3ab+20b-10b^2$

d) $(p-2q)(p+4q-r)$
$= p(p+4q-r) - 2q(p+4q-r)$
$= p(p) + p(4q) + p(-r) - 2q(p) - 2q(4q) - 2q(-r)$
$= p^2 + 4pq - pr - 2pq - 8q^2 + 2qr$ Collect like terms.
$= p^2 + 4pq - 2pq - 8q^2 - pr + 2qr$ Combine like terms.
$= p^2 + 2pq - 8q^2 - pr + 2qr$

11. There are 2 errors in the third line of the solution:
· the product of $2r$ and $-5s$ should be $-10rs$, not $-5rs$.
· the product of $-3s$ and $-5s$ should be $+15s^2$, not $-15s^2$.
There is an error in the fourth line of the solution:
· $-15s^2$ and $-18s$ were added to get $-33s^2$. These two terms cannot be added because they are not like terms.
A correct solution is:
$(2r-3s)(r-5s+6)$
$= 2r(r-5s+6) - 3s(r-5s+6)$
$= 2r(r) + 2r(-5s) + 2r(6) - 3s(r) - 3s(-5s) - 3s(6)$
$= 2r^2 - 10rs + 12r - 3rs + 15s^2 - 18s$ Collect like terms.
$= 2r^2 - 10rs - 3rs + 12r + 15s^2 - 18s$ Combine like terms.
$= 2r^2 - 13rs + 12r + 15s^2 - 18s$

12. The formula for the volume of a right rectangular prism is:
$V = Ah$, where A represents the area of the base and h represents the height.
Substitute: $A = x^2 + 3x + 2$ and $h = x + 7$ into the formula, then simplify.
$V = Ah$
$= (x^2 + 3x + 2)(x + 7)$
$= x^2(x+7) + 3x(x+7) + 2(x+7)$
$= x^2(x) + x^2(7) + 3x(x) + 3x(7) + 2(x) + 2(7)$
$= x^3 + 7x^2 + 3x^2 + 21x + 2x + 14$ Combine like terms.
$= x^3 + 10x^2 + 23x + 14$
An expression for the volume of the prism is: $x^3 + 10x^2 + 23x + 14$

13. a) $(r^2 + 3r + 2)(4r^2 + r + 1)$
$= r^2(4r^2 + r + 1) + 3r(4r^2 + r + 1) + 2(4r^2 + r + 1)$
$= r^2(4r^2) + r^2(r) + r^2(1) + 3r(4r^2) + 3r(r) + 3r(1) + 2(4r^2) + 2(r) + 2(1)$
$= 4r^4 + r^3 + r^2 + 12r^3 + 3r^2 + 3r + 8r^2 + 2r + 2$ Collect like terms.
$= 4r^4 + r^3 + 12r^3 + r^2 + 3r^2 + 8r^2 + 3r + 2r + 2$ Combine like terms.
$= 4r^4 + 13r^3 + 12r^2 + 5r + 2$

To check, substitute $r = 2$ into the trinomial product and its simplification.

$\left(r^2 + 3r + 2\right)\left(4r^2 + r + 1\right) = 4r^4 + 13r^3 + 12r^2 + 5r + 2$

Left side: $\left(r^2 + 3r + 2\right)\left(4r^2 + r + 1\right) = \left[(2)^2 + 3(2) + 2\right]\left[4(2)^2 + 2 + 1\right]$

$= (4 + 6 + 2)(16 + 3)$

$= (12)(19)$

$= 228$

Right side: $4r^4 + 13r^3 + 12r^2 + 5r + 2 = 4(2)^4 + 13(2)^3 + 12(2)^2 + 5(2) + 2$

$= 64 + 104 + 48 + 10 + 2$

$= 228$

Since the left side equals the right side, the product is likely correct.

b) $\left(2d^2 + 2d + 1\right)\left(d^2 + 6d + 3\right)$

$= 2d^2\left(d^2 + 6d + 3\right) + 2d\left(d^2 + 6d + 3\right) + 1\left(d^2 + 6d + 3\right)$

$= 2d^2\left(d^2\right) + 2d^2(6d) + 2d^2(3) + 2d\left(d^2\right) + 2d(6d) + 2d(3) + 1\left(d^2\right) + 1(6d) + 1(3)$

$= 2d^4 + 12d^3 + 6d^2 + 2d^3 + 12d^2 + 6d + d^2 + 6d + 3$ Collect like terms.

$= 2d^4 + 12d^3 + 2d^3 + 6d^2 + 12d^2 + d^2 + 6d + 6d + 3$ Combine like terms.

$= 2d^4 + 14d^3 + 19d^2 + 12d + 3$

To check, substitute $d = 2$ into the trinomial product and its simplification.

$\left(2d^2 + 2d + 1\right)\left(d^2 + 6d + 3\right) = 2d^4 + 14d^3 + 19d^2 + 12d + 3$

Left side: $\left(2d^2 + 2d + 1\right)\left(d^2 + 6d + 3\right) = \left[2(2)^2 + 2(2) + 1\right]\left[(2)^2 + 6(2) + 3\right]$

$= (8 + 4 + 1)(4 + 12 + 3)$

$= (13)(19)$

$= 247$

Right side: $2d^4 + 14d^3 + 19d^2 + 12d + 3 = 2(2)^4 + 14(2)^3 + 19(2)^2 + 12(2) + 3$

$= 32 + 112 + 76 + 24 + 3$

$= 247$

Since the left side equals the right side, the product is likely correct.

c) $\left(4c^2 - 2c - 3\right)\left(-c^2 + 6c + 2\right)$

$= 4c^2\left(-c^2 + 6c + 2\right) - 2c\left(-c^2 + 6c + 2\right) - 3\left(-c^2 + 6c + 2\right)$

$= 4c^2\left(-c^2\right) + 4c^2(6c) + 4c^2(2) - 2c\left(-c^2\right) - 2c(6c) - 2c(2) - 3\left(-c^2\right) - 3(6c) - 3(2)$

$= -4c^4 + 24c^3 + 8c^2 + 2c^3 - 12c^2 - 4c + 3c^2 - 18c - 6$ Collect like terms.

$= -4c^4 + 24c^3 + 2c^3 + 8c^2 - 12c^2 + 3c^2 - 4c - 18c - 6$ Combine like terms.

$= -4c^4 + 26c^3 - c^2 - 22c - 6$

To check, substitute $c = 2$ into the trinomial product and its simplification.

$(4c^2 - 2c - 3)(-c^2 + 6c + 2) = -4c^4 + 26c^3 - c^2 - 22c - 6$

Left side: $(4c^2 - 2c - 3)(-c^2 + 6c + 2) = [4(2)^2 - 2(2) - 3][-(2)^2 + 6(2) + 2]$

$= (16 - 4 - 3)(-4 + 12 + 2)$

$= (9)(10)$

$= 90$

Right side: $-4c^4 + 26c^3 - c^2 - 22c - 6 = -4(2)^4 + 26(2)^3 - (2)^2 - 22(2) - 6$

$= -64 + 208 - 4 - 44 - 6$

$= 90$

Since the left side equals the right side, the product is likely correct.

d)

$(-4n^2 - n + 3)(-2n^2 + 5n - 1)$

$= -4n^2(-2n^2 + 5n - 1) - n(-2n^2 + 5n - 1) + 3(-2n^2 + 5n - 1)$

$= -4n^2(-2n^2) - 4n^2(5n) - 4n^2(-1) - n(-2n^2) - n(5n) - n(-1) + 3(-2n^2) + 3(5n) + 3(-1)$

$= 8n^4 - 20n^3 + 4n^2 + 2n^3 - 5n^2 + n - 6n^2 + 15n - 3$ Collect like terms.

$= 8n^4 - 20n^3 + 2n^3 + 4n^2 - 5n^2 - 6n^2 + n + 15n - 3$ Combine like terms.

$= 8n^4 - 18n^3 - 7n^2 + 16n - 3$

To check, substitute $n = 2$ into the trinomial product and its simplification.

$(-4n^2 - n + 3)(-2n^2 + 5n - 1) = 8n^4 - 18n^3 - 7n^2 + 16n - 3$

Left side: $(-4n^2 - n + 3)(-2n^2 + 5n - 1) = [-4(2)^2 - 2 + 3][-2(2)^2 + 5(2) - 1]$

$= (-16 + 1)(-8 + 10 - 1)$

$= (-15)(1)$

$= -15$

Right side: $8n^4 - 18n^3 - 7n^2 + 16n - 3 = 8(2)^4 - 18(2)^3 - 7(2)^2 + 16(2) - 3$

$= 128 - 144 - 28 + 32 - 3$

$= -15$

Since the left side equals the right side, the product is likely correct.

14. There are 3 errors in the second line of the solution:

- the product of $4g$ and $-g$ should be $-4g^2$, not $+4g^2$.
- the product of $4g$ and 4 should be $16g$, not $8g$.
- the product of –2 and 4 should be –8, not +8.

There is an error in the third line of the solution:

- the sum of $-3g^3$ and $-4g^3$ should be $-7g^3$, not $5g^3$.
- the sum of $12g^2$, $4g^2$, and $2g^2$ should be $18g^2$, not $6g^2$.

A correct solution is:

$$(3g^2+4g-2)(-g^2-g+4)$$
$$=3g^2(-g^2-g+4)+4g(-g^2-g+4)-2(-g^2-g+4)$$
$$=3g^2(-g^2)+3g^2(-g)+3g^2(4)+4g(-g^2)+4g(-g)+4g(4)-2(-g^2)-2(-g)-2(4)$$
$$=-3g^4-3g^3+12g^2-4g^3-4g^2+16g+2g^2+2g-8 \quad \text{Collect like terms.}$$
$$=-3g^4-3g^3-4g^3+12g^2-4g^2+2g^2+16g+2g-8 \quad \text{Combine like terms.}$$
$$=-3g^4-7g^3+10g^2+18g-8$$

15. a) $(3s+5)(2s+2)+(3s+7)(s+6)$
$$=3s(2s+2)+5(2s+2)+3s(s+6)+7(s+6)$$
$$=3s(2s)+3s(2)+5(2s)+5(2)+3s(s)+3s(6)+7(s)+7(6)$$
$$=6s^2+6s+10s+10+3s^2+18s+7s+42 \quad \text{Collect like terms.}$$
$$=6s^2+3s^2+6s+10s+18s+7s+10+42 \quad \text{Combine like terms.}$$
$$=9s^2+41s+52$$

b) $(2x+3)(5x+4)+(x-4)(3x-7)$
$$=2x(5x+4)+3(5x+4)+x(3x-7)-4(3x-7)$$
$$=2x(5x)+2x(4)+3(5x)+3(4)+x(3x)+x(-7)-4(3x)-4(-7)$$
$$=10x^2+8x+15x+12+3x^2-7x-12x+28 \quad \text{Collect like terms.}$$
$$=10x^2+3x^2+8x+15x-7x-12x+12+28 \quad \text{Combine like terms.}$$
$$=13x^2+4x+40$$

c)

$$(3m+4)(m-4n)+(5m-2)(3m-6n)$$
$$=3m(m-4n)+4(m-4n)+5m(3m-6n)-2(3m-6n)$$
$$=3m(m)+3m(-4n)+4(m)+4(-4n)+5m(3m)+5m(-6n)-2(3m)-2(-6n)$$
$$=3m^2-12mn+4m-16n+15m^2-30mn-6m+12n \quad \text{Collect like terms.}$$
$$=3m^2+15m^2+4m-6m-12mn-30mn-16n+12n \quad \text{Combine like terms.}$$
$$=18m^2-2m-42mn-4n$$

d)

$(4y-5)(3y+2)-(3y+2)(4y-5)$

$=4y(3y+2)-5(3y+2)-3y(4y-5)-2(4y-5)$

$=4y(3y)+4y(2)-5(3y)-5(2)-3y(4y)-3y(-5)-2(4y)-2(-5)$

$=12y^2+8y-15y-10-12y^2+15y-8y+10$ Collect like terms.

$=12y^2-12y^2+8y-15y+15y-8y-10+10$ Combine like terms.

$=0$

e)

$(3x-2)^2-(2x+6)(3x-1)$

$=(3x-2)(3x-2)-(2x+6)(3x-1)$

$=3x(3x-2)-2(3x-2)-2x(3x-1)-6(3x-1)$

$=3x(3x)+3x(-2)-2(3x)-2(-2)-2x(3x)-2x(-1)-6(3x)-6(-1)$

$=9x^2-6x-6x+4-6x^2+2x-18x+6$ Collect like terms.

$=9x^2-6x^2-6x-6x+2x-18x+4+6$ Combine like terms.

$=3x^2-28x+10$

f) $(2a+1)(4a-3)-(a-2)^2$

$=(2a+1)(4a-3)-(a-2)(a-2)$

$=2a(4a-3)+1(4a-3)-a(a-2)-(-2)(a-2)$

$=2a(4a)+2a(-3)+4a-3-a(a)-a(-2)-(-2)(a)-(-2)(-2)$

$=8a^2-6a+4a-3-a^2+2a+2a-4$ Collect like terms.

$=8a^2-a^2-6a+4a+2a+2a-3-4$ Combine like terms.

$=7a^2+2a-7$

16. a) The cardboard has length 20 cm.
A length of x cm is cut from each of its ends.
So, a polynomial that represents the length of the box in centimetres is:
$20-x-x$, or $20-2x$

b) The cardboard has width 10 cm.
A length of x cm is cut from each of its ends.
So, a polynomial that represents the width of the box in centimetres is:
$10-x-x$, or $10-2x$

c) The base of the box is a rectangle.
The formula for the area of a rectangle is: $A = lw$, where l is the length of the rectangle and w is its width.
Use the formula $A = lw$. Substitute: $l = 20 - 2x$ and $w = 10 - 2x$
A polynomial that represents the area of the base of the box is:
$A = (20 - 2x)(10 - 2x)$
Expand and simplify.

$$\begin{aligned} A &= (20 - 2x)(10 - 2x) \\ &= 20(10 - 2x) - 2x(10 - 2x) \\ &= 20(10) + 20(-2x) - 2x(10) - 2x(-2x) \\ &= 200 - 40x - 20x + 4x^2 && \text{Combine like terms.} \\ &= 200 - 60x + 4x^2 && \text{Arrange in descending order.} \\ &= 4x^2 - 60x + 200 \end{aligned}$$

So, the area of the base of the box in square centimetres is: $4x^2 - 60x + 200$

d) The formula for the volume of a rectangular prism is: $V = Ah$, where A is the area of the base and h is its height.
Each side that is folded up has height x.
Use the formula $V = Ah$. Substitute: $A = 4x^2 - 60x + 200$ and $h = x$
A polynomial that represents the volume of the box is:
$V = (4x^2 - 60x + 200)x$
Expand and simplify.

$$\begin{aligned} V &= (4x^2 - 60x + 200)x \\ &= 4x^2(x) - 60x(x) + 200(x) \\ &= 4x^3 - 60x^2 + 200x \end{aligned}$$

So, the volume of the box in cubic centimetres is: $4x^3 - 60x^2 + 200x$

17. a) The area of the shaded region is:
area of large rectangle – area of small rectangle
The formula for the area of a rectangle is: $A = lw$, where l is the length of the rectangle and w is its width.

Area of large rectangle:
Use the formula $A = lw$. Substitute: $l = 5x + 8$ and $w = 6x + 2$
$A_L = (5x + 8)(6x + 2)$

Area of small rectangle:
Use the formula $A = lw$. Substitute: $l = x + 5$ and $w = 3x$
$A_S = (x + 5)(3x)$

So, area of the shaded region is:

$$\begin{aligned}A &= A_L - A_S\\ &= (5x+8)(6x+2) - (x+5)(3x)\\ &= 5x(6x+2) + 8(6x+2) - x(3x) - 5(3x)\\ &= 5x(6x) + 5x(2) + 8(6x) + 8(2) - 3x^2 - 15x\\ &= 30x^2 + 10x + 48x + 16 - 3x^2 - 15x \quad \text{Collect like terms.}\\ &= 30x^2 - 3x^2 + 10x + 48x - 15x + 16 \quad \text{Combine like terms.}\\ &= 27x^2 + 43x + 16\end{aligned}$$

A polynomial that represents the area of the shaded region is:

$27x^2 + 43x + 16$

b) The area of the shaded region is:
area of large rectangle – area of small rectangle
The formula for the area of a rectangle is: $A = lw$, where l is the length of the rectangle and w is its width.

Area of large rectangle:
Use the formula $A = lw$. Substitute: $l = 2x - 2$ and $w = x + 1$

$A_L = (2x-2)(x+1)$

Area of small rectangle:
Use the formula $A = lw$. Substitute: $l = x$ and $w = x - 2$

$A_S = x(x-2)$

So, area of the shaded region is:

$$\begin{aligned}A &= A_L - A_S\\ &= (2x-2)(x+1) - x(x-2)\\ &= 2x(x+1) - 2(x+1) - x(x) - x(-2)\\ &= 2x(x) + 2x(1) - 2(x) - 2(1) - x^2 + 2x\\ &= 2x^2 + 2x - 2x - 2 - x^2 + 2x \quad \text{Collect like terms.}\\ &= 2x^2 - x^2 + 2x - 2x + 2x - 2 \quad \text{Combine like terms.}\\ &= x^2 + 2x - 2\end{aligned}$$

A polynomial that represents the area of the shaded region is:

$x^2 + 2x - 2$

C

18. a)

$$\begin{aligned}(x-2)^3 &= (x-2)(x-2)(x-2)\\ &= [(x-2)(x-2)](x-2)\\ &= [x(x-2)-2(x-2)](x-2)\\ &= [x(x)+x(-2)-2(x)-2(-2)](x-2)\\ &= (x^2-2x-2x+4)(x-2)\\ &= (x^2-4x+4)(x-2)\\ &= x^2(x-2)-4x(x-2)+4(x-2)\\ &= x^2(x)+x^2(-2)-4x(x)-4x(-2)+4(x)+4(-2)\\ &= x^3-2x^2-4x^2+8x+4x-8\\ &= x^3-6x^2+12x-8\end{aligned}$$

b)

$$\begin{aligned}(2y+5)^3 &= (2y+5)(2y+5)(2y+5)\\ &= [(2y+5)(2y+5)](2y+5)\\ &= [2y(2y+5)+5(2y+5)](2y+5)\\ &= [2y(2y)+2y(5)+5(2y)+5(5)](2y+5)\\ &= (4y^2+10y+10y+25)(2y+5)\\ &= (4y^2+20y+25)(2y+5)\\ &= 4y^2(2y+5)+20y(2y+5)+25(2y+5)\\ &= 4y^2(2y)+4y^2(5)+20y(2y)+20y(5)+25(2y)+25(5)\\ &= 8y^3+20y^2+40y^2+100y+50y+125\\ &= 8y^3+60y^2+150y+125\end{aligned}$$

c)

$$\begin{aligned}(4a-3b)^3 &= (4a-3b)(4a-3b)(4a-3b)\\
&= \left[(4a-3b)(4a-3b)\right](4a-3b)\\
&= \left[4a(4a-3b)-3b(4a-3b)\right](4a-3b)\\
&= \left[4a(4a)+4a(-3b)-3b(4a)-3b(-3b)\right](4a-3b)\\
&= \left(16a^2-12ab-12ab+9b^2\right)(4a-3b)\\
&= \left(16a^2-24ab+9b^2\right)(4a-3b)\\
&= 16a^2(4a-3b)-24ab(4a-3b)+9b^2(4a-3b)\\
&= 16a^2(4a)+16a^2(-3b)-24ab(4a)-24ab(-3b)+9b^2(4a)+9b^2(-3b)\\
&= 64a^3-48a^2b-96a^2b+72ab^2+36ab^2-27b^3\\
&= 64a^3-144a^2b+108ab^2-27b^3\end{aligned}$$

d)

$$\begin{aligned}(c+d)^3 &= (c+d)(c+d)(c+d)\\
&= \left[(c+d)(c+d)\right](c+d)\\
&= \left[c(c+d)+d(c+d)\right](c+d)\\
&= \left[c(c)+c(d)+d(c)+d(d)\right](c+d)\\
&= \left(c^2+cd+cd+d^2\right)(c+d)\\
&= \left(c^2+2cd+d^2\right)(c+d)\\
&= c^2(c+d)+2cd(c+d)+d^2(c+d)\\
&= c^2(c)+c^2(d)+2cd(c)+2cd(d)+d^2(c)+d^2(d)\\
&= c^3+c^2d+2c^2d+2cd^2+cd^2+d^3\\
&= c^3+3c^2d+3cd^2+d^3\end{aligned}$$

19. a)

$$\begin{aligned}&2a(2a-1)(3a+2)\\
&= 2a\left[2a(3a+2)-1(3a+2)\right]\\
&= 2a\left[2a(3a)+2a(2)-1(3a)-1(2)\right]\\
&= 2a\left(6a^2+4a-3a-2\right)\\
&= 2a\left(6a^2+a-2\right)\\
&= 2a\left(6a^2\right)+2a(a)+2a(-2)\\
&= 12a^3+2a^2-4a\end{aligned}$$

b)

$$\begin{aligned}
&-3r(r-1)(2r+1)\\
&=-3r[r(2r+1)-1(2r+1)]\\
&=-3r[r(2r)+r(1)-1(2r)-1(1)]\\
&=-3r(2r^2+r-2r-1)\\
&=-3r(2r^2-r-1)\\
&=-3r(2r^2)-3r(-r)-3r(-1)\\
&=-6r^3+3r^2+3r
\end{aligned}$$

c)

$$\begin{aligned}
&5x^2(2x-1)(4x-3)\\
&=5x^2[2x(4x-3)-1(4x-3)]\\
&=5x^2[2x(4x)+2x(-3)-1(4x)-1(-3)]\\
&=5x^2(8x^2-6x-4x+3)\\
&=5x^2(8x^2-10x+3)\\
&=5x^2(8x^2)+5x^2(-10x)+5x^2(3)\\
&=40x^4-50x^3+15x^2
\end{aligned}$$

d)

$$\begin{aligned}
&-xy(2x+5)(4x-5)\\
&=-xy[2x(4x-5)+5(4x-5)]\\
&=-xy[2x(4x)+2x(-5)+5(4x)+5(-5)]\\
&=-xy(8x^2-10x+20x-25)\\
&=-xy(8x^2+10x-25)\\
&=-xy(8x^2)-xy(10x)-xy(-25)\\
&=-8x^3y-10x^2y+25xy
\end{aligned}$$

e)

$$
\begin{aligned}
&2b(2b-c)(b+c)\\
&=2b[2b(b+c)-c(b+c)]\\
&=2b[2b(b)+2b(c)-c(b)-c(c)]\\
&=2b(2b^2+2bc-bc-c^2)\\
&=2b(2b^2+bc-c^2)\\
&=2b(2b^2)+2b(bc)+2b(-c^2)\\
&=4b^3+2b^2c-2bc^2
\end{aligned}
$$

f)

$$
\begin{aligned}
&y^2(y^2+1)(y^2-1)\\
&=y^2[y^2(y^2-1)+1(y^2-1)]\\
&=y^2[y^2(y^2)+y^2(-1)+y^2-1]\\
&=y^2(y^4-y^2+y^2-1)\\
&=y^2(y^4-1)\\
&=y^2(y^4)+y^2(-1)\\
&=y^6-y^2
\end{aligned}
$$

20. a) A cube with edge length e has volume: $V=e^3$
Substitute $e=2x+3$, then simplify.

$$
\begin{aligned}
V&=(2x+3)^3\\
&=(2x+3)(2x+3)(2x+3)\\
&=[(2x+3)(2x+3)](2x+3)\\
&=[2x(2x+3)+3(2x+3)](2x+3)\\
&=[2x(2x)+2x(3)+3(2x)+3(3)](2x+3)\\
&=(4x^2+6x+6x+9)(2x+3)\\
&=(4x^2+12x+9)(2x+3)\\
&=4x^2(2x+3)+12x(2x+3)+9(2x+3)\\
&=4x^2(2x)+4x^2(3)+12x(2x)+12x(3)+9(2x)+9(3)\\
&=8x^3+12x^2+24x^2+36x+18x+27\\
&=8x^3+36x^2+54x+27
\end{aligned}
$$

b) A cube has 6 congruent square faces.
Each face has area: $(2x + 3)(2x + 3)$

So, the surface area of the cube is: $6(2x + 3)(2x + 3)$, or $6(2x + 3)^2$
Simplify the expression.

$$\begin{aligned}
&6(2x + 3)^2 \\
&= 6(2x + 3)(2x + 3) \\
&= 6[2x(2x + 3) + 3(2x + 3)] \\
&= 6[2x(2x) + 2x(3) + 3(2x) + 3(3)] \\
&= 6(4x^2 + 6x + 6x + 9) \\
&= 6(4x^2 + 12x + 9) \\
&= 6(4x^2) + 6(12x) + 6(9) \\
&= 24x^2 + 72x + 54
\end{aligned}$$

21. a)

$$\begin{aligned}
&(3x + 4)(x - 5)(2x + 8) \\
&= [3x(x - 5) + 4(x - 5)](2x + 8) \\
&= [3x(x) + 3x(-5) + 4(x) + 4(-5)](2x + 8) \\
&= (3x^2 - 15x + 4x - 20)(2x + 8) \\
&= (3x^2 - 11x - 20)(2x + 8) \\
&= 3x^2(2x + 8) - 11x(2x + 8) - 20(2x + 8) \\
&= 3x^2(2x) + 3x^2(8) - 11x(2x) - 11x(8) - 20(2x) - 20(8) \\
&= 6x^3 + 24x^2 - 22x^2 - 88x - 40x - 160 \\
&= 6x^3 + 2x^2 - 128x - 160
\end{aligned}$$

b)

$$\begin{aligned}
&(b - 7)(b + 8)(3b - 4) \\
&= [b(b + 8) - 7(b + 8)](3b - 4) \\
&= [b(b) + b(8) - 7(b) - 7(8)](3b - 4) \\
&= (b^2 + 8b - 7b - 56)(3b - 4) \\
&= (b^2 + b - 56)(3b - 4) \\
&= b^2(3b - 4) + b(3b - 4) - 56(3b - 4) \\
&= b^2(3b) + b^2(-4) + b(3b) + b(-4) - 56(3b) - 56(-4) \\
&= 3b^3 - 4b^2 + 3b^2 - 4b - 168b + 224 \\
&= 3b^3 - b^2 - 172b + 224
\end{aligned}$$

c)

$$(2x-5)(3x+4)^2$$
$$=(2x-5)(3x+4)(3x+4)$$
$$=\left[2x(3x+4)-5(3x+4)\right](3x+4)$$
$$=\left[2x(3x)+2x(4)-5(3x)-5(4)\right](3x+4)$$
$$=\left(6x^2+8x-15x-20\right)(3x+4)$$
$$=\left(6x^2-7x-20\right)(3x+4)$$
$$=6x^2(3x+4)-7x(3x+4)-20(3x+4)$$
$$=6x^2(3x)+6x^2(4)-7x(3x)-7x(4)-20(3x)-20(4)$$
$$=18x^3+24x^2-21x^2-28x-60x-80$$
$$=18x^3+3x^2-88x-80$$

d)

$$(5a-3)^2(2a-7)$$
$$=(5a-3)(5a-3)(2a-7)$$
$$=\left[5a(5a-3)-3(5a-3)\right](2a-7)$$
$$=\left[5a(5a)+5a(-3)-3(5a)-3(-3)\right](2a-7)$$
$$=\left(25a^2-15a-15a+9\right)(2a-7)$$
$$=\left(25a^2-30a+9\right)(2a-7)$$
$$=25a^2(2a-7)-30a(2a-7)+9(2a-7)$$
$$=25a^2(2a)+25a^2(-7)-30a(2a)-30a(-7)+9(2a)+9(-7)$$
$$=50a^3-175a^2-60a^2+210a+18a-63$$
$$=50a^3-235a^2+228a-63$$

e)

$$(2k-3)(2k+3)^2$$
$$=(2k-3)(2k+3)(2k+3)$$
$$=\left[2k(2k+3)-3(2k+3)\right](2k+3)$$
$$=\left[2k(2k)+2k(3)-3(2k)-3(3)\right](2k+3)$$
$$=\left(4k^2+6k-6k-9\right)(2k+3)$$
$$=\left(4k^2-9\right)(2k+3)$$
$$=4k^2(2k+3)-9(2k+3)$$
$$=4k^2(2k)+4k^2(3)-9(2k)-9(3)$$
$$=8k^3+12k^2-18k-27$$

22. a)

$$(x+y+1)^3$$
$$=(x+y+1)(x+y+1)(x+y+1)$$
$$=\left[x(x+y+1)+y(x+y+1)+1(x+y+1)\right](x+y+1)$$
$$=\left[x(x)+x(y)+x(1)+y(x)+y(y)+y(1)+x+y+1\right](x+y+1)$$
$$=(x^2+xy+x+xy+y^2+y+x+y+1)(x+y+1)$$
$$=(x^2+2x+2xy+2y+y^2+1)(x+y+1)$$
$$=x^2(x+y+1)+2x(x+y+1)+2xy(x+y+1)+2y(x+y+1)+y^2(x+y+1)+x+y+1$$
$$=x^2(x)+x^2(y)+x^2(1)+2x(x)+2x(y)+2x(1)+2xy(x)+2xy(y)+2xy(1)+2y(x)+2y(y)+$$
$$2y(1)+y^2(x)+y^2(y)+y^2(1)+x+y+1$$
$$=x^3+x^2y+x^2+2x^2+2xy+2x+2x^2y+2xy^2+2xy+2xy+2y^2+2y+xy^2+y^3+y^2+x+y+1$$
$$=x^3+3x^2y+3xy^2+y^3+3x^2+6xy+3y^2+3x+3y+1$$

b)

$$(x-y-1)^3$$
$$=(x-y-1)(x-y-1)(x-y-1)$$
$$=\left[x(x-y-1)-y(x-y-1)-1(x-y-1)\right](x-y-1)$$
$$=\left[x(x)+x(-y)+x(-1)-y(x)-y(-y)-y(-1)-x+y+1\right](x-y-1)$$
$$=(x^2-xy-x-xy+y^2+y-x+y+1)(x-y-1)$$
$$=(x^2-2x-2xy+2y+y^2+1)(x-y-1)$$
$$=x^2(x-y-1)-2x(x-y-1)-2xy(x-y-1)+2y(x-y-1)+y^2(x-y-1)+x-y-1$$
$$=x^2(x)+x^2(-y)+x^2(-1)-2x(x)-2x(-y)-2x(-1)-2xy(x)-2xy(-y)-2xy(-1)+2y(x)+$$
$$2y(-y)+2y(-1)+y^2(x)+y^2(-y)+y^2(-1)+x-y-1$$
$$=x^3-x^2y-x^2-2x^2+2xy+2x-2x^2y+2xy^2+2xy+2xy-2y^2-2y+xy^2-y^3-y^2+x-y-1$$
$$=x^3-3x^2y+3xy^2-y^3-3x^2+6xy-3y^2+3x-3y-1$$

c)

$$
\begin{aligned}
&(x+y+z)^3\\
&=(x+y+z)(x+y+z)(x+y+z)\\
&=[x(x+y+z)+y(x+y+z)+z(x+y+z)](x+y+z)\\
&=[x(x)+x(y)+x(z)+y(x)+y(y)+y(z)+z(x)+z(y)+z(z)](x+y+z)\\
&=(x^2+xy+xz+xy+y^2+yz+xz+yz+z^2)(x+y+z)\\
&=(x^2+2xy+y^2+2yz+z^2+2xz)(x+y+z)\\
&=x^2(x+y+z)+2xy(x+y+z)+y^2(x+y+z)+2yz(x+y+z)+z^2(x+y+z)+2xz(x+y+z)\\
&=x^2(x)+x^2(y)+x^2(z)+2xy(x)+2xy(y)+2xy(z)+y^2(x)+y^2(y)+y^2(z)+2yz(x)+\\
&\quad 2yz(y)+2yz(z)+z^2(x)+z^2(y)+z^2(z)+2xz(x)+2xz(y)+2xz(z)\\
&=x^3+x^2y+x^2z+2x^2y+2xy^2+2xyz+xy^2+y^3+y^2z+2xyz+2y^2z+2yz^2+xz^2+\\
&\quad yz^2+z^3+2x^2z+2xyz+2xz^2\\
&=x^3+3x^2y+3xy^2+y^3+3x^2z+6xyz+3y^2z+3xz^2+3yz^2+z^3
\end{aligned}
$$

d)

$$
\begin{aligned}
&(x-y-z)^3\\
&=(x-y-z)(x-y-z)(x-y-z)\\
&=[x(x-y-z)-y(x-y-z)-z(x-y-z)](x-y-z)\\
&=[x(x)+x(-y)+x(-z)-y(x)-y(-y)-y(-z)-z(x)-z(-y)-z(-z)](x-y-z)\\
&=(x^2-xy-xz-xy+y^2+yz-xz+yz+z^2)(x-y-z)\\
&=(x^2-2xy+y^2+2yz+z^2-2xz)(x-y-z)\\
&=x^2(x-y-z)-2xy(x-y-z)+y^2(x-y-z)+2yz(x-y-z)+z^2(x-y-z)-2xz(x-y-z)\\
&=x^2(x)+x^2(-y)+x^2(-z)-2xy(x)-2xy(-y)-2xy(-z)+y^2(x)+y^2(-y)+y^2(-z)+2yz(x)+\\
&\quad 2yz(-y)+2yz(-z)+z^2(x)+z^2(-y)+z^2(-z)-2xz(x)-2xz(-y)-2xz(-z)\\
&=x^3-x^2y-x^2z-2x^2y+2xy^2+2xyz+xy^2-y^3-y^2z+2xyz-2y^2z-2yz^2+xz^2-\\
&\quad yz^2-z^3-2x^2z+2xyz+2xz^2\\
&=x^3-3x^2y+3xy^2-y^3-3x^2z+6xyz-3y^2z+3xz^2-3yz^2-z^3
\end{aligned}
$$

Lesson 3.8 **Factoring Special Polynomials** **Exercises (pages 194–195)**

A

4. a) $(x+2)^2 = (x+2)(x+2)$
$= x(x+2) + 2(x+2)$
$= x(x) + x(2) + 2(x) + 2(2)$
$= x^2 + 2x + 2x + 4$
$= x^2 + 4x + 4$

b) $(3-y)^2 = (3-y)(3-y)$
$= 3(3-y) - y(3-y)$
$= 3(3) + 3(-y) - y(3) - y(-y)$
$= 9 - 3y - 3y + y^2$
$= 9 - 6y + y^2$

c) $(5+d)^2 = (5+d)(5+d)$
$= 5(5+d) + d(5+d)$
$= 5(5) + 5(d) + d(5) + d(d)$
$= 25 + 5d + 5d + d^2$
$= 25 + 10d + d^2$

d) $(7-f)^2 = (7-f)(7-f)$
$= 7(7-f) - f(7-f)$
$= 7(7) + 7(-f) - f(7) - f(-f)$
$= 49 - 7f - 7f + f^2$
$= 49 - 14f + f^2$

e) $(x+2)(x-2) = x(x-2) + 2(x-2)$
$= x(x) + x(-2) + 2(x) + 2(-2)$
$= x^2 - 2x + 2x - 4$
$= x^2 - 4$

f) $(3-y)(3+y) = 3(3+y) - y(3+y)$
$= 3(3) + 3(y) - y(3) - y(y)$
$= 9 + 3y - 3y - y^2$
$= 9 - y^2$

g) $(5+d)(5-d) = 5(5-d) + d(5-d)$
$= 5(5) + 5(-d) + d(5) + d(-d)$
$= 25 - 5d + 5d - d^2$
$= 25 - d^2$

h) $(7-f)(7+f) = 7(7+f) - f(7+f)$
$= 7(7) + 7(f) - f(7) - f(f)$
$= 49 + 7f - 7f - f^2$
$= 49 - f^2$

5. a) $25 - t^2$
This polynomial is a difference of squares because it is of the form $a^2 - b^2$, and $25 = (5)(5)$ and $t^2 = (t)(t)$.

b) $16m^2 + 49n^2$
This polynomial is not a perfect square trinomial because it has only 2 terms.
It is not a difference of squares because it is not of the form $a^2 - b^2$.
So, this polynomial is neither a perfect square trinomial nor a difference of squares.

c) $4x^2 - 24xy + 9y^2$
This polynomial is not a difference of squares because it is not of the form $a^2 - b^2$.
It has 3 terms, so it may be a perfect square trinomial.
Check: $4x^2 = (2x)(2x)$ and $9y^2 = (3y)(3y)$
Since $2(2x)(3y) \neq 24xy$, the polynomial is not a perfect square trinomial.
So, this polynomial is neither a perfect square trinomial nor a difference of squares.

d) $9m^2 - 24mn + 16n^2$
This polynomial is not a difference of squares because it is not of the form $a^2 - b^2$.
It has 3 terms, so it may be a perfect square trinomial.
Check: $9m^2 = (3m)(3m)$ and $16n^2 = (4n)(4n)$
Since $2(3m)(4n) = 24mn$, the polynomial is a perfect square trinomial.

6. a) $x^2 - 49$
Write each term as a perfect square.
$x^2 - 49 = (x)^2 - (7)^2$ Write these terms in binomial factors.
$= (x+7)(x-7)$

b) $b^2 - 121$

Write each term as a perfect square.

$b^2 - 121 = (b)^2 - (11)^2$ Write these terms in binomial factors.

$= (b + 11)(b - 11)$

c) $1 - q^2$

Write each term as a perfect square.

$1 - q^2 = (1)^2 - (q)^2$ Write these terms in binomial factors.

$= (1 + q)(1 - q)$

d) $36 - c^2$

Write each term as a perfect square.

$36 - c^2 = (6)^2 - (c)^2$ Write these terms in binomial factors.

$= (6 + c)(6 - c)$

B

7. a) i) $a^2 + 10a + 25$

The first term is a perfect square since $a^2 = (a)(a)$.

The third term is a perfect square since $25 = (5)(5)$.

The second term is twice the product of *a* and 5:

$10a = 2(a)(5)$

Since the second term is positive, the operations in the binomial factors must be addition.

So, the trinomial is a perfect square trinomial and its factors are:

$(a + 5)(a + 5)$, or $(a + 5)^2$

ii) $b^2 - 12b + 36$

The first term is a perfect square since $b^2 = (b)(b)$.

The third term is a perfect square since $36 = (6)(6)$.

The second term is twice the product of *b* and 6:

$12b = 2(b)(6)$

Since the second term is negative, the operations in the binomial factors must be subtraction.

So, the trinomial is a perfect square trinomial and its factors are:

$(b - 6)(b - 6)$, or $(b - 6)^2$

iii) $c^2 + 14c + 49$

The first term is a perfect square since $c^2 = (c)(c)$.

The third term is a perfect square since $49 = (7)(7)$.

The second term is twice the product of *c* and 7:
$14c = 2(c)(7)$

Since the second term is positive, the operations in the binomial factors must be addition.
So, the trinomial is a perfect square trinomial and its factors are:
$(c + 7)(c + 7)$, or $(c + 7)^2$

iv) $d^2 - 16d + 64$

The first term is a perfect square since $d^2 = (d)(d)$.

The third term is a perfect square since $64 = (8)(8)$.

The second term is twice the product of *d* and 8:
$16d = 2(d)(8)$

Since the second term is negative, the operations in the binomial factors must be subtraction.
So, the trinomial is a perfect square trinomial and its factors are:
$(d - 8)(d - 8)$, or $(d - 8)^2$

v) $e^2 + 18e + 81$

The first term is a perfect square since $e^2 = (e)(e)$.

The third term is a perfect square since $81 = (9)(9)$.

The second term is twice the product of *e* and 9:
$18e = 2(e)(9)$

Since the second term is positive, the operations in the binomial factors must be addition.
So, the trinomial is a perfect square trinomial and its factors are:
$(e + 9)(e + 9)$, or $(e + 9)^2$

vi) $f^2 - 20f + 100$

The first term is a perfect square since $f^2 = (f)(f)$.

The third term is a perfect square since $100 = (10)(10)$.

The second term is twice the product of *f* and 10:
$20f = 2(f)(10)$

Since the second term is negative, the operations in the binomial factors must be subtraction.
So, the trinomial is a perfect square trinomial and its factors are:
$(f - 10)(f - 10)$, or $(f - 10)^2$

b) The variables in the trinomials and binomial factors are consecutive letters of the alphabet, starting with *a*.

Trinomials:
The first term in the trinomials is the square of the variable.
The coefficient of the second term starts at 10 and increases by two each time. The variable has degree one. The signs alternate.
The third terms in the trinomials are consecutive perfect squares, starting at 25.

Binomial factors:
The first term is the variable.
The second terms are consecutive whole numbers, starting at 5. The sign alternates.

Patterns that relate the trinomial to its factors:
The first term in the trinomial is the square of the first term in the binomial.
The third term in the trinomial is the square of the second term in the binomial.
The second term in the trinomial is twice the product of the first and second terms in the binomial.
The operation in the binomial factors is the same as the sign of the second term in the trinomial.

The next 4 trinomials in the pattern and their factors are:

$g^2 + 22g + 121 = (g + 11)^2$

$h^2 - 24h + 144 = (h - 12)^2$

$i^2 + 26i + 169 = (i + 13)^2$

$j^2 - 28j + 196 = (j - 14)^2$

8. a) $4x^2 - 12x + 9$

The first term is a perfect square since $4x^2 = (2x)(2x)$.

The third term is a perfect square since $9 = (3)(3)$.

The second term is twice the product of $2x$ and 3:

$12x = 2(2x)(3)$

Since the second term is negative, the operations in the binomial factors must be subtraction.

So, the trinomial is a perfect square trinomial and its factors are:

$(2x - 3)(2x - 3)$, or $(2x - 3)^2$

To verify, multiply:

$$\begin{aligned}(2x-3)(2x-3) &= 2x(2x-3) - 3(2x-3)\\ &= 4x^2 - 6x - 6x + 9\\ &= 4x^2 - 12x + 9\end{aligned}$$

Since the trinomial is the same as the original trinomial, the factors are correct.

b) $9 + 30n + 25n^2$

The first term is a perfect square since $9 = (3)(3)$.

The third term is a perfect square since $25n^2 = (5n)(5n)$.

The second term is twice the product of 3 and $5n$:

$30n = 2(3)(5n)$

Since the second term is positive, the operations in the binomial factors must be addition.
So, the trinomial is a perfect square trinomial and its factors are:

$(3 + 5n)(3 + 5n)$, or $(3 + 5n)^2$

To verify, multiply:

$$\begin{aligned}(3 + 5n)(3 + 5n) &= 3(3 + 5n) + 5n(3 + 5n)\\ &= 9 + 15n + 15n + 25n^2\\ &= 9 + 30n + 25n^2\end{aligned}$$

Since the trinomial is the same as the original trinomial, the factors are correct.

c) $81 - 36v + 4v^2$

The first term is a perfect square since $81 = (9)(9)$.

The third term is a perfect square since $4v^2 = (2v)(2v)$.

The second term is twice the product of 9 and $2v$:

$36v = 2(9)(2v)$

Since the second term is negative, the operations in the binomial factors must be subtraction.
So, the trinomial is a perfect square trinomial and its factors are:

$(9 - 2v)(9 - 2v)$, or $(9 - 2v)^2$

To verify, multiply:

$$\begin{aligned}(9 - 2v)(9 - 2v) &= 9(9 - 2v) - 2v(9 - 2v)\\ &= 81 - 18v - 18v + 4v^2\\ &= 81 - 36v + 4v^2\end{aligned}$$

Since the trinomial is the same as the original trinomial, the factors are correct.

d) $25 + 40h + 16h^2$

The first term is a perfect square since $25 = (5)(5)$.

The third term is a perfect square since $16h^2 = (4h)(4h)$.

The second term is twice the product of 5 and $4h$:

$40h = 2(5)(4h)$

Since the second term is positive, the operations in the binomial factors must be addition.
So, the trinomial is a perfect square trinomial and its factors are:

$(5 + 4h)(5 + 4h)$, or $(5 + 4h)^2$

To verify, multiply:

$$\begin{aligned}(5+4h)(5+4h) &= 5(5+4h)+4h(5+4h)\\ &= 25+20h+20h+16h^2\\ &= 25+40h+16h^2\end{aligned}$$

Since the trinomial is the same as the original trinomial, the factors are correct.

e) $9g^2+48g+64$

The first term is a perfect square since $9g^2=(3g)(3g)$.

The third term is a perfect square since $64=(8)(8)$.

The second term is twice the product of $3g$ and 8:

$48g=2(3g)(8)$

Since the second term is positive, the operations in the binomial factors must be addition.
So, the trinomial is a perfect square trinomial and its factors are:

$(3g+8)(3g+8)$, or $(3g+8)^2$

To verify, multiply:

$$\begin{aligned}(3g+8)(3g+8) &= 3g(3g+8)+8(3g+8)\\ &= 9g^2+24g+24g+64\\ &= 9g^2+48g+64\end{aligned}$$

Since the trinomial is the same as the original trinomial, the factors are correct.

f) $49r^2-28r+4$

The first term is a perfect square since $49r^2=(7r)(7r)$.

The third term is a perfect square since $4=(2)(2)$.

The second term is twice the product of $7r$ and 2:

$28r=2(7r)(2)$

Since the second term is negative, the operations in the binomial factors must be subtraction.
So, the trinomial is a perfect square trinomial and its factors are:

$(7r-2)(7r-2)$, or $(7r-2)^2$

To verify, multiply:

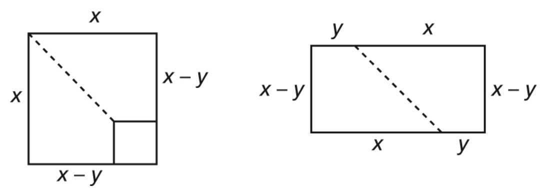
$$\begin{aligned}(7r-2)(7r-2) &= 7r(7r-2)-2(7r-2)\\ &= 49r^2-14r-14r+4\\ &= 49r^2-28r+4\end{aligned}$$

Since the trinomial is the same as the original trinomial, the factors are correct.

9. a) The formula for the area of a square, A, is: $A=s^2$, where s is the side length of the square.
The larger square has side length x.
So, an expression that represents its area is: x^2
The smaller square has side length y.
So, an expression that represents its area is: y^2

The area of the piece that remains is the area of the larger square minus the area of the smaller square.
So, an expression for the area of the piece that remains is: $x^2 - y^2$

b)



From the diagrams above, the rectangle has dimensions $x - y$ and $x + y$.
The formula for the area, A, of a rectangle is:
$A = lw$, where l is the length of the rectangle and w is its width.
Substitute: $l = x + y$ and $w = x - y$

$$\begin{aligned} A &= (x + y)(x - y) \\ &= x(x - y) + y(x - y) \\ &= x^2 - xy + xy - y^2 \\ &= x^2 - y^2 \end{aligned}$$

The area of the rectangle is represented by the expression: $x^2 - y^2$

c) The two expressions represent the area of the same region, so the expressions are equal:
$x^2 - y^2 = (x + y)(x - y)$
This is the difference of squares.

10. a) $9d^2 - 16f^2$

Write each term as a perfect square.

$$\begin{aligned} 9d^2 - 16f^2 &= (3d)^2 - (4f)^2 \\ &= (3d + 4f)(3d - 4f) \end{aligned}$$

Write these terms in binomial factors.

To verify, multiply:

$$\begin{aligned} (3d + 4f)(3d - 4f) &= 3d(3d - 4f) + 4f(3d - 4f) \\ &= 9d^2 - 12df + 12df - 16f^2 \\ &= 9d^2 - 16f^2 \end{aligned}$$

Since the binomial is the same as the original binomial, the factors are correct.

b) $25s^2 - 64t^2$

Write each term as a perfect square.

$$\begin{aligned} 25s^2 - 64t^2 &= (5s)^2 - (8t)^2 \\ &= (5s + 8t)(5s - 8t) \end{aligned}$$

Write these terms in binomial factors.

To verify, multiply:

$$\begin{aligned} (5s + 8t)(5s - 8t) &= 5s(5s - 8t) + 8t(5s - 8t) \\ &= 25s^2 - 40st + 40st - 64t^2 \\ &= 25s^2 - 64t^2 \end{aligned}$$

Since the binomial is the same as the original binomial, the factors are correct.

c) $144a^2 - 9b^2$

The terms have a common factor 9. Remove this common factor.

$144a^2 - 9b^2 = 9(16a^2 - b^2)$ Write each term in the binomial as a perfect square.

$= 9[(4a)^2 - (b)^2]$ Write these terms in binomial factors.

$= 9(4a + b)(4a - b)$

To verify, multiply:

$$\begin{aligned} 9(4a+b)(4a-b) &= 9[4a(4a-b) + b(4a-b)] \\ &= 9(16a^2 - 4ab + 4ab - b^2) \\ &= 9(16a^2 - b^2) \\ &= 144a^2 - 9b^2 \end{aligned}$$

Since the binomial is the same as the original binomial, the factors are correct.

d) $121m^2 - n^2$

Write each term as a perfect square.

$121m^2 - n^2 = (11m)^2 - (n)^2$ Write these terms in binomial factors.

$= (11m + n)(11m - n)$

To verify, multiply:

$$\begin{aligned} (11m+n)(11m-n) &= 11m(11m-n) + n(11m-n) \\ &= 121m^2 - 11mn + 11mn - n^2 \\ &= 121m^2 - n^2 \end{aligned}$$

Since the binomial is the same as the original binomial, the factors are correct.

e) $81k^2 - 49m^2$

Write each term as a perfect square.

$81k^2 - 49m^2 = (9k)^2 - (7m)^2$ Write these terms in binomial factors.

$= (9k + 7m)(9k - 7m)$

To verify, multiply:

$$\begin{aligned} (9k+7m)(9k-7m) &= 9k(9k-7m) + 7m(9k-7m) \\ &= 81k^2 - 63km + 63km - 49m^2 \\ &= 81k^2 - 49m^2 \end{aligned}$$

Since the binomial is the same as the original binomial, the factors are correct.

f) $100y^2 - 81z^2$

Write each term as a perfect square.

$100y^2 - 81z^2 = (10y)^2 - (9z)^2$ Write these terms in binomial factors.

$= (10y + 9z)(10y - 9z)$

To verify, multiply:

$$\begin{aligned}(10y+9z)(10y-9z) &= 10y(10y-9z)+9z(10y-9z)\\ &= 100y^2-90yz+90yz-81z^2\\ &= 100y^2-81z^2\end{aligned}$$

Since the binomial is the same as the original binomial, the factors are correct.

g) v^2-36t^2

Write each term as a perfect square.

$$\begin{aligned}v^2-36t^2 &= (v)^2-(6t)^2 \quad \text{Write these terms in binomial factors.}\\ &= (v+6t)(v-6t)\end{aligned}$$

To verify, multiply:

$$\begin{aligned}(v+6t)(v-6t) &= v(v-6t)+6t(v-6t)\\ &= v^2-6tv+6tv-36t^2\\ &= v^2-36t^2\end{aligned}$$

Since the binomial is the same as the original binomial, the factors are correct.

h) $4j^2-225h^2$

Write each term as a perfect square.

$$\begin{aligned}4j^2-225h^2 &= (2j)^2-(15h)^2 \quad \text{Write these terms in binomial factors.}\\ &= (2j+15h)(2j-15h)\end{aligned}$$

To verify, multiply:

$$\begin{aligned}(2j+15h)(2j-15h) &= 2j(2j-15h)+15h(2j-15h)\\ &= 4j^2-30hj+30hj-225h^2\\ &= 4j^2-225h^2\end{aligned}$$

Since the binomial is the same as the original binomial, the factors are correct.

11. a) $y^2+7yz+10z^2$

Use decomposition. Since the 3rd term in the trinomial is positive, the operations in the binomial factors will be the same. Since the second term is positive, these operations will be addition.

The two binomials will have the form: $(?y+?z)(?y+?z)$

The product of the coefficients of y^2 and z^2 is: (1)(10) = 10

Write 7*yz* as a sum of two terms whose coefficients have a product of 10.

Factors of 10	Sum of Factors
1, 10	1 + 10 = 11
2, 5	2 + 5 = 7

The two coefficients are 2 and 5, so write the trinomial $y^2+7yz+10z^2$ as $y^2+2yz+5yz+10z^2$.

Remove a common factor from the 1st pair of terms, and from the 2nd pair of terms:

$y^2 + 2yz + 5yz + 10z^2$

$= y(y + 2z) + 5z(y + 2z)$ Remove the common binomial factor.

$= (y + 2z)(y + 5z)$

So, $y^2 + 7yz + 10z^2 = (y + 2z)(y + 5z)$

To verify, multiply:

$$(y + 2z)(y + 5z) = y(y + 5z) + 2z(y + 5z)$$
$$= y^2 + 5yz + 2yz + 10z^2$$
$$= y^2 + 7yz + 10z^2$$

Since this trinomial is the same as the original trinomial, the factors are correct.

b) $4w^2 - 8wx - 21x^2$

Use decomposition. Since the 3rd term in the trinomial is negative, the operations in the binomial factors will be addition and subtraction.

The two binomials will have the form: $(?w + ?x)(?w - ?x)$

The product of the coefficients of w^2 and x^2 is: (4)(–21) = –84

Write $-8wx$ as a sum of two terms whose coefficients have a product of –84.

Factors of –84	**Sum of Factors**
4, –21	4 – 21 = –17
–4, 21	–4 + 21 = 17
6, –14	6 – 14 = –8
–6, 14	–6 + 14 = 8

The two coefficients are 6 and –14, so write the trinomial $4w^2 - 8wx - 21x^2$ as $4w^2 + 6wx - 14wx - 21x^2$.

Remove a common factor from the 1st pair of terms, and from the 2nd pair of terms:

$4w^2 + 6wx - 14wx - 21x^2$

$= 2w(2w + 3x) - 7x(2w + 3x)$ Remove the common binomial factor.

$= (2w + 3x)(2w - 7x)$

So, $4w^2 - 8wx - 21x^2 = (2w + 3x)(2w - 7x)$

To verify, multiply:

$$(2w + 3x)(2w - 7x) = 2w(2w - 7x) + 3x(2w - 7x)$$
$$= 4w^2 - 14wx + 6wx - 21x^2$$
$$= 4w^2 - 8wx - 21x^2$$

Since this trinomial is the same as the original trinomial, the factors are correct.

c) $12s^2 - 7su + u^2$

Use decomposition. Since the 3rd term in the trinomial is positive, the operations in the binomial factors will be the same. Since the second term is negative, these operations will be subtraction.

The two binomials will have the form: $(?s - ?u)(?s - ?u)$

The product of the coefficients of s^2 and u^2 is: (12)(1) = 12

Write –7*su* as a sum of two terms whose coefficients have a product of 12.

Factors of 12	Sum of Factors
–1, –12	–1 – 12 = –13
–2, –6	–2 – 6 = –8
–3, –4	–3 – 4 = –7

The two coefficients are –3 and –4, so write the trinomial $12s^2 - 7su + u^2$ as $12s^2 - 3su - 4su + u^2$.

Remove a common factor from the 1st pair of terms, and from the 2nd pair of terms:

$12s^2 - 3su - 4su + u^2$

$= 3s(4s - u) - u(4s - u)$ Remove the common binomial factor.

$= (4s - u)(3s - u)$

So, $12s^2 - 7su + u^2 = (4s - u)(3s - u)$

To verify, multiply:

$(4s - u)(3s - u) = 4s(3s - u) - u(3s - u)$

$= 12s^2 - 4su - 3su + u^2$

$= 12s^2 - 7su + u^2$

Since this trinomial is the same as the original trinomial, the factors are correct.

d) $3t^2 - 7tv + 4v^2$

Use decomposition. Since the 3rd term in the trinomial is positive, the operations in the binomial factors will be the same. Since the second term is negative, these operations will be subtraction.

The two binomials will have the form: $(?t - ?v)(?t - ?v)$

The product of the coefficients of t^2 and v^2 is: (3)(4) = 12

Write –7*tv* as a sum of two terms whose coefficients have a product of 12.

Factors of 12	Sum of Factors
–1, –12	–1 – 12 = –13
–2, –6	–2 – 6 = –8
–3, –4	–3 – 4 = –7

The two coefficients are –3 and –4, so write the trinomial $3t^2 - 7tv + 4v^2$ as $3t^2 - 3tv - 4tv + 4v^2$.

Remove a common factor from the 1st pair of terms, and from the 2nd pair of terms:

$3t^2 - 3tv - 4tv + 4v^2$

$= 3t(t - v) - 4v(t - v)$ Remove the common binomial factor.

$= (t - v)(3t - 4v)$

So, $3t^2 - 7tv + 4v^2 = (t - v)(3t - 4v)$

To verify, multiply:

$(t - v)(3t - 4v) = t(3t - 4v) - v(3t - 4v)$

$= 3t^2 - 4tv - 3tv + 4v^2$

$= 3t^2 - 7tv + 4v^2$

Since this trinomial is the same as the original trinomial, the factors are correct.

e) $10r^2 + 9rs - 9s^2$

Use decomposition. Since the 3rd term in the trinomial is negative, the operations in the binomial factors will be addition and subtraction.

The two binomials will have the form: $(?r + ?s)(?r - ?s)$

The product of the coefficients of r^2 and s^2 is: (10)(–9) = –90

Write $9rs$ as a sum of two terms whose coefficients have a product of –90.

Factors of –90	Sum of Factors
5, –18	5 – 18 = –13
–5, 18	–5 + 18 = 13
6, –15	6 – 15 = –9
–6, 15	–6 + 15 = 9

The two coefficients are –6 and 15, so write the trinomial $10r^2 + 9rs - 9s^2$ as $10r^2 - 6rs + 15rs - 9s^2$.

Remove a common factor from the 1st pair of terms, and from the 2nd pair of terms:

$10r^2 - 6rs + 15rs - 9s^2$

$= 2r(5r - 3s) + 3s(5r - 3s)$ Remove the common binomial factor.

$= (5r - 3s)(2r + 3s)$

So, $10r^2 + 9rs - 9s^2 = (5r - 3s)(2r + 3s)$

To verify, multiply:

$(5r - 3s)(2r + 3s) = 5r(2r + 3s) - 3s(2r + 3s)$

$= 10r^2 + 15rs - 6rs - 9s^2$

$= 10r^2 + 9rs - 9s^2$

Since this trinomial is the same as the original trinomial, the factors are correct.

f) $8p^2 + 18pq - 35q^2$

Use decomposition. Since the 3rd term in the trinomial is negative, the operations in the binomial factors will be addition and subtraction.

The two binomials will have the form: $(?r + ?s)(?r - ?s)$

The product of the coefficients of p^2 and q^2 is: (8)(–35) = –280

Write 18*pq* as a sum of two terms whose coefficients have a product of –280.

Factors of –280	Sum of Factors
8, –35	8 – 35 = –27
–8, 35	–8 + 35 = 27
10, –28	10 – 28 = –18
–10, 28	–10 + 28 = 18

The two coefficients are –10 and 28, so write the trinomial $8p^2 + 18pq - 35q^2$ as $8p^2 - 10pq + 28pq - 35q^2$.

Remove a common factor from the 1st pair of terms, and from the 2nd pair of terms:

$8p^2 - 10pq + 28pq - 35q^2$

$= 2p(4p - 5q) + 7q(4p - 5q)$ Remove the common binomial factor.

$= (4p - 5q)(2p + 7q)$

So, $8p^2 + 18pq - 35q^2 = (4p - 5q)(2p + 7q)$

To verify, multiply:

$(4p - 5q)(2p + 7q) = 4p(2p + 7q) - 5q(2p + 7q)$

$= 8p^2 + 28pq - 10pq - 35q^2$

$= 8p^2 + 18pq - 35q^2$

Since this trinomial is the same as the original trinomial, the factors are correct.

12. a) $4x^2 + 28xy + 49y^2$

The first term is a perfect square since $4x^2 = (2x)(2x)$.

The third term is a perfect square since $49y^2 = (7y)(7y)$.

The second term is twice the product of 2*x* and 7*y*:

$28xy = 2(2x)(7y)$

Since the second term is positive, the operations in the binomial factors must be addition.
So, the trinomial is a perfect square trinomial and its factors are:

$(2x + 7y)(2x + 7y)$, or $(2x + 7y)^2$

b) $15m^2 + 7mn - 4n^2$

The first term is not a perfect square, so the trinomial is not a perfect square.
So, I'll factor by decomposition.
Since the 3rd term in the trinomial is negative, the operations in the binomial factors will be addition and subtraction.

The two binomials will have the form: $(?m + ?n)(?m - ?n)$

The product of the coefficients of m^2 and n^2 is: (15)(–4) = –60

Write $7mn$ as a sum of two terms whose coefficients have a product of –60.

Factors of –60	Sum of Factors
4, –15	4 – 15 = –11
–4, 15	–4 + 15 = 11
5, –12	5 – 12 = –7
–5, 12	–5 + 12 = 7

The two coefficients are –5 and 12, so write the trinomial $15m^2 + 7mn - 4n^2$ as $15m^2 - 5mn + 12mn - 4n^2$.
Remove a common factor from the 1st pair of terms, and from the 2nd pair of terms:
$15m^2 - 5mn + 12mn - 4n^2$
$= 5m(3m - n) + 4n(3m - n)$ Remove the common binomial factor.
$= (3m - n)(5m + 4n)$
So, $15m^2 + 7mn - 4n^2 = (3m - n)(5m + 4n)$

c) $16r^2 + 8rt + t^2$
The first term is a perfect square since $16r^2 = (4r)(4r)$.
The third term is a perfect square since $t^2 = (t)(t)$.
The second term is twice the product of $4r$ and t:
$8rt = 2(4r)(t)$
Since the second term is positive, the operations in the binomial factors must be addition.
So, the trinomial is a perfect square trinomial and its factors are:
$(4r + t)(4r + t)$, or $(4r + t)^2$

d) $9a^2 - 42ab + 49b^2$
The first term is a perfect square since $9a^2 = (3a)(3a)$.
The third term is a perfect square since $49b^2 = (7b)(7b)$.
The second term is twice the product of $3a$ and $7b$:
$42ab = 2(3a)(7b)$
Since the second term is negative, the operations in the binomial factors must be subtraction.
So, the trinomial is a perfect square trinomial and its factors are:
$(3a - 7b)(3a - 7b)$, or $(3a - 7b)^2$

e) $12h^2 + 25hk + 12k^2$
The first term is not a perfect square, so the trinomial is not a perfect square.
So, I'll factor by decomposition.
Since the 3rd term in the trinomial is positive, the operations in the binomial factors will be the same. Since the second term is positive, these operations will be addition.
The two binomials will have the form: $(?h + ?k)(?h + ?k)$
The product of the coefficients of h^2 and k^2 is: (12)(12) = 144

Write $25hk$ as a sum of two terms whose coefficients have a product of 144.

Factors of 144	Sum of Factors
4, 36	4 + 36 = 40
6, 24	6 + 24 = 30
8, 18	8 + 18 = 26
9, 16	9 + 16 = 25

The two coefficients are 9 and 16, so write the trinomial $12h^2 + 25hk + 12k^2$ as $12h^2 + 9hk + 16hk + 12k^2$.
Remove a common factor from the 1st pair of terms, and from the 2nd pair of terms:
$12h^2 + 9hk + 16hk + 12k^2$
$= 3h(4h + 3k) + 4k(4h + 3k)$ Remove the common binomial factor.
$= (4h + 3k)(3h + 4k)$
So, $12h^2 + 25hk + 12k^2 = (4h + 3k)(3h + 4k)$

f) $15f^2 - 31fg + 10g^2$
The first term is not a perfect square, so the trinomial is not a perfect square.
So, I'll factor by decomposition.
Since the 3rd term in the trinomial is positive, the operations in the binomial factors will be the same. Since the second term is negative, these operations will be subtraction.
The two binomials will have the form: $(?f - ?g)(?f - ?g)$
The product of the coefficients of f^2 and g^2 is: (15)(10) = 150
Write $-31fg$ as a sum of two terms whose coefficients have a product of 150.

Factors of 150	Sum of Factors
–3, –50	–3 – 50 = –53
–5, –30	–5 – 30 = –35
–6, –25	–6 – 25 = –31

The two coefficients are –6 and –25, so write the trinomial $15f^2 - 31fg + 10g^2$ as $15f^2 - 6fg - 25fg + 10g^2$.
Remove a common factor from the 1st pair of terms, and from the 2nd pair of terms:
$15f^2 - 6fg - 25fg + 10g^2$
$= 3f(5f - 2g) - 5g(5f - 2g)$ Remove the common binomial factor.
$= (5f - 2g)(3f - 5g)$
So, $15f^2 - 31fg + 10g^2 = (5f - 2g)(3f - 5g)$

13. a) $8m^2 - 72n^2$

As written, each term of the binomial is not a perfect square. But the terms have a common factor 8. Remove this common factor.

$8m^2 - 72n^2 = 8(m^2 - 9n^2)$ Write each term in the binomial as a perfect square.

$= 8[(m)^2 - (3n)^2]$ Write these terms in binomial factors.

$= 8(m + 3n)(m - 3n)$

b) $8z^2 + 8yz + 2y^2$

As written, each term of the trinomial has a common factor 2.
Remove this common factor.

$8z^2 + 8yz + 2y^2 = 2(4z^2 + 4yz + y^2)$

Look at the trinomial $4z^2 + 4yz + y^2$.

The first term is a perfect square since $4z^2 = (2z)(2z)$.

The third term is a perfect square since $y^2 = (y)(y)$.

The second term is twice the product of $2z$ and y:

$4yz = 2(2z)(y)$

Since the second term is positive, the operations in the binomial factors must be addition.
So, the trinomial is a perfect square trinomial and its factors are:

$(2z + y)(2z + y)$, or $(2z + y)^2$

So, $8z^2 + 8yz + 2y^2 = 2(2z + y)^2$

c) $12x^2 - 27y^2$

As written, each term of the binomial is not a perfect square. But the terms have a common factor 3. Remove this common factor.

$12x^2 - 27y^2 = 3(4x^2 - 9y^2)$ Write each term in the binomial as a perfect square.

$= 3[(2x)^2 - (3y)^2]$ Write these terms in binomial factors.

$= 3(2x + 3y)(2x - 3y)$

d) $8p^2 + 40pq + 50q^2$

As written, each term of the trinomial has a common factor 2.
Remove this common factor.

$8p^2 + 40pq + 50q^2 = 2(4p^2 + 20pq + 25q^2)$

Look at the trinomial $4p^2 + 20pq + 25q^2$.

The first term is a perfect square since $4p^2 = (2p)(2p)$.

The third term is a perfect square since $25q^2 = (5q)(5q)$.

The second term is twice the product of $2p$ and $5q$:

$20pq = 2(2p)(5q)$

Since the second term is positive, the operations in the binomial factors must be addition.
So, the trinomial is a perfect square trinomial and its factors are:

$(2p+5q)(2p+5q)$, or $(2p+5q)^2$

So, $8p^2+40pq+50q^2 = 2(2p+5q)^2$

e) $-24u^2-6uv+9v^2$

As written, each term of the trinomial has a common factor –3.
Remove this common factor.

$-24u^2-6uv+9v^2 = -3(8u^2+2uv-3v^2)$

Look at the trinomial $8u^2+2uv-3v^2$.

The first term is not a perfect square, so the trinomial is not a perfect square.
So, I'll factor by decomposition.
Since the 3rd term in the trinomial is negative, the operations in the binomial factors will be addition and subtraction.

The two binomials will have the form: $(?u+?v)(?u-?v)$

The product of the coefficients of u^2 and v^2 is: (8)(–3) = –24
Write $2uv$ as a sum of two terms whose coefficients have a product of –24.

Factors of –24	Sum of Factors
3, –8	3 – 8 = –5
–3, 8	–3 + 8 = 5
4, –6	4 – 6 = –2
–4, 6	–4 + 6 = 2

The two coefficients are –4 and 6, so write the trinomial $8u^2+2uv-3v^2$ as

$8u^2-4uv+6uv-3v^2$.

Remove a common factor from the 1st pair of terms, and from the 2nd pair of terms:

$8u^2-4uv+6uv-3v^2$

$=4u(2u-v)+3v(2u-v)$ Remove the common binomial factor.

$=(2u-v)(4u+3v)$

So, $8u^2+2uv-3v^2 = (2u-v)(4u+3v)$

And, $-24u^2-6uv+9v^2 = -3(2u-v)(4u+3v)$

f) $-18b^2+128c^2$

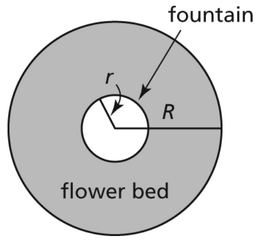
As written, each term of the binomial is not a perfect square. But the terms have a common factor –2. Remove this common factor.

$-18b^2+128c^2 = -2(9b^2-64c^2)$ Write each term in the binomial as a perfect square.

$= -2\left[(3b)^2-(8c)^2\right]$ Write these terms in binomial factors.

$= -2(3b+8c)(3b-8c)$

14. a)



b) The flower bed and the circular fountain form a large circle with radius R.
The area of the flower bed = the area of the large circle minus the area of the small circle.
The formula for the area of a circle, A, is $A = \pi r^2$, where r is the radius of the circle.

The area of the large circle is: $A = \pi R^2$

The area of the small circle is: $A = \pi r^2$

So, the area of the flower bed is: $\pi R^2 - \pi r^2$
As written, each term of the binomial is not a perfect square. But the terms have a common factor, π. Remove this common factor.

$\pi R^2 - \pi r^2 = \pi\left(R^2 - r^2\right)$ Write each term in the binomial as a perfect square.

$= \pi\left[(R)^2 - (r)^2\right]$ Write these terms in binomial factors.

$= \pi(R + r)(R - r)$

An expression for the area of the flower bed is: $\pi(R + r)(R - r)$

c) Use the expression: $\pi(R + r)(R - r)$
Substitute: $R = 350$ and $r = 150$

$$\begin{aligned}\pi(R + r)(R - r) &= \pi(350 + 150)(350 - 150)\\ &= \pi(500)(200)\\ &= 100\ 000\pi\\ &= 314\ 159.2654...\end{aligned}$$

So, the area of the flower bed is approximately 314 159 cm^2.

15. a) i) $x^2 + \square x + 49$

The first term is a perfect square since $x^2 = (x)(x)$.

The third term is a perfect square since $49 = (7)(7)$ or $49 = (-7)(-7)$.

For the trinomial to be a perfect square, the second term must be twice the product of x and 7 or twice the product of x and –7:

$\square x = 2(x)(7)$ $\quad$ $\square x = 2(x)(-7)$

$= 14x$ $\quad$ $= -14x$

So, the integers that can replace $\square$ are 14 and –14.

ii) $4a^2 + 20ab + \square b^2$

For the trinomial to be a perfect square, the coefficient of the second term, 20, must be twice the product of the square root of 4 and the square root of the number represented by the box.
I know the square root of 4 is 2.
Let the square root of the number represented by the box be x.

So, $20 = 2(2)(x)$
$20 = 4x$
$x = 5$
So, $\square = 5^2$, or 25

iii) $\square c^2 - 24cd + 16d^2$

For the trinomial to be a perfect square, the coefficient of the second term, –24, must be negative two times the product of the square root of 16 and the square root of the number represented by the box.
I know the square root of 16 is 4.
Let the square root of the number represented by the box be x.

So, $-24 = -2(4)(x)$
$-24 = -8x$
$x = 3$
So, $\square = 3^2$, or 9

b) i) 2 integers; the middle term can be positive or negative. It can only have one value: twice the product of the square roots of its first and last terms.

ii) 1 integer; the last term in a perfect square is positive. It can only have one value: the square of half the value of the middle term divided by the square root of the first term.

iii) 1 integer; the first term in a perfect square is positive. It can only have one value: the square of half the value of the middle term divided by the square root of the last term.

16. Let's look at a few examples.

$(x+1)^2 = x^2 + 2x + 1$

$(2x+3)^2 = 4x^2 + 12x + 9$

When all the terms in the trinomial are positive, b is greater than c and b is greater than a.
This is not possible when a, b, and c are consecutive positive integers.

$(x-1)^2 = x^2 - 2x + 1$

$(2x-3)^2 = 4x^2 - 12x + 9$

When the middle terms in the trinomial is negative, b is less than c and b is less than a.
This is not possible when a, b, and c are consecutive negative integers.
So, there are three possibilities for a, b, and c when they are not all positive integers and not all negative integers:
$a = 0, b = 1, c = 2$
$a = -1, b = 0, c = 1$
$a = -2, b = -1, c = 0$
Check each possibility to determine whether the trinomial can be factored:

$0x^2 + x + 2 = x + 2$ This cannot be factored.

$$\begin{aligned}-x^2 + 0x + 1 &= -x^2 + 1\\ &= -(x^2 - 1)\\ &= -(x-1)(x+1)\end{aligned}$$

This trinomial can be factored.

$$\begin{aligned}-2x^2 - x + 0 &= -2x^2 - x\\ &= -x(2x+1)\end{aligned}$$

This trinomial can be factored.
So, there are 2 possibilities for consecutive integers *a*, *b*, and *c*:
–1, 0, 1 and –2, –1, 0

17. To determine (199)(201), I can use the difference of squares.
I know 199 = 200 – 1 and 201 = 200 + 1

So, $(199)(201) = (200-1)(200+1)$ I know $(a-b)(a+b) = a^2 - b^2$

$$\begin{aligned}&= 200^2 - 1^2\\ &= 40\,000 - 1\\ &= 39\,999\end{aligned}$$

(199)(201) = 39 999

18. The area of the shaded region is:
area of large square – area of small square
The formula for the area of a square is: $A = s^2$, where *s* is the side length of the square.

Area of large square:
Use the formula $A = s^2$. Substitute: $s = 3x + 5$

$$A_L = (3x+5)^2$$

Area of small rectangle:
Use the formula $A = s^2$. Substitute: $s = 2x - 1$

$$A_L = (2x-1)^2$$

So, area of the shaded region is:

$$\begin{aligned}A &= A_L - A_S\\ &= (3x+5)^2 - (2x-1)^2\\ &= (3x+5)(3x+5) - (2x-1)(2x-1)\\ &= 3x(3x+5) + 5(3x+5) - [2x(2x-1) - 1(2x-1)]\\ &= 9x^2 + 15x + 15x + 25 - (4x^2 - 2x - 2x + 1)\\ &= 9x^2 + 30x + 25 - 4x^2 + 4x - 1\\ &= 5x^2 + 34x + 24\end{aligned}$$

A polynomial that represents the area of the shaded region is:
$5x^2 + 34x + 24$

C

19. a) i) $\left(x^2+5\right)^2$

This expression is not a difference of squares because it is not of the form a^2-b^2.
It is not a perfect square trinomial because it does not have three terms.
So, this expression is neither a perfect square trinomial nor a difference of squares.
It is the square of a binomial, or a perfect square trinomial in factored form.

ii) $-100+r^2$

This expression is a difference of squares because it can be written in the form a^2-b^2: r^2-100, and $r^2=(r)(r)$ and $100=(10)(10)$

iii) $81a^2b^2-1$

This expression is a difference of squares because it is written in the form a^2-b^2, and $81a^2b^2=(9ab)(9ab)$ and $1=(1)(1)$.

iv) $16s^4+8s^2+1$

This expression is not a difference of squares because it is not of the form a^2-b^2.
It has 3 terms, so it may be a perfect square trinomial.
Check: $16s^4=(4s^2)(4s^2)$ and $1=(1)(1)$
Since $2\left(4s^2\right)(1)=8s^2$, the expression is a perfect square trinomial.

b) i) This expression represents a perfect square trinomial in factored form, and it cannot be factored further.

ii) $-100+r^2$ can be written as: r^2-100
This is a difference of squares so it can be factored.
Write each term as a perfect square.

$$\begin{aligned} r^2-100 &= (r)^2-(10)^2 \quad \text{Write these terms in binomial factors.} \\ &= (r+10)(r-10) \end{aligned}$$

iii) $81a^2b^2-1$ is a difference of squares so it can be factored.
Write each term as a perfect square.

$$\begin{aligned} 81a^2b^2-1 &= (9ab)^2-(1)^2 \quad \text{Write these terms in binomial factors.} \\ &= (9ab+1)(9ab-1) \end{aligned}$$

iv) $16s^4+8s^2+1$ is a perfect square trinomial, so it can be factored.
The first term is a perfect square since $16s^4=\left(4s^2\right)\left(4s^2\right)$.
The third term is a perfect square since $1=(1)(1)$.
The second term is twice the product of $4s^2$ and 1: $8s^2=2\left(4s^2\right)(1)$
Since the second term is positive, the operations in the binomial factors must be addition.
So, its factors are: $\left(4s^2+1\right)\left(4s^2+1\right)$, or $\left(4s^2+1\right)^2$

20. a) $x^4 - 13x^2 + 36$

The first term is a perfect square since $x^4 = (x^2)(x^2)$.

The third term is a perfect square since $36 = (6)(6)$.

The second term is not twice the product of x^2 and 6 so the trinomial is not a perfect square.
So, I'll factor by decomposition.
Since the 3rd term in the trinomial is positive, the operations in the binomial factors will be the same. Since the second term is negative, these operations will be subtraction.
The two binomials will have the form: $(?x^2 - ?)(?x^2 - ?)$

The product of the coefficient of x^4 and the constant term is: (1)(36) = 36
Write $-13x^2$ as a sum of two terms whose coefficients have a product of 36.

Factors of 36	Sum of Factors
–2, –18	–2 – 18 = –20
–3, –12	–3 – 12 = –15
–4, –9	–4 – 9 = –13

The two coefficients are –4 and –9, so write the trinomial $x^4 - 13x^2 + 36$ as $x^4 - 4x^2 - 9x^2 + 36$.
Remove a common factor from the 1st pair of terms, and from the 2nd pair of terms:
$x^4 - 4x^2 - 9x^2 + 36$
$= x^2(x^2 - 4) - 9(x^2 - 4)$ Remove the common binomial factor.
$= (x^2 - 4)(x^2 - 9)$
So, $x^4 - 13x^2 + 36 = (x^2 - 4)(x^2 - 9)$
This can be factored further. Both $x^2 - 4$ and $x^2 - 9$ are difference of squares.
$(x^2 - 4)(x^2 - 9) = \left[(x)^2 - (2)^2\right]\left[(x)^2 - (3)^2\right]$
$= (x + 2)(x - 2)(x + 3)(x - 3)$
So, $x^4 - 13x^2 + 36 = (x + 2)(x - 2)(x + 3)(x - 3)$

b) $a^4 - 17a^2 + 16$

The first term is a perfect square since $a^4 = (a^2)(a^2)$.

The third term is a perfect square since $16 = (4)(4)$.

The second term is not twice the product of a^2 and 4 so the trinomial is not a perfect square.
So, I'll factor by decomposition.
Since the 3rd term in the trinomial is positive, the operations in the binomial factors will be the same. Since the second term is negative, these operations will be subtraction.
The two binomials will have the form: $(?a^2 - ?)(?a^2 - ?)$

The product of the coefficient of a^4 and the constant term is: (1)(16) = 16
Write $-17a^2$ as a sum of two terms whose coefficients have a product of 16.

Factors of 16	Sum of Factors
–1, –16	–1 – 16 = –17

The two coefficients are –1 and –16, so write the trinomial $a^4 - 17a^2 + 16$ as
$a^4 - a^2 - 16a^2 + 16$.
Remove a common factor from the 1st pair of terms, and from the 2nd pair of terms:
$a^4 - a^2 - 16a^2 + 16$
$= a^2(a^2 - 1) - 16(a^2 - 1)$ Remove the common binomial factor.
$= (a^2 - 1)(a^2 - 16)$
So, $a^4 - 17a^2 + 16 = (a^2 - 1)(a^2 - 16)$
This can be factored further. Both $a^2 - 1$ and $a^2 - 16$ are difference of squares.
$(a^2 - 1)(a^2 - 16) = [(a)^2 - (1)^2][(a)^2 - (4)^2]$
$= (a + 1)(a - 1)(a + 4)(a - 4)$
So, $a^4 - 17a^2 + 16 = (a + 1)(a - 1)(a + 4)(a - 4)$

c) $y^4 - 5y^2 + 4$
The first term is a perfect square since $y^4 = (y^2)(y^2)$.
The third term is a perfect square since $4 = (2)(2)$.
The second term is not twice the product of y^2 and 2 so the trinomial is not a perfect square.
So, I'll factor by decomposition.
Since the 3rd term in the trinomial is positive, the operations in the binomial factors will be the same. Since the second term is negative, these operations will be subtraction.
The two binomials will have the form: $(?y^2 - ?)(?y^2 - ?)$
The product of the coefficient of y^4 and the constant term is: (1)(4) = 4
Write $-5y^2$ as a sum of two terms whose coefficients have a product of 4.

Factors of 4	Sum of Factors
–1, –4	–1 – 4 = –5

The two coefficients are –1 and –4, so write the trinomial $y^4 - 5y^2 + 4$ as
$y^4 - y^2 - 4y^2 + 4$.
Remove a common factor from the 1st pair of terms, and from the 2nd pair of terms:
$y^4 - y^2 - 4y^2 + 4$
$= y^2(y^2 - 1) - 4(y^2 - 1)$ Remove the common binomial factor.
$= (y^2 - 1)(y^2 - 4)$

So, $y^4 - 5y^2 + 4 = (y^2 - 1)(y^2 - 4)$

This can be factored further. Both $y^2 - 1$ and $y^2 - 4$ are difference of squares.

$$(y^2 - 1)(y^2 - 4) = \left[(y)^2 - (1)^2\right]\left[(y)^2 - (2)^2\right]$$

$$= (y + 1)(y - 1)(y + 2)(y - 2)$$

So, $y^4 - 5y^2 + 4 = (y + 1)(y - 1)(y + 2)(y - 2)$

21. a) $8d^2 - 32e^2$ Remove the common factor, 8.

$8d^2 - 32e^2 = 8(d^2 - 4e^2)$

$d^2 - 4e^2$ is a difference of squares.

$$8d^2 - 32e^2 = 8\left((d)^2 - (2e)^2\right)$$

$$= 8(d + 2e)(d - 2e)$$

b) $25m^2 - \frac{1}{4}n^2$ This is a difference of squares.

$$= \left((5m)^2 - \left(\frac{1}{2}n\right)^2\right)$$

$$= \left(5m + \frac{1}{2}n\right)\left(5m - \frac{1}{2}n\right)$$

c) $18x^2y^2 - 50y^4$ Remove the common factor, $2y^2$.

$18x^2y^2 - 50y^4 = 2y^2(9x^2 - 25y^2)$

$9x^2 - 25y^2$ is a difference of squares.

$$18x^2y^2 - 50y^4 = 2y^2\left((3x)^2 - (5y)^2\right)$$

$$= 2y^2(3x + 5y)(3x - 5y)$$

d) $25s^2 + 49t^2$

This binomial cannot be factored because it is a sum of squares and not a difference of squares.

e) $10a^2 - 7b^2$

This binomial is a difference of squares but it cannot be factored over whole numbers because the coefficients of a and b are not perfect squares.

f) $\frac{x^2}{16} - \frac{y^2}{49}$ This is a difference of squares.

$= \left(\left(\frac{x}{4}\right)^2 - \left(\frac{y}{7}\right)^2\right)$

$= \left(\frac{x}{4} + \frac{y}{7}\right)\left(\frac{x}{4} - \frac{y}{7}\right)$

Review (pages 198–200)

3.1

1. Use a calculator and repeated division by prime factors.

a) 594 ÷ 2 = 297
297 ÷ 3 = 99
99 ÷ 3 = 33
33 ÷ 3 = 11
11 ÷ 11 = 1
So, 594 = 2 · 3 · 3 · 3 · 11
Using powers: $594 = 2 \cdot 3^3 \cdot 11$

b) 2100 ÷ 2 = 1050
1050 ÷ 2 = 525
525 ÷ 3 = 175
175 ÷ 5 = 35
35 ÷ 5 = 7
7 ÷ 7 = 1
So, 2100 = 2 · 2 · 3 · 5 · 5 · 7
Using powers: $2100 = 2^2 \cdot 3 \cdot 5^2 \cdot 7$

c) 4875 ÷ 3 = 1625
1625 ÷ 5 = 325
325 ÷ 5 = 65
65 ÷ 5 = 13
13 ÷ 13 = 1
So, 4875 = 3 · 5 · 5 · 5 · 13
Using powers: $4875 = 3 \cdot 5^3 \cdot 13$

d) 9009 ÷ 3 = 3003
3003 ÷ 3 = 1001
1001 ÷ 7 = 143
143 ÷ 11 = 13
13 ÷ 13 = 1
So, 9009 = 3 · 3 · 7 · 11 · 13
Using powers: $9009 = 3^2 \cdot 7 \cdot 11 \cdot 13$

2. Write the prime factorization of each number.
Highlight the factors that appear in each prime factorization.

a) 120 = **2** · **2** · 2 · 3 · **5**
160 = **2** · **2** · 2 · 2 · 2 · **5**
180 = **2** · **2** · 3 · 3 · **5**
The greatest common factor is: 2 · 2 · 5 = 20

b) 245 = **5** · **7** · 7
280 = 2 · 2 · 2 · **5** · **7**
385 = **5** · **7** · 11
The greatest common factor is: 5 · 7 = 35

c) $176 = \mathbf{2} \cdot \mathbf{2} \cdot \mathbf{2} \cdot \mathbf{2} \cdot 11$
$320 = \mathbf{2} \cdot \mathbf{2} \cdot \mathbf{2} \cdot \mathbf{2} \cdot 2 \cdot 2 \cdot 5$
$368 = \mathbf{2} \cdot \mathbf{2} \cdot \mathbf{2} \cdot \mathbf{2} \cdot 23$
The greatest common factor is: $2 \cdot 2 \cdot 2 \cdot 2 = 16$

d) $484 = \mathbf{2} \cdot \mathbf{2} \cdot 11 \cdot 11$
$496 = \mathbf{2} \cdot \mathbf{2} \cdot 2 \cdot 2 \cdot 31$
$884 = \mathbf{2} \cdot \mathbf{2} \cdot 13 \cdot 17$
The greatest common factor is: $2 \cdot 2 = 4$

3. Write the prime factorization of each number. Highlight the greatest power of each prime factor in any list. The least common multiple is the product of the greatest power of each prime factor:

a) $70 = 2 \cdot \mathbf{5} \cdot 7$
$90 = 2 \cdot \mathbf{3} \cdot \mathbf{3} \cdot 5$
$140 = \mathbf{2} \cdot \mathbf{2} \cdot 5 \cdot 7$
The greatest power of 2 in any list is: $2 \cdot 2 = 2^2$
The greatest power of 3 in any list is: $3 \cdot 3 = 3^2$
The greatest power of 5 in any list is 5.
The greatest power of 7 in any list is 7.
The least common multiple is: $2^2 \cdot 3^2 \cdot 5 \cdot 7 = 1260$
So, the least common multiple of 70, 90, and 140 is 1260.

b) $120 = \mathbf{2} \cdot \mathbf{2} \cdot \mathbf{2} \cdot \mathbf{3} \cdot \mathbf{5}$
$130 = 2 \cdot 5 \cdot \mathbf{13}$
$309 = 3 \cdot \mathbf{103}$
The greatest power of 2 in any list is: $2 \cdot 2 \cdot 2 = 2^3$
The greatest power of 3 in any list is: 3
The greatest power of 5 in any list is 5.
The greatest power of 13 in any list is 13.
The greatest power of 103 in any list is 103.
The least common multiple is: $2^3 \cdot 3 \cdot 5 \cdot 13 \cdot 103 = 160\ 680$
So, the least common multiple of 120, 130, and 309 is 160 680.

c) $200 = \mathbf{2} \cdot \mathbf{2} \cdot \mathbf{2} \cdot 5 \cdot 5$
$250 = 2 \cdot \mathbf{5} \cdot \mathbf{5} \cdot \mathbf{5}$
$500 = 2 \cdot 2 \cdot 5 \cdot 5 \cdot 5$
The greatest power of 2 in any list is: $2 \cdot 2 \cdot 2 = 2^3$
The greatest power of 5 in any list is: $5 \cdot 5 \cdot 5 = 5^3$
The least common multiple is: $2^3 \cdot 5^3 = 1000$
So, the least common multiple of 200, 250, and 500 is 1000.

d) $180 = 2 \cdot 2 \cdot \mathbf{3} \cdot \mathbf{3} \cdot \mathbf{5}$
$240 = \mathbf{2} \cdot \mathbf{2} \cdot \mathbf{2} \cdot \mathbf{2} \cdot 3 \cdot 5$
$340 = 2 \cdot 2 \cdot 5 \cdot \mathbf{17}$
The greatest power of 2 in any list is: $2 \cdot 2 \cdot 2 \cdot 2 = 2^4$
The greatest power of 3 in any list is: $3 \cdot 3 = 3^2$
The greatest power of 5 in any list is: 5
The greatest power of 17 in any list is: 17
The least common multiple is: $2^4 \cdot 3^2 \cdot 5 \cdot 17 = 12\ 240$
So, the least common multiple of 180, 240, and 340 is 12 240.

4. The least number of beads on any strand is 1 more than the least common multiple of 6, 4, and 10.
Factor each number.
$6 = 2 \cdot \mathbf{3}$
$4 = \mathbf{2} \cdot \mathbf{2}$
$10 = 2 \cdot \mathbf{5}$
The least common multiple of 6, 4, and 10 is: $2 \cdot 2 \cdot 3 \cdot 5 = 60$
So, the least number of beads on each strand is 61.

5. For parts a, b, and c, a fraction is simplified when its numerator and denominator have no common factors. So, divide the numerator and denominator of each fraction by their greatest common factor.

a) $\frac{1015}{1305}$
Write the prime factorization of the numerator and of the denominator.
Highlight the prime factors that appear in both lists.
$1015 = \mathbf{5} \cdot 7 \cdot \mathbf{29}$
$1305 = 3 \cdot 3 \cdot \mathbf{5} \cdot \mathbf{29}$
The greatest common factor is: $5 \cdot 29 = 145$

$$\frac{1015}{1305} = \frac{1015 \div 145}{1305 \div 145} = \frac{7}{9}$$

b) $\frac{2475}{3825}$
Write the prime factorization of the numerator and of the denominator.
Highlight the prime factors that appear in both lists.
$2475 = \mathbf{3} \cdot \mathbf{3} \cdot \mathbf{5} \cdot \mathbf{5} \cdot 11$
$3825 = \mathbf{3} \cdot \mathbf{3} \cdot \mathbf{5} \cdot \mathbf{5} \cdot 17$
The greatest common factor is: $3 \cdot 3 \cdot 5 \cdot 5 = 225$

$$\frac{2475}{3825} = \frac{2475 \div 225}{3825 \div 225} = \frac{11}{17}$$

c) $\dfrac{6656}{7680}$

Write the prime factorization of the numerator and of the denominator.
Highlight the prime factors that appear in both lists.
$6656 = \mathbf{2} \cdot \mathbf{2} \cdot \mathbf{2} \cdot \mathbf{2} \cdot \mathbf{2} \cdot \mathbf{2} \cdot \mathbf{2} \cdot \mathbf{2} \cdot \mathbf{2} \cdot 13$
$7680 = \mathbf{2} \cdot \mathbf{2} \cdot \mathbf{2} \cdot \mathbf{2} \cdot \mathbf{2} \cdot \mathbf{2} \cdot \mathbf{2} \cdot \mathbf{2} \cdot \mathbf{2} \cdot 2 \cdot 3 \cdot 5$
The greatest common factor is: $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 512$

$$\frac{6656}{7680} = \frac{6656 \div 512}{7680 \div 512} = \frac{13}{15}$$

For parts d, e, and f, add, divide, or subtract fractions with a common denominator by adding, dividing, or subtracting their numerators. So, write the fractions in each part with denominator equal to the least common multiple of the original denominators.

d) $\dfrac{7}{36} + \dfrac{15}{64}$

Write the prime factorization of the denominators.
Highlight the greatest power of each prime factor in either list.
$36 = 2 \cdot 2 \cdot \mathbf{3} \cdot \mathbf{3}$
$64 = \mathbf{2} \cdot \mathbf{2} \cdot \mathbf{2} \cdot \mathbf{2} \cdot \mathbf{2} \cdot \mathbf{2}$
The least common multiple is: $2^6 \cdot 3^2 = 576$

$$\frac{7}{36} + \frac{15}{64} = \frac{112}{576} + \frac{135}{576} = \frac{112 + 135}{576} = \frac{247}{576}$$

e) $\dfrac{5}{9} \div \dfrac{3}{4}$

Write the prime factorization of the denominators.
Highlight the greatest power of each prime factor in either list.
$9 = \mathbf{3} \cdot \mathbf{3}$
$4 = \mathbf{2} \cdot \mathbf{2}$
The least common multiple is: $3^2 \cdot 2^2 = 36$

$$\frac{5}{9} \div \frac{3}{4} = \frac{20}{36} \div \frac{27}{36} = \frac{20}{27}$$

f) $\frac{28}{128} - \frac{12}{160}$

Both fractions can be simplified before they are subtracted.

For $\frac{28}{128}$, the numerator and denominator have a common factor of 4, so divide numerator and denominator by 4 to get $\frac{7}{32}$.

For $\frac{12}{160}$, the numerator and denominator have a common factor of 4, so divide numerator and denominator by 4 to get $\frac{3}{40}$.

The expression becomes: $\frac{7}{32} - \frac{3}{40}$

Write the prime factorization of the denominators.
Highlight the greatest power of each prime factor in either list.

$32 = \mathbf{2} \cdot \mathbf{2} \cdot \mathbf{2} \cdot \mathbf{2} \cdot \mathbf{2}$

$40 = 2 \cdot 2 \cdot 2 \cdot \mathbf{5}$

The least common multiple is: $2^5 \cdot 5 = 160$

$$\frac{7}{32} - \frac{3}{40} = \frac{35}{160} - \frac{12}{160}$$
$$= \frac{23}{160}$$

3.2

6. Determine the prime factors of each area. If the factors can be arranged in 2 equal groups, then the area is a perfect square. The side length of the square is the square root of the area.

a) $784 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 7 \cdot 7$
$= (2 \cdot 2 \cdot 7)(2 \cdot 2 \cdot 7)$ This is a perfect square.
$= 28 \cdot 28$

So, $\sqrt{784} = 28$

The side length of the square is 28 in.

b) $1024 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$
$= (2 \cdot 2 \cdot 2 \cdot 2 \cdot 2)(2 \cdot 2 \cdot 2 \cdot 2 \cdot 2)$ This is a perfect square.
$= 32 \cdot 32$

So, $\sqrt{1024} = 32$

The side length of the square is 32 cm.

7. Determine the prime factors of each volume. If the factors can be arranged in 3 equal groups, then the volume is a perfect cube. The edge length of the cube is the cube root of the volume.

a) $1728 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot 3$
$= (2 \cdot 2 \cdot 3)(2 \cdot 2 \cdot 3)(2 \cdot 2 \cdot 3)$
$= 12 \cdot 12 \cdot 12$

So, $\sqrt[3]{1728} = 12$
The edge length of the cube is 12 cm.

b) $2744 = 2 \cdot 2 \cdot 2 \cdot 7 \cdot 7 \cdot 7$
$= (2 \cdot 7)(2 \cdot 7)(2 \cdot 7)$
$= 14 \cdot 14 \cdot 14$
So, $\sqrt[3]{2744} = 14$
The edge length of the cube is 14 ft.

8. Write each number as a product of its prime factors. If the factors can be arranged in 2 equal groups, then the number is a perfect square. If the factors can be arranged in 3 equal groups, then the number is a perfect cube.

a) $256 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$
$= (2 \cdot 2 \cdot 2 \cdot 2)(2 \cdot 2 \cdot 2 \cdot 2)$
$= 16 \cdot 16$
The factors can be arranged in 2 equal groups, so 256 is a perfect square.
$\sqrt{256} = 16$

b) $324 = 2 \cdot 2 \cdot 3 \cdot 3 \cdot 3 \cdot 3$
$= (2 \cdot 3 \cdot 3)(2 \cdot 3 \cdot 3)$
$= 18 \cdot 18$
The factors can be arranged in 2 equal groups, so 324 is a perfect square.
$\sqrt{324} = 18$

c) $729 = 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3$
$= (3 \cdot 3 \cdot 3)(3 \cdot 3 \cdot 3)$, or $27 \cdot 27$
$= (3 \cdot 3)(3 \cdot 3)(3 \cdot 3)$, or $9 \cdot 9 \cdot 9$
The factors can be arranged in 2 equal groups and 3 equal groups, so 729 is both a perfect square and a perfect cube.
$\sqrt{729} = 27$
$\sqrt[3]{729} = 9$

d) $1298 = 2 \cdot 11 \cdot 59$
The factors cannot be arranged in 2 equal groups or 3 equal groups, so 1298 is neither a perfect square nor a perfect cube.

e) $1936 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 11 \cdot 11$
$= (2 \cdot 2 \cdot 11)(2 \cdot 2 \cdot 11)$
$= 44 \cdot 44$
The factors can be arranged in 2 equal groups, so 1936 is a perfect square.
$\sqrt{1936} = 44$

f) $9261 = 3 \cdot 3 \cdot 3 \cdot 7 \cdot 7 \cdot 7$
$= (3 \cdot 7)(3 \cdot 7)(3 \cdot 7)$
$= 21 \cdot 21 \cdot 21$
The factors can be arranged 3 equal groups, so 9261 is a perfect cube.
$\sqrt[3]{9261} = 21$

9. The area of a square is 18 225 square feet.
The side length of the square is the square root of the area.
$18\ 225 = 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 5 \cdot 5$
$= (3 \cdot 3 \cdot 3 \cdot 5)(3 \cdot 3 \cdot 3 \cdot 5)$
$= 135 \cdot 135$
$\sqrt{18\ 225} = 135$
The side length of the square is 135 ft.
The perimeter of the square is: 4(135 ft.) = 540 ft.

10. The surface area of a cube is 11 616 cm^2.
So, the area of one face of the cube is: $\dfrac{11\ 616}{6}$ cm = 1936
The edge length of the cube is the square root of the area of one face.
$1936 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 11 \cdot 11$
$= (2 \cdot 2 \cdot 11)(2 \cdot 2 \cdot 11)$
$= 44 \cdot 44$
So, $\sqrt{1936} = 44$
The edge length of the cube is 44 cm.

3.3

11. For each binomial, factor each term then identify the greatest common factor. Write each term as the product of the greatest common factor and another monomial. Then use the distributive property to write the expression as a product.

a) $8m = \mathbf{2} \cdot \mathbf{2} \cdot 2 \cdot \boldsymbol{m}$
$4m^2 = \mathbf{2} \cdot \mathbf{2} \cdot \boldsymbol{m} \cdot m$
The greatest common factor is: $2 \cdot 2 \cdot m = 4m$
$8m - 4m^2 = 4m(2) - 4m(m)$
$= 4m(2 - m)$
I could arrange algebra tiles as a rectangle; the rectangle would have length $2 - m$ and width $4m$. I would have to be careful with the signs of the factors because the rectangle would show a width of $-4m$.

b) $3 = \mathbf{3}$
$9g^2 = \mathbf{3} \cdot 3 \cdot g \cdot g$
The greatest common factor is: 3
$-3 + 9g^2 = 3(-1) + 3(3g^2)$
$= 3(-1 + 3g^2)$ Remove -1 as a common factor.
$= -3(1 - 3g^2)$
I could arrange algebra tiles into equal groups. There would be 3 equal groups with 1 negative 1-tile and 3 g^2-tiles in each group.

c) $28a^2 = 2 \cdot 2 \cdot \mathbf{7} \cdot \boldsymbol{a} \cdot \boldsymbol{a}$
$7a^3 = \mathbf{7} \cdot \boldsymbol{a} \cdot \boldsymbol{a} \cdot a$
The greatest common factor is: $7 \cdot a \cdot a = 7a^2$
$28a^2 - 7a^3 = 7a^2(4) - 7a^2(a)$
$= 7a^2(4 - a)$
I could not use algebra tiles because I do not have an a^3-tile.

d) $6a^2b^3c = 2 \cdot \mathbf{3} \cdot \boldsymbol{a} \cdot \boldsymbol{a} \cdot \boldsymbol{b} \cdot \boldsymbol{b} \cdot b \cdot \boldsymbol{c}$
$15a^2b^2c^2 = \mathbf{3} \cdot 5 \cdot \boldsymbol{a} \cdot \boldsymbol{a} \cdot \boldsymbol{b} \cdot \boldsymbol{b} \cdot \boldsymbol{c} \cdot c$
The greatest common factor is: $3 \cdot a \cdot a \cdot b \cdot b \cdot c = 3a^2b^2c$
$6a^2b^3c - 15a^2b^2c^2 = 3a^2b^2c(2b) - 3a^2b^2c(5c)$
$= 3a^2b^2c(2b - 5c)$
I could not use algebra tiles because I do not have tiles for terms with more than one variable.

e) $24m^2n = 2 \cdot \mathbf{2} \cdot 2 \cdot \mathbf{3} \cdot \boldsymbol{m} \cdot m \cdot \boldsymbol{n}$
$6mn^2 = \mathbf{2} \cdot \mathbf{3} \cdot \boldsymbol{m} \cdot \boldsymbol{n} \cdot n$
The greatest common factor is: $2 \cdot 3 \cdot m \cdot n = 6mn$
$-24m^2n - 6mn^2 = 6mn(-4m) + 6mn(-n)$
$= 6mn(-4m - n)$ Remove –1 as a common factor.
$= -6mn(4m + n)$
I could not use algebra tiles because I do not have tiles for terms with more than one variable.

f) $14b^3c^2 = 2 \cdot \mathbf{7} \cdot \boldsymbol{b} \cdot \boldsymbol{b} \cdot b \cdot c \cdot c$
$21a^3b^2 = 3 \cdot \mathbf{7} \cdot a \cdot a \cdot a \cdot \boldsymbol{b} \cdot \boldsymbol{b}$
The greatest common factor is: $7 \cdot b \cdot b = 7b^2$
$14b^3c^2 - 21a^3b^2 = 7b^2(2bc^2) - 7b^2(3a^3)$
$= 7b^2(2bc^2 - 3a^3)$
I could not use algebra tiles because I do not have tiles for terms with more than one variable.

12. For each trinomial, factor each term then identify the greatest common factor. Write each term as the product of the greatest common factor and another monomial. Then use the distributive property to write the expression as a product. Expand to verify.

a) $12 = 2 \cdot 2 \cdot \mathbf{3}$
$6g = 2 \cdot \mathbf{3} \cdot g$
$3g^2 = \mathbf{3} \cdot g \cdot g$
The greatest common factor is: 3
$12 + 6g - 3g^2 = 3(4) + 3(2g) - 3(g^2)$
$= 3(4 + 2g - g^2)$
Check: $3(4 + 2g - g^2) = 3(4) + 3(2g) - 3(g^2)$
$= 12 + 6g - 3g^2$

b) $3c^2d = 3 \cdot c \cdot c \cdot \boldsymbol{d}$
$10cd = 2 \cdot 5 \cdot c \cdot \boldsymbol{d}$
$2d = 2 \cdot \boldsymbol{d}$
The greatest common factor is: d
$3c^2d - 10cd - 2d = d(3c^2) - d(10c) - d(2)$
$= d(3c^2 - 10c - 2)$
Check: $d(3c^2 - 10c - 2) = d(3c^2) - d(10c) - d(2)$
$= 3c^2d - 10cd - 2d$

c) $8mn^2 = \mathbf{2} \cdot \mathbf{2} \cdot 2 \cdot \boldsymbol{m} \cdot \boldsymbol{n} \cdot n$
$12mn = \mathbf{2} \cdot \mathbf{2} \cdot 3 \cdot \boldsymbol{m} \cdot \boldsymbol{n}$
$16m^2n = \mathbf{2} \cdot \mathbf{2} \cdot 2 \cdot 2 \cdot \boldsymbol{m} \cdot m \cdot \boldsymbol{n}$
The greatest common factor is: $2 \cdot 2 \cdot m \cdot n = 4mn$
$8mn^2 - 12mn - 16m^2n = 4mn(2n) - 4mn(3) - 4mn(4m)$

$= 4mn(2n - 3 - 4m)$

Check: $4mn(2n - 3 - 4m) = 4mn(2n) - 4mn(3) - 4mn(4m)$
$= 8mn^2 - 12mn - 16m^2n$

d) $y^4 = \mathbf{y} \cdot y \cdot y \cdot y$
$12y^2 = 2 \cdot 2 \cdot 3 \cdot \mathbf{y} \cdot y$
$24y = 2 \cdot 2 \cdot 2 \cdot 3 \cdot \mathbf{y}$
The greatest common factor is: y
$y^4 - 12y^2 + 24y = y(y^3) - y(12y) + y(24)$
$= y(y^3 - 12y + 24)$
Check: $y(y^3 - 12y + 24) = y(y^3) - y(12y) + y(24)$
$= y^4 - 12y^2 + 24y$

e) $30x^2y = \mathbf{2} \cdot 3 \cdot \mathbf{5} \cdot \mathbf{x} \cdot \mathbf{x} \cdot \mathbf{y}$
$20x^2y^2 = \mathbf{2} \cdot 2 \cdot \mathbf{5} \cdot \mathbf{x} \cdot \mathbf{x} \cdot \mathbf{y} \cdot y$
$10x^3y^2 = \mathbf{2} \cdot \mathbf{5} \cdot \mathbf{x} \cdot \mathbf{x} \cdot x \cdot \mathbf{y} \cdot y$
The greatest common factor is: $2 \cdot 5 \cdot x \cdot x \cdot y = 10x^2y$
$30x^2y - 20x^2y^2 + 10x^3y^2 = 10x^2y(3) - 10x^2y(2y) + 10x^2y(xy)$
$= 10x^2y(3 - 2y + xy)$
Check: $10x^2y(3 - 2y + xy) = 10x^2y(3) - 10x^2y(2y) + 10x^2y(xy)$
$= 30x^2y - 20x^2y^2 + 10x^3y^2$

f) $8b^3 = \mathbf{2} \cdot \mathbf{2} \cdot 2 \cdot \mathbf{b} \cdot b \cdot b$
$20b^2 = \mathbf{2} \cdot \mathbf{2} \cdot 5 \cdot \mathbf{b} \cdot b$
$4b = \mathbf{2} \cdot \mathbf{2} \cdot \mathbf{b}$
The greatest common factor is: $2 \cdot 2 \cdot b = 4b$
$-8b^3 + 20b^2 - 4b = 4b(-2b^2) + 4b(5b) + 4b(-1)$
$= 4b(-2b^2 + 5b - 1)$ Remove –1 as a common factor.
$= -4b(2b^2 - 5b + 1)$
Check: $-4b(2b^2 - 5b + 1) = -4b(2b^2) - 4b(-5b) - 4b(1)$
$= -8b^3 + 20b^2 - 4b$

13. For each polynomial, factor each term then identify the greatest common factor. Write each term as the product of the greatest common factor and another monomial. Then use the distributive property to write the expression as a product. Expand to verify.

a) $8x^2 = \mathbf{2} \cdot \mathbf{2} \cdot 2 \cdot \mathbf{x} \cdot x$
$12x = \mathbf{2} \cdot \mathbf{2} \cdot 3 \cdot \mathbf{x}$
The greatest common factor is: $2 \cdot 2 \cdot x = 4x$
$8x^2 - 12x = 4x(2x) - 4x(3)$
$= 4x(2x - 3)$
Check: $4x(2x - 3) = 4x(2x) - 4x(3)$
$= 8x^2 - 12x$

b) $3y^3 = \mathbf{3} \cdot \mathbf{y} \cdot y \cdot y$
$12y^2 = 2 \cdot 2 \cdot \mathbf{3} \cdot \mathbf{y} \cdot y$
$15y = \mathbf{3} \cdot 5 \cdot \mathbf{y}$
The greatest common factor is: $3 \cdot y = 3y$
$3y^3 - 12y^2 + 15y = 3y(y^2) - 3y(4y) + 3y(5)$
$= 3y(y^2 - 4y + 5)$
Check: $3y(y^2 - 4y + 5) = 3y(y^2) - 3y(4y) + 3y(5)$
$= 3y^3 - 12y^2 + 15y$

c) $4b^3 = \mathbf{2} \cdot 2 \cdot \boldsymbol{b} \cdot b \cdot b$
$2b = \mathbf{2} \cdot \boldsymbol{b}$
$6b^2 = \mathbf{2} \cdot 3 \cdot \boldsymbol{b} \cdot b$
The greatest common factor is: $2 \cdot b = 2b$
$4b^3 - 2b - 6b^2 = 2b(2b^2) - 2b(1) - 2b(3b)$
$= 2b(2b^2 - 1 - 3b)$
Check: $2b(2b^2 - 1 - 3b) = 2b(2b^2) - 2b(1) - 2b(3b)$
$= 4b^3 - 2b - 6b^2$

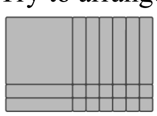
d) $6m^3 = \mathbf{2} \cdot \mathbf{3} \cdot \boldsymbol{m} \cdot m \cdot m$
$12m = \mathbf{2} \cdot 2 \cdot \mathbf{3} \cdot \boldsymbol{m}$
$24m^2 = \mathbf{2} \cdot 2 \cdot 2 \cdot \mathbf{3} \cdot \boldsymbol{m} \cdot m$
The greatest common factor is: $2 \cdot 3 \cdot m = 6m$
$6m^3 - 12m - 24m^2 = 6m(m^2) - 6m(2) - 6m(4m)$
$= 6m(m^2 - 2 - 4m)$
Check: $6m(m^2 - 2 - 4m) = 6m(m^2) - 6m(2) - 6m(4m)$
$= 6m^3 - 12m - 24m^2$

14. a) Look at the trinomial factor. Each term contains q, so the common factor of the given trinomial is $5q$. Remove q from each term in the trinomial factor. The correct solution is:
$15p^2q + 25pq^2 - 35q^3 = 5q(3p^2 + 5pq - 7q^2)$

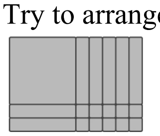
b) The common factor, –3, is correct, but it was not removed from the 2nd and 3rd terms of the given trinomial. Remove –3 from these terms. Also, when –3 was removed from the first term of the given trinomial, the sign should have been changed. The correct solution is:
$-12mn + 15m^2 + 18n^2 = -3(4mn - 5m^2 - 6n^2)$

3.4

15. a) Try to arrange 1 x^2-tile, 8 x-tiles, and twelve 1-tiles in a rectangle. A rectangle is possible.

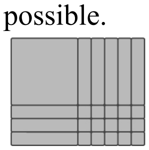


b) Try to arrange 1 x^2-tile, 7 x-tiles, and ten 1-tiles in a rectangle. A rectangle is possible.



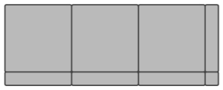
c) Try to arrange 1 x^2-tile, 4 x-tiles, and one 1-tile in a rectangle. A rectangle is not possible.

d) Try to arrange 1 x^2-tile, 8 x-tiles, and fifteen 1-tiles in a rectangle. A rectangle is possible.

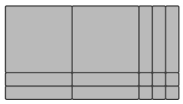


16. a) Try to arrange 2 k^2-tiles, 3 k-tiles, and two 1-tiles in a rectangle. A rectangle is not possible.

b) Try to arrange 3 g^2-tiles, 4 g-tiles, and one 1-tile in a rectangle. A rectangle is possible.



c) Try to arrange 2 t^2-tiles, 7 t-tiles, and six 1-tiles in a rectangle. A rectangle is possible.



d) Try to arrange 7 h^2-tiles, 5 h-tiles, and one 1-tile in a rectangle. A rectangle is not possible.

17. Since the number of 1-tiles is a prime number, these tiles can only be arranged in a row of 5. Arrange the tiles this way, at the bottom right corner of the x^2-tile. Complete a rectangle by adding x-tiles. Six x-tiles are needed.

3.5

18. Use the distributive property to expand. Draw a rectangle and label its sides with the binomial factors. Divide the rectangle into 4 smaller rectangles, and label each with a term in the expansion.

a)
$$\begin{aligned}(g + 5)(g - 4) &= g(g - 4) + 5(g - 4)\\ &= g^2 - 4g + 5g - 20\\ &= g^2 + g - 20\end{aligned}$$

	g	-4
g	$(g)(g) = g^2$	$(g)(-4) = -4g$
5	$(5)(g) = 5g$	$(5)(-4) = -20$

b)
$$\begin{aligned}(h + 7)(h + 7) &= h(h + 7) + 7(h + 7)\\ &= h^2 + 7h + 7h + 49\\ &= h^2 + 14h + 49\end{aligned}$$

	h	7
h	$(h)(h) = h^2$	$(h)(7) = 7h$
7	$(7)(h) = 7h$	$(7)(7) = 49$

c)
$$\begin{aligned}(k - 4)(k + 11) &= k(k + 11) - 4(k + 11)\\ &= k^2 + 11k - 4k - 44\end{aligned}$$

$= k^2 + 7k - 44$

	k	11
k	$(k)(k) = k^2$	$(k)(11) = 11k$
–4	$(-4)(k) = -4k$	$(-4)(11) = -44$

d) $(9 + s)(9 - s) = 9(9 - s) + s(9 - s)$
$= 81 - 9s + 9s - s^2$
$= 81 - s^2$

	9	$-s$
9	$(9)(9) = 81$	$(9)(-s) = -9s$
s	$(s)(9) = 9s$	$(s)(-s) = -s^2$

e) $(12 - t)(12 - t) = 12(12 - t) - t(12 - t)$
$= 144 - 12t - 12t + t^2$
$= 144 - 24t + t^2$

	12	$-t$
12	$(12)(12) = 144$	$(12)(-t) = -12t$
$-t$	$(-t)(12) = -12t$	$(-t)(-t) = t^2$

f) $(7 + r)(6 - r) = 7(6 - r) + r(6 - r)$
$= 42 - 7r + 6r - r^2$
$= 42 - r - r^2$

	6	$-r$
7	$(7)(6) = 42$	$(7)(-r) = -7r$
r	$(r)(6) = 6r$	$(r)(-r) = -r^2$

g) $(y - 3)(y - 11) = y(y - 11) - 3(y - 11)$
$= y^2 - 11y - 3y + 33$
$= y^2 - 14y + 33$

	y	–11
y	$(y)(y) = y^2$	$(y)(-11) = -11y$
–3	$(-3)(y) = -3y$	$(-3)(-11) = 33$

h) $(x - 5)(x + 5) = x(x + 5) - 5(x + 5)$
$= x^2 + 5x - 5x - 25$

$= x^2 - 25$

	x	5
x	$(x)(x) = x^2$	$(x)(5) = 5x$
-5	$(-5)(x) = -5x$	$(-5)(5) = -25$

19. For each trinomial, use mental math to find two numbers whose sum is equal to the coefficient of the middle term and whose product is equal to the constant term. These numbers are the constants in the binomial factors.

a) $q^2 + 6q + 8$
Since all the terms are positive, consider only positive factors of 8.
The factors of 8 are: 1 and 8; 2 and 4
The two factors with a sum of 6 are 2 and 4.
So, $q^2 + 6q + 8 = (q + 2)(q + 4)$
Check by expanding.
$(q + 2)(q + 4) = q(q + 4) + 2(q + 4)$
$= q^2 + 4q + 2q + 8$
$= q^2 + 6q + 8$

b) $n^2 - 4n - 45$
The factors of –45 are: 1 and –45; –1 and 45; 3 and –15; –3 and 15; 5 and –9; –5 and 9
The two factors with a sum of –4 are 5 and –9.
So, $n^2 - 4n - 45 = (n + 5)(n - 9)$
Check by expanding.
$(n + 5)(n - 9) = n(n - 9) + 5(n - 9)$
$= n^2 - 9n + 5n - 45$
$= n^2 - 4n - 45$

c) $54 - 15s + s^2$
Since the constant term is positive, the two numbers in the binomials have the same sign. Since the coefficient of s is negative, both numbers are negative. So, list only the negative factors of 54.
The negative factors of 54 are: –1 and –54; –2 and –27; –3 and –18; –6 and –9
The two factors with a sum of –15 are –6 and –9.
So, $54 - 15s + s^2 = (-6 + s)(-9 + s)$
Multiply each binomial by –1; which is the same as multiplying their product by 1, so its value is not changed.
So, the factors can be written as: $(6 - s)(9 - s)$
Check by expanding.
$(6 - s)(9 - s) = 6(9 - s) - s(9 - s)$
$= 54 - 6s - 9s + s^2$
$= 54 - 15s + s^2$

d) $k^2 - 9k - 90$
The factors of –90 are: 1 and –90; –1 and 90; 2 and –45; –2 and 45; 3 and –30; –3 and 30; 5 and –18; –5 and 18; 6 and –15; –6 and 15; 9 and –10; –9 and 10
The two factors with a sum of –9 are 6 and –15.
So, $k^2 - 9k - 90 = (k + 6)(k - 15)$
Check by expanding.

$(k + 6)(k - 15) = k(k - 15) + 6(k - 15)$
$= k^2 - 15k + 6k - 90$
$= k^2 - 9k - 90$

e) $x^2 - x - 20$
The factors of –20 are: 1 and –20; –1 and 20; 2 and –10; –2 and 10; 4 and –5; –4 and 5
The two factors with a sum of –1 are 4 and –5.
So, $x^2 - x - 20 = (x + 4)(x - 5)$
Check by expanding.
$(x + 4)(x - 5) = x(x - 5) + 4(x - 5)$
$= x^2 - 5x + 4x - 20$
$= x^2 - x - 20$

f) $12 - 7y + y^2$
Since the constant term is positive, the two numbers in the binomials have the same sign. Since the coefficient of y is negative, both numbers are negative. So, list only the negative factors of 12.
The negative factors of 12 are: –1 and –12; –2 and –6; –3 and –4
The two factors with a sum of –7 are –3 and –4.
So, $12 - 7y + y^2 = (-3 + y)(-4 + y)$
Multiply each binomial by –1; which is the same as multiplying their product by 1, so its value is not changed.
So, the factors can be written as: $(3 - y)(4 - y)$
Check by expanding.
$(3 - y)(4 - y) = 3(4 - y) - y(4 - y)$
$= 12 - 3y - 4y + y^2$
$= 12 - 7y + y^2$

20. For each trinomial, use mental math to find two numbers whose sum is equal to the coefficient of the middle term and whose product is equal to the constant term. These numbers are the constants in the binomial factors.

a) i) $m^2 + 7m + 12$
Since all the terms are positive, consider only positive factors of 12.
The factors of 12 are: 1 and 12; 2 and 6; 3 and 4
The two factors with a sum of 7 are 3 and 4.
So, $m^2 + 7m + 12 = (m + 3)(m + 4)$

a) ii) $m^2 + 8m + 12$
Use the factors of 12 from part i.
The two factors with a sum of 8 are 2 and 6.
So, $m^2 + 8m + 12 = (m + 2)(m + 6)$

a) iii) $m^2 + 13m + 12$
Use the factors of 12 from part i.
The two factors with a sum of 13 are 1 and 12.
So, $m^2 + 13m + 12 = (m + 1)(m + 12)$

a) iv) $m^2 - 7m + 12$
Since the constant term is positive, the two numbers in the binomials have the same sign. Since the coefficient of m is negative, both numbers are negative. So, list only the negative factors of 12.

The negative factors of 12 are: –1 and –12; –2 and –6; –3 and –4
The two factors with a sum of –7 are –3 and –4.
So, $m^2 - 7m + 12 = (m - 3)(m - 4)$

a) v) $m^2 - 8m + 12$
Use the factors of 12 from part iv.
The two factors with a sum of –8 are –2 and –6.
So, $m^2 - 8m + 12 = (m - 2)(m - 6)$

a) vi) $m^2 - 13m + 12$
Use the factors of 12 from part iv.
The two factors with a sum of –13 are –1 and –12.
So, $m^2 - 13m + 12 = (m - 1)(m - 12)$

b) There are no more trinomials that begin with m^2, end in +12, and can be factored. In part a, there are only 6 pairs of factors of 12. All these pairs have already been used to factor the 6 trinomials in part a.

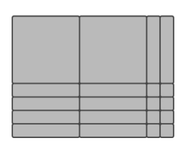
21. a) The constant terms in the binomials have a sum of 12 and a product of 27. Their sum should be –12. So, these constant terms should be –3 and –9. The correct solution is:
$u^2 - 12u + 27 = (u - 3)(u - 9)$

b) The constant terms in the binomials have a sum of 1 and a product of –20. Their sum should be –1. So, these constant terms should be 4 and –5. The correct solution is:
$v^2 - v - 20 = (v + 4)(v - 5)$

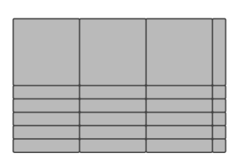
c) The constant terms in the binomials have a sum of 10 and a product of 24. Their product should be –24. So, the constant terms have opposite signs; they are factors of –24 with a sum of 10. The factors are –2 and 12. The correct solution is:
$w^2 + 10w - 24 = (w - 2)(w + 12)$

3.6

22. a) $(h + 4)(2h + 2)$
Use algebra tiles.
Make a rectangle with dimensions $h + 4$ and $2h + 2$.
Place tiles to represent each dimension,
then fill the rectangle with tiles.
The tiles that form the product represent $2h^2 + 10h + 8$.
So, $(h + 4)(2h + 2) = 2h^2 + 10h + 8$

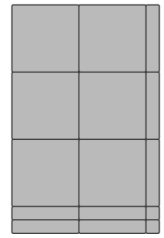


b) $(j + 5)(3j + 1)$
Use algebra tiles.
Make a rectangle with dimensions $j + 5$ and $3j + 1$.
Place tiles to represent each dimension,
then fill the rectangle with tiles.
The tiles that form the product represent $3j^2 + 16j + 5$.
So, $(j + 5)(3j + 1) = 3j^2 + 16j + 5$



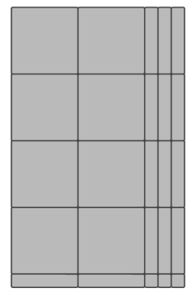
c) $(3k + 2)(2k + 1)$

Use algebra tiles.
Make a rectangle with dimensions $3k + 2$ and $2k + 1$.
Place tiles to represent each dimension,
then fill the rectangle with tiles.
The tiles that form the product represent $6k^2 + 7k + 2$.
So, $(3k + 2)(2k + 1) = 6k^2 + 7k + 2$



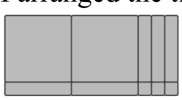
d) $(4m + 1)(2m + 3)$

Use algebra tiles.
Make a rectangle with dimensions $4m + 1$ and $2m + 3$.
Place tiles to represent each dimension,
then fill the rectangle with tiles.
The tiles that form the product represent $8m^2 + 14m + 3$.
So, $(4m + 1)(2m + 3) = 8m^2 + 14m + 3$



23. a) **i)** There are 2 x^2-tiles, 5 x-tiles, and 3 unit tiles.
So, the algebra tiles represent the trinomial: $2x^2 + 5x + 3$

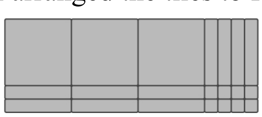
ii) I arranged the tiles to form a rectangle:



iii) I can use the rectangle to factor the trinomial.
The rectangle has side lengths $2x + 3$ and $x + 1$.
So, $2x^2 + 5x + 3 = (2x + 3)(x + 1)$

b) **i)** There are 3 x^2-tiles, 10 x-tiles, and 8 unit tiles.
So, the algebra tiles represent the trinomial: $3x^2 + 10x + 8$

ii) I arranged the tiles to form a rectangle:



iii) I can use the rectangle to factor the trinomial.
The rectangle has side lengths $3x + 4$ and $x + 2$.
So, $3x^2 + 10x + 8 = (3x + 4)(x + 2)$

24. Use the distributive property to expand. Draw a rectangle and label its sides with the binomial factors. Divide the rectangle into 4 smaller rectangles, and label each with a term in the expansion.

a) $(2r + 7)(3r + 5) = 2r(3r + 5) + 7(3r + 5)$

$= 6r^2 + 10r + 21r + 35$
$= 6r^2 + 31r + 35$

	$3r$	5
$2r$	$(2r)(3r) = 6r^2$	$(2r)(5) = 10r$
7	$(7)(3r) = 21r$	$(7)(5) = 35$

b) $(9y + 1)(y - 9) = 9y(y - 9) + 1(y - 9)$
$= 9y^2 - 81y + y - 9$
$= 9y^2 - 80y - 9$

	y	-9
$9y$	$(9y)(y) = 9y^2$	$(9y)(-9) = -81y$
1	$(1)(y) = y$	$(1)(-9) = -9$

c) $(2a - 7)(2a - 6) = 2a(2a - 6) - 7(2a - 6)$
$= 4a^2 - 12a - 14a + 42$
$= 4a^2 - 26a + 42$

	$2a$	-6
$2a$	$(2a)(2a) = 4a^2$	$(2a)(-6) = -12a$
-7	$(-7)(2a) = -14a$	$(-7)(-6) = 42$

d) $(3w - 2)(3w - 1) = 3w(3w - 1) - 2(3w - 1)$
$= 9w^2 - 3w - 6w + 2$
$= 9w^2 - 9w + 2$

	$3w$	-1
$3w$	$(3w)(3w) = 9w^2$	$(3w)(-1) = -3w$
-2	$(-2)(3w) = -6w$	$(-2)(-1) = 2$

e) $(4p + 5)(4p + 5) = 4p(4p + 5) + 5(4p + 5)$
$= 16p^2 + 20p + 20p + 25$
$= 16p^2 + 40p + 25$

	$4p$	5
$4p$	$(4p)(4p) = 16p^2$	$(4p)(5) = 20p$
5	$(5)(4p) = 20p$	$(5)(5) = 25$

f) $(-y + 1)(-3y - 1) = -y(-3y - 1) + 1(-3y - 1)$
$= 3y^2 + y - 3y - 1$
$= 3y^2 - 2y - 1$

	$-3y$	-1
$-y$	$(-y)(-3y) = 3y^2$	$(-y)(-1) = y$
1	$(1)(-3y) = -3y$	$(1)(-1) = -1$

25. Factor by decomposition. For each trinomial, write the middle term as the sum of two terms whose coefficients have a product equal to the product of the coefficient of the 1st term and the constant term. Remove common factors from the 1st two terms and from the last two terms, then remove a binomial as a common factor.

a) $4k^2 - 7k + 3$
The product is: 3(4) = 12
Consider only the negative factors of 12 because the *k*-term is negative and the constant term is positive.
The negative factors of 12 are: –1 and –12; –2 and –6; –3 and –4
The factors with a sum of –7 are –3 and –4.
So, $4k^2 - 7k + 3 = 4k^2 - 3k - 4k + 3$
$= k(4k - 3) - 1(4k - 3)$
$= (4k - 3)(k - 1)$
Check: $(4k - 3)(k - 1) = 4k(k - 1) - 3(k - 1)$
$= 4k^2 - 4k - 3k + 3$
$= 4k^2 - 7k + 3$

b) $6c^2 - 13c - 5$
The product is: 6(–5) = –30
The factors of –30 are: 1 and –30; –1 and 30; 2 and –15; –2 and 15; 3 and –10; –3 and 10; 5 and –6; –5 and 6
The two factors with a sum of –13 are: 2 and –15
So, $6c^2 - 13c - 5 = 6c^2 + 2c - 15c - 5$
$= 2c(3c + 1) - 5(3c + 1)$
$= (3c + 1)(2c - 5)$
Check: $(3c + 1)(2c - 5) = 3c(2c - 5) + 1(2c - 5)$
$= 6c^2 - 15c + 2c - 5$
$= 6c^2 - 13c - 5$

c) $4b^2 - 5b - 6$
The product is: 4(–6) = –24
The factors of –24 are: 1 and –24; –1 and 24; 2 and –12; –2 and 12; 3 and –8; –3 and 8; 4 and –6; –4 and 6
The two factors with a sum of –5 are: 3 and –8
So, $4b^2 - 5b - 6 = 4b^2 + 3b - 8b - 6$
$= b(4b + 3) - 2(4b + 3)$
$= (4b + 3)(b - 2)$
Check: $(4b + 3)(b - 2) = 4b(b - 2) + 3(b - 2)$
$= 4b^2 - 8b + 3b - 6$
$= 4b^2 - 5b - 6$

d) $6a^2 - 31a + 5$
The product is: 6(5) = 30
Consider only the negative factors of 30 because the *a*-term is negative and the constant term is positive.
The negative factors of 30 are: –1 and –30; –2 and –15; –3 and –10; –5 and –6
The factors with a sum of –31 are –1 and –30.
So, $6a^2 - 31a + 5 = 6a^2 - 1a - 30a + 5$
$= a(6a - 1) - 5(6a - 1)$
$= (6a - 1)(a - 5)$
Check: $(6a - 1)(a - 5) = 6a(a - 5) - 1(a - 5)$
$= 6a^2 - 30a - 1a + 5$
$= 6a^2 - 31a + 5$

e) $28x^2 + 9x - 4$
The product is: 28(–4) = –112
The factors of –112 are: 1 and –112; –1 and 112; 2 and –56; 4 and –28; –4 and 28; 7 and –16; –7 and 16; 8 and –14; –8 and 14
The two factors with a sum of 9 are: –7 and 16
So, $28x^2 + 9x - 4 = 28x^2 - 7x + 16x - 4$
$= 7x(4x - 1) + 4(4x - 1)$
$= (4x - 1)(7x + 4)$
Check: $(4x - 1)(7x + 4) = 4x(7x + 4) - 1(7x + 4)$
$= 28x^2 + 16x - 7x - 4$
$= 28x^2 + 9x - 4$

f) $21x^2 + 8x - 4$
The product is: 21(–4) = –84
The factors of –84 are: 1 and –84; –1 and 84; 2 and –42; –2 and 42; 3 and –28; –3 and 28; 4 and –21; –21 and 4; 6 and –14; –6 and 14; 7 and –12; –7 and 12
The two factors with a sum of 8 are: –6 and 14
So, $21x^2 + 8x - 4 = 21x^2 - 6x + 14x - 4$
$= 3x(7x - 2) + 2(7x - 2)$
$= (7x - 2)(3x + 2)$
Check: $(7x - 2)(3x + 2) = 7x(3x + 2) - 2(3x + 2)$
$= 21x^2 + 14x - 6x - 4$
$= 21x^2 + 8x - 4$

26. a) $6m^2 + 5m - 21 = (6m - 20)(m + 1)$
Consider how the mistake might have been made.
The product of the constant terms in the binomial factors is not correct.
These terms must have a product of –21, so try –21 and 1.
$(6m - 21)(m + 1) = 6m(m + 1) - 21(m + 1)$
$= 6m^2 + 6m - 21m - 21$
$= 6m^2 - 15m - 21$
This does not match the given trinomial.
Try transposing the constant terms in the binomial factors.
$(6m - 1)(m + 21) = 6m(m + 21) - 1(m + 21)$
$= 6m^2 + 126m - 1m - 21$
$= 6m^2 + 125m - 21$
This does not match the given trinomial.

Try transposing the signs in the binomial factors: $(6m + 1)(m - 21)$
The trinomial will be $6m^2 - 125m - 21$. This does not match the given trinomial.
So, factor the trinomial by decomposition.
The product is: $6(-21) = -126$
The factors of -126 are: 1 and -126; -1 and 126; 2 and -63; -2 and 63; 3 and -42; -3 and 42; 6 and -21; -21 and 6; 7 and -18; -7 and 18; 9 and -14; -9 and 14
The two factors with a sum of 5 are: -9 and 14
So, $6m^2 + 5m - 21 = 6m^2 - 9m + 14m - 21$
$= 3m(2m - 3) + 7(2m - 3)$
$= (2m - 3)(3m + 7)$ ←These factors are correct.

b) $12n^2 - 17n - 5 = (4n - 1)(3n + 5)$
The product of the constant terms is correct.
Try transposing the constant terms in the binomial factors.
$(4n - 5)(3n + 1) = 4n(3n + 1) - 5(3n + 1)$
$= 12n^2 + 4n - 15n - 5$
$= 12n^2 - 11n - 5$
This does not match the given trinomial.
Try transposing the signs in the binomial factors.
$(4n + 1)(3n - 5) = 4n(3n - 5) + 1(3n - 5)$
$= 12n^2 - 20n + 3n - 5$
$= 12n^2 - 17n - 5$
This matches the given trinomial, so the correct factors are: $(4n + 1)(3n - 5)$
Alternative solution:
Expand the given binomial factors.
$(4n - 1)(3n + 5) = 4n(3n + 5) - 1(3n + 5)$
$= 12n^2 + 20n - 3n - 5$
$= 12n^2 + 17n - 5$
Compare this trinomial with the given trinomial. The only difference is the sign of the middle term. So, transpose the signs in the binomial factors.
$(4n + 1)(3n - 5) = 4n(3n - 5) + 1(3n - 5)$
$= 12n^2 - 20n + 3n - 5$
$= 12n^2 - 17n - 5$
This matches the given trinomial, so the correct factors are: $(4n + 1)(3n - 5)$

c) $20p^2 - 9p - 20 = (4p + 4)(5p - 5)$
The product of the constant terms is correct.
Try transposing the constant terms in the binomial factors.
$(4p + 5)(5p - 4) = 4p(5p - 4) + 5(5p - 4)$
$= 20p^2 - 16p + 25p - 20$
$= 20p^2 + 9p - 20$
Compare this trinomial with the given trinomial. The only difference is the sign of the middle term. So, transpose the signs in the binomial factors.
$(4p - 5)(5p + 4) = 4p(5p + 4) - 5(5p + 4)$
$= 20p^2 + 16p - 25p - 20$
$= 20p^2 - 9p - 20$
This matches the given trinomial, so the correct factors are: $(4p - 5)(5p + 4)$

3.7

27. a) $(c+1)(c^2+3c+2)$

$= c(c^2+3c+2)+1(c^2+3c+2)$

$= c(c^2)+c(3c)+c(2)+c^2+3c+2$

$= c^3+3c^2+2c+c^2+3c+2$ Collect like terms.

$= c^3+3c^2+c^2+2c+3c+2$ Combine like terms.

$= c^3+4c^2+5c+2$

To check, substitute $c = 2$ into the trinomial product and its simplification.

$(c+1)(c^2+3c+2)=c^3+4c^2+5c+2$

Left side: $(c+1)(c^2+3c+2)=(2+1)(2^2+3(2)+2)$

$=3(4+6+2)$

$=3(12)$

$=36$

Right side: $c^3+4c^2+5c+2=2^3+4(2)^2+5(2)+2$

$=8+16+10+2$

$=36$

Since the left side equals the right side, the product is likely correct.

b) $(5-4r)(6+3r-2r^2)$

$=5(6+3r-2r^2)-4r(6+3r-2r^2)$

$=5(6)+5(3r)+5(-2r^2)-4r(6)-4r(3r)-4r(-2r^2)$

$=30+15r-10r^2-24r-12r^2+8r^3$ Collect like terms.

$=8r^3-10r^2-12r^2+15r-24r+30$ Combine like terms.

$=8r^3-22r^2-9r+30$

To check, substitute $r = 2$ into the trinomial product and its simplification.

$(5-4r)(6+3r-2r^2)=8r^3-22r^2-9r+30$

Left side: $(5-4r)(6+3r-2r^2)=(5-4(2))(6+3(2)-2(2)^2)$

$=-3(6+6-8)$

$=-3(4)$

$=-12$

Right side: $8r^3-22r^2-9r+30=8(2)^3-22(2)^2-9(2)+30$

$=64-88-18+30$

$=-12$

Since the left side equals the right side, the product is likely correct.

c) $\left(-j^2+3j+1\right)\left(2j+11\right)$

$=-j^2\left(2j+11\right)+3j\left(2j+11\right)+1\left(2j+11\right)$

$=-j^2\left(2j\right)-j^2\left(11\right)+3j\left(2j\right)+3j\left(11\right)+2j+11$

$=-2j^3-11j^2+6j^2+33j+2j+11$ Combine like terms.

$=-2j^3-5j^2+35j+11$

To check, substitute $j=2$ into the trinomial product and its simplification.

$\left(-j^2+3j+1\right)\left(2j+11\right)=-2j^3-5j^2+35j+11$

Left side: $\left(-j^2+3j+1\right)\left(2j+11\right)=\left(-(2)^2+3(2)+1\right)\left(2(2)+11\right)$

$=\left(-4+6+1\right)\left(4+11\right)$

$=(3)(15)$

$=45$

Right side: $-2j^3-5j^2+35j+11=-2(2)^3-5(2)^2+35(2)+11$

$=-16-20+70+11$

$=45$

Since the left side equals the right side, the product is likely correct.

d) $\left(3x^2+7x+2\right)\left(2x-3\right)$

$=3x^2\left(2x-3\right)+7x\left(2x-3\right)+2\left(2x-3\right)$

$=3x^2\left(2x\right)+3x^2\left(-3\right)+7x\left(2x\right)+7x\left(-3\right)+2\left(2x\right)+2\left(-3\right)$

$=6x^3-9x^2+14x^2-21x+4x-6$ Combine like terms.

$=6x^3+5x^2-17x-6$

To check, substitute $x=2$ into the trinomial product and its simplification.

$\left(3x^2+7x+2\right)\left(2x-3\right)=6x^3+5x^2-17x-6$

Left side: $\left(3x^2+7x+2\right)\left(2x-3\right)=\left(3(2)^2+7(2)+2\right)\left(2(2)-3\right)$

$=\left(12+14+2\right)(1)$

$=28$

Right side: $6x^3+5x^2-17x-6=6(2)^3+5(2)^2-17(2)-6$

$=48+20-34-6$

$=28$

Since the left side equals the right side, the product is likely correct.

28. a) $(4m - p)^2$

$(4m - p)(4m - p)$

$= 4m(4m - p) - p(4m - p)$

$= 4m(4m) + 4m(-p) - p(4m) - p(-p)$

$= 16m^2 - 4mp - 4mp + p^2$ Combine like terms.

$= 16m^2 - 8mp + p^2$

b) $(3g - 4h)^2$

$(3g - 4h)(3g - 4h)$

$= 3g(3g - 4h) - 4h(3g - 4h)$

$= 3g(3g) + 3g(-4h) - 4h(3g) - 4h(-4h)$

$= 9g^2 - 12gh - 12gh + 16h^2$ Combine like terms.

$= 9g^2 - 24gh + 16h^2$

c) $(y - 2z)(y + z - 2)$

$= y(y + z - 2) - 2z(y + z - 2)$

$= y(y) + y(z) + y(-2) - 2z(y) - 2z(z) - 2z(-2)$

$= y^2 + yz - 2y - 2yz - 2z^2 + 4z$ Collect like terms.

$= y^2 + yz - 2yz - 2z^2 - 2y + 4z$ Combine like terms.

$= y^2 - yz - 2z^2 - 2y + 4z$

d) $(3c - 4d)(7 - 6c + 5d)$

$= 3c(7 - 6c + 5d) - 4d(7 - 6c + 5d)$

$= 3c(7) + 3c(-6c) + 3c(5d) - 4d(7) - 4d(-6c) - 4d(5d)$

$= 21c - 18c^2 + 15cd - 28d + 24cd - 20d^2$ Collect like terms.

$= -18c^2 + 15cd + 24cd - 20d^2 + 21c - 28d$ Combine like terms.

$= -18c^2 + 39cd - 20d^2 + 21c - 28d$

29. a)

$(m^2 + 3m + 2)(2m^2 + m + 5)$

$= m^2(2m^2 + m + 5) + 3m(2m^2 + m + 5) + 2(2m^2 + m + 5)$

$= m^2(2m^2) + m^2(m) + m^2(5) + 3m(2m^2) + 3m(m) + 3m(5) + 2(2m^2) + 2(m) + 2(5)$

$= 2m^4 + m^3 + 5m^2 + 6m^3 + 3m^2 + 15m + 4m^2 + 2m + 10$ Collect like terms.

$= 2m^4 + m^3 + 6m^3 + 5m^2 + 3m^2 + 4m^2 + 15m + 2m + 10$ Combine like terms.

$= 2m^4 + 7m^3 + 12m^2 + 17m + 10$

To check, substitute $m = 2$ into the trinomial product and its simplification.

$(m^2 + 3m + 2)(2m^2 + m + 5) = 2m^4 + 7m^3 + 12m^2 + 17m + 10$

Left side: $(m^2 + 3m + 2)(2m^2 + m + 5) = \left[(2)^2 + 3(2) + 2\right]\left[2(2)^2 + 2 + 5\right]$

$$= (4 + 6 + 2)(8 + 7)$$
$$= (12)(15)$$
$$= 180$$

Right side: $2m^4 + 7m^3 + 12m^2 + 17m + 10 = 2(2)^4 + 7(2)^3 + 12(2)^2 + 17(2) + 10$

$$= 32 + 56 + 48 + 34 + 10$$
$$= 180$$

Since the left side equals the right side, the product is likely correct.

b) $(1 - 3x + 2x^2)(5 + 4x - x^2)$

$= 1(5 + 4x - x^2) - 3x(5 + 4x - x^2) + 2x^2(5 + 4x - x^2)$

$= 5 + 4x - x^2 - 3x(5) - 3x(4x) - 3x(-x^2) + 2x^2(5) + 2x^2(4x) + 2x^2(-x^2)$

$= 5 + 4x - x^2 - 15x - 12x^2 + 3x^3 + 10x^2 + 8x^3 - 2x^4$ Collect like terms.

$= 5 + 4x - 15x - x^2 - 12x^2 + 10x^2 + 3x^3 + 8x^3 - 2x^4$ Combine like terms.

$= 5 - 11x - 3x^2 + 11x^3 - 2x^4$

To check, substitute $x = 2$ into the trinomial product and its simplification.

$(1 - 3x + 2x^2)(5 + 4x - x^2) = 5 - 11x - 3x^2 + 11x^3 - 2x^4$

Left side: $(1 - 3x + 2x^2)(5 + 4x - x^2) = \left[1 - 3(2) + 2(2)^2\right]\left[5 + 4(2) - (2)^2\right]$

$$= (1 - 6 + 8)(5 + 8 - 4)$$
$$= (3)(9)$$
$$= 27$$

Right side:

$5 - 11x - 3x^2 + 11x^3 - 2x^4 = 5 - 11(2) - 3(2)^2 + 11(2)^3 - 2(2)^4$

$$= 5 - 22 - 12 + 88 - 32$$
$$= 27$$

Since the left side equals the right side, the product is likely correct.

c)

$(-2k^2+7k+6)(3k^2-2k-3)$
$=-2k^2(3k^2-2k-3)+7k(3k^2-2k-3)+6(3k^2-2k-3)$
$=-2k^2(3k^2)-2k^2(-2k)-2k^2(-3)+7k(3k^2)+7k(-2k)+7k(-3)+6(3k^2)+6(-2k)+6(-3)$
$=-6k^4+4k^3+6k^2+21k^3-14k^2-21k+18k^2-12k-18$ Collect like terms.
$=-6k^4+4k^3+21k^3+6k^2-14k^2+18k^2-21k-12k-18$ Combine like terms.
$=-6k^4+25k^3+10k^2-33k-18$

To check, substitute $c = 2$ into the trinomial product and its simplification.

$$(-2k^2+7k+6)(3k^2-2k-3)=-6k^4+25k^3+10k^2-33k-18$$

Left side: $(-2k^2+7k+6)(3k^2-2k-3)=[-2(2)^2+7(2)+6][3(2)^2-2(2)-3]$
$=(-8+14+6)(12-4-3)$
$=(12)(5)$
$=60$

Right side: $-6k^4+25k^3+10k^2-33k-18=-6(2)^4+25(2)^3+10(2)^2-33(2)-18$
$=-96+200+40-66-18$
$=60$

Since the left side equals the right side, the product is likely correct.

d) $(-3-5h+2h^2)(-1+h+h^2)$
$=-3(-1+h+h^2)-5h(-1+h+h^2)+2h^2(-1+h+h^2)$
$=3-3h-3h^2+5h-5h^2-5h^3-2h^2+2h^3+2h^4$ Collect like terms.
$=3-3h+5h-3h^2-5h^2-2h^2-5h^3+2h^3+2h^4$ Combine like terms.
$=3+2h-10h^2-3h^3+2h^4$

To check, substitute $h = 2$ into the trinomial product and its simplification.

$(-3-5h+2h^2)(-1+h+h^2)=3+2h-10h^2-3h^3+2h^4$

Left side: $(-3-5h+2h^2)(-1+h+h^2)=[-3-5(2)+2(2)^2][-1+2+(2)^2]$
$=(-3-10+8)(1+4)$
$=(-5)(5)$
$=-25$

Right side:
$3+2h-10h^2-3h^3+2h^4=3+2(2)-10(2)^2-3(2)^3+2(2)^4$
$=3+4-40-24+32$
$=-25$

Since the left side equals the right side, the product is likely correct.

30. a) $(5a+1)(4a+2)+(a-5)(2a-1)$

$= 5a(4a+2)+1(4a+2)+a(2a-1)-5(2a-1)$

$= 5a(4a)+5a(2)+4a+2+a(2a)+a(-1)-5(2a)-5(-1)$

$= 20a^2+10a+4a+2+2a^2-a-10a+5$ Collect like terms.

$= 20a^2+2a^2+10a+4a-a-10a+2+5$ Combine like terms.

$= 22a^2+3a+7$

b) $(6c-2)(4c+2)-(c+7)^2$

$= (6c-2)(4c+2)-(c+7)(c+7)$

$= 6c(4c+2)-2(4c+2)-c(c+7)-7(c+7)$

$= 6c(4c)+6c(2)-2(4c)-2(2)-c(c)-c(7)-7(c)-7(7)$

$= 24c^2+12c-8c-4-c^2-7c-7c-49$ Collect like terms.

$= 24c^2-c^2+12c-8c-7c-7c-4-49$ Combine like terms.

$= 23c^2-10c-53$

31. a) Suppose n represents an even integer.
Then, the next two consecutive even integers are: $n+2$, $n+4$

b) An expression for the product of the 3 integers is:
$n(n+2)(n+4)$
Simplify.

$$\begin{aligned} n(n+2)(n+4) &= n[n(n+4)+2(n+4)] \\ &= n[n(n)+n(4)+2(n)+2(4)] \\ &= n(n^2+4n+2n+8) \\ &= n(n^2+6n+8) \\ &= n^3+6n^2+8n \end{aligned}$$

3.8

32. a) $81-4b^2$
Write each term as a perfect square.

$81-4b^2 = (9)^2-(2b)^2$ Write these terms in binomial factors.

$= (9+2b)(9-2b)$

b) $16v^2-49$
Write each term as a perfect square.

$16v^2-49 = (4v)^2-(7)^2$ Write these terms in binomial factors.

$= (4v+7)(4v-7)$

c) $64g^2 - 16h^2$

As written, each term of the binomial has a common factor 16. Remove this common factor.

$64g^2 - 16h^2 = 16(4g^2 - h^2)$ Write each term in the binomial as a perfect square.

$= 16[(2g)^2 - (h)^2]$ Write these terms in binomial factors.

$= 16(2g + h)(2g - h)$

d) $18m^2 - 2n^2$

As written, each term of the binomial has a common factor 2. Remove this common factor.

$18m^2 - 2n^2 = 2(9m^2 - n^2)$ Write each term in the binomial as a perfect square.

$= 2[(3m)^2 - (n)^2]$ Write these terms in binomial factors.

$= 2(3m + n)(3m - n)$

33. a) $m^2 - 14m + 49$

The first term is a perfect square since $m^2 = (m)(m)$.

The third term is a perfect square since $49 = (7)(7)$.

The second term is twice the product of *m* and 7:

$14m = 2(m)(7)$

Since the second term is negative, the operations in the binomial factors must be subtraction.

So, the trinomial is a perfect square trinomial and its factors are:

$(m - 7)(m - 7)$, or $(m - 7)^2$

To verify, multiply:

$(m - 7)(m - 7) = m(m - 7) - 7(m - 7)$

$= m^2 - 7m - 7m + 49$

$= m^2 - 14m + 49$

Since the trinomial is the same as the original trinomial, the factors are correct.

b) $n^2 + 10n + 25$

The first term is a perfect square since $n^2 = (n)(n)$.

The third term is a perfect square since $25 = (5)(5)$.

The second term is twice the product of *n* and 5:

$10n = 2(n)(5)$

Since the second term is positive, the operations in the binomial factors must be addition.

So, the trinomial is a perfect square trinomial and its factors are:

$(n + 5)(n + 5)$, or $(n + 5)^2$

To verify, multiply:

$$\begin{aligned}(n+5)(n+5) &= n(n+5)+5(n+5)\\ &= n^2+5n+5n+25\\ &= n^2+10n+25\end{aligned}$$

Since the trinomial is the same as the original trinomial, the factors are correct.

c) $4p^2+12p+9$

The first term is a perfect square since $4p^2=(2p)(2p)$.

The third term is a perfect square since $9=(3)(3)$.

The second term is twice the product of $2p$ and 3:

$12p=2(2p)(3)$

Since the second term is positive, the operations in the binomial factors must be addition.
So, the trinomial is a perfect square trinomial and its factors are:

$(2p+3)(2p+3)$, or $(2p+3)^2$

To verify, multiply:

$$\begin{aligned}(2p+3)(2p+3) &= 2p(2p+3)+3(2p+3)\\ &= 4p^2+6p+6p+9\\ &= 4p^2+12p+9\end{aligned}$$

Since the trinomial is the same as the original trinomial, the factors are correct.

d) $16-40q+25q^2$

The first term is a perfect square since $16=(4)(4)$.

The third term is a perfect square since $25q^2=(5q)(5q)$.

The second term is twice the product of 4 and $5q$:

$40q=2(4)(5q)$

Since the second term is negative, the operations in the binomial factors must be subtraction.
So, the trinomial is a perfect square trinomial and its factors are:

$(4-5q)(4-5q)$, or $(4-5q)^2$

To verify, multiply:

$$\begin{aligned}(4-5q)(4-5q) &= 4(4-5q)-5q(4-5q)\\ &= 16-20q-20q+25q^2\\ &= 16-40q+25q^2\end{aligned}$$

Since the trinomial is the same as the original trinomial, the factors are correct.

e) $4r^2+28r+49$

The first term is a perfect square since $4r^2=(2r)(2r)$.

The third term is a perfect square since $49=(7)(7)$.

The second term is twice the product of $2r$ and 7:

$28r=2(2r)(7)$

Since the second term is positive, the operations in the binomial factors must be addition.

So, the trinomial is a perfect square trinomial and its factors are:
$(2r+7)(2r+7)$, or $(2r+7)^2$
To verify, multiply:
$$\begin{aligned}(2r+7)(2r+7) &= 2r(2r+7)+7(2r+7)\\ &= 4r^2+14r+14r+49\\ &= 4r^2+28r+49\end{aligned}$$
Since the trinomial is the same as the original trinomial, the factors are correct.

f) $36-132s+121s^2$

The first term is a perfect square since $36=(6)(6)$.
The third term is a perfect square since $121s^2=(11s)(11s)$.
The second term is twice the product of 6 and $11s$:
$132s=2(6)(11s)$
Since the second term is negative, the operations in the binomial factors must be subtraction.
So, the trinomial is a perfect square trinomial and its factors are:
$(6-11s)(6-11s)$, or $(6-11s)^2$
To verify, multiply:
$$\begin{aligned}(6-11s)(6-11s) &= 6(6-11s)-11s(6-11s)\\ &= 36-66s-66s+121s^2\\ &= 36-132s+121s^2\end{aligned}$$
Since the trinomial is the same as the original trinomial, the factors are correct.

34. a) $g^2+6gh+9h^2$

The first term is a perfect square since $g^2=(g)(g)$.
The third term is a perfect square since $9h^2=(3h)(3h)$.
The second term is twice the product of g and $3h$:
$6gh=2(g)(3h)$
Since the second term is positive, the operations in the binomial factors must be addition.
So, the trinomial is a perfect square trinomial and its factors are:
$(g+3h)(g+3h)$, or $(g+3h)^2$

b) $16j^2-24jk+9k^2$

The first term is a perfect square since $16j^2=(4j)(4j)$.
The third term is a perfect square since $9k^2=(3k)(3k)$.
The second term is twice the product of $4j$ and $3k$:
$24jk=2(4j)(3k)$
Since the second term is negative, the operations in the binomial factors must be subtraction.
So, the trinomial is a perfect square trinomial and its factors are:

$(4j - 3k)(4j - 3k)$, or $(4j - 3k)^2$

c) $25t^2 + 20tu + 4u^2$

The first term is a perfect square since $25t^2 = (5t)(5t)$.

The third term is a perfect square since $4u^2 = (2u)(2u)$.

The second term is twice the product of $5t$ and $2u$:

$20tu = 2(5t)(2u)$

Since the second term is positive, the operations in the binomial factors must be addition.
So, the trinomial is a perfect square trinomial and its factors are:

$(5t + 2u)(5t + 2u)$, or $(5t + 2u)^2$

d) $9v^2 - 48vw + 64w^2$

The first term is a perfect square since $9v^2 = (3v)(3v)$.

The third term is a perfect square since $64w^2 = (8w)(8w)$.

The second term is twice the product of $3v$ and $8w$:

$48vw = 2(3v)(8w)$

Since the second term is negative, the operations in the binomial factors must be subtraction.
So, the trinomial is a perfect square trinomial and its factors are:

$(3v - 8w)(3v - 8w)$, or $(3v - 8w)^2$

35. The area of the shaded region is:
area of large square – area of small square
The formula for the area of a square is: $A = s^2$, where s is the side length of the square.
Area of large square:
Use the formula $A = s^2$. Substitute: $s = 2x + 5$

$A_L = (2x + 5)^2$

Area of small square:
Use the formula $A = s^2$. Substitute: $s = x + 3$

$A_L = (x + 3)^2$

So, area of the shaded region is:

$$\begin{aligned} A &= A_L - A_S \\ &= (2x + 5)^2 - (x + 3)^2 \\ &= (2x + 5)(2x + 5) - (x + 3)(x + 3) \\ &= 2x(2x + 5) + 5(2x + 5) - [x(x + 3) + 3(x + 3)] \\ &= 4x^2 + 10x + 10x + 25 - (x^2 + 3x + 3x + 9) \\ &= 4x^2 + 20x + 25 - x^2 - 6x - 9 \\ &= 3x^2 + 14x + 16 \end{aligned}$$

A polynomial that represents the area of the shaded region is:

$3x^2 + 14x + 16$

Practice Test (page 201)

1. **A.** This is not true because 64 has these factors: 1, 2, 4, 8, 16, 32, and 64
So, **A** is the correct answer.

2. Expand each product until the trinomial matches the given trinomial.
A. $(2x + 1)(x + 6) = 2x^2 + 12x + 1x + 6$
$= 2x^2 + 13x + 6$
B. $(2x + 2)(x + 3) = 2x^2 + 6x + 2x + 6$
$= 2x^2 + 8x + 6$
C. $(2x + 3)(x + 2) = 2x^2 + 4x + 3x + 6$
$= 2x^2 + 7x + 6$
This trinomial matches the given trinomial, so **C** is the correct answer.

3. $20 = 2 \cdot 2 \cdot 5$
$45 = 3 \cdot 3 \cdot 5$
$50 = 2 \cdot 5 \cdot 5$
The least common multiple is the product of the greatest prime factor in each set of factors:
$2^2 \cdot 3^2 \cdot 5^2 = 900$
The least common multiple is 900.
The greatest common factor is the factor that occurs in every set of factors: 5
The greatest common factor is 5.

4. **a)** **i)** For a perfect square; its prime factors can be arranged into 2 equal groups.
Use the factors from question 3.

$20 = 2 \cdot 2 \cdot 5$
Multiply 20 by 5 to get 2 equal groups of factors: $2 \cdot 2 \cdot 5 \cdot 5$
$20 \cdot 5 = 100$
100 is a perfect square.
To get more perfect squares, multiply 20 by 5 times any perfect square, such as 20(5)(4) to get 400; and 20(5)(9) to get 900.

$45 = 3 \cdot 3 \cdot 5$
Multiply 45 by 5 to get 2 equal groups of factors: $3 \cdot 3 \cdot 5 \cdot 5$
$45 \cdot 5 = 225$
225 is a perfect square.
To get more perfect squares, multiply 45 by 5 times any perfect square, such as 45(5)(4) to get 900; and 45(5)(9) to get 2025.

$50 = 2 \cdot 5 \cdot 5$
Multiply 50 by 2 to get 2 equal groups of factors: $2 \cdot 2 \cdot 5 \cdot 5$
$50 \cdot 2 = 100$
100 is a perfect square.
To get more perfect squares, multiply 50 by 2 times any perfect square, such as 50(2)(4) to get 400; and 50(2)(9) to get 900.

ii) For a perfect cube, its prime factors can be arranged in 3 equal groups.
Use the factors from question 3.

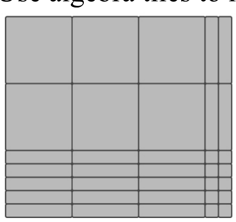
$20 = 2 \cdot 2 \cdot 5$
Multiply 20 by $2 \cdot 5 \cdot 5 = 50$ to get 3 equal groups of factors: $2 \cdot 2 \cdot 2 \cdot 5 \cdot 5 \cdot 5$
$20 \cdot 50 = 100$
1000 is a perfect cube.
To get more perfect cubes, multiply 20 by 50 times any perfect cube, such as 20(50)(8) to get 8000; and 20(50)(27) to get 27 000.

$45 = 3 \cdot 3 \cdot 5$
Multiply 45 by $3 \cdot 5 \cdot 5 = 75$ to get 3 equal groups of factors: $3 \cdot 3 \cdot 3 \cdot 5 \cdot 5 \cdot 5$
$45 \cdot 75 = 3375$
3375 is a perfect cube.
To get more perfect cubes, multiply 45 by 75 times any perfect cube, such as 45(75)(8) to get 27 000; and 45(75)(27) to get 91 125.

$50 = 2 \cdot 5 \cdot 5$
Multiply 50 by $2 \cdot 2 \cdot 5 = 20$ to get 3 equal groups of factors: $2 \cdot 2 \cdot 2 \cdot 5 \cdot 5 \cdot 5$
$50 \cdot 20 = 1000$
1000 is a perfect cube.
To get more perfect cubes, multiply 50 by 20 times any perfect cube, such as 50(20)(8) to get 8000; and 50(20)(27) to get 27 000.

b) There is more than one answer in each of part a because a perfect square can be generated by multiplying any two perfect squares; and a perfect cube can be generated by multiplying any 2 perfect cubes.

5. **a)** $(2c + 5)(3c + 2)$
Use algebra tiles to make a rectangle with width $2c + 5$ and length $3c + 2$.



The tiles that form the product are: 6 c^2-tiles, 19 c-tiles, and ten 1-tiles
So, $(2c + 5)(3c + 2) = 6c^2 + 19c + 10$

b) $(9 + 4r)(8 + 6r)$
Sketch a rectangle with dimensions $8 + 6r$ and $9 + 4r$. Divide it into 4 smaller rectangles and calculate the area of each.

	8	$6r$
9	$(9)(8) = 72$	$(9)(6r) = 54r$
$4r$	$(4r)(8) = 32r$	$(4r)(6r) = 24r^2$

From the diagram:
$(9 + 4r)(8 + 6r) = 72 + 54r + 32r + 24r^2$
$= 72 + 86r + 24r^2$

c) $(4t – 5)(3t + 7)$
Sketch a rectangle. Label its dimensions $4t – 5$ and $3t + 7$. Divide it into 4 smaller rectangles and label each one.

	$3t$	7
$4t$	$(4t)(3t) = 12t^2$	$(4t)(7) = 28t$
$–5$	$(–5)(3t) = –15t$	$(–5)(7) = –35$

From the diagram:
$(4t – 5)(3t + 7) = 12t^2 + 28t – 15t – 35$
$= 12t^2 + 13t – 35$

6. Use the distributive property.
a) $(2p – 1)(p^2 + 2p – 7) = 2p(p^2 + 2p – 7) – 1(p^2 + 2p – 7)$
$= 2p^3 + 4p^2 – 14p – p^2 – 2p + 7$
$= 2p^3 + 4p^2 – p^2 – 14p – 2p + 7$
$= 2p^3 + 3p^2 – 16p + 7$

b) $(e + 2f)(2f^2 + 5f + 3e^2) = e(2f^2 + 5f + 3e^2) + 2f(2f^2 + 5f + 3e^2)$
$= 2ef^2 + 5ef + 3e^3 + 4f^3 + 10f^2 + 6e^2f$

c) $(3y + 2z)(y + 4z) – (5y – 3z)(2y – 8z)$
$= 3y(y + 4z) + 2z(y + 4z) – [5y(2y – 8z) – 3z(2y – 8z)]$
$= 3y^2 + 12yz + 2yz + 8z^2 – [10y^2 – 40yz – 6yz + 24z^2]$
$= 3y^2 + 12yz + 2yz + 8z^2 – 10y^2 + 40yz + 6yz – 24z^2$
$= 3y^2 – 10y^2 + 12yz + 2yz + 40yz + 6yz – 24z^2 + 8z^2$
$= –7y^2 + 60yz – 16z^2$

7. a) $f^2 + 17f + 16$
Find two numbers whose sum is 17 and whose product is 16.
The numbers are 1 and 16.

So, $f^2 + 17f + 16 = (f + 1)(f + 16)$
I could use these algebra tiles: 1 f^2-tile, 17 f-tiles, and sixteen 1-tiles to make a rectangle with length $f + 16$ and width $f + 1$.

b) $c^2 - 13c + 22$
Find two numbers whose sum is –13 and whose product is 22.
Since the constant term is positive and the c-term is negative, the numbers are negative.
The numbers are –2 and –11.
So, $c^2 - 13c + 22 = (c - 2)(c - 11)$
I could use these algebra tiles: 1 c^2-tile, 13 negative c-tiles, and twenty-two 1-tiles to make a rectangle with length $c - 11$ and width $c - 2$.

c) $4t^2 + 9t - 28$
Use decomposition.
Multiply: $4(-28) = -112$
Find factors of –112 that have a sum of 9.
Factors of –112 are: 1 and –112; –1 and 112; 2 and –56; –2 and 56; 4 and –28; –4 and 28; 7 and –16; –7 and 16
The factors of –112 that have a sum of 9 are –7 and 16.
So,
$$\begin{aligned} 4t^2 + 9t - 28 &= 4t^2 - 7t + 16t - 28 \\ &= t(4t - 7) + 4(4t - 7) \\ &= (4t - 7)(t + 4) \end{aligned}$$
I would not use algebra tiles to factor. I would need to use guess and check to find a combination of positive and negative t-tiles to form a rectangle with 4 t^2-tiles, and 28 negative 1-tiles.

d) $4r^2 + 20rs + 25s^2$
This is a perfect square trinomial because:
the 1st term is a perfect square: $4r^2 = (2r)(2r)$
the 3rd term is a perfect square: $25s^2 = (5s)(5s)$
and the 2nd term is: $20rs = 2(2r)(5s)$
So, $4r^2 + 20rs + 25s^2 = (2r + 5s)(2r + 5s)$, or $(2r + 5s)^2$
I could not use algebra tiles to factor the given trinomial because I do not have tiles for more than one variable. I could use tiles to factor $4r^2 + 20r + 25$, then include the variable s when I write the factors.

e) $6x^2 - 17xy + 5y^2$
Use decomposition.
Multiply: $6(5) = 30$
Find factors of 30 that have a sum of –17.
Since the coefficient of y^2 is positive and the xy-term is negative, the numbers are negative.
List negative factors of 30: –1 and –30; –1 and –30; –2 and –15
The factors of 30 that have a sum of –17 are –2 and –15.
So,
$$\begin{aligned} 6x^2 - 17xy + 5y^2 &= 6x^2 - 2xy - 15xy + 5y^2 \\ &= 2x(3x - y) - 5y(3x - y) \\ &= (3x - y)(2x - 5y) \end{aligned}$$
I could not use algebra tiles to factor the trinomial because I do not have tiles for more than one variable.

f) $h^2 - 25j^2$
This is a difference of squares.
$h^2 = (h)(h)$
$25j^2 = (5j)(5j)$
So, $h^2 - 25j^2 = (h + 5j)(h - 5j)$
I could not use algebra tiles to factor the binomial because I do not have tiles for more than one variable.

8. The remaining volume is the difference between the volume of the cube and the volume of the prism.
The volume of the cube is: $(2r + 1)^3$
The volume of the prism is: $r(r)(2r + 1)$
The remaining volume is:

$$\begin{aligned}(2r + 1)^3 - r(r)(2r + 1) &= (2r + 1)(2r + 1)^2 - [r^2(2r + 1)]\\ &= (2r + 1)(4r^2 + 4r + 1) - [2r^3 + r^2]\\ &= 2r(4r^2 + 4r + 1) + 1(4r^2 + 4r + 1) - 2r^3 - r^2\\ &= 8r^3 + 8r^2 + 2r + 4r^2 + 4r + 1 - 2r^3 - r^2\\ &= 8r^3 - 2r^3 + 8r^2 + 4r^2 - r^2 + 2r + 4r + 1\\ &= 6r^3 + 11r^2 + 6r + 1\end{aligned}$$

The volume that remains is: $6r^3 + 11r^2 + 6r + 1$

9. All the trinomials that begin with $8t^2$ and end with +3 have t-terms with coefficients that are the sum of the factors of 8(3) = 24.
The factors of 24 are: 1 and 24; –1 and –24; 2 and 12; –2 and –12; 3 and 8; –3 and –8; 4 and 6; –4 and –6
The sums of the factors are: 25, –25; 14, –14; 11; –11; 10; –10
So, the possible trinomials are:
$8t^2 + 25t + 3$; $8t^2 - 25t + 3$; $8t^2 + 14t + 3$; $8t^2 - 14t + 3$; $8t^2 + 11t + 3$; $8t^2 - 11t + 3$; $8t^2 + 10t + 3$; $8t^2 - 10t + 3$
I have found all the trinomials because there are no other pairs of factors of 24.