Lesson 2.1 The Tangent Ratio Exercises (pages 75–77)

A

3. a) The acute angles are ∠A and ∠C.

$$\tan A = \frac{\text{length of side opposite } \angle A}{\text{length of side adjacent to } \angle A}$$

$$\tan A = \frac{BC}{AB}$$

BC is opposite ∠A.
AB is adjacent to ∠A.

$$\tan A = \frac{6}{7}$$

$$\tan C = \frac{\text{length of side opposite } \angle C}{\text{length of side adjacent to } \angle C}$$

$$\tan C = \frac{AB}{BC}$$

AB is opposite ∠C.
BC is adjacent to ∠C.

$$\tan C = \frac{7}{6}$$

b) The acute angles are ∠D and ∠F.

$$\tan D = \frac{\text{length of side opposite } \angle D}{\text{length of side adjacent to } \angle D}$$

$$\tan D = \frac{EF}{DE}$$

EF is opposite ∠D.
DE is adjacent to ∠D.

$$\tan D = \frac{6}{4}$$

$$\tan D = \frac{3}{2}$$

$$\tan F = \frac{\text{length of side opposite } \angle F}{\text{length of side adjacent to } \angle F}$$

$$\tan F = \frac{DE}{EF}$$

DE is opposite ∠F.
EF is adjacent to ∠F.

$$\tan F = \frac{4}{6}$$

$$\tan F = \frac{2}{3}$$

c) The acute angles are $\angle$H and $\angle$J.

$$\tan \mathrm{H} = \frac{\text{length of side opposite } \angle \mathrm{H}}{\text{length of side adjacent to } \angle \mathrm{H}}$$

$$\tan \mathrm{H} = \frac{\mathrm{GJ}}{\mathrm{GH}}$$

GJ is opposite $\angle$H.
GH is adjacent to $\angle$H.

$$\tan \mathrm{H} = \frac{10}{8}$$

$$\tan \mathrm{H} = \frac{5}{4}$$

$$\tan \mathrm{J} = \frac{\text{length of side opposite } \angle \mathrm{J}}{\text{length of side adjacent to } \angle \mathrm{J}}$$

$$\tan \mathrm{J} = \frac{\mathrm{GH}}{\mathrm{GJ}}$$

GH is opposite $\angle$J.
GJ is adjacent to $\angle$J.

$$\tan \mathrm{J} = \frac{8}{10}$$

$$\tan \mathrm{J} = \frac{4}{5}$$

d) The acute angles are $\angle$K and $\angle$M.

$$\tan \mathrm{K} = \frac{\text{length of side opposite } \angle \mathrm{K}}{\text{length of side adjacent to } \angle \mathrm{K}}$$

$$\tan \mathrm{K} = \frac{\mathrm{MN}}{\mathrm{KN}}$$

MN is opposite $\angle$K.
KN is adjacent to $\angle$K.

$$\tan \mathrm{K} = \frac{5}{7}$$

$$\tan \mathrm{M} = \frac{\text{length of side opposite } \angle \mathrm{M}}{\text{length of side adjacent to } \angle \mathrm{M}}$$

$$\tan \mathrm{M} = \frac{\mathrm{KN}}{\mathrm{MN}}$$

KN is opposite $\angle$M.
MN is adjacent to $\angle$M.

$$\tan \mathrm{M} = \frac{7}{5}$$

4. Use a calculator to determine the measure of each angle.

a) $\tan X = 0.25$

$$\angle X = \tan^{-1}(0.25)$$
$$= 14.0362...$$

So, $\angle X \doteq 14°$

b) $\tan X = 1.25$

$$\angle X = \tan^{-1}(1.25)$$
$$= 51.3401...$$

So, $\angle X \doteq 51°$

c) $\tan X = 2.50$

$$\angle X = \tan^{-1}(2.50)$$
$$= 68.1985...$$

So, $\angle X \doteq 68°$

d) $\tan X = 20$

$$\angle X = \tan^{-1}(20)$$
$$= 87.1375...$$

So, $\angle X \doteq 87°$

5. a) Use the tangent ratio.

$$\frac{\text{opposite}}{\text{adjacent}} = \frac{3}{6}$$
$$= \frac{1}{2}$$

Use a calculator to determine the measure of the angle.

$$\tan^{-1}\left(\frac{1}{2}\right) = 26.5650...°$$

So, the angle measure is approximately 27°.

b) Use the tangent ratio.

$$\frac{\text{opposite}}{\text{adjacent}} = \frac{14}{14}$$
$$= 1$$

Use a calculator to determine the measure of the angle.

$\tan^{-1}(1) = 45°$

So, the angle measure is 45°.

c) Use the tangent ratio.

$$\frac{\text{opposite}}{\text{adjacent}} = \frac{9}{5}$$

Use a calculator to determine the measure of the angle.

$$\tan^{-1}\left(\frac{9}{5}\right) = 60.9453...^\circ$$

So, the angle measure is approximately 61°.

d) Use the tangent ratio.

$$\frac{\text{opposite}}{\text{adjacent}} = \frac{8}{3}$$

Use a calculator to determine the measure of the angle.

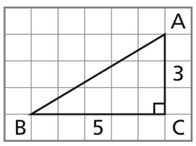
$$\tan^{-1}\left(\frac{8}{3}\right) = 69.4439...^\circ$$

So, the angle measure is approximately 69°.

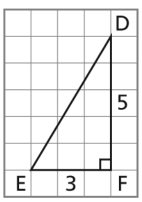
B

6. Sketches will vary. For example:

a)

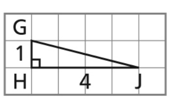


b)

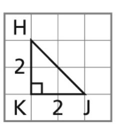


c)

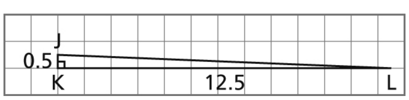
d)



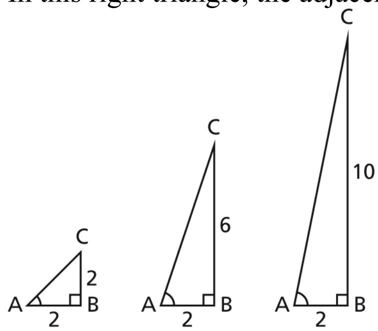
e)



f)

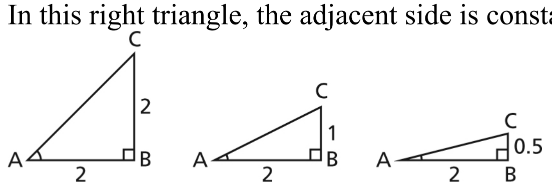


7. a) In this right triangle, the adjacent side is constant.



When the opposite and adjacent sides are equal, tan A = 1 and ∠A = 45°. As ∠A increases, the opposite side increases, so the ratio $\frac{\text{opposite}}{\text{adjacent}}$ increases, and tan A increases. So, tan 60° is greater than tan 45°, which is 1.

b) In this right triangle, the adjacent side is constant.



As ∠A decreases, the opposite side decreases, so the ratio $\frac{\text{opposite}}{\text{adjacent}}$ decreases, and tan A decreases. So, tan 30° is less than tan 45°, which is 1.

8. a) First, use the Pythagorean Theorem in right ΔEFG to determine the length of the side adjacent to ∠F.

$$EF^2 = EG^2 + FG^2 \qquad \text{Isolate the unknown.}$$
$$FG^2 = EF^2 - EG^2$$
$$FG^2 = 5.9^2 - 3.5^2$$
$$= 22.56$$
$$FG = \sqrt{22.56}$$

Then, use the tangent ratio in right ΔEFG to determine the length of the side opposite ∠J.

$$\tan F = \frac{EG}{FG}$$

EG is opposite ∠F.
FG is adjacent to ∠F.

$$\tan F = \frac{3.5}{\sqrt{22.56}}$$
$$\tan F = 0.7368\ldots$$
$$\angle F \doteq 36.4°$$

b) First, use the Pythagorean Theorem in right ΔHJK to determine the length of the side opposite ∠J.

$JK^2 = HJ^2 + HK^2$ Isolate the unknown.

$HK^2 = JK^2 - HJ^2$

$HK^2 = 6.4^2 - 2.4^2$

$= 35.2$

$HK = \sqrt{35.2}$

Then, use the tangent ratio in right ΔHJK.

$\tan J = \dfrac{HK}{HJ}$ HK is opposite ∠J. HJ is adjacent to ∠J.

$\tan J = \dfrac{\sqrt{35.2}}{2.4}$

$\tan J = 2.4720\ldots$

$\angle J \doteq 68.0°$

9. a) i) In right △ABC,

$\tan B = \dfrac{AC}{BC}$ AC is opposite ∠B. BC is adjacent to ∠B.

$\tan B = \dfrac{8}{4}$

$\tan B = 2$

ii) In right △DEF,

$\tan D = \dfrac{EF}{DE}$ EF is opposite ∠D. DE is adjacent to ∠D.

$\tan D = \dfrac{4}{2}$

$\tan D = 2$

iii) In right △GHJ,

$\tan G = \dfrac{HJ}{GJ}$ HJ is opposite ∠G. GJ is adjacent to ∠G.

$\tan G = \dfrac{9.0}{4.5}$

$\tan G = 2$

Each right triangle has an acute angle with a tangent ratio of 2.
So, △ABC, △FDE, and △HGJ are similar triangles.

b) i) From the answer to part a i,

$\tan B = 2$

$\angle B \doteq 63.4°$

The sum of the angles in a triangle is 180°, so:

$\angle A \doteq 180° - 63.4° - 90°$

$\doteq 26.6°$

ii) $\triangle$FDE is similar to $\triangle$ABC, so

$$\begin{aligned}\angle D &= \angle B\\ &\doteq 63.4°\end{aligned}$$

and

$$\begin{aligned}\angle F &= \angle A\\ &\doteq 26.6°\end{aligned}$$

iii) $\triangle$HGJ is similar to $\triangle$ABC, so

$$\begin{aligned}\angle G &= \angle B\\ &\doteq 63.4°\end{aligned}$$

and

$$\begin{aligned}\angle H &= \angle A\\ &\doteq 26.6°\end{aligned}$$

c) No. First I determined the measures of the angles in one triangle. Then, I used the fact that corresponding angles in similar triangles have equal measures to determine the angle measures in the other triangles.

10. a) Use the tangent ratio. Let the angle of inclination be $\angle$A.

$$\begin{aligned}\tan A &= \frac{\text{opposite}}{\text{adjacent}}\\ &= \frac{4.0}{5.5} \qquad \text{Use a calculator.}\\ \angle A &= \tan^{-1}\left(\frac{4.0}{5.5}\right)\\ &= 36.0273...\end{aligned}$$

The angle of inclination is approximately 36°.

b) Use the tangent ratio. Let the angle of inclination be $\angle$A.

$$\begin{aligned}\tan A &= \frac{\text{opposite}}{\text{adjacent}}\\ &= \frac{7.5}{6.5} \qquad \text{Use a calculator.}\\ \angle A &= \tan^{-1}\left(\frac{7.5}{6.5}\right)\\ &= 49.0856...\end{aligned}$$

The angle of inclination is approximately 49.1°.

c) Use the tangent ratio. Let the angle of inclination be $\angle A$.

$$\tan A = \frac{\text{opposite}}{\text{adjacent}}$$

$$= \frac{3.4}{9.2} \qquad \text{Use a calculator.}$$

$$\angle A = \tan^{-1}\left(\frac{3.4}{9.2}\right)$$

$$= 20.2825...$$

The angle of inclination is approximately 20.3°.

d) Use the tangent ratio. Let the angle of inclination be $\angle A$.

$$\tan A = \frac{\text{opposite}}{\text{adjacent}}$$

$$= \frac{8.2}{1.1} \qquad \text{Use a calculator.}$$

$$\angle A = \tan^{-1}\left(\frac{8.2}{1.1}\right)$$

$$= 82.3595...$$

The angle of inclination is approximately 82.4°.

11. a) Use the tangent ratio. Let the angle of inclination be $\angle A$.

$$\tan A = \frac{\text{opposite}}{\text{adjacent}}$$

$$= \frac{20}{100} \qquad \text{Use a calculator.}$$

$$\angle A = \tan^{-1}\left(\frac{20}{100}\right)$$

$$= 11.3099...$$

The angle of inclination is approximately 11°.

b) Use the tangent ratio. Let the angle of inclination be $\angle A$.

$$\tan A = \frac{\text{opposite}}{\text{adjacent}}$$

$$= \frac{25}{100} \qquad \text{Use a calculator.}$$

$$\angle A = \tan^{-1}\left(\frac{25}{100}\right)$$

$$= 14.0362...$$

The angle of inclination is approximately 14°.

c) Use the tangent ratio. Let the angle of inclination be $\angle A$.

$$\tan A = \frac{\text{opposite}}{\text{adjacent}}$$

$$= \frac{10}{100} \qquad \text{Use a calculator.}$$

$$\angle A = \tan^{-1}\left(\frac{10}{100}\right)$$

$$= 5.7105...$$

The angle of inclination is approximately 6°.

d) Use the tangent ratio. Let the angle of inclination be $\angle A$.

$$\tan A = \frac{\text{opposite}}{\text{adjacent}}$$

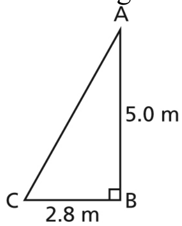
$$= \frac{15}{100} \qquad \text{Use a calculator.}$$

$$\angle A = \tan^{-1}\left(\frac{15}{100}\right)$$

$$= 8.5307...$$

The angle of inclination is approximately 9°.

12. The solar panel will work best when the angle of inclination of the roof is approximately equal to the latitude of the house.
Draw a right triangle to represent the cross-section of the roof and solar panel.



$\angle C$ is the angle of inclination. In right $\triangle ABC$,

$$\tan C = \frac{AB}{BC}$$

AB is opposite $\angle C$.
BC is adjacent to $\angle C$.

$$\tan C = \frac{5.0}{2.8}$$

$$\angle C = \tan^{-1}\left(\frac{5.0}{2.8}\right) \doteq 60.8°$$

The latitude of Whitehorse is approximately 60.8°. It is the only location whose latitude is within 1° of the angle of inclination of the solar panel.

13. The acute angles in the diagram are: $\angle P$, $\angle R$, $\angle PQS$, and $\angle RQS$.
In right $\triangle PQR$,

$$\tan P = \frac{QR}{PQ}$$

QR is opposite $\angle P$.
PQ is adjacent to $\angle P$.

$$\tan P = \frac{12}{5}$$

$$\angle P \doteq 67.4°$$

Use the fact that the sum of the angles in a triangle is 180°.
In $\triangle PQR$, $\angle Q = 90°$ and $\angle P \doteq 67.4°$, so:

$$\angle R \doteq 180° - 67.4° - 90° \doteq 22.6°$$

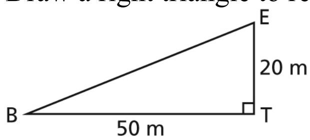
In $\triangle PQS$, $\angle S = 90°$ and $\angle P \doteq 67.4°$, so:

$$\angle PQS \doteq 180° - 67.4° - 90° \doteq 22.6°$$

In $\triangle RQS$, $\angle S = 90°$ and $\angle R \doteq 22.6°$, so:

$$\angle RQS \doteq 180° - 22.6° - 90° \doteq 67.4°$$

14. Draw a right triangle to represent the birdwatcher, the base of the tree, and the eagle.



∠B is the angle of inclination. In right △BTE,

$\tan B = \dfrac{ET}{BT}$ ET is opposite ∠B. BT is adjacent to ∠B.

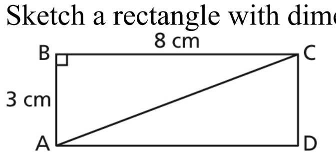
$\tan B = \dfrac{20}{50}$

$\tan B = 0.4$

$\angle B = 21.8014\ldots°$

So, the birdwatcher must incline his camera about 22° to take a photograph of the eagle.

15. Sketch a rectangle with dimensions 3 cm by 8 cm.



A diagonal of the rectangle divides it into two congruent right triangles. So, to determine the angles the diagonal makes with the sides of the rectangle, determine the measures of ∠A and ∠C in right △ABC.

To determine the angle the diagonal makes with the shorter side, determine the measure of ∠A:

$\tan A = \dfrac{BC}{AB}$ BC is opposite ∠A. AB is adjacent to ∠A.

$\tan A = \dfrac{8}{3}$

$\angle A = 69.4439\ldots°$

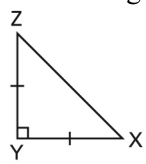
Use the fact that the sum of angles in a triangle is 180°.

In ΔABC, ∠B = 90° and ∠A ≐ 69.4°, so:

$$\begin{aligned}\angle C &\doteq 180° - 90° - 69.4° \\ &\doteq 20.6°\end{aligned}$$

So, the diagonal makes an angle of approximately 69.4° with the shorter side of the rectangle and an angle of approximately 20.6° with the longer side.

16. Sketch a right isosceles triangle.



The acute angles in right △XYZ are ∠X and ∠Z.

$$\tan X = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan X = \frac{YZ}{XY} \qquad XY = YZ$$

$$\tan X = \frac{YZ}{YZ}$$

$$\tan X = 1$$

Similarly,

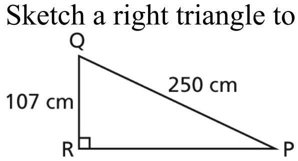
$$\tan Z = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan Z = \frac{XY}{YZ} \qquad XY = YZ$$

$$\tan Z = \frac{YZ}{YZ}$$

$$\tan Z = 1$$

17. Sketch a right triangle to represent the slide.



The length of the slide is the hypotenuse of the right triangle.
To use the tangent ratio to determine ∠P, we need to know the length of PR.
Use the Pythagorean Theorem.

$$PQ^2 = QR^2 + PR^2 \qquad \text{Isolate the unknown.}$$

$$PR^2 = PQ^2 - QR^2$$

$$PR^2 = 250^2 - 107^2$$

$$= 51\,051$$

$$PR = \sqrt{51\,051}$$

Use the tangent ratio in right △PQR.

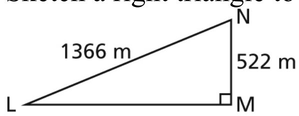
$$\tan P = \frac{QR}{PR} \qquad \begin{array}{l}QR \text{ is opposite } \angle P.\\ PR \text{ is adjacent to } \angle P.\end{array}$$

$$\tan P = \frac{107}{\sqrt{51\,051}}$$

$$\angle P \doteq 25^\circ$$

The angle the slide makes with the ground is approximately 25°.

18. Sketch a right triangle to represent the ski lift.



The length of the ski lift is the hypotenuse of the right triangle.
To use the tangent ratio to determine ∠L, we need to know the length of LM.
Use the Pythagorean Theorem.

$\text{LN}^2 = \text{LM}^2 + \text{MN}^2$ Isolate the unknown.

$\text{LM}^2 = \text{LN}^2 - \text{MN}^2$

$\text{LM}^2 = 1366^2 - 522^2$

$= 1\,593\,472$

$\text{LM} = \sqrt{1\,593\,472}$

Use the tangent ratio in right △LMN.

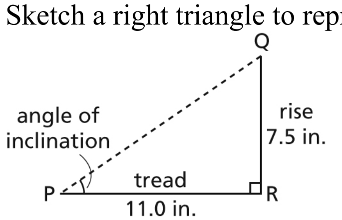
$\tan \text{L} = \dfrac{\text{MN}}{\text{LM}}$ MN is opposite ∠L. LM is adjacent to ∠L.

$\tan \text{L} = \dfrac{522}{\sqrt{1\,593\,472}}$

$\angle \text{L} \doteq 22°$

The angle of inclination of the ski lift is approximately 22°.

19. Sketch a right triangle to represent the stairs.



Calculate the angle of inclination of the stairs.

$\tan \text{P} = \dfrac{\text{rise}}{\text{tread}}$

$\tan \text{P} = \dfrac{\text{QR}}{\text{PR}}$ QR is opposite ∠P. PR is adjacent to ∠P.

$\tan \text{P} = \dfrac{7.5}{11.0}$

$\angle \text{P} \doteq 34°$

The angle to be cut and the angle of inclination form a straight line.
So, the angle to be cut is approximately:
$180° - 34° \doteq 146°$
The carpenter should cut the board at an angle of approximately 146°.

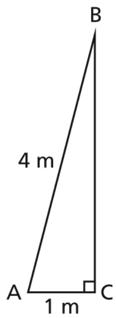
20. Sketch a right triangle to represent a ladder, AB, and its distance, AC, from the wall.

The distance AC must be less than or equal to $\frac{1}{4}$ AB.

That is, $AC \le \frac{1}{4}AB$

So, $AB \ge 4AC$

Assume $AB = 4AC$. Let AC be 1 m and AB be 4 m.



To use the tangent ratio to determine ∠A, we need to know the length of BC.

Use the Pythagorean Theorem.

$AB^2 = AC^2 + BC^2$ Isolate the unknown.

$BC^2 = AB^2 - AC^2$

$BC^2 = 4^2 - 1^2$

$= 15$

$BC = \sqrt{15}$

Use the tangent ratio in right △ABC.

$\tan A = \frac{BC}{AC}$ BC is opposite ∠A. AC is adjacent to ∠A.

$\tan A = \frac{\sqrt{15}}{1}$

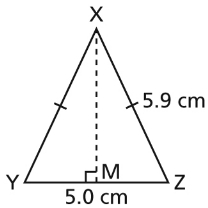
$\tan A = \sqrt{15}$

$\angle A \doteq 76°$

The least allowed angle of inclination of a ladder is approximately 76°.

C

21. Sketch △XYZ and draw the perpendicular bisector of YZ through point M.



Line MX divides △XYZ into 2 congruent right triangles: △MXY and △MXZ
To use the tangent ratio to determine ∠Y, first determine the length of MX.
Use the Pythagorean Theorem in right △MXY.

$XY^2 = MX^2 + MY^2$ Isolate the unknown.

$$MX^2 = XY^2 - MY^2$$

$$MX^2 = 5.9^2 - \left(\frac{5.0}{2}\right)^2$$

$$= 28.56$$

$$MX = \sqrt{28.56}$$

Use the tangent ratio in right △MXY.

$\tan Y = \dfrac{MX}{MY}$ MX is opposite ∠Y. MY is adjacent to ∠Y.

$$\tan Y = \frac{\sqrt{28.56}}{2.5}$$

$$\angle Y = 64.9297\ldots°$$

$$\angle Y \doteq 64.9°$$

Since △XYZ is isosceles and XY = XZ, ∠Z = ∠Y. So, $\angle Z \doteq 64.9°$

The sum of the angles in a triangle is 180°, so in ΔXYZ:

$$\angle X \doteq 180° - 64.9° - 64.9°$$

$$\doteq 50.2$$

So, the angle measures in △XYZ are: $\angle X \doteq 50.2°$, $\angle Y \doteq 64.9°$, and $\angle Z \doteq 64.9°$

22. a) There is no least possible value. The tangent can be arbitrarily close to 0. Consider a right triangle with acute ∠A and suppose the side opposite ∠A has length 1 unit. Then

$\tan A = \dfrac{1 \text{ unit}}{\text{length of side adjacent to } \angle A}$

When the side adjacent to ∠A is much longer than the side opposite ∠A, tan A is very close to 0.

b) There is no greatest possible value. The tangent can be arbitrarily large. Consider a right triangle with acute ∠A and suppose the side adjacent to ∠A has length 1 unit. Then

$\tan A = \dfrac{\text{length of side opposite } \angle A}{1 \text{ unit}}$

When the side opposite ∠A is much longer than the side adjacent to ∠A, tan A is very large.

23. a) In each right triangle, the side opposite the angle at the centre of the spiral is 1 unit long. For the 1st triangle, the length, in units, of the adjacent side is: 1

So, the tangent of the angle for the 1st triangle is: $\dfrac{1}{1}$, or 1

The hypotenuse of each right triangle is the side adjacent to the angle at the centre of the spiral in the next right triangle. So, use the Pythagorean Theorem to determine the lengths of the adjacent sides in the 2nd to 5th triangles.

For the 2nd triangle, the length, in units, of the adjacent side is: $\sqrt{1^2 + 1^2} = \sqrt{2}$

So, the tangent of the angle for the 2nd triangle is: $\dfrac{1}{\sqrt{2}}$

For the 3rd triangle, the length, in units, of the adjacent side is: $\sqrt{1^2 + \left(\sqrt{2}\right)^2} = \sqrt{3}$

So, the tangent of the angle for the 3rd triangle is: $\dfrac{1}{\sqrt{3}}$

For the 4th triangle, the length, in units, of the adjacent side is: $\sqrt{1^2 + \left(\sqrt{3}\right)^2} = \sqrt{4}$, or 2

So, the tangent of the angle for the 4th triangle is: $\dfrac{1}{\sqrt{4}}$, or $\dfrac{1}{2}$

For the 5th triangle, the length, in units, of the adjacent side is: $\sqrt{1^2 + 1^2} = \sqrt{5}$

So, the tangent of the angle for the 5th triangle is: $\dfrac{1}{\sqrt{5}}$

b) The pattern is the tangent of the angle at the centre of the spiral for the *n*th triangle is:

$\dfrac{1}{\sqrt{n}}$

So, the tangent of the angle at the centre of the spiral for the 100th triangle is:

$\dfrac{1}{\sqrt{100}}$, or $\dfrac{1}{10}$

Lesson 2.2 Using the Tangent Ratio to Calculate Lengths Exercises (pages 82–83)

A

3. a) In right △ABC, AC is the side opposite ∠B and BC is the side adjacent to ∠B.
Use the tangent ratio to write an equation.

$$\tan B = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan B = \frac{\text{AC}}{\text{BC}}$$

$$\tan 27° = \frac{b}{5.0} \qquad \text{Solve for } b.$$

$$5.0 \times \tan 27° = \frac{b}{5.0} \times 5.0$$

$$5.0 \tan 27° = b$$

$$b = 2.5476\ldots$$

So, AC is approximately 2.5 cm long.

b) In right △DEF, EF is the side opposite ∠D and DE is the side adjacent to ∠D.
Use the tangent ratio to write an equation.

$$\tan D = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan D = \frac{\text{EF}}{\text{DE}}$$

$$\tan 35° = \frac{d}{2.0} \qquad \text{Solve for } d.$$

$$2.0 \times \tan 35° = \frac{d}{2.0} \times 2.0$$

$$2.0 \tan 35° = d$$

$$d = 1.4004\ldots$$

So, EF is approximately 1.4 cm long.

c) In right △GHJ, HJ is the side opposite ∠G and GH is the side adjacent to ∠G.
Use the tangent ratio to write an equation.

$$\tan G = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan G = \frac{\text{HJ}}{\text{GH}}$$

$$\tan 59° = \frac{g}{3.0} \qquad \text{Solve for } g.$$

$$3.0 \times \tan 59° = \frac{g}{3.0} \times 3.0$$

$$3.0 \tan 59° = g$$

$$g = 4.9928\ldots$$

So, HJ is approximately 5.0 cm long.

d) In right △KMN, KN is the side opposite ∠M and KM is the side adjacent to ∠M.
Use the tangent ratio to write an equation.

$$\tan M = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan M = \frac{KN}{KM}$$

$$\tan 43° = \frac{m}{8.0} \qquad \text{Solve for } m.$$

$$8.0 \times \tan 43° = \frac{m}{8.0} \times 8.0$$

$$8.0 \tan 43° = m$$

$$m = 7.4601\ldots$$

So, KN is approximately 7.5 cm long.

4. a) In right △NPQ, NP is the side opposite ∠Q and NQ is the side adjacent to ∠Q.

$$\tan Q = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan Q = \frac{NP}{NQ}$$

$$\tan 64° = \frac{4.5}{p} \qquad \text{Solve for } p.$$

$$p \times \tan 64° = \frac{4.5}{p} \times p$$

$$p \times \tan 64° = 4.5$$

$$\frac{p \times \tan 64°}{\tan 64°} = \frac{4.5}{\tan 64°}$$

$$p = \frac{4.5}{\tan 64°}$$

$$p = 2.1947\ldots$$

So, NQ is approximately 2.2 cm long.

b) In right △RST, RS is the side opposite ∠T and ST is the side adjacent to ∠T.

$$\tan \mathrm{T} = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan \mathrm{T} = \frac{\mathrm{RS}}{\mathrm{ST}}$$

$$\tan 72° = \frac{8.7}{r} \qquad \text{Solve for } r.$$

$$r \times \tan 72° = \frac{8.7}{r} \times r$$

$$r \times \tan 72° = 8.7$$

$$\frac{r \times \tan 72°}{\tan 72°} = \frac{8.7}{\tan 72°}$$

$$r = \frac{8.7}{\tan 72°}$$

$$r = 2.8268\ldots$$

So, ST is approximately 2.8 cm long.

c) In right △UVW, VW is the side opposite ∠U and UW is the side adjacent to ∠U.

$$\tan \mathrm{U} = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan \mathrm{U} = \frac{\mathrm{VW}}{\mathrm{UW}}$$

$$\tan 23° = \frac{1.2}{v} \qquad \text{Solve for } v.$$

$$v \times \tan 23° = \frac{1.2}{v} \times v$$

$$v \times \tan 23° = 1.2$$

$$\frac{v \times \tan 23°}{\tan 23°} = \frac{1.2}{\tan 23°}$$

$$v = \frac{1.2}{\tan 23°}$$

$$v = 2.8270\ldots$$

So, UW is approximately 2.8 cm long.

5. a) In right △PQR, QR is the side opposite ∠P and PQ is the side adjacent to ∠P.

$$\tan P = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan P = \frac{\text{QR}}{\text{PQ}}$$

$$\tan 15° = \frac{1.5}{r} \qquad \text{Solve for } r.$$

$$r \times \tan 15° = \frac{1.5}{r} \times r$$

$$r \times \tan 15° = 1.5$$

$$\frac{r \times \tan 15°}{\tan 15°} = \frac{1.5}{\tan 15°}$$

$$r = \frac{1.5}{\tan 15°}$$

$$r = 5.5980\ldots$$

So, PQ is approximately 5.6 cm long.

b) In right △STU, SU is the side opposite ∠T and ST is the side adjacent to ∠T.

$$\tan T = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan T = \frac{\text{SU}}{\text{ST}}$$

$$\tan 61° = \frac{7.4}{u} \qquad \text{Solve for } u.$$

$$u \times \tan 61° = \frac{7.4}{u} \times u$$

$$u \times \tan 61° = 7.4$$

$$\frac{u \times \tan 61°}{\tan 61°} = \frac{7.4}{\tan 61°}$$

$$u = \frac{7.4}{\tan 61°}$$

$$u = 4.1018\ldots$$

So, ST is approximately 4.1 cm long.

c) In right △VWX, VX is the side opposite ∠W and WX is the side adjacent to ∠W.

$$\tan W = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan W = \frac{VX}{WX}$$

$$\tan 32° = \frac{2.4}{v} \qquad \text{Solve for } v.$$

$$v \times \tan 32° = \frac{2.4}{v} \times v$$

$$v \times \tan 32° = 2.4$$

$$\frac{v \times \tan 32°}{\tan 32°} = \frac{2.4}{\tan 32°}$$

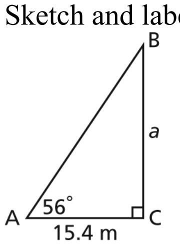
$$v = \frac{2.4}{\tan 32°}$$

$$v = 3.8408\ldots$$

So, WX is approximately 3.8 cm long.

B

6. Sketch and label a diagram to represent the information in the problem.



In right △ABC, BC is the side opposite ∠A and AC is the side adjacent to ∠A.

$$\tan A = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan A = \frac{BC}{AC}$$

$$\tan 56° = \frac{a}{15.4} \qquad \text{Solve for } a.$$

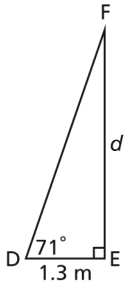
$$15.4 \times \tan 56° = \frac{a}{15.4} \times 15.4$$

$$15.4 \tan 56° = a$$

$$a = 22.8314\ldots$$

The wire reaches approximately 22.8 m up the tower.

7. Sketch and label a diagram to represent the information in the problem.



In right △DEF, EF is the side opposite ∠D and DE is the side adjacent to ∠D.

$$\tan D = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan D = \frac{EF}{DE}$$

$$\tan 71° = \frac{d}{1.3} \qquad \text{Solve for } d.$$

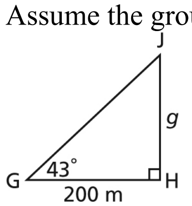
$$1.3 \times \tan 71° = \frac{d}{1.3} \times 1.3$$

$$1.3 \tan 71° = d$$

$$d = 3.7754\ldots$$

The ladder reaches approximately 3.8 m up the wall.

8. Sketch and label a diagram to represent the information in the problem.
Assume the ground is horizontal.



In right △GHJ, HJ is the side opposite ∠G and GH is the side adjacent to ∠G.

$$\tan G = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan G = \frac{HJ}{GH}$$

$$\tan 43° = \frac{g}{200} \qquad \text{Solve for } g.$$

$$200 \times \tan 43° = \frac{g}{200} \times 200$$

$$200 \tan 43° = d$$

$$d = 186.5030\ldots$$

The helicopter is approximately 187 m high.

9. Calculate the unknown length of the leg of the right triangle using the tangent ratio. Then use the Pythagorean Theorem to determine the length of the hypotenuse.

a) In right △FGH, FG is the side opposite ∠H and GH is the side adjacent to ∠H.

$$\tan H = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan H = \frac{FG}{GH}$$

$$\tan 42° = \frac{FG}{2.7} \qquad \text{Solve for FG.}$$

$$2.7 \times \tan 42° = \frac{FG}{2.7} \times 2.7$$

$$2.7 \tan 42° = FG$$

$$FG = 2.4310\ldots$$

Use the Pythagorean Theorem in right △FGH.

$$FH^2 = FG^2 + GH^2$$

$$FH^2 = (2.4310\ldots)^2 + 2.7^2$$

$$= 13.2002\ldots$$

$$FH = \sqrt{13.2002\ldots}$$

$$= 3.6332\ldots$$

The length of the hypotenuse is approximately 3.6 cm.

b) In right △JKM, KM is the side opposite ∠J and JM is the side adjacent to ∠J.

$$\tan J = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan J = \frac{KM}{JM}$$

$$\tan 36° = \frac{5.9}{JM} \qquad \text{Solve for JM.}$$

$$JM \times \tan 36° = \frac{5.9}{JM} \times JM$$

$$JM \times \tan 36° = 5.9$$

$$\frac{JM \times \tan 36°}{\tan 36°} = \frac{5.9}{\tan 36°}$$

$$JM = \frac{5.9}{\tan 36°}$$

$$JM = 8.1206\ldots$$

Use the Pythagorean Theorem in right △JKM.

$$\mathrm{JK}^2 = \mathrm{JM}^2 + \mathrm{MK}^2$$
$$\mathrm{JK}^2 = (8.1206\ldots)^2 + 5.9^2$$
$$= 100.7550\ldots$$
$$\mathrm{JK} = \sqrt{100.7550\ldots}$$
$$= 10.0376\ldots$$

The length of the hypotenuse is approximately 10.0 cm.

10. Sketch and label a diagram to represent the information in the problem.
Assume the ground is horizontal.

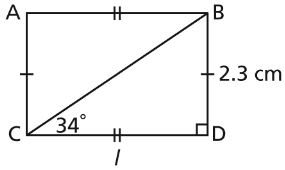
In right △KLM, LM is the side opposite ∠K and KL is the side adjacent to ∠K.

$$\tan \mathrm{K} = \frac{\text{opposite}}{\text{adjacent}}$$
$$\tan \mathrm{K} = \frac{\mathrm{LM}}{\mathrm{KL}}$$
$$\tan 81° = \frac{191}{m} \qquad \text{Solve for } m.$$
$$m \times \tan 81° = \frac{191}{m} \times m$$
$$m \times \tan 81° = 191$$
$$\frac{m \times \tan 81°}{\tan 81°} = \frac{191}{\tan 81°}$$
$$m = \frac{191}{\tan 81°}$$
$$m = 30.2514\ldots$$

Claire was approximately 30 m from the tower.

The distance is approximate because I assumed the ground was horizontal and I didn't consider the height of Claire's eye above the ground.

11. a) Sketch and label the rectangle. Let the length of CD be l.



b) In right △BCD, BD is the side opposite ∠C and CD is the side adjacent to ∠C.

$$\tan C = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan C = \frac{BD}{CD}$$

$$\tan 34° = \frac{2.3}{l} \qquad \text{Solve for } l.$$

$$l \times \tan 34° = \frac{2.3}{l} \times l$$

$$l \times \tan 34° = 2.3$$

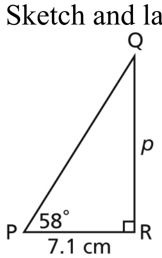
$$\frac{l \times \tan 34°}{\tan 34°} = \frac{2.3}{\tan 34°}$$

$$l = \frac{2.3}{\tan 34°}$$

$$l = 3.4098\ldots$$

The length of the rectangle is approximately 3.4 cm.

12. Sketch and label a diagram to represent the information in the problem.



To determine the area of a triangle, we need to know its base and height.
In right △PQR, QR is the side opposite ∠P and PR is the side adjacent to ∠P.

$$\tan P = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan P = \frac{QR}{PR}$$

$$\tan 58° = \frac{p}{7.1} \qquad \text{Solve for } p.$$

$$7.1 \times \tan 58° = \frac{p}{7.1} \times 7.1$$

$$7.1 \tan 58° = p$$

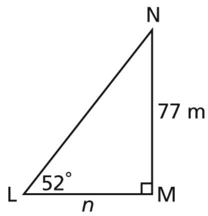
$$p = 11.3623\ldots$$

The area of the triangle is:

$$\frac{1}{2}(\text{PR})(\text{QR}) = \frac{1}{2}(7.1)(11.3623\ldots)$$
$$= 40.3364\ldots$$

The area of the triangle is approximately 40.3 cm^2.

13. Sketch and label a diagram to represent the information in the problem.

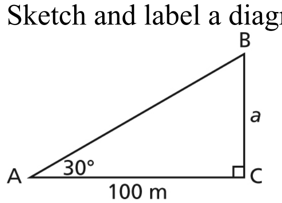


In the diagram, n represents Liam's distance from a point on the ground vertically below the statue. In right △LMN, MN is the side opposite ∠L and LM is the side adjacent to ∠L.

$$\tan \text{L} = \frac{\text{opposite}}{\text{adjacent}}$$
$$\tan \text{L} = \frac{\text{MN}}{\text{LM}}$$
$$\tan 52° = \frac{77}{n} \qquad \text{Solve for } n.$$
$$n \times \tan 52° = \frac{77}{n} \times n$$
$$n \times \tan 52° = 77$$
$$\frac{n \times \tan 52°}{\tan 52°} = \frac{77}{\tan 52°}$$
$$n = \frac{77}{\tan 52°}$$
$$n = 60.1589\ldots$$

Liam is approximately 60 m from a point on the ground vertically below the statue.

14. Sketch and label a diagram to represent the information in the problem.



In right △ABC, BC is the side opposite ∠A and AC is the side adjacent to ∠A.

$$\tan A = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan A = \frac{BC}{AC}$$

$$\tan 30° = \frac{a}{100} \qquad \text{Solve for } a.$$

$$100 \times \tan 30° = \frac{a}{100} \times 100$$

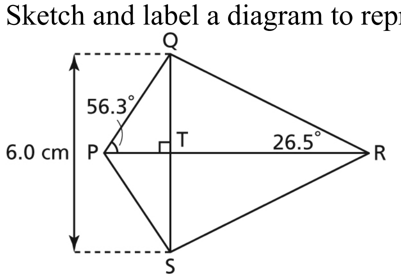
$$100 \tan 30° = a$$

$$a = 57.7350\ldots$$

The balloon is approximately 58 m high.
I am assuming that the ground between Janelle and the store is horizontal.

C

15. Sketch and label a diagram to represent the information in the problem.



The diagonal PR divides the kite into two congruent triangles. So:
$\angle SPT = \angle QPT = 56.3°$
$\angle SRT = \angle QRT = 26.5°$

$$\begin{aligned}\angle QPS &= \angle QPT + \angle SPT \\ &= 56.3° + 56.3° \\ &= 112.6°\end{aligned}$$

$$\begin{aligned}\angle QRS &= \angle QRT + \angle SRT \\ &= 26.5° + 26.5° \\ &= 53.0°\end{aligned}$$

In $\triangle PQR$, the sum of the angles is 180°. So:

$$\begin{aligned}\angle PQR &= 180° - \angle P - \angle R \\ &= 180° - 56.3° - 26.5° \\ &= 97.2°\end{aligned}$$

$\angle PSR = \angle PQR = 97.2°$
In $\triangle PQT$, the sum of the angles is 180°. So:

$$\begin{aligned}\angle PQT &= 180° - \angle QPT - \angle PTQ \\ &= 180° - 56.3° - 90° \\ &= 33.7°\end{aligned}$$

$\angle PST = \angle PQT = 33.7°$
$\angle PTQ$ and $\angle QTR$ form a straight angle. So:

$$\begin{aligned}\angle QTR &= 180° - \angle PTQ\\ &= 180° - 90°\\ &= 90°\end{aligned}$$

$\angle PTS = \angle QTP = 90°$

$\angle RTS = \angle QTR = 90°$

In $\triangle QRT$, the sum of the angles is 180°. So:

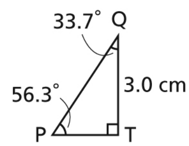
$$\begin{aligned}\angle RQT &= 180° - \angle QRT - \angle QTR\\ &= 180° - 26.5° - 90°\\ &= 63.5°\end{aligned}$$

$\angle RST = \angle RQT = 63.5°$

In right $\triangle PQT$, PT is the side opposite $\angle Q$ and QT is the side adjacent to $\angle Q$.

$$\tan Q = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan Q = \frac{PT}{QT}$$



$$\tan 33.7° = \frac{PT}{3.0}$$ Solve for PT.

$$3.0 \times \tan 33.7° = \frac{PT}{3.0} \times 3.0$$

$$3.0 \tan 33.7° = PT$$

$$PT = 2.0007\ldots$$

Use the Pythagorean Theorem in right $\triangle PQT$.

$$PQ^2 = PT^2 + QT^2$$

$$\begin{aligned}PQ^2 &= (2.0007\ldots)^2 + 3.0^2\\ &= 13.0030\ldots\end{aligned}$$

$$\begin{aligned}PQ &= \sqrt{13.0030\ldots}\\ &= 3.6059\ldots\end{aligned}$$

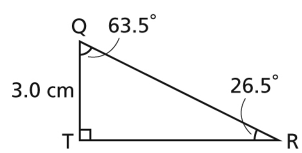
$PQ \doteq 3.6$ cm

$PS = PQ \doteq 3.6$ cm

In right $\triangle QRT$, RT is the side opposite $\angle Q$ and QT is the side adjacent to $\angle Q$.

$$\tan Q = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan Q = \frac{RT}{QT}$$



$$\tan 63.5° = \frac{RT}{3.0}$$ Solve for RT.

$$3.0 \times \tan 63.5° = \frac{RT}{3.0} \times 3.0$$

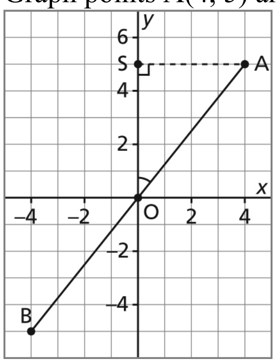
$$3.0 \tan 63.5° = RT$$

$$RT = 6.0170\ldots$$

Use the Pythagorean Theorem in right $\triangle QRT$.

$$QR^2 = RT^2 + QT^2$$
$$QR^2 = (6.0170\ldots)^2 + 3.0^2$$
$$= 45.2051\ldots$$
$$QR = \sqrt{45.2051\ldots}$$
$$= 6.7234\ldots$$
$$QR \doteq 6.7 \text{ cm}$$
$$SR = QR \doteq 6.7 \text{ cm}$$

16. a) Graph points A(4, 5) and B(–4, –5) on a coordinate grid.

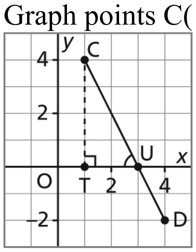


Mark a point with the same *y*-coordinate as A on the *y*-axis: S(0, 5)
Then △AOS is a right triangle with ∠S = 90°.
In right △AOS, ∠O is the measure of the acute angle between AB and the *y*-axis.
Use the tangent ratio in right △AOS.

$$\tan O = \frac{AS}{OS}$$ AS is opposite ∠O. OS is adjacent to ∠O.

$$\tan O = \frac{4}{5}$$
$$\tan O = 0.8$$
$$\angle O \doteq 38.7°$$

b) Graph points C(1, 4) and D(4, –2) on a coordinate grid.



Mark a point with the same *x*-coordinate as C on the *x*-axis: T(1, 0)
Mark the point where CD intersects the *x*-axis: U(3, 0)
Then △CTU is a right triangle with ∠T = 90°.
In right △CTU, ∠U is the measure of the acute angle between CD and the *x*-axis.

Use the tangent ratio in right $\triangle$CTU.

$\tan U = \dfrac{CT}{TU}$ CT is opposite $\angle$U. TU is adjacent to $\angle$U.

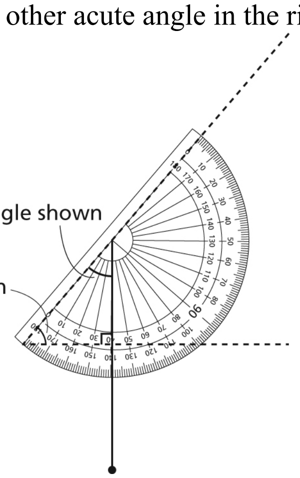
$\tan U = \dfrac{4}{2}$

$\tan U = 2$

$\angle U \doteq 63.4°$

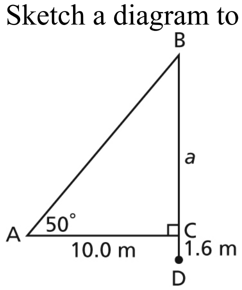
Lesson 2.3 **Math Lab: Measuring an Inaccessible Height** **Assess Your Understanding (page 86)**

1. The base of the protractor, the horizontal line that forms one arm of the angle of inclination, and the vertical piece of string form a right triangle. The angle of inclination is the acute angle formed by the base of the protractor and the horizontal line. The angle shown on the protractor is the other acute angle in the right triangle.



So, the sum of the angle of inclination and the angle shown on the protractor is 90°.

2. The angle of inclination is:
$90° - 40° = 50°$
Sketch a diagram to represent the situation.



In right △ABC, BC is the side opposite ∠A and AC is the side adjacent to ∠A.
Use the tangent ratio in right △ABC.

$$\tan A = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan A = \frac{BC}{AC}$$

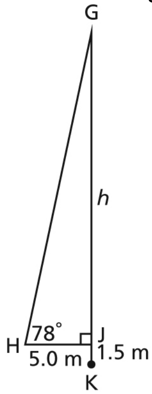
$$\tan 50° = \frac{a}{10.0} \qquad \text{Solve for } a.$$

$$10.0 \tan 50° = a$$

$$a = 11.9175\ldots$$

The height of the tree is:
1.6 m + 11.9175… m = 13.5175… m
The height of the tree is approximately 13.5 m.

3. The angle of inclination is:
$90° - 12° = 78°$
Sketch a diagram to represent the situation.



In right △GHJ, GJ is the side opposite ∠H and HJ is the side adjacent to ∠H.
Use the tangent ratio in right △GHJ.

$$\tan H = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan H = \frac{GJ}{HJ}$$

$$\tan 78° = \frac{h}{5.0} \qquad \text{Solve for } h.$$

$$5.0 \tan 78° = h$$

$$h = 23.5231\ldots$$

The height of the totem pole is:
1.5 m + 23.5231… m = 25.0231… m
The height of the totem pole is approximately 25 m.

Checkpoint 1 **Assess Your Understanding (page 88)**

2.1

1. a) In right △ABC:

$$\tan C = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan C = \frac{AB}{BC}$$ AB is opposite ∠C. BC is adjacent to ∠C.

$$\tan C = \frac{2}{8}$$

$$\angle C \doteq 14°$$

b) In right △DEF:

$$\tan F = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan F = \frac{DE}{EF}$$ DE is opposite ∠F. EF is adjacent to ∠F.

$$\tan F = \frac{6}{4}$$

$$\angle F \doteq 56°$$

c) First, use the Pythagorean Theorem in right △GHJ to determine the length of GJ.

$$HJ^2 = GH^2 + GJ^2$$ Isolate the unknown.

$$GJ^2 = HJ^2 - GH^2$$

$$GJ^2 = 20^2 - 16^2$$

$$= 144$$

$$GJ = \sqrt{144}$$

$$= 12$$

Use the tangent ratio in right △GHJ.

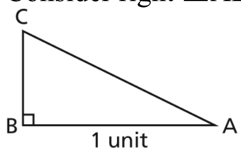
$$\tan J = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan J = \frac{GH}{GJ}$$ GH is opposite ∠J. GJ is adjacent to ∠J.

$$\tan J = \frac{16}{12}$$

$$\angle J \doteq 53°$$

2. Consider right $\triangle$ABC shown below.



In right $\triangle$ABC:

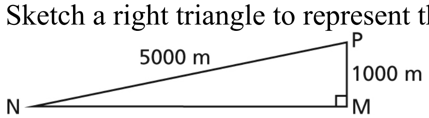
$$\tan A = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan A = \frac{BC}{AB}$$ BC is opposite $\angle$A. AB is adjacent to $\angle$A.

$$\tan A = \frac{BC}{1}$$

As $\angle$A increases, BC increases, which means that tan A also increases.

3. Sketch a right triangle to represent the plane.



In right $\triangle$MNP, MP is the side opposite $\angle$N and MN is the side adjacent to $\angle$N.
To use the tangent ratio to determine $\angle$N, we need to know the length of MN.
Use the Pythagorean Theorem.

$$NP^2 = MN^2 + MP^2$$ Isolate the unknown.

$$MN^2 = NP^2 - MP^2$$

$$MN^2 = 5000^2 - 1000^2$$
$$= 24\,000\,000$$

$$MN = \sqrt{24\,000\,000}$$

Use the tangent ratio in right $\triangle$MNP.

$$\tan N = \frac{PM}{MN}$$ PM is opposite $\angle$N. MN is adjacent to $\angle$N.

$$\tan N = \frac{1000}{\sqrt{24\,000\,000}}$$

$$\angle N \doteq 11.5°$$

The angle between the ground and the line of sight from an observer at the beginning of the landing strip is approximately 11.5°.

2.2

4. a) In right $\triangle$KMN:

$$\tan N = \frac{\text{opposite}}{\text{adjacent}}$$
$$\tan N = \frac{KM}{KN}$$
$$\tan 61° = \frac{n}{6.2} \qquad \text{Solve for } n.$$
$$6.2 \times \tan 61° = \frac{n}{6.2} \times 6.2$$
$$6.2 \tan 61° = n$$
$$n = 11.1850\ldots$$

KM is approximately 11.2 cm long.

b) In right $\triangle$PQR:

$$\tan R = \frac{\text{opposite}}{\text{adjacent}}$$
$$\tan R = \frac{PQ}{QR}$$
$$\tan 56° = \frac{10.8}{p} \qquad \text{Solve for } p.$$
$$p \times \tan 56° = \frac{10.8}{p} \times p$$
$$p \times \tan 56° = 10.8$$
$$\frac{p \times \tan 56°}{\tan 56°} = \frac{10.8}{\tan 56°}$$
$$p = \frac{10.8}{\tan 56°}$$
$$p = 7.2846\ldots$$

QR is approximately 7.3 cm long.

c) First, use the tangent ratio in right △STV to determine ST.

$$\tan T = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan T = \frac{SV}{ST}$$

$$\tan 27° = \frac{5.3}{ST} \qquad \text{Solve for ST.}$$

$$ST \times \tan 27° = \frac{5.3}{ST} \times ST$$

$$ST \times \tan 27° = 5.3$$

$$\frac{ST \times \tan 27°}{\tan 27°} = \frac{5.3}{\tan 27°}$$

$$ST = \frac{5.3}{\tan 27°}$$

$$ST = 10.4018\ldots$$

Use the Pythagorean Theorem in right △STV to determine *s*.

$$TV^2 = ST^2 + SV^2$$

$$s^2 = (10.4018\ldots)^2 + 5.3^2$$

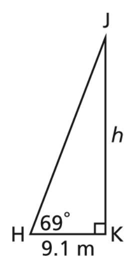
$$= 136.2881\ldots$$

$$s = \sqrt{136.2881\ldots}$$

$$= 11.6742\ldots$$

TV is approximately 11.7 cm long.

5. Sketch a right triangle to represent the situation.
In right △HJK, JK is the side opposite ∠H and HK is the side adjacent to ∠H.
Use the tangent ratio in right △HJK.



$$\tan H = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan H = \frac{JK}{HK}$$

$$\tan 69° = \frac{h}{9.1} \qquad \text{Solve for } h.$$

$$9.1 \times \tan 69° = \frac{h}{9.1} \times 9.1$$

$$9.1 \tan 69° = h$$

$$h = 23.7063\ldots$$

The top of the hoodoo was approximately 23.7 m above the level ground.

Lesson 2.4 The Sine and Cosine Ratios Exercises (pages 95–96)

A

4. **a)** **i)** In right △AGH, GH is opposite ∠A, AG is adjacent to ∠A, and AH is the hypotenuse.

ii) In right △AKT, KT is opposite ∠A, AK is adjacent to ∠A, and AT is the hypotenuse.

b) **i)**

$$\sin A = \frac{\text{opposite}}{\text{hypotenuse}}$$
$$\sin A = \frac{GH}{AH}$$
$$\sin A = \frac{6}{10}$$
$$\sin A = 0.60$$

$$\cos A = \frac{\text{adjacent}}{\text{hypotenuse}}$$
$$\cos A = \frac{AG}{AH}$$
$$\cos A = \frac{8}{10}$$
$$\cos A = 0.80$$

b) **ii)**

$$\sin A = \frac{\text{opposite}}{\text{hypotenuse}}$$
$$\sin A = \frac{KT}{AT}$$
$$\sin A = \frac{7}{25}$$
$$\sin A = 0.28$$

$$\cos A = \frac{\text{adjacent}}{\text{hypotenuse}}$$
$$\cos A = \frac{AK}{AT}$$
$$\cos A = \frac{24}{25}$$
$$\cos A = 0.96$$

5. Use a calculator.

a) $\sin 57° \doteq 0.84$
$\cos 57° \doteq 0.54$

b) $\sin 5° \doteq 0.09$
$\cos 5° \doteq 1.00$

c) $\sin 19° \doteq 0.33$
$\cos 19° \doteq 0.95$

d) $\sin 81° \doteq 0.99$
$\cos 81° \doteq 0.16$

6. Use a calculator.

a) $\sin X = 0.25$

$$\begin{aligned}\angle X &= \sin^{-1}(0.25)\\ &\doteq 14°\end{aligned}$$

b) $\cos X = 0.64$

$$\begin{aligned}\angle X &= \cos^{-1}(0.64)\\ &\doteq 50°\end{aligned}$$

c) $\sin X = \dfrac{6}{11}$

$$\begin{aligned}\angle X &= \sin^{-1}\left(\frac{6}{11}\right)\\ &\doteq 33°\end{aligned}$$

d) $\cos X = \dfrac{7}{9}$

$$\begin{aligned}\angle X &= \cos^{-1}\left(\frac{7}{9}\right)\\ &\doteq 39°\end{aligned}$$

B

7. a) In right $\triangle BCD$, the length of the side opposite $\angle C$ and the length of the hypotenuse are given.

$$\sin C = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin C = \frac{BD}{BC}$$

$$\sin C = \frac{5}{9}$$

$$\angle C \doteq 34°$$

b) In right $\triangle$EFG, the length of the side opposite $\angle$E and the length of the hypotenuse are given.

$$\sin \mathrm{E} = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin \mathrm{E} = \frac{\mathrm{FG}}{\mathrm{EG}}$$

$$\sin \mathrm{E} = \frac{4}{7}$$

$$\angle \mathrm{E} \doteq 35°$$

c) In right $\triangle$HJK, the length of the side opposite $\angle$H and the length of the hypotenuse are given.

$$\sin \mathrm{H} = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin \mathrm{H} = \frac{\mathrm{JK}}{\mathrm{HJ}}$$

$$\sin \mathrm{H} = \frac{10}{16}$$

$$\angle \mathrm{H} \doteq 39°$$

d) In right $\triangle$MNP, the length of the side opposite $\angle$N and the length of the hypotenuse are given.

$$\sin \mathrm{N} = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin \mathrm{N} = \frac{\mathrm{MP}}{\mathrm{NP}}$$

$$\sin \mathrm{N} = \frac{6}{11}$$

$$\angle \mathrm{N} \doteq 33°$$

8. a) In right $\triangle$QRS, the length of the side adjacent to $\angle$Q and the length of the hypotenuse are given.

$$\cos \mathrm{Q} = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos \mathrm{Q} = \frac{\mathrm{QR}}{\mathrm{QS}}$$

$$\cos \mathrm{Q} = \frac{18}{24}$$

$$\angle \mathrm{Q} \doteq 41°$$

b) In right △TUV, the length of the side adjacent to ∠U and the length of the hypotenuse are given.

$$\cos U = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos U = \frac{TU}{UV}$$

$$\cos U = \frac{5}{24}$$

$$\angle U \doteq 78°$$

c) In right △WYX, the length of the side adjacent to ∠Y and the length of the hypotenuse are given.

$$\cos Y = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos Y = \frac{XY}{WY}$$

$$\cos Y = \frac{9}{10}$$

$$\angle Y \doteq 26°$$

d) In right △ABZ, the length of the side adjacent to ∠A and the length of the hypotenuse are given.

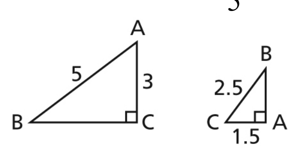
$$\cos A = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos A = \frac{AB}{AZ}$$

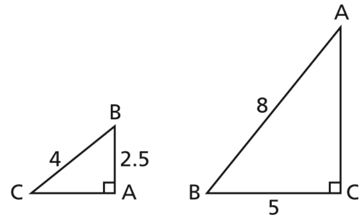
$$\cos A = \frac{2}{5}$$

$$\angle A \doteq 66°$$

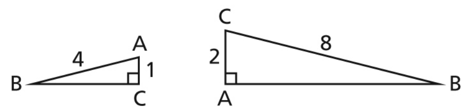
9. **a)** Sketch the triangles so that the ratio of the length of the side opposite ∠B to the length of the hypotenuse is $\frac{3}{5}$. For example:



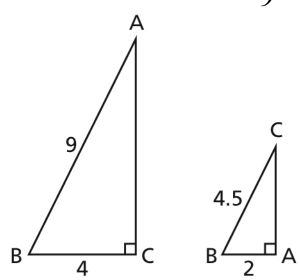
b) Sketch the triangles so that the ratio of the length of the side adjacent to $\angle$B to the length of the hypotenuse is $\frac{5}{8}$. For example:



c) Sketch the triangles so that the ratio of the length of the side opposite $\angle$B to the length of the hypotenuse is $\frac{1}{4}$. For example:



d) Sketch the triangles so that the ratio of the length of the side adjacent to $\angle$B to the length of the hypotenuse is $\frac{4}{9}$. For example:



10. I could use the sine and cosine ratios to determine the measures of both acute angles, or I could use the sine or cosine ratio to determine the measure of one angle and then use the fact that the sum of the angles in a triangle is 180° to determine the measure of the other angle.

a) In right $\triangle$CDE:

$$\cos C = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos C = \frac{CD}{CE}$$

$$\cos C = \frac{2.4}{2.5}$$

$$\angle C \doteq 16.3°$$

$$\sin \mathrm{E} = \frac{\text{opposite}}{\text{hypotenuse}}$$
$$\sin \mathrm{E} = \frac{\mathrm{CD}}{\mathrm{CE}}$$
$$\sin \mathrm{E} = \frac{2.4}{2.5}$$
$$\angle \mathrm{E} \doteq 73.7°$$

b) In right $\triangle$FGH:

$$\cos \mathrm{F} = \frac{\text{adjacent}}{\text{hypotenuse}}$$
$$\cos \mathrm{F} = \frac{\mathrm{FG}}{\mathrm{FH}}$$
$$\cos \mathrm{F} = \frac{1.1}{2.5}$$
$$\angle \mathrm{F} \doteq 63.9°$$

$$\sin \mathrm{H} = \frac{\text{opposite}}{\text{hypotenuse}}$$
$$\sin \mathrm{H} = \frac{\mathrm{FG}}{\mathrm{FH}}$$
$$\sin \mathrm{H} = \frac{1.1}{2.5}$$
$$\angle \mathrm{H} \doteq 26.1°$$

c) In right $\triangle$JKM:

$$\cos \mathrm{J} = \frac{\text{adjacent}}{\text{hypotenuse}}$$
$$\cos \mathrm{J} = \frac{\mathrm{JM}}{\mathrm{JK}}$$
$$\cos \mathrm{J} = \frac{2.6}{3.3}$$
$$\angle \mathrm{J} \doteq 38.0°$$

$$\angle \mathrm{K} = 180° - 90° - \angle \mathrm{J}$$
$$\angle \mathrm{K} \doteq 180° - 90° - 38.0°$$
$$\doteq 52.0°$$

d) In right △PQR:

$$\sin P = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin P = \frac{QR}{PQ}$$

$$\sin P = \frac{2.2}{2.9}$$

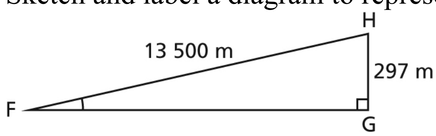
$$\angle P \doteq 49.3°$$

$$\angle Q = 180° - 90° - \angle P$$

$$\angle Q \doteq 180° - 90° - 49.3°$$

$$\doteq 40.7°$$

11. Sketch and label a diagram to represent the information in the problem.



∠F is the angle of inclination of the track.
In right △FGH:

$$\sin F = \frac{\text{opposite}}{\text{hypotenuse}}$$

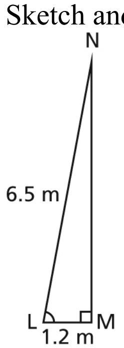
$$\sin F = \frac{GH}{FH}$$

$$\sin F = \frac{297}{13\,500}$$

$$\angle F = 1.2606\ldots°$$

The angle of inclination of the track is approximately 1.3°.

12. Sketch and label a diagram to represent the information in the problem.

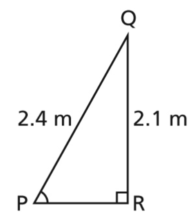


∠L is the angle of inclination of the ladder.
LN is the length of the ladder.
LM is the distance from the base of the ladder to the wall.
In right △LMN:

$$\cos L = \frac{\text{adjacent}}{\text{hypotenuse}}$$
$$\cos L = \frac{LM}{LN}$$
$$\cos L = \frac{1.2}{6.5}$$
$$\angle L = 79.3612\ldots°$$

The angle of inclination of the ladder is approximately 79.4°.

13. Sketch and label a diagram to represent the information in the problem.

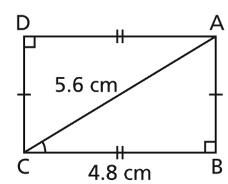


∠P is the angle of inclination of the rope.
PQ is the length of the rope.
QR is the height of the end of the rope that is attached to the tent.
In right △PQR:

$$\sin P = \frac{\text{opposite}}{\text{hypotenuse}}$$
$$\sin P = \frac{QR}{PQ}$$
$$\sin P = \frac{2.1}{2.4}$$
$$\angle P = 61.0449\ldots°$$

The angle of inclination of the rope is approximately 61°.

14. Sketch and label a diagram to represent the information in the problem.



In right △ABC:

$$\cos C = \frac{\text{adjacent}}{\text{hypotenuse}}$$
$$\cos C = \frac{BC}{AC}$$
$$\cos C = \frac{4.8}{5.6}$$
$$\angle C = 31.0027\ldots°$$

The angle between a diagonal and the longest side of the rectangle is approximately 31°.

15. a) **i)** sin 10° = 0.1736…

ii) sin 20° = 0.3420…

iii) sin 40° = 0.6427…

iv) sin 50° = 0.7660…

v) sin 60° = 0.8660…

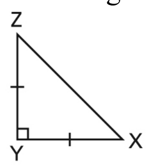
vi) $\sin 80° = 0.9848\ldots$

b) Consider a right triangle with acute $\angle A$ and hypotenuse length 1 unit. Then:

$$\sin A = \frac{\text{length of side opposite } \angle A}{1 \text{ unit}}$$

As $\angle A$ increases, the side opposite $\angle A$ increases, so the ratio sin A increases.

16. Sketch right $\triangle XYZ$:



$$\cos X = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos X = \frac{XY}{XZ}$$

and

$$\sin X = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin X = \frac{YZ}{XZ}$$

Since XY = YZ, $\cos X = \frac{YZ}{XZ}$ and sin X = cos X.

Similarly, sin Z = cos Z

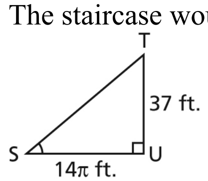
C

17. Suppose the staircase were straightened out.
The staircase would cover a horizontal distance equal to the circumference of the silo:

$C = \pi d$

$C = \pi(14)$

The circumference of the silo is 14π ft.
The staircase would look like the diagram below.



$\angle S$ is the angle of inclination.
SU is the horizontal distance of the staircase.
TU is the height of the silo.
In right $\triangle STU$:

$$\tan S = \frac{TU}{SU}$$
$$\tan S = \frac{37}{14\pi}$$
$$\angle S = 40.0721\ldots°$$

The angle of inclination of the staircase is approximately 40°.

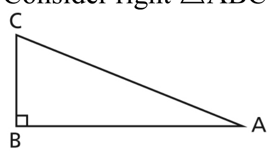
18. a) **i)** sin 90° = 1

ii) sin 0° = 0

iii) cos 90° = 0

iv) cos 0° = 1

b) Consider right △ABC with acute ∠A.



In right △ABC:

$$\sin A = \frac{\text{opposite}}{\text{hypotenuse}} \qquad \text{and} \qquad \cos A = \frac{\text{adjacent}}{\text{hypotenuse}}$$
$$\sin A = \frac{BC}{AC} \qquad \qquad \cos A = \frac{AB}{AC}$$

As vertex A moves closer to vertex B, ∠A increases.
When ∠A is close to 90°, the length of the hypotenuse, AC, is close to the length of BC.
Then, sin A is close to $\frac{BC}{BC}$, or 1.
When ∠A is close to 90°, the length of the side adjacent to ∠A, AB, is close to 0.
Then, cos A is close to $\frac{0}{AC}$, or 0.

As vertex C moves closer to vertex B, ∠A decreases.
When ∠A is close to 0°, the length of the hypotenuse, AC, is close to the length of AB.
Then, cos A is close to $\frac{AB}{AB}$, or 1.
When ∠A is close to 0°, the length of the side opposite ∠A, BC, is close to 0.
Then, sin A is close to $\frac{0}{AC}$, or 0.

Lesson 2.5 **Using the Sine and Cosine Ratios to Calculate Lengths** **Exercises (pages 101–102)**

A

3. a) In right △XYZ, XZ is the hypotenuse and YZ is the side opposite ∠X.
Use the sine ratio.

$$\sin X = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin X = \frac{YZ}{XZ}$$

$$\sin 50° = \frac{x}{4.0} \qquad \text{Solve for } x.$$

$$4.0 \sin 50° = \frac{x(4.0)}{4.0}$$

$$4.0 \sin 50° = x$$

$$x = 3.0641\ldots$$

YZ is approximately 3.1 cm long.

b) In right △UVW, VW is the hypotenuse and UV is the side opposite ∠W.
Use the sine ratio.

$$\sin W = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin W = \frac{UV}{VW}$$

$$\sin 30° = \frac{w}{3.0} \qquad \text{Solve for } w.$$

$$3.0 \sin 30° = \frac{w(3.0)}{3.0}$$

$$3.0 \sin 30° = w$$

$$w = 1.5$$

UV is 1.5 cm long.

c) In right $\triangle$RST, RT is the hypotenuse and ST is the side opposite $\angle$R.
Use the sine ratio.

$$\sin \mathrm{R} = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin \mathrm{R} = \frac{\mathrm{ST}}{\mathrm{RT}}$$

$$\sin 25° = \frac{r}{3.5} \qquad \text{Solve for } r.$$

$$3.5 \sin 25° = \frac{r(3.5)}{3.5}$$

$$3.5 \sin 25° = r$$

$$r = 1.4791\ldots$$

ST is approximately 1.5 cm long.

d) In right $\triangle$PNQ, NQ is the hypotenuse and PQ is the side opposite $\angle$N.
Use the sine ratio.

$$\sin \mathrm{N} = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin \mathrm{N} = \frac{\mathrm{PQ}}{\mathrm{NQ}}$$

$$\sin 55° = \frac{n}{4.5} \qquad \text{Solve for } n.$$

$$4.5 \sin 55° = \frac{n(4.5)}{4.5}$$

$$4.5 \sin 55° = n$$

$$n = 3.6861\ldots$$

PQ is approximately 3.7 cm long.

4. a) In right $\triangle$KMN, KM is the hypotenuse and KN is the side adjacent to $\angle$K.
Use the cosine ratio.

$$\cos \mathrm{K} = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos \mathrm{K} = \frac{\mathrm{KN}}{\mathrm{KM}}$$

$$\cos 72° = \frac{m}{5.5} \qquad \text{Solve for } m.$$

$$5.5 \cos 72° = \frac{m(5.5)}{5.5}$$

$$5.5 \cos 72° = m$$

$$m = 1.6995\ldots$$

KN is approximately 1.7 cm long.

b) In right $\triangle$GHJ, JH is the hypotenuse and GH is the side adjacent to $\angle$H.
Use the cosine ratio.

$$\cos H = \frac{\text{adjacent}}{\text{hypotenuse}}$$
$$\cos H = \frac{GH}{JH}$$
$$\cos 41° = \frac{j}{4.2} \qquad \text{Solve for } j.$$
$$4.2\cos 41° = \frac{j(4.2)}{4.2}$$
$$4.2\cos 41° = j$$
$$j = 3.1697\ldots$$

GH is approximately 3.2 cm long.

c) In right $\triangle$DEF, DF is the hypotenuse and DE is the side adjacent to $\angle$D.
Use the cosine ratio.

$$\cos D = \frac{\text{adjacent}}{\text{hypotenuse}}$$
$$\cos D = \frac{DE}{DF}$$
$$\cos 62° = \frac{f}{11.4} \qquad \text{Solve for } f.$$
$$11.4\cos 62° = \frac{f(11.4)}{11.4}$$
$$11.4\cos 62° = f$$
$$f = 5.3519\ldots$$

DE is approximately 5.4 cm long.

d) In right $\triangle$ABC, AB is the hypotenuse and BC is the side adjacent to $\angle$B.
Use the cosine ratio.

$$\cos B = \frac{\text{adjacent}}{\text{hypotenuse}}$$
$$\cos B = \frac{BC}{AB}$$
$$\cos 23° = \frac{a}{8.6} \qquad \text{Solve for } a.$$
$$8.6\cos 23° = \frac{a(8.6)}{8.6}$$
$$8.6\cos 23° = a$$
$$a = 7.9163\ldots$$

BC is approximately 7.9 cm long.

B

5. a) In right △MNP, NP is the hypotenuse and MP is the side adjacent to ∠P.
Use the cosine ratio.

$$\cos P = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos P = \frac{\text{MP}}{\text{NP}}$$

$$\cos 57° = \frac{13.8}{m} \qquad \text{Solve for } m.$$

$$m\cos 57° = \frac{13.8m}{m}$$

$$m\cos 57° = 13.8$$

$$\frac{m\cos 57°}{\cos 57°} = \frac{13.8}{\cos 57°}$$

$$m = \frac{13.8}{\cos 57°}$$

$$m = 25.3378\ldots$$

NP is approximately 25.3 cm long.

b) In right △HJK, HK is the hypotenuse and HJ is the side opposite ∠K.
Use the sine ratio.

$$\sin K = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin K = \frac{\text{HJ}}{\text{HK}}$$

$$\sin 51° = \frac{6.2}{j} \qquad \text{Solve for } j.$$

$$j\sin 51° = \frac{6.2j}{j}$$

$$j\sin 51° = 6.2$$

$$\frac{j\sin 51°}{\sin 51°} = \frac{6.2}{\sin 51°}$$

$$j = \frac{6.2}{\sin 51°}$$

$$j = 7.9779\ldots$$

HK is approximately 8.0 cm long.

c) In right △EFG, EG is the hypotenuse and EF is the side adjacent to ∠E.
Use the cosine ratio.

$$\cos E = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos E = \frac{EF}{EG}$$

$$\cos 28° = \frac{6.8}{f} \qquad \text{Solve for } f.$$

$$f\cos 28° = \frac{6.8f}{f}$$

$$f\cos 28° = 6.8$$

$$\frac{f\cos 28°}{\cos 28°} = \frac{6.8}{\cos 28°}$$

$$f = \frac{6.8}{\cos 28°}$$

$$f = 7.7014\ldots$$

EG is approximately 7.7 cm long.

d) In right △BCD, BD is the hypotenuse and BC is the side opposite ∠D.
Use the sine ratio.

$$\sin D = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin D = \frac{BC}{BD}$$

$$\sin 42° = \frac{8.3}{c} \qquad \text{Solve for } c.$$

$$c\sin 42° = \frac{8.3c}{c}$$

$$c\sin 42° = 8.3$$

$$\frac{c\sin 42°}{\sin 42°} = \frac{8.3}{\sin 42°}$$

$$c = \frac{8.3}{\sin 42°}$$

$$c = 12.4041\ldots$$

BD is approximately 12.4 cm long.

6. In the diagram, DF is the height that the ladder can reach up the wall of the building.
In right △DEF, DE is the hypotenuse and DF is the side opposite ∠E.

$$\sin E = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin E = \frac{DF}{DE}$$

$$\sin 77° = \frac{DF}{30.5} \qquad \text{Solve for DF.}$$

$$30.5 \sin 77° = \frac{DF(30.5)}{30.5}$$

$$30.5 \sin 77° = DF$$

$$DF = 29.7182\ldots$$

The ladder can reach approximately 29.7 m up the wall of the apartment building.

7. a) In right △CDE, CE is the hypotenuse and DE is the side adjacent to ∠E.

$$\cos E = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos E = \frac{DE}{CE}$$

$$\cos 58.5° = \frac{25.23}{CE} \qquad \text{Solve for CE.}$$

$$CE \cos 58.5° = \frac{25.23(CE)}{CE}$$

$$CE \cos 58.5° = 25.23$$

$$\frac{CE \cos 58.5°}{\cos 58.5°} = \frac{25.23}{\cos 58.5°}$$

$$CE = \frac{25.23}{\cos 58.5°}$$

$$CE = 48.2872\ldots$$

The distance from C to E is approximately 48.3 m.

b) If the distance from C to E is not known, the surveyor could use the tangent ratio since CD is opposite ∠E and the length of the side adjacent to ∠E is known.
If the distance from C to E is known, the surveyor could use the Pythagorean Theorem or the sine ratio to calculate the distance from C to D. For example, using the sine ratio:

$$\sin E = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin E = \frac{CD}{CE}$$

$$\sin 58.5° = \frac{CD}{48.2872\ldots} \qquad \text{Solve for CD.}$$

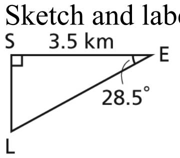
$$(48.2872\ldots)\sin 58.5° = \frac{CD(48.2872\ldots)}{48.2872\ldots}$$

$$(48.2872\ldots)\sin 58.5° = CD$$

$$CD = 41.1716\ldots$$

The distance from C to D is approximately 41.2 m.

8. Sketch and label a diagram to represent the information in the problem.



ES is the distance travelled by the ship.
EL is the distance from the ship to the lighthouse.
∠E is the angle between the ship's path and the line of sight to the lighthouse.
In right △ELS, EL is the hypotenuse and ES is the side adjacent to ∠E.

$$\cos E = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos E = \frac{ES}{EL}$$

$$\cos 28.5° = \frac{3.5}{EL} \qquad \text{Solve for EL.}$$

$$EL\cos 28.5° = \frac{3.5(EL)}{EL}$$

$$EL\cos 28.5° = 3.5$$

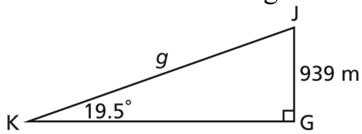
$$\frac{EL\cos 28.5°}{\cos 28.5°} = \frac{3.5}{\cos 28.5°}$$

$$EL = \frac{3.5}{\cos 28.5°}$$

$$EL = 3.9826\ldots$$

The ship is approximately 4.0 km from the lighthouse.

9. Sketch and label a diagram to represent the information in the problem.



GJ is the height of the plane.
KJ is the distance from the plane to the airport.
∠K is the angle of elevation of the plane measured from the airport.
In right △GKJ, JK is the hypotenuse and GJ is the side opposite ∠K.

$$\sin \mathrm{K} = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin \mathrm{K} = \frac{\mathrm{GJ}}{\mathrm{JK}}$$

$$\sin 19.5° = \frac{939}{g} \qquad \text{Solve for } g.$$

$$g \sin 19.5° = \frac{939g}{g}$$

$$g \sin 19.5° = 939$$

$$\frac{g \sin 19.5°}{\sin 19.5°} = \frac{939}{\sin 19.5°}$$

$$g = \frac{939}{\sin 19.5°}$$

$$g = 2813.0039\ldots$$

The plane is approximately 2813 m from the airport.

10. In right $\triangle$CDE, CE is the hypotenuse and CD is the side adjacent to $\angle$C.

$$\cos C = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos C = \frac{CD}{CE}$$

$$\cos 76^\circ = \frac{CD}{18.9} \qquad \text{Solve for CD.}$$

$$18.9\cos 76^\circ = \frac{CD(18.9)}{18.9}$$

$$18.9\cos 76^\circ = CD$$

$$CD = 4.5723\ldots$$

In right $\triangle$CDE, CE is the hypotenuse and DE is the side opposite $\angle$C.

$$\sin C = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin C = \frac{DE}{CE}$$

$$\sin 76^\circ = \frac{DE}{18.9} \qquad \text{Solve for DE.}$$

$$18.9\sin 76^\circ = \frac{DE(18.9)}{18.9}$$

$$18.9\sin 76^\circ = DE$$

$$DE = 18.3385\ldots$$

So, the dimensions of the rectangle are approximately 4.6 cm by 18.3 cm.

11. a) The required length of the bookcase is AB.
In right $\triangle$ABC, AB is the hypotenuse and AC is the side adjacent to $\angle$A.
Use the cosine ratio.

$$\cos A = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos A = \frac{AC}{AB}$$

$$\cos 40^\circ = \frac{3.24}{AB} \qquad \text{Solve for AB.}$$

$$AB\cos 40^\circ = \frac{3.24(AB)}{AB}$$

$$AB\cos 40^\circ = 3.24$$

$$\frac{AB\cos 40^\circ}{\cos 40^\circ} = \frac{3.24}{\cos 40^\circ}$$

$$AB = \frac{3.24}{\cos 40^\circ}$$

$$AB = 4.2295\ldots$$

The length of the top of the bookcase is approximately 4.23 m.

b) The greatest height of the bookcase is side BC.
In right $\triangle$ABC, AB is the hypotenuse and BC is the side opposite $\angle$A.
Use the sine ratio.

$$\sin A = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin A = \frac{BC}{AB}$$

$$\sin 40° = \frac{BC}{4.2295\ldots} \qquad \text{Solve for BC.}$$

$$(4.2295\ldots)\sin 40° = \frac{BC(4.2295\ldots)}{4.2295\ldots}$$

$$(4.2295\ldots)\sin 40° = BC$$

$$BC = 2.7186\ldots$$

The greatest height of the bookcase is approximately 2.72 m.

12. a) i) In right $\triangle$CDE, DE is the hypotenuse and CD is the side opposite $\angle$E.

$$\sin E = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin E = \frac{CD}{DE}$$

$$\sin 34° = \frac{CD}{8.8} \qquad \text{Solve for CD.}$$

$$8.8 \sin 34° = \frac{CD(8.8)}{8.8}$$

$$8.8 \sin 34° = CD$$

$$CD = 4.9208\ldots$$

In right $\triangle$CDE, DE is the hypotenuse and CE is the side adjacent to $\angle$E.

$$\cos E = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos E = \frac{CE}{DE}$$

$$\cos 34° = \frac{CE}{8.8} \qquad \text{Solve for CE.}$$

$$8.8 \cos 34° = \frac{CE(8.8)}{8.8}$$

$$8.8 \cos 34° = CE$$

$$CE = 7.2955\ldots$$

Perimeter of $\triangle$CDE:
4.9208… cm + 7.2955… cm + 8.8 cm = 21.0164… cm
The perimeter of $\triangle$CDE is approximately 21.0 cm.

ii) In right $\triangle$JKM, JM is the hypotenuse and JK is the side opposite $\angle$M.

$$\sin M = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin M = \frac{JK}{JM}$$

$$\sin 63° = \frac{JK}{5.6} \qquad \text{Solve for JK.}$$

$$5.6\sin 63° = \frac{JK(5.6)}{5.6}$$

$$5.6\sin 63° = JK$$

$$JK = 4.9896\ldots$$

In right $\triangle$JKM, JM is the hypotenuse and KM is the side adjacent to $\angle$M.

$$\cos M = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos M = \frac{KM}{JM}$$

$$\cos 63° = \frac{KM}{5.6} \qquad \text{Solve for KM.}$$

$$5.6\cos 63° = \frac{KM(5.6)}{5.6}$$

$$5.6\cos 63° = KM$$

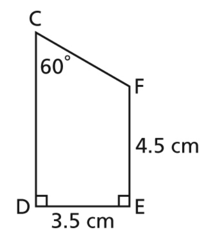
$$KM = 2.5423\ldots$$

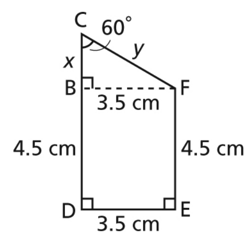
Perimeter of the rectangle is:
2(4.9896… cm) + 2(2.5423… cm) = 15.0639… cm
The perimeter of the rectangle is approximately 15.1 cm.

b) In part a, I used the sine and cosine ratios to determine the side lengths. In each case, I could have used the tangent ratio or the Pythagorean Theorem to determine the second side length.

C

13. Sketch and label a diagram to represent the information in the problem.
The trapezoid is made up of a right triangle and a rectangle.





In right △BCF, CF is the hypotenuse and BF is the side opposite ∠C.

$$\sin C = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin C = \frac{\text{BF}}{\text{CF}}$$

$$\sin 60° = \frac{3.5}{y} \qquad \text{Solve for } y.$$

$$y \sin 60° = \frac{3.5y}{y}$$

$$y \sin 60° = 3.5$$

$$\frac{y \sin 60°}{\sin 60°} = \frac{3.5}{\sin 60°}$$

$$y = \frac{3.5}{\sin 60°}$$

$$y = 4.0414\ldots$$

In right △BCF, CF is the hypotenuse and BC is the side adjacent to ∠C.

$$\cos C = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos C = \frac{\text{BC}}{\text{CF}}$$

$$\cos 60° = \frac{x}{4.0414\ldots} \qquad \text{Solve for } x.$$

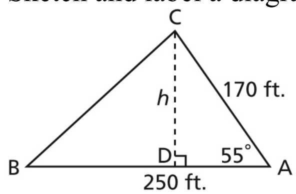
$$(4.0414\ldots)\cos 60° = \frac{(4.0414\ldots)x}{4.0414\ldots}$$

$$(4.0414\ldots)\cos 60° = x$$

$$x = 2.0207\ldots$$

The perimeter of trapezoid CDEF is:
2(4.5 cm) + 3.5 cm + 4.0414… cm + 2.0207… cm = 18.5621… cm
The perimeter of trapezoid CDEF is approximately 18.6 cm.

14. a) Sketch and label a diagram to represent the information in the problem.



In right $\triangle$ACD, AC is the hypotenuse and CD is the side opposite $\angle$A.

$$\sin A = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin A = \frac{CD}{AC}$$

$$\sin 55° = \frac{h}{170} \qquad \text{Solve for } h.$$

$$170 \sin 55° = \frac{h(170)}{170}$$

$$170 \sin 55° = h$$

$$h = 139.2558\ldots$$

The height of the triangle is approximately 139 ft.

b) The area of the triangle is:

$$\frac{1}{2}(250)(139.2558\ldots) = 17\ 406.9809\ldots$$

The area of the lot is approximately 17 407 square feet.

Checkpoint 2 **Assess Your Understanding (page 104)**

2.4

1. **a)** In right △PQR, PR is opposite ∠Q and PQ is the hypotenuse.

$$\sin Q = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin Q = \frac{PR}{PQ}$$

$$\sin Q = \frac{4}{8}$$

$$\angle Q = 30°$$

b) In right △STU, SU is adjacent to ∠S and ST is the hypotenuse.

$$\cos S = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos S = \frac{SU}{ST}$$

$$\cos S = \frac{6}{9}$$

$$\angle S \doteq 48°$$

c) In right △JKM, JM is opposite ∠K and MK is the hypotenuse.

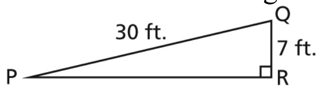
$$\sin K = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin K = \frac{JM}{MK}$$

$$\sin K = \frac{9.5}{11.5}$$

$$\angle K \doteq 56°$$

2. Sketch and label a diagram to represent the information in the problem.



$\angle$P is the angle of inclination of the conveyor.
PQ is the length of the conveyor.
QR is the height of the loading dock.
In right $\triangle$PQR:

$$\sin P = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin P = \frac{QR}{PQ}$$

$$\sin P = \frac{7}{30}$$

$$\angle P = 13.4934\ldots°$$

The angle of inclination of the conveyor is approximately 13°.

3. **a)** **i)** cos 10° = 0.9848…

ii) cos 20° = 0.9396…

iii) cos 30° = 0.8660…

iv) cos 40° = 0.7660…

v) cos 50° = 0.6427…

vi) cos 60° = 0.5

vii) cos 70° = 0.3420…

viii) cos 80° = 0.1736…

b) Consider a right triangle with acute $\angle$A and hypotenuse length 1 unit. Then:

$$\cos A = \frac{\text{length of side adjacent to } \angle A}{1 \text{ unit}}$$

As $\angle$A increases, the side adjacent to $\angle$A decreases, so the ratio cos A decreases.

2.5

4. a) In right △NPQ, PQ is adjacent to ∠Q and NQ is the hypotenuse.

$$\cos Q = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos Q = \frac{PQ}{NQ}$$

$$\cos 65^\circ = \frac{n}{10.0} \qquad \text{Solve for } n.$$

$$10.0\cos 65^\circ = \frac{n(10.0)}{10.0}$$

$$10.0\cos 65^\circ = n$$

$$n = 4.2261\ldots$$

PQ is approximately 4.2 cm long.

b) In right △RST, ST is opposite ∠R and RS is the hypotenuse.

$$\sin R = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin R = \frac{ST}{RS}$$

$$\sin 20^\circ = \frac{r}{8.0} \qquad \text{Solve for } r.$$

$$8.0\sin 20^\circ = \frac{r(8.0)}{8.0}$$

$$8.0\sin 20^\circ = r$$

$$r = 2.7361\ldots$$

ST is approximately 2.7 cm long.

c) In right △UVW, UV is adjacent to ∠U and UW is the hypotenuse.

$$\cos U = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos U = \frac{UV}{UW}$$

$$\cos 50^\circ = \frac{9.0}{v} \qquad \text{Solve for } v.$$

$$v\cos 50^\circ = \frac{9.0v}{v}$$

$$v\cos 50^\circ = 9.0$$

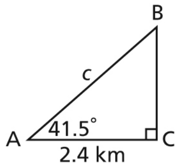
$$\frac{v\cos 50^\circ}{\cos 50^\circ} = \frac{9.0}{\cos 50^\circ}$$

$$v = \frac{9.0}{\cos 50^\circ}$$

$$v = 14.0015\ldots$$

UW is approximately 14.0 cm long.

5. Sketch and label a diagram to represent the information in the problem.



AB is the distance from the ship to the beacon.
AC is the distance the ship travelled due west.
∠A is the angle between the ship's path and the line of sight to the beacon.
In right △ABC:

$$\cos A = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos A = \frac{AC}{AB}$$

$$\cos 41.5° = \frac{2.4}{c} \qquad \text{Solve for } c.$$

$$c\cos 41.5° = \frac{2.4c}{c}$$

$$c\cos 41.5° = 2.4$$

$$\frac{c\cos 41.5°}{\cos 41.5°} = \frac{2.4}{\cos 41.5°}$$

$$c = \frac{2.4}{\cos 41.5°}$$

$$c = 3.2044\ldots$$

The ship is approximately 3.2 km from the beacon.

Lesson 2.6 Applying the Trigonometric Ratios Exercises (pages 111–112)

A

3. a) In right △ABC, the lengths of AC and BC are given.
AC is the hypotenuse and BC is the side opposite ∠A.
So, I would use the sine ratio.

b) In right △DEF, the lengths of DE and EF are given.
DE is the side adjacent to ∠D and EF is the side opposite ∠D.
So, I would use the tangent ratio.

c) In right △GHJ, the lengths of GH and GJ are given.
GJ is the hypotenuse and GH is the side adjacent to ∠G.
So, I would use the cosine ratio.

d) In right △XYZ, the lengths of XZ and YZ are given.
YZ is the side adjacent to ∠Y and XZ is the side opposite ∠Y.
So, I would use the tangent ratio.

4. a) In right △KMN, the length of KM is given and I need to determine the length of KN.
KM is the hypotenuse and KN is the side adjacent to ∠K.
So, I will use the cosine ratio.

$$\cos \mathrm{K} = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos \mathrm{K} = \frac{\mathrm{KN}}{\mathrm{KM}}$$

$$\cos 37° = \frac{m}{5.8} \qquad \text{Solve for } m.$$

$$5.8\cos 37° = \frac{m(5.8)}{5.8}$$

$$5.8\cos 37° = m$$

$$m = 4.6320\ldots$$

KN is approximately 4.6 cm long.

b) In right $\triangle$PQR, the length of QR is given and I need to determine the length of PQ.
QR is the side adjacent to $\angle$R and PQ is the side opposite $\angle$R.
So, I will use the tangent ratio.

$$\tan R = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan R = \frac{PQ}{QR}$$

$$\tan 52° = \frac{r}{3.7} \qquad \text{Solve for } r.$$

$$3.7 \times \tan 52° = \frac{r}{3.7} \times 3.7$$

$$3.7 \tan 52° = r$$

$$r = 4.7357\ldots$$

PQ is approximately 4.7 cm long.

c) In right $\triangle$AYZ, the length of YZ is given and I need to determine the length of AY.
AY is the hypotenuse and YZ is the side opposite $\angle$A.
So, I will use the sine ratio.

$$\sin A = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin A = \frac{YZ}{AY}$$

$$\sin 62° = \frac{10.4}{z} \qquad \text{Solve for } z.$$

$$z \sin 62° = \frac{10.4z}{z}$$

$$z \sin 62° = 10.4$$

$$\frac{z \sin 62°}{\sin 62°} = \frac{10.4}{\sin 62°}$$

$$z = \frac{10.4}{\sin 62°}$$

$$z = 11.7787\ldots$$

AY is approximately 11.8 cm long.

d) In right △BCD, the length of CD is given and I need to determine the length of BD.
BD is the hypotenuse and CD is the side adjacent to ∠D.
So, I will use the cosine ratio.

$$\cos D = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos D = \frac{\text{CD}}{\text{BD}}$$

$$\cos 55° = \frac{8.3}{c} \qquad \text{Solve for } c.$$

$$c\cos 55° = \frac{8.3c}{c}$$

$$c\cos 55° = 8.3$$

$$\frac{c\cos 55°}{\cos 55°} = \frac{8.3}{\cos 55°}$$

$$c = \frac{8.3}{\cos 55°}$$

$$c = 14.4706\ldots$$

BD is approximately 14.5 cm long.

5. a) In right △EFG, the lengths of the legs, EF and FG, are given.
So, I would use the Pythagorean Theorem to determine the length of the hypotenuse, EG.

b) In right △HJK, the length of HK and the measure of ∠J are given.
HK is the side opposite ∠J, and HJ is the hypotenuse.
So, I would use the sine ratio to determine the length of HJ.

c) In right △MNP, the lengths of one leg, MN, and the hypotenuse, MP, are given. So, I would use the Pythagorean Theorem to determine the length of the other leg, PN.

d) In right △QRS, the lengths of the legs, RS and QS, are given.
So, I would use the Pythagorean Theorem to determine the length of the hypotenuse, QR.

B

6. a) Determine the measure of ∠T.

$$\begin{aligned}\angle T &= 90° - \angle U\\ &= 90° - 33°\\ &= 57°\end{aligned}$$

Determine the length of UV.

$$\tan T = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan T = \frac{UV}{TV}$$

$$\tan 57° = \frac{UV}{12.5} \qquad \text{Solve for UV.}$$

$$12.5 \tan 57° = UV$$

$$UV = 19.2483\ldots$$

Determine the length of TU. Use the Pythagorean Theorem.

$$\begin{aligned}TU^2 &= TV^2 + UV^2\\ TU^2 &= 12.5^2 + (19.2483\ldots)^2\\ &= 526.7475\ldots\\ TU &= \sqrt{526.7475\ldots}\\ &= 22.9509\ldots\end{aligned}$$

∠T is 57°, TU is approximately 23.0 cm, and UV is approximately 19.2 cm.

b) Determine the length of XY.

$$\tan \mathrm{W} = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan \mathrm{W} = \frac{\mathrm{XY}}{\mathrm{WX}}$$

$$\tan 47° = \frac{\mathrm{XY}}{5.9} \qquad \text{Solve for XY.}$$

$$5.9 \tan 47° = \mathrm{XY}$$

$$\mathrm{XY} = 6.3269\ldots$$

Determine the length of WY. Use the Pythagorean Theorem.

$$\mathrm{WY}^2 = \mathrm{WX}^2 + \mathrm{XY}^2$$

$$\begin{aligned}\mathrm{WY}^2 &= 5.9^2 + (6.3269\ldots)^2\\ &= 74.8406\ldots\end{aligned}$$

$$\begin{aligned}\mathrm{WY} &= \sqrt{74.8406\ldots}\\ &= 8.6510\ldots\end{aligned}$$

Determine the measure of $\angle$Y.

$$\begin{aligned}\angle \mathrm{Y} &= 90° - \angle \mathrm{W}\\ &= 90° - 47°\\ &= 43°\end{aligned}$$

$\angle$Y is 43°, XY is approximately 6.3 cm, and WY is approximately 8.7 cm.

c) Determine the length of BZ. Use the Pythagorean Theorem.

$$\begin{aligned} BZ^2 &= AB^2 + AZ^2 \\ BZ^2 &= 5.6^2 + 9.8^2 \\ &= 127.4 \\ BZ &= \sqrt{127.4} \\ &= 11.2871\ldots \end{aligned}$$

Determine the measure of ∠B.

$$\tan B = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan B = \frac{AZ}{AB}$$

$$\tan B = \frac{9.8}{5.6}$$

$$\angle B = 60.2551\ldots^\circ$$

Determine the measure of ∠Z.

$$\begin{aligned} \angle Z &= 90^\circ - \angle B \\ &= 90^\circ - 60.2551\ldots^\circ \\ &= 29.7448\ldots^\circ \end{aligned}$$

∠B is approximately 60.3°, ∠Z is approximately 29.7°, and BZ is approximately 11.3 cm.

d) Determine the measure of ∠E.

$$\begin{aligned}\angle E &= 90° - \angle D\\ &= 90° - 29°\\ &= 61°\end{aligned}$$

Determine the length of CD.

$$\cos D = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos D = \frac{CD}{DE}$$

$$\cos 29° = \frac{CD}{13.7} \qquad \text{Solve for CD.}$$

$$13.7\cos 29° = CD$$

$$CD = 11.9822\ldots$$

Determine the length of CE.

$$\sin D = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin D = \frac{CE}{DE}$$

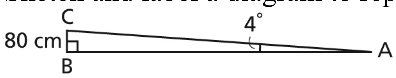
$$\sin 29° = \frac{CE}{13.7} \qquad \text{Solve for CE.}$$

$$13.7\sin 29° = CE$$

$$CE = 6.6418\ldots$$

∠E is 61°, CD is approximately 12.0 cm, and CE is approximately 6.6 cm.

7. Sketch and label a diagram to represent the information in the problem.



AC is the length of the ramp.
AB is the horizontal distance the ramp will take up.
BC is the maximum height of the ramp.
∠A is the angle of elevation of the ramp.

a) In right △ABC, AC is the hypotenuse and BC is the side opposite ∠A.

$$\sin A = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin A = \frac{BC}{AC}$$

$$\sin 4° = \frac{80}{AC} \qquad \text{Solve for AC.}$$

$$AC \sin 4° = 80$$

$$AC = \frac{80}{\sin 4°}$$

$$AC = 1146.8469\ldots$$

The length of the ramp is approximately 1147 cm.

b) In right △ABC, AC is the hypotenuse and BC and AB are the legs.
Use the Pythagorean Theorem.

$$AC^2 = AB^2 + BC^2 \qquad \text{Solve for the unknown.}$$

$$AB^2 = AC^2 - BC^2$$

$$AB^2 = (1146.8469\ldots)^2 - 80^2$$

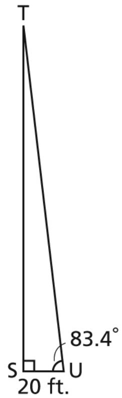
$$= 1\,308\,857.954\ldots$$

$$AB = \sqrt{1\,308\,857.954\ldots}$$

$$= 1144.0533\ldots$$

The ramp will take up a horizontal distance of approximately 1144 cm.

8. Sketch and label a diagram to represent the information in the problem.



Assume the ground is horizontal.
ST is the height of the totem pole.
SU is the distance from the base of the totem pole.
$\angle$U is the angle of elevation of the top of the pole.
In right $\triangle$STU, ST is the side opposite $\angle$U and SU is side adjacent to $\angle$U.

$$\tan \mathrm{U} = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan \mathrm{U} = \frac{\mathrm{ST}}{\mathrm{SU}}$$

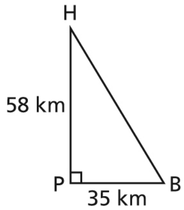
$$\tan 83.4° = \frac{\mathrm{ST}}{20} \qquad \text{Solve for ST.}$$

$$20 \tan 83.4° = \mathrm{ST}$$

$$\mathrm{ST} = 172.8549\ldots$$

The totem pole is approximately 173 ft. tall.

9. Sketch and label a diagram to represent the information in the problem.



BP is the distance from the base to the sick person.
PH is the distance from the sick person to the hospital.
BH is the distance from the base to the hospital.
$\angle$H is the angle between the path the helicopter took due north and the path it will take to return directly to its base.

a) In right $\triangle$BHP, BH is the hypotenuse and BP and PH are the legs.
Use the Pythagorean Theorem.

$$\begin{aligned} \mathrm{BH}^2 &= \mathrm{BP}^2 + \mathrm{PH}^2 \\ \mathrm{BH}^2 &= 35^2 + 58^2 \\ &= 4589 \\ \mathrm{BH} &= \sqrt{4589} \\ &= 67.7421\ldots \end{aligned}$$

The distance between the hospital and the base is approximately 68 km.

b) In right $\triangle$BHP, BP is the side opposite $\angle$H and PH is the side adjacent to $\angle$H.

$$\tan \mathrm{H} = \frac{\text{opposite}}{\text{adjacent}}$$

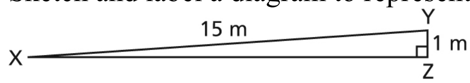
$$\tan \mathrm{H} = \frac{\mathrm{BP}}{\mathrm{PH}}$$

$$\tan \mathrm{H} = \frac{35}{58}$$

$$\angle \mathrm{H} = 31.1088\ldots°$$

The angle between the path the helicopter took due north and the path it will take to return directly to its base is approximately 31°.

10. Sketch and label a diagram to represent the information in the problem.



XY is the distance travelled along the road.
YZ is the rise of the road.
XZ is the horizontal distance travelled.
∠X is the angle of inclination of the road.

a) In right △XYZ, YZ is the side opposite ∠X and XY is the hypotenuse.

$$\sin X = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin X = \frac{YZ}{XY}$$

$$\sin X = \frac{1}{15}$$

$$\angle X = 3.8225\ldots^\circ$$

The angle of inclination of the road is approximately 4°.

b) In right △XYZ, XY is the hypotenuse and XZ and YZ are the legs.
Use the Pythagorean Theorem.

$$XY^2 = XZ^2 + YZ^2 \qquad \text{Isolate the unknown.}$$

$$XZ^2 = XY^2 - YZ^2$$

$$XZ^2 = 15^2 - 1^2$$

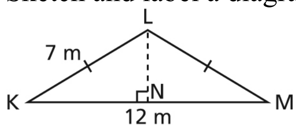
$$= 224$$

$$XZ = \sqrt{224}$$

$$= 14.9666\ldots$$

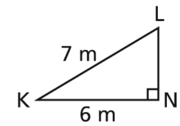
The horizontal distance travelled is approximately 15.0 m.

11. Sketch and label a diagram to represent the information in the problem.



$\angle$K is the angle of inclination of the roof.
$\angle$KLM is the angle at the peak of the roof.

a) In right $\triangle$KLN, KN is the side adjacent to $\angle$K and it is $\frac{1}{2}$KM = 6 m; and KL is the hypotenuse.



$$\cos K = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos K = \frac{KN}{KL}$$

$$\cos K = \frac{6}{7}$$

$$\angle K = 31.0027\ldots°$$

The angle of inclination of the roof is approximately 31°.

b) In isosceles $\triangle$KLM, $\angle M = \angle K \doteq 31°$
The sum of the angles in a triangle is 180°.
So, $\angle L = 180° - 31° - 31°$

$$= 118°$$

The measure of the angle at the peak of the roof is approximately 118°.

12. a) In right △CDE, DE is the hypotenuse and CE is the length of the side opposite ∠D.

$$\sin D = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin D = \frac{CE}{DE}$$

$$\sin 45° = \frac{CE}{5.6} \qquad \text{Solve for CE.}$$

$$5.6 \sin 45° = CE$$

$$CE = 3.9597\ldots$$

$$\begin{aligned}\angle E &= 90° - \angle D\\ &= 90° - 45°\\ &= 45°\end{aligned}$$

Since ∠D = ∠E, △CDE is an isosceles right triangle with CD = CE.

Perimeter of △CDE:
5.6 cm + 2(3.9597… cm) = 13.5195… cm

Area of △CDE:

$$\begin{aligned}\frac{1}{2}(CE)(CD) &= \frac{1}{2}(CE)^2\\ &= \frac{1}{2}(3.9597\ldots)^2\\ &= 7.84\end{aligned}$$

The perimeter of △CDE is approximately 13.5 cm and its area is approximately 7.8 cm^2.

b) In right △FGH, FH is the hypotenuse and GH is the side opposite ∠F.

$$\sin F = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin F = \frac{\text{GH}}{\text{FH}}$$

$$\sin 28° = \frac{\text{GH}}{10.7} \qquad \text{Solve for GH.}$$

$$10.7 \sin 28° = \text{GH}$$

$$\text{GH} = 5.0233\ldots$$

In right △FGH, FH is the hypotenuse and FG is the side adjacent to ∠F.

$$\cos F = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos F = \frac{\text{FG}}{\text{FH}}$$

$$\cos 28° = \frac{\text{FG}}{10.7} \qquad \text{Solve for FG.}$$

$$10.7 \cos 28° = \text{FG}$$

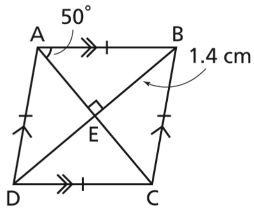
$$\text{FG} = 9.4475\ldots$$

Perimeter of rectangle:
2(5.0233… cm + 9.4475… cm) = 28.9417… cm

Area of rectangle:
(5.0233… cm)(9.4475… cm) = 47.4582… cm^2

The perimeter of rectangle FGHJ is approximately 28.9 cm and its area is approximately 47.5 cm^2.

13. Sketch and label a copy of the rhombus.



In right $\triangle$ABE, AB is the hypotenuse and BE is the side opposite $\angle$A.

$$\sin A = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin A = \frac{BE}{AB}$$

$$\sin 50° = \frac{1.4}{AB} \qquad \text{Solve for AB.}$$

$$AB \sin 50° = 1.4$$

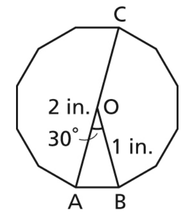
$$AB = \frac{1.4}{\sin 50°}$$

$$AB = 1.8275\ldots$$

Perimeter of rhombus:
4(1.8275… cm) = 7.3102… cm

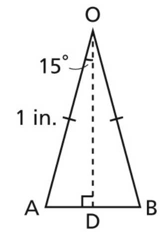
The perimeter of rhombus ABCD is approximately 7.3 cm.

14. a) Sketch and label the base of the candle.
The base is a regular 12-sided polygon and the distance from one vertex to the opposite vertex is 2 in., so the distance from the centre of the polygon to a vertex is 1 in.
The polygon can be divided into 12 congruent isosceles triangles.
In each triangle, the angle at the centre of the polygon is:



$$\frac{360°}{12} = 30°$$

$\triangle$ABO is an isosceles triangle.
So, draw the perpendicular bisector OD to form two congruent right triangles.
OD bisects $\angle$AOB, so $\angle$AOD = 15°
In right $\triangle$AOD, AO is the hypotenuse and AD is the side opposite $\angle$O.



$$\sin O = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin O = \frac{AD}{AO}$$

$$\sin 15° = \frac{AD}{1}$$ Solve for AD.

$$AD = \sin 15°$$

So:

$$\begin{aligned} AB &= 2(AD) \\ &= 2\sin 15° \end{aligned}$$

In right $\triangle$AOD, AO is the hypotenuse and DO is the side adjacent to $\angle$O.

$$\cos O = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos O = \frac{DO}{AO}$$

$$\cos 15° = \frac{DO}{1}$$ Solve for DO.

$$DO = \cos 15°$$

The area of $\triangle$ABO is:

$$\begin{aligned} \frac{1}{2}(AB)(DO) &= \frac{1}{2}(2\sin 15°)(\cos 15°) \\ &= (\sin 15°)(\cos 15°) \end{aligned}$$

The 12-sided polygon is made up of 12 triangles congruent to $\triangle$ABO.
So, the area of the polygon, in square inches, is:
$12 \times (\sin 15°)(\cos 15°) = 3$

The area of the base is 3 square inches.

b) The volume of wax in the candle, in cubic inches, is:

$$\begin{aligned}\text{volume} &= (\text{base area})(\text{height})\\ &= (3)(5)\\ &= 15\end{aligned}$$

The volume of wax in the candle is 15 cubic inches.

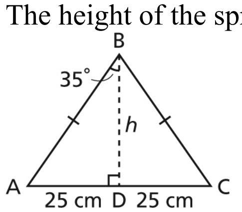
C

15. Sketch and label a diagram to represent the information in the problem.
Each nozzle should spray water to at least one-half the distance between it and the next nozzle. Assume the ground is horizontal and the nozzles spray an equal distance to the left and right.
Triangle ABC is isosceles, so AB = BC and BD bisects ∠ABC and bisects AC.

So, AD is $\frac{1}{2}(50\text{ cm}) = 25\text{ cm}$

And, ∠ABD is $\frac{1}{2}(70°) = 35°$

The height of the sprayer above the crops is the length of BD.



In right △ABD, AD is the side opposite ∠B and BD is the side adjacent to ∠B.

$$\tan A = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan A = \frac{\text{AD}}{\text{BD}}$$

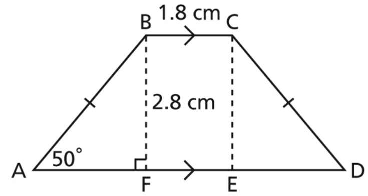
$$\tan 35° = \frac{25}{h} \qquad \text{Solve for } h.$$

$$h \tan 35° = 25$$

$$h = \frac{25}{\tan 35°}$$

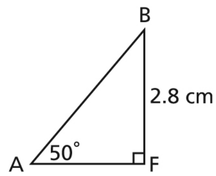
$$h = 35.7037\ldots$$

The sprayer should be placed at least 36 cm above the crops.



16. Sketch and label the trapezoid.
The trapezoid is isosceles, so AF = DE and AB = CD.

In right △ABF, AB is the hypotenuse and BF is the side opposite ∠A.



$$\sin A = \frac{\text{opposite}}{\text{hypotenuse}}$$
$$\sin A = \frac{BF}{AB}$$
$$\sin 50° = \frac{2.8}{AB} \qquad \text{Solve for AB.}$$
$$AB \sin 50° = 2.8$$
$$AB = \frac{2.8}{\sin 50°}$$
$$AB = 3.6551\ldots$$

In right △ABF, BF is the side opposite ∠A and AF is the side adjacent to ∠A.

$$\tan A = \frac{\text{opposite}}{\text{adjacent}}$$
$$\tan A = \frac{BF}{AF}$$
$$\tan 50° = \frac{2.8}{AF} \qquad \text{Solve for AF.}$$
$$AF \tan 50° = 2.8$$
$$AF = \frac{2.8}{\tan 50°}$$
$$AF = 2.3494\ldots$$

Perimeter of the trapezoid:
2(1.8 cm) + 2(3.6551… cm) + 2(2.3494… cm) = 15.6092… cm

The area of the trapezoid is the sum of the areas of 2 congruent right triangles with base 2.3494… cm and height 2.8 cm, and the area of a rectangle with width 1.8 cm and height 2.8 cm.
Area of trapezoid:

$$2 \times \frac{1}{2}(2.3494\ldots \text{ cm})(2.8 \text{ cm}) + (1.8 \text{ cm})(2.8 \text{ cm}) = 11.6185\ldots \text{ cm}^2$$

The perimeter of the trapezoid is approximately 15.6 cm and its area is approximately 11.6 cm^2.

Lesson 2.7 Solving Problems Involving More than One Right Triangle Exercises (pages 118–121)

A

3. a) JK = JM + KM

In right $\triangle$JMN, JM is opposite $\angle$N and MN is adjacent to $\angle$N.
Use the tangent ratio in $\triangle$JMN.

$$\tan N = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan N = \frac{JM}{MN}$$

$$\tan 40° = \frac{JM}{5.0} \qquad \text{Solve for JM.}$$

$$5.0 \tan 40° = JM$$

$$JM = 4.1954\ldots$$

In right $\triangle$KMN, KM is opposite $\angle$N and MN is adjacent to $\angle$N.
Use the tangent ratio in $\triangle$KMN.

$$\tan N = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan N = \frac{KM}{MN}$$

$$\tan 20° = \frac{KM}{5.0} \qquad \text{Solve for KM.}$$

$$5.0 \tan 20° = KM$$

$$KM = 1.8198\ldots$$

$$\begin{aligned} JK &= JM + KM \\ &= 4.1954\ldots + 1.8198\ldots \\ &= 6.0153\ldots \end{aligned}$$

JK is approximately 6.0 cm long.

b) $JK = AJ + AK$

In right $\triangle ABJ$, AJ is opposite $\angle B$ and AB is adjacent to $\angle B$.
Use the tangent ratio in right $\triangle ABJ$.

$$\tan B = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan B = \frac{AJ}{AB}$$

$$\tan 15° = \frac{AJ}{3.0} \qquad \text{Solve for AJ.}$$

$$3.0 \tan 15° = AJ$$

$$AJ = 0.8038\ldots$$

In right $\triangle ABK$, AK is opposite $\angle B$ and AB is adjacent to $\angle B$.
Use the tangent ratio in $\triangle ABK$.

$$\tan B = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan B = \frac{AK}{AB}$$

$$\tan 60° = \frac{AK}{3.0} \qquad \text{Solve for AK.}$$

$$3.0 \tan 60° = AK$$

$$AK = 5.1961\ldots$$

$$\begin{aligned} JK &= AJ + AK \\ &= 0.8038\ldots + 5.1961\ldots \\ &= 6 \end{aligned}$$

JK is 6.0 cm long.

c) In right $\triangle$CDK, CK is opposite $\angle$D and CD is adjacent to $\angle$D.
Use the tangent ratio in right $\triangle$CDK.

$$\tan D = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan D = \frac{\text{CK}}{\text{CD}}$$

$$\tan 35° = \frac{\text{CK}}{3.0} \qquad \text{Solve for CK.}$$

$$3.0 \tan 35° = \text{CK}$$

$$\text{CK} = 2.1006\ldots$$

In right $\triangle$CDJ, CJ is opposite $\angle$D and CD is adjacent to $\angle$D.
$\angle D = 35° + 30°$, or $65°$
Use the tangent ratio in $\triangle$CDJ.

$$\tan D = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan D = \frac{\text{CJ}}{\text{CD}}$$

$$\tan 65° = \frac{\text{CJ}}{3.0} \qquad \text{Solve for CJ.}$$

$$3.0 \tan 65° = \text{CJ}$$

$$\text{CJ} = 6.4335\ldots$$

$$\begin{aligned} \text{JK} &= \text{CJ} - \text{CK} \\ &= 6.4335\ldots - 2.1006\ldots \\ &= 4.3328\ldots \end{aligned}$$

JK is approximately 4.3 cm.

d) JK = EK – EJ
In right △EFJ, EF is opposite ∠J and EJ is adjacent to ∠J.
Use the tangent ratio in right △EFJ.

$$\tan J = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan J = \frac{EF}{EJ}$$

$$\tan 60° = \frac{4.2}{EJ} \qquad \text{Solve for EJ.}$$

$$EJ \tan 60° = 4.2$$

$$EJ = \frac{4.2}{\tan 60°}$$

$$EJ = 2.4248\ldots$$

In right △EFK, EF is opposite ∠K and EK is adjacent to ∠K.
Use the tangent ratio in right △EFK.

$$\tan K = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan K = \frac{EF}{EK}$$

$$\tan 35° = \frac{4.2}{EK} \qquad \text{Solve for EK.}$$

$$EK \tan 35° = 4.2$$

$$EK = \frac{4.2}{\tan 35°}$$

$$EK = 5.9982\ldots$$

$$\begin{aligned} JK &= EK - EJ \\ &= 5.9982\ldots - 2.4248\ldots \\ &= 3.5733\ldots \end{aligned}$$

JK is approximately 3.6 cm.

4. a) To determine the length of GH, first determine the length of GD.
In right $\triangle$ADG, AD is opposite $\angle$G and GD is the hypotenuse.
Use the sine ratio in right $\triangle$ADG.

$$\sin G = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin G = \frac{AD}{GD}$$

$$\sin 46° = \frac{4.5}{GD} \qquad \text{Solve for GD.}$$

$$GD \sin 46° = 4.5$$

$$GD = \frac{4.5}{\sin 46°}$$

$$GD = 6.2557\ldots$$

In right $\triangle$DGH, GH is opposite $\angle$D and GD is the hypotenuse.
Use the sine ratio in right $\triangle$DGH.

$$\sin D = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin D = \frac{GH}{GD}$$

$$\sin 66° = \frac{GH}{6.2557\ldots} \qquad \text{Solve for GH.}$$

$$(6.2557\ldots)\sin 66° = GH$$

$$GH = 5.7148\ldots$$

GH is approximately 5.7 cm long.

b) To determine the length of GH, first determine the length of EH.
In right △EFH, FH is adjacent to ∠H
and EH is the hypotenuse.
Use the cosine ratio in right △EFH.

$$\cos H = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos H = \frac{\text{FH}}{\text{EH}}$$

$$\cos 59° = \frac{3.4}{\text{EH}} \qquad \text{Solve for EH.}$$

$$\text{EH} \cos 59° = 3.4$$

$$\text{EH} = \frac{3.4}{\cos 59°}$$

$$\text{EH} = 6.6014\ldots$$

In right △EGH, GH is adjacent to ∠H and EH is the hypotenuse.
Use the cosine ratio in right △EGH.

$$\cos H = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos H = \frac{\text{GH}}{\text{EH}}$$

$$\cos 42° = \frac{\text{GH}}{6.6014\ldots} \qquad \text{Solve for GH.}$$

$$(6.6014\ldots)\cos 42° = \text{GH}$$

$$\text{GH} = 4.9058\ldots$$

GH is approximately 4.9 cm long.

c) To determine the length of GH, first determine the length of GD.
In right $\triangle$CDG, CG is adjacent to $\angle$G and GD is the hypotenuse.
Use the cosine ratio in right $\triangle$CDG.

$$\cos G = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos G = \frac{CG}{GD}$$

$$\cos 52° = \frac{3.9}{GD}$$ Solve for GD.

$$GD \cos 52° = 3.9$$

$$GD = \frac{3.9}{\cos 52°}$$

$$GD = 6.3346\ldots$$

In right $\triangle$DGH, GH is opposite $\angle$D and GD is the hypotenuse.
Use the sine ratio in right $\triangle$DGH.

$$\sin D = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin D = \frac{GH}{GD}$$

$$\sin 64° = \frac{GH}{6.3346\ldots}$$ Solve for GH.

$$(6.3346\ldots)\sin 64° = GH$$

$$GH = 5.6935\ldots$$

GH is approximately 5.7 cm long.

B

5. a) $\angle XYZ = \angle WYX + \angle WYZ$

In right $\triangle WXY$, WX is opposite $\angle Y$ and WY is adjacent to $\angle Y$.
Use the tangent ratio in right $\triangle WXY$.

$$\tan Y = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan Y = \frac{WX}{WY}$$

$$\tan Y = \frac{3}{4}$$

$$\angle Y = 36.8698\ldots°$$

In right $\triangle WYZ$, WZ is opposite $\angle Y$ and WY is adjacent to $\angle Y$.
Use the tangent ratio in right $\triangle WYZ$.

$$\tan Y = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan Y = \frac{WZ}{WY}$$

$$\tan Y = \frac{6}{4}$$

$$\angle Y = 56.3099\ldots°$$

$$\begin{aligned}\angle XYZ &= \angle WYX + \angle WYZ\\ &= 36.8698\ldots° + 56.3099\ldots°\\ &= 93.1798\ldots°\end{aligned}$$

$\angle XYZ$ is approximately 93.2°.

b) $\angle XYZ = \angle VYX + \angle VYZ$
Determine $\angle VYZ$ first because two sides of $\triangle VYZ$ are known.
In right $\triangle VYZ$, VZ is opposite $\angle Y$ and YZ is the hypotenuse.
Use the sine ratio in right $\triangle VYZ$.

$$\sin Y = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin Y = \frac{VZ}{YZ}$$

$$\sin Y = \frac{8}{10}$$

$$\angle Y = 53.1301\ldots°$$

Use the Pythagorean Theorem in right $\triangle VYZ$.

$$YZ^2 = VY^2 + VZ^2 \qquad \text{Isolate the unknown.}$$

$$VY^2 = YZ^2 - VZ^2$$

$$VY^2 = 10^2 - 8^2$$

$$= 36$$

$$VY = \sqrt{36}$$

$$= 6$$

In right $\triangle VXY$, VX is opposite $\angle Y$ and VY is adjacent to $\angle Y$.
Use the tangent ratio in right $\triangle VXY$.

$$\tan Y = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan Y = \frac{VX}{VY}$$

$$\tan Y = \frac{17}{6}$$

$$\angle Y = 70.5599\ldots°$$

$$\angle XYZ = \angle VYZ + \angle VYX$$

$$= 53.1301\ldots° + 70.5599\ldots°$$

$$= 123.6900\ldots°$$

$\angle XYZ$ is approximately 123.7°.

c) $\angle XYZ = \angle XYU - \angle ZYU$

In right $\triangle UXY$, UX is opposite $\angle Y$ and UY is adjacent to $\angle Y$.
Use the tangent ratio in right $\triangle UXY$.

$$\tan Y = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan Y = \frac{UX}{UY}$$

$$\tan Y = \frac{3+5}{12}$$

$$\angle Y = 33.6900\ldots°$$

In right $\triangle UYZ$, UZ is opposite $\angle Y$ and UY is adjacent to $\angle Y$.
Use the tangent ratio in right $\triangle UYZ$.

$$\tan Y = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan Y = \frac{UZ}{UY}$$

$$\tan Y = \frac{5}{12}$$

$$\angle Y = 22.6198\ldots°$$

$$\begin{aligned}\angle XYZ &= \angle XYU - \angle ZYU \\ &= 33.6900\ldots° - 22.6198\ldots° \\ &= 11.0702\ldots°\end{aligned}$$

$\angle XYZ$ is approximately 11.1°.

d) $\angle XYZ = \angle TYZ - \angle TYX$
Determine $\angle TYX$ first because two sides of $\triangle TXY$ are known.
In right $\triangle TXY$, TY is adjacent to $\angle Y$ and XY is the hypotenuse.
Use the cosine ratio in right $\triangle TXY$.

$$\cos Y = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos Y = \frac{TY}{XY}$$

$$\cos Y = \frac{15}{17}$$

$$\angle Y = 28.0724\ldots°$$

Use the Pythagorean Theorem in right $\triangle TXY$.

$XY^2 = TX^2 + TY^2$ Isolate the unknown.

$$TX^2 = XY^2 - TY^2$$

$$TX^2 = 17^2 - 15^2$$

$$= 64$$

$$TX = \sqrt{64}$$

$$= 8$$

So, the length of TZ is:

$$TZ = TX + XZ$$

$$= 8 + 6$$

$$= 14$$

In right $\triangle TYZ$, TZ is opposite $\angle Y$ and TY is adjacent to $\angle Y$.
Use the tangent ratio in right $\triangle TYZ$.

$$\tan Y = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan Y = \frac{TZ}{TY}$$

$$\tan Y = \frac{14}{15}$$

$$\angle Y = 43.0250\ldots°$$

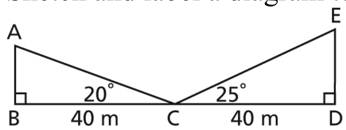
$$\angle XYZ = \angle TYZ - \angle TYX$$

$$= 43.0250\ldots° - 28.0724\ldots°$$

$$= 14.9525\ldots°$$

$\angle XYZ$ is approximately 15.0°.

6. The point from which the angles are measured is halfway between the trees, so its distance from each tree is:
80 m ÷ 2 = 40 m
Sketch and label a diagram to represent the information in the problem.



In right △ABC, AB is opposite ∠C and BC is adjacent to ∠C.
Use the tangent ratio in right △ABC.

$$\tan C = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan C = \frac{AB}{BC}$$

$$\tan 20° = \frac{AB}{40} \qquad \text{Solve for AB.}$$

$$40 \tan 20° = AB$$

$$AB = 14.5588\ldots$$

In right △CDE, DE is opposite ∠C and CD is adjacent to ∠C.
Use the tangent ratio in right △CDE.

$$\tan C = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan C = \frac{DE}{CD}$$

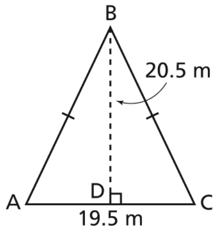
$$\tan 25° = \frac{DE}{40} \qquad \text{Solve for DE.}$$

$$40 \tan 25° = DE$$

$$DE = 18.6523\ldots$$

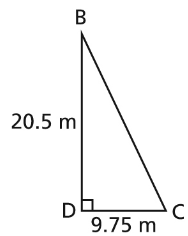
The heights of the trees are approximately 15 m and 19 m.

7. Sketch and label a diagram to represent one face of the pyramid.



$\triangle$ABC is an isosceles triangle with AB = BC and AC is 19.5 m, so CD is:
19.5 m ÷ 2 = 9.75 m

In right $\triangle$BCD, BD is opposite $\angle$C and CD is adjacent to $\angle$C.
Use the tangent ratio in right $\triangle$BCD.



$$\tan C = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan C = \frac{\text{BD}}{\text{CD}}$$

$$\tan C = \frac{20.5}{9.75}$$

$$\angle C = 64.5637\ldots°$$

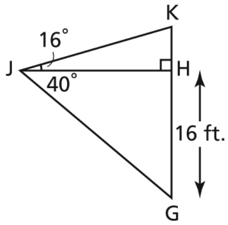
$\triangle$ABC is an isosceles triangle with AB = BC, so $\angle$A = $\angle$C.

The sum of the angles in a triangle is 180°, so in right $\triangle$ABC:

$$\angle B = 180° - 2(64.5637\ldots°)$$
$$= 50.8724\ldots°$$

Since the sum of the angles is 180°, round $\angle$B down to 50°.
So, the measures of the angles in a triangular face of the pyramid are approximately 65°, 65°, and 50°.

8. Sketch and label a diagram to represent the information in the problem.



HJ represents the horizontal distance between the student and the tree.
GK represents the height of the tree.

a) In right △GHJ, GH is opposite ∠J and HJ is adjacent to ∠J.
Use the tangent ratio in right △GHJ.

$$\tan J = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan J = \frac{GH}{HJ}$$

$$\tan 40° = \frac{16}{HJ} \qquad \text{Solve for HJ.}$$

$$HJ \tan 40° = 16$$

$$HJ = \frac{16}{\tan 40°}$$

$$HJ = 19.0680\ldots$$

The horizontal distance between the student and the tree is approximately 19 ft.

b) In right △HJK, KH is opposite ∠J and HJ is adjacent to ∠J.
Use the tangent ratio in right △HJK.

$$\tan J = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan J = \frac{HK}{HJ}$$

$$\tan 16° = \frac{HK}{19.0680\ldots} \qquad \text{Solve for HK.}$$

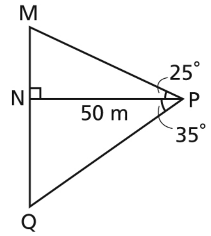
$$(19.0680\ldots)\tan 16° = HK$$

$$HK = 5.4676\ldots$$

$$\begin{aligned} GK &= HK + GH \\ &= 5.4676\ldots + 16 \\ &= 21.4676\ldots \end{aligned}$$

The height of the tree is approximately 21 ft.

9. Sketch and label a diagram to represent the information in the problem.



Assume the ground is horizontal.
NQ represents the height of the shorter tower.
MQ represents the height of the taller tower.

In right △NPQ, NQ is opposite ∠P and NP is adjacent to ∠P.
Use the tangent ratio in right △NPQ.

$$\tan P = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan P = \frac{NQ}{NP}$$

$$\tan 35° = \frac{NQ}{50} \qquad \text{Solve for NQ.}$$

$$50 \tan 35° = NQ$$

$$NQ = 35.0103\ldots$$

In right △MNP, MN is opposite ∠P and NP is adjacent to ∠P.
Use the tangent ratio in right △MNP.

$$\tan P = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan P = \frac{MN}{NP}$$

$$\tan 25° = \frac{MN}{50} \qquad \text{Solve for MN.}$$

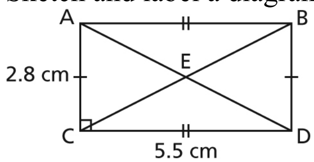
$$50 \tan 25° = MN$$

$$MN = 23.3153\ldots$$

$$\begin{aligned} MQ &= MN + NQ \\ &= 23.3153\ldots + 35.0103\ldots \\ &= 58.3257\ldots \end{aligned}$$

The shorter tower is approximately 35 m high and the taller tower is approximately 58 m high.

10. Sketch and label a diagram to represent the rectangle.



The angles at E cannot be determined directly because the 4 triangles that contain angles at E are not right triangles.
In right △ACD, CD is opposite ∠A and AC is adjacent to ∠A.
Use the tangent ratio in right △ACD.

$$\tan A = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan A = \frac{CD}{AC}$$

$$\tan A = \frac{5.5}{2.8}$$

$$\angle A = 63.0197\ldots°$$

△ACE is an isosceles triangle with AE = CE, so ∠C = ∠A.
The sum of the angles in a triangle is 180°, so in △ACE:

$$\begin{aligned}\angle E &= 180° - 2(63.0197\ldots°)\\ &= 53.9604\ldots°\end{aligned}$$

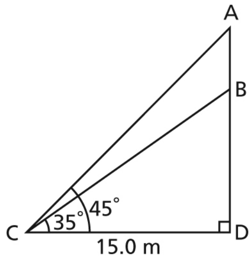
∠AEC and ∠CED form a straight angle, so:

$$\begin{aligned}\angle CED &= 180° - \angle AEC\\ &= 180° - 53.9604\ldots°\\ &= 126.0395\ldots°\end{aligned}$$

Opposite angles are equal, so:
$\angle BED = \angle AEC \doteq 54°$
And $\angle AEB = \angle CED \doteq 126°$
So, the angles at the point where the diagonals intersect are approximately 54°, 54°, 126°, and 126°.

I could also have started by determining ∠D in △ACD, and then used a similar strategy in △CDE to determine the angles. I could also have divided each isosceles triangle in half to form 2 congruent right triangles and determined the angles in these triangles.

11. Sketch and label a diagram to represent the information in the problem.



AB represents the distance between the 2 carvings.

AB = AD – BD
In right ΔACD, ∠C = 45°

$\angle A = 90° - \angle C$

$\angle A = 90° - 45°$

$\quad = 45°$

So, △ACD is an isosceles right triangle, and CD = AD.
So, AD is 15.0 m.

In right △BCD, BD is opposite ∠C and CD is adjacent to ∠C.
Use the tangent ratio in right △BCD.

$$\tan C = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan C = \frac{BD}{CD}$$

$$\tan 35° = \frac{BD}{15.0} \qquad \text{Solve for BD.}$$

$$15.0 \tan 35° = BD$$

$$BD = 10.5031\ldots$$

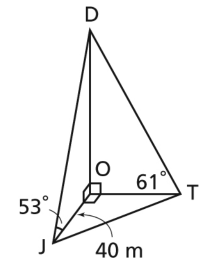
$$AB = AD - BD$$

$$AB = 15.0 - 10.5031\ldots$$

$$= 4.4968\ldots$$

The distance between the carvings is approximately 4.5 m.

12. Sketch and label a diagram to represent the information in the problem.



DO represents the height of the top of the dome.
OT represents the distance between Troy and a point directly beneath the dome.
JT represents the distance between Janelle and Troy.

a) In right △DJO, JO is adjacent to ∠J and DO is opposite ∠J.
Use the tangent ratio in right △DJO.

$$\tan J = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan J = \frac{DO}{JO}$$

$$\tan 53° = \frac{DO}{40} \qquad \text{Solve for DO.}$$

$$40 \tan 53° = DO$$

$$DO = 53.0817\ldots$$

The dome is approximately 53 m high.

b) In right △DOT, OT is adjacent to ∠T and DO is opposite ∠T.
Use the tangent ratio in right △DOT.

$$\tan T = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan T = \frac{DO}{OT}$$

$$\tan 61° = \frac{53.0817\ldots}{OT} \qquad \text{Solve for OT.}$$

$$OT \tan 61° = 53.0817\ldots$$

$$OT = \frac{53.0817\ldots}{\tan 61°}$$

$$OT = 29.4237\ldots$$

Troy is approximately 29 m from a point directly beneath the dome.

c) Use the Pythagorean Theorem in right △JOT.

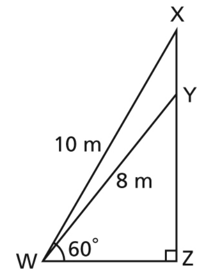
$$JT^2 = JO^2 + OT^2$$

$$JT^2 = 40^2 + (29.4237\ldots)^2$$

$$JT = \sqrt{40^2 + (29.4237\ldots)^2}$$

$$= 49.6563\ldots$$

Janelle and Troy are approximately 50 m apart.



13. Sketch and label a diagram to represent the information in the problem.
WZ represents the distance between the base of the tower and the point where the wires are attached to the ground.
∠YWZ represents the angle of inclination of the shorter guy wire.
XY represents the distance between the points where the guy wires are attached to the tower.

a) In right △WXZ, WZ is adjacent to ∠W and WX is the hypotenuse.
Use the cosine ratio in right △WXZ.

$$\cos W = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos W = \frac{WZ}{WX}$$

$$\cos 60° = \frac{WZ}{10} \qquad \text{Solve for WZ.}$$

$$10\cos 60° = WZ$$

$$WZ = 5$$

The wires are attached on the ground 5.0 m from the base of the tower.

b) In right △WYZ, WZ is adjacent to ∠W and WY is the hypotenuse.
Use the cosine ratio in right △WYZ.

$$\cos W = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos W = \frac{WZ}{WY}$$

$$\cos W = \frac{5}{8}$$

$$\angle W = 51.3178\ldots$$

The angle of inclination of the shorter guy wire is approximately 51.3°.

c) Use the Pythagorean Theorem.

In right △WXZ:

$$WX^2 = WZ^2 + XZ^2$$

$$XZ^2 = WX^2 - WZ^2$$

$$XZ^2 = 10^2 - 5^2$$

$$XZ = \sqrt{75}$$

In right △WYZ:

$$WY^2 = WZ^2 + YZ^2$$

$$YZ^2 = WY^2 - WZ^2$$

$$YZ^2 = 8^2 - 5^2$$

$$YZ = \sqrt{39}$$

$$XY = XZ - YZ$$

$$XY = \sqrt{75} - \sqrt{39}$$

$$= 2.4152\ldots$$

The points where the guy wires are attached to the tower are about 2.4 m apart.

14. a) In right △EFG, EF is adjacent to ∠E and GF is opposite ∠E.
Use the tangent ratio in right △EFG.

$$\tan E = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan E = \frac{GF}{EF}$$

$$\tan 22° = \frac{GF}{56} \qquad \text{Solve for GF.}$$

$$56 \tan 22° = GF$$

$$GF = 22.6254\ldots$$

The width of the gorge is approximately 23 m.

b) In right △FGH, GF is adjacent to ∠F and GH is opposite ∠F.
Use the tangent ratio in right △FGH.

$$\tan F = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan F = \frac{GH}{GF}$$

$$\tan 41° = \frac{GH}{22.6254\ldots} \qquad \text{Solve for GH.}$$

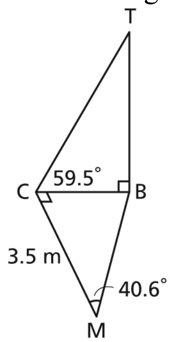
$$(22.6254\ldots)\tan 41° = GH$$

$$GH = 19.6680\ldots$$

The depth of the gorge is approximately 20 m.

15. The surveyor can identify a spot directly below the cliff along the same horizontal as her eye, then use her clinometer to find a point along her side of the river that is along a path perpendicular to her line of sight to the spot. She can then measure the distance to the point, and use the clinometer to measure the angle between her path to the point and her new line of sight to the spot below the cliff. She can use the tangent ratio to determine the distance across the river.
The surveyor can then use the clinometer to measure the angle of elevation of the top of the cliff. Since she now knows the distance across the river, she can use the tangent ratio to determine the height of the cliff.

16. a) Sketch a diagram.



BT represents the height of the tower.

b) The height cannot be calculated directly because no lengths are known in $\triangle$BCT.
So, determine the length of BC first.
In right $\triangle$BCM, CM is adjacent to $\angle$M and BC is opposite $\angle$M.
Use the tangent ratio in right $\triangle$BCM.

$$\tan \mathrm{M} = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan \mathrm{M} = \frac{\mathrm{BC}}{\mathrm{CM}}$$

$$\tan 40.6° = \frac{\mathrm{BC}}{3.5} \qquad \text{Solve for BC.}$$

$$3.5 \tan 40.6° = \mathrm{BC}$$

$$\mathrm{BC} = 2.9998\ldots$$

In right $\triangle$BCT, BC is adjacent to $\angle$C and BT is opposite $\angle$C.
Use the tangent ratio in right $\triangle$BCT.

$$\tan \mathrm{C} = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan \mathrm{C} = \frac{\mathrm{BT}}{\mathrm{BC}}$$

$$\tan 59.5° = \frac{\mathrm{BT}}{2.9998\ldots} \qquad \text{Solve for BT.}$$

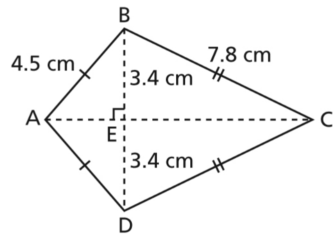
$$(2.9998\ldots)\tan 59.5° = \mathrm{BT}$$

$$\mathrm{BT} = 5.0927\ldots$$

The height of the tower is approximately 5.1 m.

C

17. Sketch and label a diagram. The longer diagonal is the perpendicular bisector of the shorter diagonal, so BE = ED = 3.4 cm



a) In right △ABE, BE is adjacent to ∠B and AB is the hypotenuse.
Use the cosine ratio in right △ABE.

$$\cos B = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos B = \frac{BE}{AB}$$

$$\cos B = \frac{3.4}{4.5}$$

$$\angle B = 40.9260\ldots°$$

In isosceles △ABD, ∠B = ∠D
The sum of the angles in a triangle is 180°, so:

$$\angle A = 180° - 2\angle B$$

$$\begin{aligned}\angle A &= 180° - 2(40.9260\ldots°)\\ &= 98.1478\ldots°\end{aligned}$$

In right △BCE, BE is adjacent to ∠B and BC is the hypotenuse.
Use the cosine ratio in right △BCE.

$$\cos B = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos B = \frac{BE}{BC}$$

$$\cos B = \frac{3.4}{7.8}$$

$$\angle B = 64.1575\ldots°$$

In isosceles △BCD, ∠B = ∠D
The sum of the angles in a triangle is 180°, so:

$$\angle C = 180° - 2\angle B$$

$$\begin{aligned}\angle C &= 180° - 2(64.1575\ldots°)\\ &= 51.6848\ldots°\end{aligned}$$

$$\angle ABC = \angle ABE + \angle EBC$$

$$\begin{aligned}\angle ABC &= 40.9260\ldots° + 64.1575\ldots°\\ &= 105.0836\ldots°\end{aligned}$$

In kite ABCD, ∠B = ∠D. So, ∠D = 105.0836…°

So, in kite ABCD, ∠A is approximately 98.1°, ∠B is approximately 105.1°, ∠C is approximately 51.7°, and ∠D is approximately 105.1°.

b) In right $\triangle$ABE, use the Pythagorean Theorem.

$$\begin{aligned}AB^2 &= AE^2 + BE^2\\ AE^2 &= AB^2 - BE^2\\ AE^2 &= 4.5^2 - 3.4^2\\ AE &= \sqrt{4.5^2 - 3.4^2}\\ &= 2.9478\ldots\end{aligned}$$

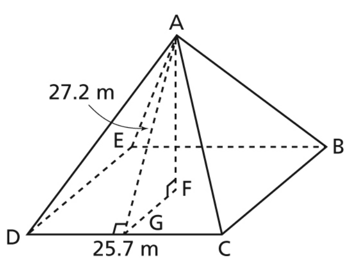
In right $\triangle$BCE, use the Pythagorean Theorem.

$$\begin{aligned}BC^2 &= BE^2 + CE^2\\ CE^2 &= BC^2 - BE^2\\ CE^2 &= 7.8^2 - 3.4^2\\ CE &= \sqrt{7.8^2 - 3.4^2}\\ &= 7.0199\ldots\end{aligned}$$

$$\begin{aligned}AC &= AE + CE\\ AC &= 2.9478\ldots + 7.0199\ldots\\ &= 9.9678\ldots\end{aligned}$$

The longer diagonal is approximately 10.0 cm long.

18. a) Sketch and label the pyramid. Its height is AF.



b) Since the 4 triangular faces are congruent, all of them have the same base length; so, BC = CD = 25.7 m

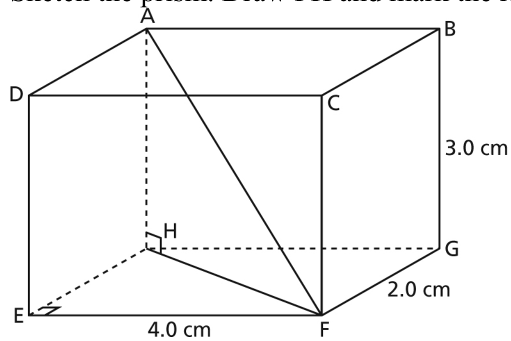
$$\begin{aligned}FG &= \frac{1}{2}BC\\ FG &= \frac{1}{2}(25.7)\\ &= 12.85\end{aligned}$$

In right $\triangle$AFG, use the Pythagorean Theorem.

$$\begin{aligned}AG^2 &= AF^2 + FG^2\\ AF^2 &= AG^2 - FG^2\\ AF^2 &= 27.2^2 - 12.85^2\\ AF &= \sqrt{27.2^2 - 12.85^2}\\ &= 23.9732\ldots\end{aligned}$$

The height of the pyramid is approximately 24.0 m.

19. a) Sketch the prism. Draw FH and mark the right angles.



To determine the length of AF, first determine the length of FH, then use the Pythagorean Theorem in △AFH.

In right △EFH, use the Pythagorean Theorem.

$$FH^2 = EF^2 + EH^2$$
$$FH^2 = 4.0^2 + 2.0^2$$
$$FH = \sqrt{4.0^2 + 2.0^2}$$
$$= 4.4721\ldots$$

In right △AFH, use the Pythagorean Theorem.

$$AF^2 = AH^2 + FH^2$$
$$AF^2 = 3.0^2 + (4.4721\ldots)^2$$
$$AF = \sqrt{3.0^2 + (4.4721\ldots)^2}$$
$$= 5.3851\ldots$$

The length of the body diagonal is approximately 5.4 cm.

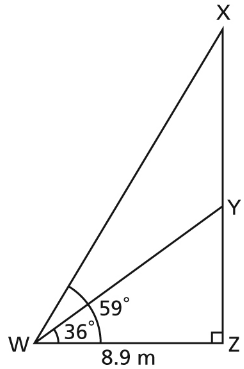
b) In right △AFH, FH is adjacent to ∠F and AF is the hypotenuse.

Use the cosine ratio in right △AFH.

$$\cos F = \frac{\text{adjacent}}{\text{hypotenuse}}$$
$$\cos F = \frac{FH}{AF}$$
$$\cos F = \frac{4.4721\ldots}{5.3851\ldots}$$
$$\angle F = 33.8545\ldots°$$

The measure of ∠AFH is approximately 33.9°.

20. Sketch and label a diagram.



XY is the distance between the guy wire anchor and the top of the tower.

In right △WYZ, WZ is adjacent to ∠W and YZ is opposite ∠W.
Use the tangent ratio in right △WYZ.

$$\tan W = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan W = \frac{YZ}{WZ}$$

$$\tan 36° = \frac{YZ}{8.9} \qquad \text{Solve for YZ.}$$

$$8.9 \tan 36° = YZ$$

$$YZ = 6.4662\ldots$$

In right △WXZ, WZ is adjacent to ∠W and XZ is opposite ∠W.
Use the tangent ratio in right △WXZ.

$$\tan W = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan W = \frac{XZ}{WZ}$$

$$\tan 59° = \frac{XZ}{8.9} \qquad \text{Solve for XZ.}$$

$$8.9 \tan 59° = XZ$$

$$XZ = 14.8120\ldots$$

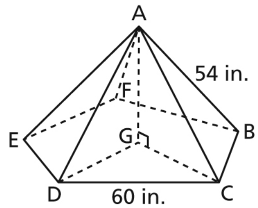
$$XY = XZ - YZ$$

$$XY = 14.8120\ldots - 6.4662\ldots$$

$$= 8.3458\ldots$$

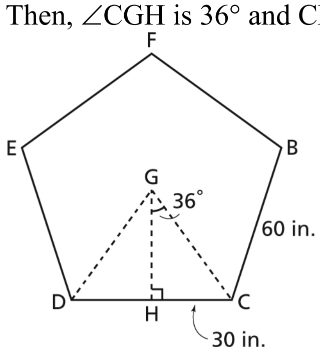
The guy wire is attached to the tower approximately 8.3 m below the top of the tower.

21. Sketch and label a diagram of the pyramid.



AG is the height of the pyramid.

Sketch the base of the pyramid.
In the base of the pyramid, ∠CGD is 360° ÷ 5 = 72°.
△CDG is isosceles, so draw the perpendicular bisector GH to divide it into two congruent right triangles: △CGH and △DGH
Then, ∠CGH is 36° and CH is 30 in.



To determine the height of the pyramid, use right △AGC. First determine the length of GC using right △CGH.
In right △CGH, CH is opposite ∠G and GC is the hypotenuse.
Use the sine ratio in right △CGH.

$$\sin G = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin G = \frac{CH}{CG}$$

$$\sin 36° = \frac{30}{CG} \qquad \text{Solve for CG.}$$

$$CG \sin 36° = 30$$

$$CG = \frac{30}{\sin 36°}$$

$$CG = 51.0390\ldots$$

In right △ACG, use the Pythagorean Theorem.

$$AC^2 = AG^2 + GC^2$$

$$AG^2 = AC^2 - GC^2$$

$$AG^2 = 54^2 - (51.0390\ldots)^2$$

$$AG = \sqrt{54^2 - (51.0390\ldots)^2}$$

$$= 17.6356\ldots$$

The height of the pyramid is approximately 18 in.

Review (pages 124–126)

2.1

1. a) In right △CDE, CE is opposite ∠D and CD is adjacent to ∠D.
Use the tangent ratio in right △CDE.

$$\tan D = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan D = \frac{CE}{CD}$$

$$\tan D = \frac{7}{10}$$

$$\angle D = 34.9920\ldots°$$

∠D is approximately 35°.

b) In right △FGH, FG is opposite ∠H and GH is adjacent to ∠H.
Use the tangent ratio in right △FGH.

$$\tan H = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan H = \frac{FG}{GH}$$

$$\tan H = \frac{3.2}{1.5}$$

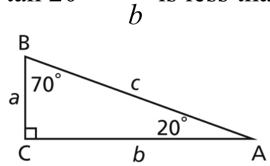
$$\angle H = 64.8851\ldots°$$

∠H is approximately 65°.

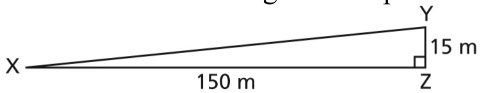
2. a) tan 20° = 0.3639…, so tan 20° <1.

b) tan 70° = 2.7474…, so tan 70° > 1.

c) I know that as the measure of an acute angle in a right triangle increases, the length of the side opposite the angle also increases. In the triangle below, side b is opposite ∠B = 70°, so it is longer than side a, which is opposite ∠A = 20°. Since b is greater than a, the ratio $\tan 20° = \frac{a}{b}$ is less than 1 and the ratio $\tan 70° = \frac{b}{a}$ is greater than 1.



3. Sketch and label a diagram to represent the information in the problem.



$\angle$X is the angle of inclination of the road.

In right $\triangle$XYZ, YZ is opposite $\angle$X and XZ is adjacent to $\angle$X.

$$\tan X = \frac{\text{opposite}}{\text{adjacent}}$$

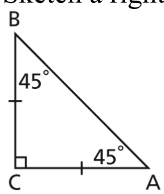
$$\tan X = \frac{YZ}{XZ}$$

$$\tan X = \frac{15}{150}$$

$$\angle X = 5.7105\ldots°$$

The angle of inclination of the road is approximately 6°.

4. Sketch a right triangle with $\angle$A = 45°.



In right $\triangle$ABC, $\angle$A = $\angle$B, so the triangle is isosceles and BC = AC.
So:

$$\tan A = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan A = \frac{BC}{AC}$$

$$\tan 45° = \frac{AC}{AC}$$

$$\tan 45° = 1$$

The triangle is an isosceles right triangle.

2.2

5. a) i) In right △JKM, JM is opposite ∠K and KM is adjacent to ∠K.

$$\tan K = \frac{\text{opposite}}{\text{adjacent}}$$
$$\tan K = \frac{\text{JM}}{\text{KM}}$$
$$\tan 63° = \frac{k}{1.9}$$
$$1.9 \tan 63° = k$$
$$k = 3.7289\ldots$$

JM is approximately 3.7 cm long.

ii) In right △NPQ, PQ is opposite ∠N and NQ is adjacent to ∠N.

$$\tan N = \frac{\text{opposite}}{\text{adjacent}}$$
$$\tan N = \frac{\text{PQ}}{\text{NQ}}$$
$$\tan 42° = \frac{2.7}{p}$$
$$p \tan 42° = 2.7$$
$$p = \frac{2.7}{\tan 42°}$$
$$p = 2.9986\ldots$$

NQ is approximately 3.0 cm long.

b) i) Use the Pythagorean Theorem in right △JKM.

$$\text{JK}^2 = \text{KM}^2 + \text{JM}^2$$
$$\text{JK}^2 = 1.9^2 + (3.7289\ldots)^2$$
$$\text{JK} = \sqrt{1.9^2 + (3.7289\ldots)^2}$$
$$= 4.1851\ldots$$

JK is approximately 4.2 cm long.

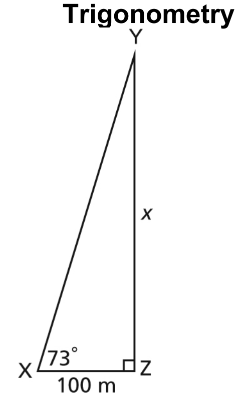
ii) Use the Pythagorean Theorem in right △NPQ.

$$\text{NP}^2 = \text{NQ}^2 + \text{PQ}^2$$
$$\text{NP}^2 = (2.9986\ldots)^2 + 2.7^2$$
$$\text{NP} = \sqrt{(2.9986\ldots)^2 + 2.7^2}$$
$$= 4.0350\ldots$$

NP is approximately 4.0 cm long.

I could also have used the sine or cosine ratios to determine the length of each hypotenuse.

6. Sketch and label a diagram to represent the information in the problem. YZ represents the height of the Eiffel tower.



In right △XYZ, YZ is opposite ∠X and XZ is adjacent to ∠X.

$$\tan X = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan X = \frac{YZ}{XZ}$$

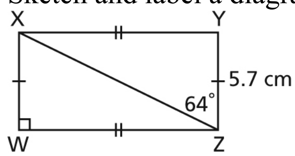
$$\tan 73° = \frac{x}{100}$$

$$100 \tan 73° = x$$

$$x = 327.0852\ldots$$

The tower is approximately 327 m tall.

7. Sketch and label a diagram to represent the information in the problem.



a) In right △XYZ, XY is opposite ∠Z and YZ is adjacent to ∠Z.

$$\tan Z = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan Z = \frac{XY}{YZ}$$

$$\tan 64° = \frac{XY}{5.7}$$

$$5.7 \tan 64° = XY$$

$$XY = 11.6867\ldots$$

The rectangle is approximately 11.7 cm long.

b) In right △XYZ, use the Pythagorean Theorem.

$$XZ^2 = XY^2 + YZ^2$$

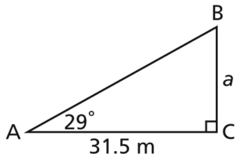
$$XZ^2 = (11.6867\ldots)^2 + 5.7^2$$

$$XZ = \sqrt{(11.6867\ldots)^2 + 5.7^2}$$

$$= 13.0026\ldots$$

The diagonal is approximately 13.0 cm long.

8. Sketch and label a diagram to represent the information in the problem.



BC represents the height of the tree.

In right △ABC, BC is opposite ∠A and AC is adjacent to ∠A.

$$\tan A = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan A = \frac{BC}{AC}$$

$$\tan 29° = \frac{a}{31.5}$$

$$31.5 \tan 29° = a$$

$$a = 17.4607\ldots$$

The tree is approximately 17.5 m tall.

9. Sketch and label a diagram to represent the information in the problem.
PR represents Aidan's distance from the base of the Lookout.

In right △PQR, QR is opposite ∠P
and PR is adjacent to ∠P.

$$\tan P = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan P = \frac{QR}{PR}$$

$$\tan 77° = \frac{130}{q}$$

$$q \tan 77° = 130$$

$$q = \frac{130}{\tan 77°}$$

$$q = 30.0128\ldots$$

Aidan is approximately 30 m from the base of the Lookout.

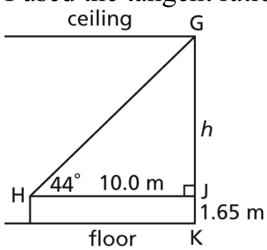
2.3

10. I measured 10 m from the wall. Then, I measured the angle of elevation of the point where the ceiling and the wall meet. The angle shown on the clinometer was 46°, so the angle of elevation was:

$90° – 46° = 44°$

My eye was approximately 1.65 m above the ground.

I used the tangent ratio to determine the height of the ceiling, h.



In right △GHJ, GJ is opposite ∠H and HJ is adjacent to ∠H.

$$\tan H = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan H = \frac{GJ}{HJ}$$

$$\tan 44° = \frac{h}{10}$$

$$10 \tan 44° = h$$

$$h = 9.6568\ldots$$

The height of the gym is:

$1.65 \text{ m} + 9.6568\ldots \text{ m} = 11.3068\ldots \text{ m}$

The height of the gymnasium is approximately 11.3 m.

2.4

11. a) In right △RST, RS is adjacent to ∠S and ST is the hypotenuse. So, use the cosine ratio in right △RST.

$$\cos S = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos S = \frac{\text{RS}}{\text{ST}}$$

$$\cos S = \frac{3}{10}$$

$$\angle S = 72.5423\ldots°$$

∠S is approximately 73°.

b) In right △UVW, VW is opposite ∠U and UV is the hypotenuse. So, use the sine ratio in right △UVW.

$$\sin U = \frac{\text{opposite}}{\text{hypotenuse}}$$

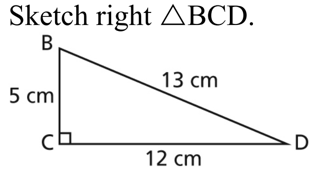
$$\sin U = \frac{\text{VW}}{\text{UV}}$$

$$\sin U = \frac{3.7}{8.0}$$

$$\angle U = 27.5485\ldots°$$

∠U is approximately 28°.

12. Sketch right △BCD.



a) i)

$$\sin D = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin D = \frac{\text{BC}}{\text{BD}}$$

$$\sin D = \frac{5}{13}$$

ii)

$$\sin B = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin B = \frac{\text{CD}}{\text{BD}}$$

$$\sin B = \frac{12}{13}$$

iii)

$$\cos B = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos B = \frac{\text{BC}}{\text{BD}}$$

$$\cos B = \frac{5}{13}$$

iv)

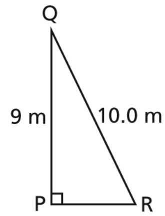
$$\cos D = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos D = \frac{\text{CD}}{\text{BD}}$$

$$\cos D = \frac{12}{13}$$

b) sin D = cos B and sin B = cos D
This occurs because the side opposite ∠B is adjacent to ∠D, and the side adjacent to ∠B is opposite ∠D.

13. Sketch and label a diagram to represent the information in the problem.



$\angle$R represents the angle of inclination of the pole.

In right $\triangle$PQR, PQ is opposite $\angle$R and QR is the hypotenuse.
So, use the sine ratio in right $\triangle$PQR.

$$\sin R = \frac{\text{opposite}}{\text{hypotenuse}}$$

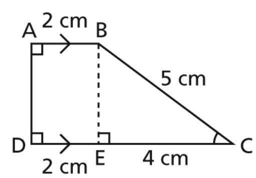
$$\sin R = \frac{PQ}{QR}$$

$$\sin R = \frac{9}{10.0}$$

$$\angle R = 64.1580\ldots°$$

The angle of inclination of the pole is approximately 64.2°.

14. Sketch and label a diagram to represent the trapezoid.
ABED is a rectangle, so DE = AB = 2 cm



$$CE = CD - DE$$

$$CE = 6 \text{ cm} - 2 \text{ cm}$$
$$= 4 \text{ cm}$$

In right $\triangle$BCE, CE is adjacent to $\angle$C and BC is the hypotenuse.
Use the cosine ratio in right $\triangle$BCE.

$$\cos C = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos C = \frac{CE}{BC}$$

$$\cos C = \frac{4}{5}$$

$$\angle C = 36.8698\ldots°$$

$\angle$C is approximately 36.9°.

2.5

15. a) In right $\triangle$EFG, FG is adjacent to $\angle$G and EG is hypotenuse.
So, use the cosine ratio in right $\triangle$EFG.

$$\cos G = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos G = \frac{\text{FG}}{\text{EG}}$$

$$\cos 33° = \frac{e}{4.7}$$

$$4.7 \cos 33° = e$$

$$e = 3.9417\ldots$$

FG is approximately 3.9 cm long.

b) In right $\triangle$HJK, HJ is opposite $\angle$K and JK is the hypotenuse.
So, use the sine ratio in right $\triangle$HJK.

$$\sin K = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin K = \frac{\text{HJ}}{\text{JK}}$$

$$\sin 48° = \frac{k}{5.9}$$

$$5.9 \sin 48° = k$$

$$k = 4.3845\ldots$$

HJ is approximately 4.4 cm long.

c) In right $\triangle$MNP, MP is opposite $\angle$N and NP is the hypotenuse.
So, use the sine ratio in right $\triangle$MNP.

$$\sin N = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin N = \frac{\text{MP}}{\text{NP}}$$

$$\sin 52° = \frac{3.7}{m}$$

$$m \sin 52° = 3.7$$

$$m = \frac{3.7}{\sin 52°}$$

$$m = 4.6953\ldots$$

NP is approximately 4.7 cm long.

d) In right $\triangle$QRS, RS is adjacent to $\angle$S and QS is the hypotenuse.
So, use the cosine ratio in right $\triangle$QRS.

$$\cos S = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos S = \frac{RS}{QS}$$

$$\cos 65° = \frac{1.9}{r}$$

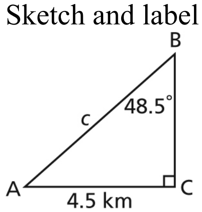
$$r\cos 65° = 1.9$$

$$r = \frac{1.9}{\cos 65°}$$

$$r = 4.4957\ldots$$

QS is approximately 4.5 cm long.

16. Sketch and label a diagram to represent the information in the problem.



AB represents the distance between the ship and Arviat.

In right $\triangle$ABC, AC is opposite $\angle$B and AB is the hypotenuse.
So, use the sine ratio in right $\triangle$ABC.

$$\sin B = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin B = \frac{AC}{AB}$$

$$\sin 48.5° = \frac{4.5}{c}$$

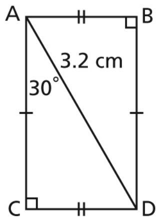
$$c\sin 48.5° = 4.5$$

$$c = \frac{4.5}{\sin 48.5°}$$

$$c = 6.0083\ldots$$

The ship is approximately 6.0 km from Arviat.

17. Sketch and label a diagram to represent the information in the problem.



In right △ACD, CD is opposite ∠A and AD is the hypotenuse.
So, use the sine ratio in right △ACD.

$$\sin A = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin A = \frac{CD}{AD}$$

$$\sin 30° = \frac{CD}{3.2}$$

$$3.2 \sin 30° = CD$$

$$CD = 1.6$$

In right △ACD, AC is adjacent to ∠A and AD is the hypotenuse.
So, use the cosine ratio in right △ACD.

$$\cos A = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos A = \frac{AC}{AD}$$

$$\cos 30° = \frac{AC}{3.2}$$

$$3.2 \cos 30° = AC$$

$$AC = 2.7712\ldots$$

The dimensions of the rectangle are 1.6 cm by approximately 2.8 cm.

2.6

18. a) In right △CDE, DE is opposite ∠C and CD is adjacent to ∠C.
Use the tangent ratio in right △CDE.

$$\tan C = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan C = \frac{DE}{CD}$$

$$\tan C = \frac{2.7}{4.2}$$

$$\angle C = 32.7352\ldots^\circ$$

The sum of the acute angles in a right triangle is 90°, so:

$$\angle E = 90^\circ - \angle C$$

$$\angle E = 90^\circ - 32.7352\ldots^\circ$$

$$= 57.2647\ldots^\circ$$

Use the Pythagorean Theorem in right △CDE.

$$CE^2 = CD^2 + DE^2$$

$$CE^2 = 4.2^2 + 2.7^2$$

$$CE = \sqrt{4.2^2 + 2.7^2}$$

$$= 4.9929\ldots$$

CE is approximately 5.0 cm, ∠C is approximately 32.7°, and ∠E is approximately 57.3°.

b) The sum of the acute angles in a right triangle is 90°, so:

$$\begin{aligned}\angle H &= 90° - \angle F\\ \angle H &= 90° - 38°\\ &= 52°\end{aligned}$$

In right △FGH, GH is opposite ∠F and FG is adjacent to ∠F.
Use the tangent ratio in right △FGH.

$$\begin{aligned}\tan F &= \frac{\text{opposite}}{\text{adjacent}}\\ \tan F &= \frac{GH}{FG}\\ \tan 38° &= \frac{GH}{3.4}\\ 3.4\tan 38° &= GH\\ GH &= 2.6563\ldots\end{aligned}$$

Use the Pythagorean Theorem in right △FGH.

$$\begin{aligned}FH^2 &= FG^2 + GH^2\\ FH^2 &= 3.4^2 + (2.6563\ldots)^2\\ FH &= \sqrt{3.4^2 + (2.6563\ldots)^2}\\ &= 4.3146\ldots\end{aligned}$$

GH is approximately 2.7 cm, FH is approximately 4.3 cm, and ∠H is 52°.

c) The sum of the acute angles in a right triangle is 90°, so:

$$\angle K = 90° - \angle J$$
$$\angle K = 90° - 27°$$
$$= 63°$$

In right △JKM, KM is opposite ∠J and JK is the hypotenuse.
Use the sine ratio in right △JKM.

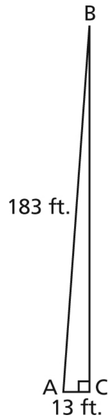
$$\sin J = \frac{\text{opposite}}{\text{hypotenuse}}$$
$$\sin J = \frac{KM}{JK}$$
$$\sin 27° = \frac{KM}{4.4}$$
$$4.4 \sin 27° = KM$$
$$KM = 1.9975\ldots$$

In right △JKM, JM is adjacent to ∠J and JK is the hypotenuse.
Use the cosine ratio in right △JKM.

$$\cos J = \frac{\text{adjacent}}{\text{hypotenuse}}$$
$$\cos J = \frac{JM}{JK}$$
$$\cos 27° = \frac{JM}{4.4}$$
$$4.4 \cos 27° = JM$$
$$JM = 3.9204\ldots$$

KM is approximately 2.0 cm, JM is approximately 3.9 cm, and ∠K is 63°.

19. Sketch and label a diagram to represent the information in the problem.



The angle of inclination is $\angle A$.
In right $\triangle ABC$, AC is adjacent to $\angle A$ and AB is the hypotenuse.
Use the cosine ratio in right $\triangle ABC$.

$$\cos A = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos A = \frac{AC}{AB}$$

$$\cos A = \frac{13}{183}$$

$$\angle A = 85.9263\ldots°$$

The angle of inclination is approximately 85.9°.

20. a) In right $\triangle PQR$, QR is the hypotenuse and PR is opposite $\angle Q$.
Use the sine ratio in right $\triangle PQR$.

$$\sin Q = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin Q = \frac{PR}{QR}$$

$$\sin 54° = \frac{PR}{14.8}$$

$$14.8 \sin 54° = PR$$

$$PR = 11.9734\ldots$$

In right $\triangle PQR$, QR is the hypotenuse and PQ is adjacent to $\angle Q$.
Use the cosine ratio in right $\triangle PQR$.

$$\cos Q = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos Q = \frac{PQ}{QR}$$

$$\cos 54° = \frac{PQ}{14.8}$$

$$14.8 \cos 54° = PQ$$

$$PQ = 8.6992\ldots$$

Perimeter of $\triangle PQR$:
14.8 cm + 11.9734… cm + 8.6992… cm = 35.4726… cm

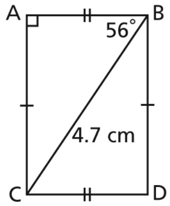
Area of $\triangle PQR$:

$$\frac{1}{2}(PQ)(PR) = \frac{1}{2}(11.9734\ldots)(8.6992\ldots)$$

$$= 52.0798\ldots$$

The perimeter of $\triangle PQR$ is approximately 35.5 cm and its area is approximately 52.1 cm^2.

b) Sketch and label a diagram of the rectangle.



In right △ABC, BC is the hypotenuse and AC is opposite ∠B.
Use the sine ratio in right △ABC.

$$\sin B = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin B = \frac{AC}{BC}$$

$$\sin 56° = \frac{AC}{4.7}$$

$$4.7 \sin 56° = AC$$

$$AC = 3.8964\ldots$$

In right △ABC, BC is the hypotenuse and AB is adjacent to ∠B.
Use the cosine ratio in right △ABC.

$$\cos B = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos B = \frac{AB}{BC}$$

$$\cos 56° = \frac{AB}{4.7}$$

$$4.7 \cos 56° = AB$$

$$AB = 2.6282\ldots$$

Perimeter of rectangle:
2(3.8964… cm + 2.6282… cm) = 13.0493… cm

Area of rectangle:
(3.8964… cm)(2.6282… cm) = 10.2407… cm^2

The perimeter of the rectangle is approximately 13.0 cm and its area is approximately 10.2 cm^2.

21. a) In right △ABC, AB is the hypotenuse and AC is opposite ∠B.
Use the sine ratio in right △ABC.

$$\sin B = \frac{\text{opposite}}{\text{hypotenuse}}$$
$$\sin B = \frac{AC}{AB}$$
$$\sin 60° = \frac{2.8}{AB}$$
$$AB \sin 60° = 2.8$$
$$AB = \frac{2.8}{\sin 60°}$$
$$AB = 3.2331\ldots$$

AB is approximately 3.2 m.

b) In right △ABC, AC is opposite ∠B and BC is adjacent to ∠B.
Use the tangent ratio in right △ABC.

$$\tan B = \frac{\text{opposite}}{\text{adjacent}}$$
$$\tan B = \frac{AC}{BC}$$
$$\tan 60° = \frac{2.8}{BC}$$
$$BC \tan 60° = 2.8$$
$$BC = \frac{2.8}{\tan 60°}$$
$$BC = 1.6165\ldots$$

$$BD = BC + DC$$
$$BD = 1.6165\ldots + 6.6$$
$$= 8.2165\ldots$$

BD is approximately 8.2 m.

2.7

22. a) Use the Pythagorean Theorem in right $\triangle$GJK.

$$\text{KJ}^2 = \text{GK}^2 + \text{GJ}^2$$
$$\text{KJ}^2 = 10.8^2 + (3.2 + 5.1)^2$$
$$\text{KJ} = \sqrt{10.8^2 + (3.2 + 5.1)^2}$$
$$= 13.6209\ldots$$

KJ is approximately 13.6 cm long.

b) Use the Pythagorean Theorem in right $\triangle$GHK.

$$\text{HK}^2 = \text{GK}^2 + \text{GH}^2$$
$$\text{HK}^2 = 10.8^2 + 3.2^2$$
$$\text{HK} = \sqrt{10.8^2 + 3.2^2}$$
$$= 11.2641\ldots$$

HK is approximately 11.3 cm long.

c) $\angle\text{HKJ} = \angle\text{GKJ} - \angle\text{HKJ}$

In right $\triangle$GJK, GJ is opposite $\angle$K and GK is adjacent to $\angle$K.
Use the tangent ratio in right $\triangle$GJK long.

$$\tan \text{K} = \frac{\text{opposite}}{\text{adjacent}}$$
$$\tan \text{K} = \frac{\text{GJ}}{\text{GK}}$$
$$\tan \text{K} = \frac{3.2 + 5.1}{10.8}$$
$$\tan \text{K} = \frac{8.3}{10.8}$$
$$\angle\text{K} = 37.5429\ldots°$$

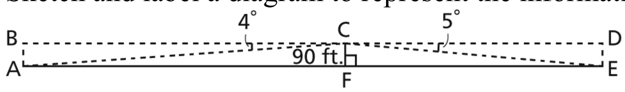
In right $\triangle$GHK, GH is opposite $\angle$K and GK is adjacent to $\angle$K.
Use the tangent ratio in right $\triangle$GHK.

$$\tan \text{K} = \frac{\text{opposite}}{\text{adjacent}}$$
$$\tan \text{K} = \frac{\text{GH}}{\text{GK}}$$
$$\tan \text{K} = \frac{3.2}{10.8}$$
$$\angle\text{K} = 16.5043\ldots°$$

$$\angle\text{HKJ} = \angle\text{GKJ} - \angle\text{HKJ}$$
$$\angle\text{HKJ} = 37.5429\ldots° - 16.5043\ldots°$$
$$= 21.0385\ldots°$$

$\angle$HKJ is approximately 21.0°.

23. Sketch and label a diagram to represent the information in the problem.



AE represents the distance between the fires.

AE = AF + FE

In right △ABC, AB is opposite ∠C and BC is adjacent to ∠C.

Use the tangent ratio in right △ABC.

$$\tan C = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan C = \frac{AB}{BC}$$

$$\tan 4° = \frac{90}{BC}$$

$$BC \tan 4° = 90$$

$$BC = \frac{90}{\tan 4°}$$

$$BC = 1287.0599\ldots$$

Since ABCF is a rectangle, AF = BC.

In right △CDE, DE is opposite ∠C and CD is adjacent to ∠C.

Use the tangent ratio in right △CDE.

$$\tan C = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan C = \frac{DE}{CD}$$

$$\tan 5° = \frac{90}{CD}$$

$$CD \tan 5° = 90$$

$$CD = \frac{90}{\tan 5°}$$

$$CD = 1028.7047\ldots$$

Since CDEF is a rectangle, EF = CD.

$$AE = AF + FE$$

$$AE = 1287.0599\ldots + 1028.7047\ldots$$

$$= 2315.7646\ldots$$

The fires are approximately 2316 ft. apart.

Practice Test (page 127)

1. In △PQR:

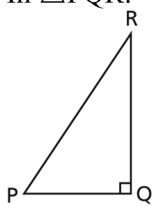
$\tan Q = \frac{\text{opposite}}{\text{adjacent}}$ $\sin P = \frac{\text{opposite}}{\text{hypotenuse}}$ $\sin Q = \frac{\text{opposite}}{\text{hypotenuse}}$ $\tan P = \frac{\text{opposite}}{\text{adjacent}}$

$\tan Q = \frac{PR}{QR}$ $\sin P = \frac{QR}{PQ}$ $\sin Q = \frac{PR}{PQ}$ $\tan P = \frac{QR}{PR}$

$\tan Q = \frac{3}{4}$ $\sin P = \frac{4}{5}$ $\sin Q = \frac{3}{5}$ $\tan P = \frac{4}{3}$

3 statements are true.
Answer B is correct.

2. In △PQR:



∠P increases when QR increases and PQ remains the same length.
Then PR also increases. So:

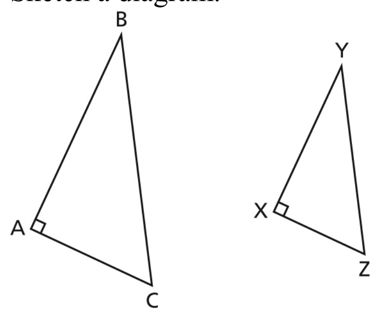
$\tan P = \frac{QR}{PQ}$ increases and $\cos P = \frac{PQ}{PR}$ decreases

∠P also increases when QR remains the same length and PQ decreases.
Then PR also decreases. So:

$\sin P = \frac{QR}{PR}$ increases

Answer C is correct.

3. Sketch a diagram.



In $\triangle$ABC:

$$\sin B = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin B = \frac{AC}{BC}$$

In $\triangle$XYZ:

$$\sin Y = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin Y = \frac{XZ}{YZ}$$

Since the triangles are similar, corresponding sides are in the same ratio:

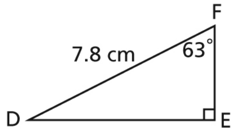
$$\frac{AC}{XZ} = \frac{BC}{YZ}$$ Multiply each side by XZ.

$$\frac{AC}{XZ}XZ = \frac{BC}{YZ}XZ$$ Divide each side by BC.

$$\frac{AC}{BC} = \frac{XZ}{YZ}$$

So, sin B = sin Y

4. Sketch and label the triangle.



The sum of the acute angles in a right triangle is 90°, so:

$$\angle D = 90° - \angle F$$
$$\angle D = 90° - 63°$$
$$= 27°$$

In right $\triangle$DEF, DE is opposite $\angle$F
and DF is the length of the hypotenuse.
Use the sine ratio in right $\triangle$DEF.

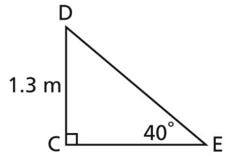
$$\sin F = \frac{\text{opposite}}{\text{hypotenuse}}$$
$$\sin F = \frac{DE}{DF}$$
$$\sin 63° = \frac{DE}{7.8}$$
$$7.8 \sin 63° = DE$$
$$DE = 6.9498\ldots$$

In right $\triangle$DEF, EF is adjacent to $\angle$F and DF is the hypotenuse.
Use the cosine ratio in right $\triangle$DEF.

$$\cos F = \frac{\text{adjacent}}{\text{hypotenuse}}$$
$$\cos F = \frac{EF}{DF}$$
$$\cos 63° = \frac{EF}{7.8}$$
$$7.8 \cos 63° = EF$$
$$EF = 3.5411\ldots$$

DE is approximately 6.9 cm, EF is approximately 3.5 cm, and $\angle$D is 27°.

5. Sketch and label a diagram to represent the information in the problem.



The shortest possible length of the ramp is represented by DE.

In right $\triangle$CDE, CD is opposite $\angle$E and DE is the hypotenuse.
Use the sine ratio in right $\triangle$CDE.

$$\sin E = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin E = \frac{CD}{DE}$$

$$\sin 40° = \frac{1.3}{DE}$$

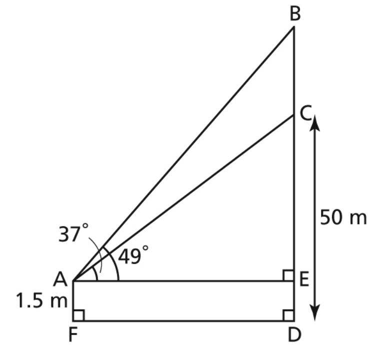
$$DE \sin 40° = 1.3$$

$$DE = \frac{1.3}{\sin 40°}$$

$$DE = 2.0224\ldots$$

Round up since the angle of inclination of the ramp should be less than 40°.
To the nearest centimetre, the shortest possible length of the ramp is 2.03 m.

6. Sketch and label a diagram to represent the information in the problem.
BD represents the height of the tower.



Assume the ground is horizontal.
Then quadrilateral AEDF is a rectangle and DE = AF = 1.5 m.

$$\begin{aligned} CE &= CD - DE \\ CE &= 50 - 1.5 \\ &= 48.5 \end{aligned}$$

In right $\triangle ACE$, CE is opposite $\angle A$ and AE is adjacent to $\angle A$.
Use the tangent ratio in right $\triangle ACE$.

$$\tan A = \frac{\text{opposite}}{\text{adjacent}}$$
$$\tan A = \frac{CE}{AE}$$
$$\tan 37° = \frac{48.5}{AE}$$
$$AE \tan 37° = 48.5$$
$$AE = \frac{48.5}{\tan 37°}$$
$$AE = 64.3616\ldots$$

In right $\triangle ABE$, BE is opposite $\angle A$ and AE is adjacent to $\angle A$.
Use the tangent ratio in right $\triangle ABE$.

$$\tan A = \frac{\text{opposite}}{\text{adjacent}}$$
$$\tan A = \frac{BE}{AE}$$
$$\tan 49° = \frac{BE}{64.3616\ldots}$$
$$(64.3616\ldots)\tan 49° = BE$$
$$BE = 74.0396\ldots$$

$$\begin{aligned} BD &= BE + DE \\ BD &= 74.0396\ldots + 1.5 \\ &= 75.5396\ldots \end{aligned}$$

To the nearest tenth of a metre, the tower is 75.5 m high.