Lesson 1.1 **Imperial Measures of Length** **Exercises (pages 11–12)**

A

3. Answers may vary. For example:

a) Foot; because my desk is higher than 1 ft., but not as high as 1 yd.

b) Inch; because a mattress is thicker than 1 in., but not quite as thick as 1 ft.

c) Foot; because the measurement would be more accurate in feet rather than yards

d) Inch; because the measurement would be more accurate in inches rather than feet

e) Mile; because a mile is closer to the distance from the school to my home than an inch, a foot, or a yard

4. **a)** Inch; because the length of a piece of paper is less than 1 ft.

b) I used my thumb as a referent for one inch. I aligned the tip of my thumb with one corner of a piece of notepaper. I marked a point at my knuckle along the length of the paper. I moved my thumb so the tip aligned with the marked point. I continued until I got to the end of the length of the piece of paper. The paper was about eleven thumb units long. The actual measurement was 11 in., so my estimate was reasonable.

5. **a)** Foot; because this unit is large enough to measure the door, but small enough to get an accurate fraction if the measurement is not a whole number

b) I used my shoe as a referent for one foot. I aligned the heel of my shoe with the bottom of the door. I marked a point at the tip of the shoe up the door with a piece of tape. I moved my shoe so the heel aligned with the marked point. I continued until I got to the top of the door. The door was about seven shoes high. The actual measurement was about $6\frac{1}{2}$ ft., so my estimate was reasonable.

6. Estimates may vary. For example:

a) About 9 in.

b) About 12 ft.

c) About 10 yd.

d) About 1 mi.

7. **a)** Since 1 ft. = 12 in., to convert feet to inches multiply by 12.
3 ft. = 3(12 in.)
3 ft. = 36 in.

b) Since 1 yd. = 3 ft., to convert yards to feet multiply by 3.
63 yd. = 63(3 ft.)
63 yd. = 189 ft.

c) Since 12 in. = 1 ft., to convert inches to feet, divide by 12.

48 in. = $\frac{48}{12}$ ft.

48 in. = 4 ft.

B

8. **a)** Since 1 mi. = 5280 ft., to convert miles to feet multiply by 5280.
2 mi. = 2(5280 ft.)
2 mi. = 10 560 ft.

b) Since 12 in. = 1 ft., to convert inches to feet, divide by 12.

574 in. = $\frac{574}{12}$ ft. Write the improper fraction as a mixed number.

574 in. = $47\frac{10}{12}$ ft.

574 in. = 47 ft. 10 in.

Since 3 ft. = 1 yd., divide 47 ft. by 3 to convert to yards.

47 ft. = $\frac{47}{3}$ yd.

47 ft. = $15\frac{2}{3}$ yd.

47 ft. = 15 yd. 2 ft.
So, 574 in. = 15 yd. 2 ft. 10 in.

c) Since 1 mi. = 5280 ft., subtract 5280 ft. from 7390 ft.
7390 ft – 5280 ft. = 2110 ft.
So, 7390 ft. = 1 mi. 2110 ft.

Since 3 ft. = 1 yd., to convert feet to yards divide by 3.

2110 ft. = $\frac{2110}{3}$ yd. Write this improper fraction as a mixed number.

2110 ft. = $703\frac{1}{3}$ yd.

2110 ft. = 703 yd. 1 ft.
So, 7390 ft. = 1 mi. 703 yd. 1 ft.

9. First convert 165 in. to feet.
Since 12 in. = 1 ft., to convert inches to feet, divide by 12.

165 in. = $\frac{165}{12}$ ft.

Write the improper fraction as a mixed number.

$\frac{165}{12}$ ft. $= 13\frac{9}{12}$ ft.

$\frac{165}{12}$ ft. = 13 ft. 9 in.

Since 3 ft. = 1 yd., divide by 3 to convert feet to yards.

13 ft. $= \frac{13}{3}$ yd.

Write the improper fraction as a mixed number.

$\frac{13}{3}$ yd. $= 4\frac{1}{3}$ yd.

$\frac{13}{3}$ yd. = 4 yd. 1 ft.

So, 165 in. = 4 yd. 1 ft. 9 in.

10. a) Let x represent the perimeter in yards.
The ratio of x yards to feet is equal to the ratio of 1 yd. to 3 ft.
Write a proportion.

$$\frac{x}{52} = \frac{1}{3}$$

$$52\left(\frac{x}{52}\right) = 52\left(\frac{1}{3}\right)$$

$$x = \frac{52}{3}$$

$$x = 17\frac{1}{3}$$

$17\frac{1}{3}$ is 17 yd. 1 ft.

So, the perimeter of the pen is 17 yd. 1 ft.

b) The perimeter of the pen is greater than 17 yd., so Carolyn must buy 18 yd. of fencing material.
The cost, C, is:
C = 18($10.99)
C = $197.82
Before taxes, the fencing material costs $197.82.

11. a) Since the mats are measured in inches, convert the length of material to inches.
Convert 10 yd. to inches.
1 yd. = 3 ft.
10 yd. = 10(3 ft.)
10 yd. = 30 ft.

1 ft. = 12 in.
30 ft. = 30(12 in.)
30 ft. = 360 in.

The number of mats is: $\dfrac{360}{15} = 24$
David can make 24 mats.

b) To convert yards to inches, first convert yards to feet, then convert feet to inches.
Write a conversion factor for yards and feet, with feet in the numerator: $\dfrac{3\text{ ft.}}{1\text{ yd.}}$

Write a conversion factor for feet and inches, with inches in the numerator: $\dfrac{12\text{ in.}}{1\text{ ft.}}$

Then, $10\text{ yd.} \times \dfrac{3\text{ ft.}}{1\text{ yd.}} \times \dfrac{12\text{ in.}}{1\text{ ft.}} = \dfrac{10\ \cancel{\text{yd.}}}{1} \times \dfrac{3\ \cancel{\text{ft.}}}{1\ \cancel{\text{yd.}}} \times \dfrac{12\text{ in.}}{1\ \cancel{\text{ft.}}}$
$= (10 \times 3 \times 12)\text{ in.}$
$= 360\text{ in.}$
Since this measurement is equal to the measurement in part a, the conversion is verified.

12. To check if Pierre-Marc's answer is correct, convert 21 ft. 9 in. into yards, feet, and inches.
3 ft. = 1 yd.
So, to convert feet to yards divide by 3.
21 ft. = $\dfrac{21}{3}$ yd.
21 ft. = 7 yd.
So, 21 ft. 9 in. = 7 yd. 9 in.
Pierre-Marc's answer was incorrect.

13. First, convert the heights to inches.
1 ft. = 12 in., so to convert feet to inches multiply by 12.

For Sandy Allen's height of 7 ft. 7 in.:
7 ft. = 7(12 in.)
7 ft. = 84 in.
So, Sandy is 84 in. + 7 in. = 91 in. tall

For Leonid Stadnyk's height of 8 ft. 5 in.:
8 ft. = 8(12 in.)
8 ft. = 96 in.
So, Leonid is 96 in. + 5 in. = 101 in. tall.

Subtract Sandy's height from Leonid's height to determine the difference:
101 in. – 91 in. = 10 in.
So, Sandy is 10 in. shorter than Leonid.

14. a) Determine the perimeter, P, of the bedroom minus the width of the doorway.
P = 2(12 ft. 9 in. + 8 ft. 1 in.) – 2 ft. 6 in.
P = 2(20 ft. 10 in.) – 2 ft. 6 in.
P = 40 ft. 20 in. – 2 ft. 6 in.
P = 38 ft. 14 in.
Since 1 ft. = 12 in.,
14 in. = 1 ft. 2 in.

So, P = 39 ft. 2 in.
The total length of border needed is 39 ft. 2 in.

b) The border is purchased in 12-ft. rolls.
The number of rolls is approximately $\frac{39 \text{ ft.}}{12 \text{ ft.}} = 3\frac{3}{12}$
Since only whole rolls can be bought, 4 rolls are needed.

c) The cost, C, of 4 rolls is:
C = 4($12.49)
C = $49.96
Before taxes, the border will cost $49.96.

15. a) Determine the perimeter, P, of the room minus the width of the doorway.
Since the room is square,
P = 4(18 ft. 4 in.) – 3 ft.
P = 72 ft. 16 in. – 3 ft.
P = 69 ft. 16 in.
Since 1 ft. = 12 in.,
16 in. = 1 ft. 4 in.
So, P = 70 ft. 4 in.
The perimeter of the room is greater than 70 ft., so the contractor must buy 71 ft. of trim.

The cost, C, is:
C = 71($1.69)
C = $119.99
So, the cost of the trim for the room, before taxes, is $119.99.

b) Determine the perimeter, P, of the window.
P = 2(40 in. + 26 in.)
P = 2(66 in.)
P = 132 in.
Since the trim is measured in feet, convert 132 in. to feet.
12 in. = 1 ft., so to convert inches to feet, divide by 12.
132 in. = $\frac{132}{12}$ ft.
132 in. = 11 ft.

The cost, C, is:
C = 11($1.69)
C = $18.59
The cost of the trim for the window, before taxes, is $18.59.

16. The puzzle scale is 1 in. represents 360 in.
$35\frac{2}{5}$ in. represents $35\frac{2}{5}$(360 in.) = 12 744 in.
1 ft. = 12 in.
Divide by 12 to convert 12 744 in. to feet:

$\dfrac{12\ 744}{12} = 1062$
So, the height of the Eiffel Tower is 1062 ft.

17. The map scale is 1 in. represents 1 500 000.
$2\frac{5}{8}$ represents $2\frac{5}{8}(1\ 500\ 000 \text{ in.}) = 3\ 937\ 500$ in.
1 ft. = 12 in.
Divide by 12 to convert 3 937 500 in. to feet:
$\dfrac{3\ 937\ 500}{12} = 328\ 125$
1 mi. = 5280 ft.
Divide by 5280 to convert 328 125 ft. to miles.
$\dfrac{328\ 125}{5280} = 62.144\ldots$
The distance between Trois-Rivières and Québec City is approximately 62 mi.

18. Convert 18 ft. to inches.
1 ft. = 12 in.
18 ft. = 18(12 in.)
18 ft. = 216 in.
Since Erica is planting tulips 8 in. apart, divide 216 in. by 8.
$\dfrac{216 \text{ in.}}{8 \text{ in.}} = 27$
Erica begins and ends the row with tulip bulbs; so, she will need 27 + 1, or 28 tulip bulbs.

C

19. 1 h = 60 min = 3600 s
So, in 1 h, the student could walk $\dfrac{3600 \text{ s}}{10 \text{ s}}(30 \text{ ft.}) = 10\ 800$ ft.
Convert 10 800 ft. to miles and yards.
5280 ft. = 1 mi.
$10\ 800 \text{ ft.} = \dfrac{10\ 800}{5280} \text{ mi.}$
$10\ 800 \text{ ft.} = 2\dfrac{240}{5280} \text{ mi.}$
10 800 ft. = 2 mi. 240 ft.

Convert 240 ft. to yards.
3 ft. = 1 yd.
$240 \text{ ft.} = \dfrac{240}{3} \text{ yd.}$
240 ft. = 80 yd.

So, the student could walk 2 mi. 80 yd. in 1 h.

20. Convert 95 mi. to inches.
1 mi. = 5280 ft. and 1 ft. = 12 in.
So, 95 mi. = 95(5280)(12) in.
95 mi. = 6 019 200 in.

To determine the map scale, divide 6 019 200 in. by $2\frac{9}{16}$ in., or $\frac{41}{16}$ in.

So, $\dfrac{6\,019\,200}{\frac{41}{16}} = 2\,348\,956.098\ldots$

The map scale, to the nearest thousand, is 1:2 349 000.

21. a) Since 20 reams of paper form a stack 40 in. high, then the height of 1 ream is:
$\dfrac{40 \text{ in.}}{20} = 2$ in.
Convert the height of Mount Logan to inches.
1 ft. = 12 in.
19 500 ft. = 19 500(12 in.)
19 500 ft. = 234 000 in.

Each ream is 2 in. high, so the height of Mount Logan is:
$\dfrac{234\,000}{2} = 117\,000$ reams
Each ream costs \$3, so the value of the stack is: \$3(117 000) = \$351 000
The value of the stack is \$351 000.

b) I estimate by rounding the height of Mount Logan to 20 000 ft.
Then, I use mental math to convert the height to inches:
20 000 ft. = 20 000(12 in.)
20 000 ft. = 240 000 in.
Each ream is 2 in. high, so I use mental math again to determine the approximate number of reams in the height:
$\dfrac{240\,000}{2} = 120\,000$
Since each ream is \$3, I use mental math to determine the approximate cost:
\$3(120 000) = \$360 000
Since \$351 000 is close to \$360 000, the answer is reasonable.

22. Convert 100 mi. to inches.
1 mi. = 5280 ft. and 1 ft. = 12 in.
100 mi. = 100(5280)(12) in.
100 mi. = 6 336 000 in.

Think: How many stacks of 5 toonies would fit in this distance?
Divide by $\frac{2}{5}$. This is the same as multiplying by $\frac{5}{2}$.

$6\,336\,000\left(\dfrac{5}{2}\right) = 15\,840\,000$

Each stack is 5 toonies, or 5($2) = $10.
So, the value of the stack is: $10(15 840 000) = $158 400 000

Lesson 1.2 **Math Lab: Measuring Length and Distance** **Assess Your Understanding (page 15)**

1. Referents may vary. For example:
a) The height of a door knob from the ground

b) The width of a finger

c) The thickness of a pencil lead

2. Referents may vary. For example: The distance walked in 15 min.

3. Answers may vary. For example: Calipers cannot be used for large measures, or to measure irregular objects accurately.

4. Referents may vary.
a) I would walk around the can placing my feet heel to toe, to get an estimate in feet.
I would wrap a piece of string around the garbage can, then measure the length of the string.

b) I would use the width of a finger to estimate the thickness of my hand, in centimetres.
I would use a caliper to measure the thickness on my hand.

c) I would use a pedometer to count the number of steps. Since each step is about 3 ft., I'd multiply the number of steps by 3 to get an estimate in feet.
I would measure the distance walked in 10 s, then see how long it takes me to walk from home to the store. Then I'd divide the number of seconds by 10 and multiply by the distance I can walk in 10 s, to get the distance from home to the store.

d) I would use the width of a finger to estimate the distance on the map, in centimetres;
I would use a ruler to measure the distance between the two cities on the map, then I could use the map scale to calculate the distance.

e) I would measure the distance walked in 10 s, then see how long it takes me to walk around the track. Then I'd divide the number of seconds by 10 and multiply by the distance I can walk in 10 s, to get the distance around the track.

f) I would use the thickness of a pencil lead as a referent to estimate the length in millimetres.
I would use a ruler to measure the length in millimetres.

5. To estimate the length in SI units, I know that one of my strides is about 1 m long. I would stride across the bridge and count my strides to get an estimate in metres.
To estimate the length in imperial units, I know that one of my steps is about 1 yd. long. I would walk across the bridge and count my steps to get an estimate in yards.
To measure the length in metres, I would ask my parents to drive across the bridge and use the car odometer to measure the distance in decimals of a kilometre.
To measure the length in yards, I would use a trundle wheel that measures in imperial units.

Lesson 1.3 Relating SI and Imperial Units Exercises (pages 22–23)

A

Answers will vary, depending on the conversion ratios used.

4. a) 1 in. = 2.54 cm
So, 16 in. = 16(2.54 cm)
16 in. = 40.64 cm
16 in. $\doteq$ 40.6 cm

b) 1 ft. $\doteq$ 0.3 m
So, 4 ft. $\doteq$ 4(0.3 m)
4 ft. $\doteq$ 1.2 m

c) 1 yd. $\doteq$ 0.9 m
So, 5 yd. $\doteq$ 5(0.9 m)
5 yd. $\doteq$ 4.5 m

d) 1 yd. $\doteq$ 0.9 m
So, 1650 yd. $\doteq$ 1650(0.9 m)
1650 yd. $\doteq$ 1485 m   Divide by 1000 to convert metres to kilometres.
1650 yd. $\doteq \dfrac{1485 \text{ km}}{1000}$
1650 yd. $\doteq$ 1.485 km
1650 yd. $\doteq$ 1.5 km

e) Use exact conversions.
1 yd. = 91.44 cm
So, 1 yd. = 0.9144 m   Divide by 1000 to convert metres to kilometres.
1 yd. = $\dfrac{0.9144}{1000}$ km
1 yd. = 0.000 914 4 km

1 mi. = 1760 yd.
So, 6 mi. = 6(1760 yd.)
6 mi. = 10 560 yd.
From above, 1 yd. = 0.000 914 4 km
So, 6 mi. = 10 560(0.000 914 4 km)
6 mi. $\doteq$ 9.656 km
6 mi. $\doteq$ 9.7 km

f) 1 in. = 2.54 cm
1 cm = 10 mm; so, 1 in. = 25.4 mm
2 in. = 2(25.4 mm)
2 in. = 50.8 mm

5. a) 25 mm = 2.5 cm
2.5 cm $\doteq$ 1 in.

b) $1 \text{ m} \doteq 3\frac{1}{4} \text{ ft.}$

So, $2.5 \text{ m} \doteq 2.5(3\frac{1}{4} \text{ ft.})$

$2.5 \text{ m} \doteq 8.125 \text{ ft.}$
$2.5 \text{ m} \doteq 8 \text{ ft.}$

c) $0.9 \text{ m} \doteq 1 \text{ yd.}$

So, $10 \text{ m} \doteq \frac{10}{0.9} \text{ yd.}$

$10 \text{ m} \doteq 11.\bar{1} \text{ yd.}$
$10 \text{ m} \doteq 11 \text{ yd.}$

d) $1 \text{ km} \doteq \frac{6}{10} \text{ mi.}$

So, $150 \text{ km} \doteq 150(\frac{6}{10} \text{ mi.})$

$150 \text{ km} \doteq 90 \text{ mi.}$

6. a) 1 ft. = 12 in.
So, 1 ft. 10 in. = 12 in. + 10 in.
1 ft. 10 in. = 22 in.

1 in. = 2.54 cm
So, 22 in. = 22(2.54 cm)
22 in. = 55.88 cm
22 in. $\doteq$ 55.9 cm

b) 1 yd. = 36 in.
So, 2 yd. = 2(36 in.)
2 yd. = 72 in.
And, 1 ft. = 12 in.
So, 2 ft. = 2(12 in.)
2 ft. = 24 in.
So, 2 yd. 2 ft. 5 in. = 72 in. + 24 in. + 5 in.
2 yd. 2 ft. 5 in. = 101 in.

1 in. = 2.54 cm
So, 101 in. = 101(2.54 cm)
101 in. = 256.54 cm
101 in. $\doteq$ 256.5 cm

c) 1 yd. = 36 in.
So, 10 yd. = 10(36 in.)
10 yd. = 360 in.
And, 1 ft. = 12 in.
So, 10 yd. 1 ft. 7 in. = 360 in. + 12 in. + 7 in.
10 yd. 1 ft. 7 in. = 379 in.

1 in. = 2.54 cm
1 cm = 0.01 m; so, 1 in. = 0.0254 m
379 in. = 379(0.0254 m)
379 in. = 9.6266 m
379 in. $\doteq$ 9.6 m

B

7. a) i) 2.54 cm = 1 in.

$75 \text{ cm} = \dfrac{75}{2.54} \text{ in.}$

75 cm = 29.527… in.
75 cm $\doteq$ 30 in.

12 in. = 1 ft.

So, $30 \text{ in.} = \dfrac{30}{12} \text{ ft.}$

$30 \text{ in.} = 2\dfrac{6}{12} \text{ ft.}$

So, 75 cm $\doteq$ 2 ft. 6 in.

ii) 91.44 cm = 1 yd.

So, $274 \text{ cm} = \dfrac{274}{91.44} \text{ yd.}$

274 cm = 2.996… yd.
274 cm $\doteq$ 3 yd.

iii) 10 000 m = 10 km

$1 \text{ km} \doteq \dfrac{6}{10} \text{ mi.}$

So, $10 \text{ km} \doteq 10(\dfrac{6}{10} \text{ mi.})$

10 km $\doteq$ 6 mi.

b) Answers may vary.
i) To check:
1 ft. $\doteq$ 30 cm
2 ft. $\doteq$ 60 cm
3 ft. $\doteq$ 90 cm
Since 75 cm is halfway between 60 cm and 90 cm, then 75 cm is halfway between 2 ft. and 3 ft., which is 2 ft. 6 in.
So, the answer is reasonable.

ii) To check:
274 cm = 2.74 m
Since 1 m $\doteq$ 1 yd.,
then 2.74 m $\doteq$ 2.74 yd.
So, the answer is reasonable.

iii) To check:
1 mi. $\doteq$ 1.6 km
Since 5 mi. = (5 × 1.6 km)
= 9 km
then 6 mi. is a bit more than 9 km.
So, the answer is reasonable.

8. Convert 110 yd. and 60 yd. to metres.
1 yd. = 91.44 cm, or 0.9144 m
So, 110 yd. = 110(0.9144 m)
110 yd. = 100.584 m
To the nearest tenth of a metre, 110 yd. $\doteq$ 100.6 m
Then, 60 yd. = 60(0.9144 m)
60 yd. = 54.864 m
To the nearest tenth of a metre, 60 yd. $\doteq$ 54.9 m
The dimensions of a lacrosse field are approximately 100.6 m by 54.9 m.

9. To compare distances, convert one measurement so the units are the same.
1 mi. $\doteq$ 1.6 km
So, 886 mi. $\doteq$ 886(1.6 km)
886 mi. $\doteq$ 1417.6 km
Since 1417.6 km > 1375 km, the Tennessee River is longer than the Fraser River.

10. Convert 87 mi. to kilometres.
1 mi. $\doteq$ 1.6 km
So, 87 mi. $\doteq$ 87(1.6) km
87 mi. $\doteq$ 139.2 km
139.2 km is close to 142 km, so the odometer is probably accurate.

11. a) Convert one measurement so the units are the same.
Use the conversion: 1 yd. = 91.44 cm, or 0.9144 m.

$$\frac{\$0.89}{1\text{ yd.}} = \frac{\$0.89}{0.9144\text{ m}}$$ Divide numerator and denominator by 0.9144.

$$\frac{\$0.89}{1\text{ yd.}} \doteq \frac{\$0.9733...}{1\text{ m}}$$

$$\frac{\$0.89}{1\text{ yd.}} \doteq \$0.97/\text{m}$$

Since \$0.93/m < \$0.97/m, the warehouse has the better price.

b) 1 yd. is approximately 90 cm, or 0.9 m.
\$0.89/m is close to \$0.90/m.

So, the cost per metre is approximately: $\dfrac{\$0.90}{0.90\text{ m}} = \$1/\text{m}$

\$1/m is close to \$0.97/m; so, my answer is reasonable.

12. a) To compare distances, convert one measurement so the units are the same.
Convert the distance Jean-Luc ran to metres.
Use the conversion: 1 yd. = 0.9144 m
So, 400 yd. = 400(0.9144 m)
400 yd. = 365.76 m
Jean-Luc ran two laps, or: 2(365.76 m) = 731.52 m
Michael ran: 7(110 m) = 770 m
Since 770 m > 731.52 m, Michael ran farther.

b) Write a conversion factor for yards and metres, with metres in the numerator: $\dfrac{0.9144\text{ m}}{1\text{ yd.}}$

So, $400\text{ yd.} \times \dfrac{0.9144\text{ m}}{1\text{ yd.}} = \dfrac{400\ \cancel{\text{yd.}}}{1} \times \dfrac{0.9144\text{ m}}{1\ \cancel{\text{yd.}}}$

= 400(0.9144 m)
= 365.76 m
Since this measurement is equal to the measurement in part a, the conversion is verified.

13. a) To determine the height of the CN Tower in feet, convert metres to inches, then convert inches to feet.

1 in. = 2.54 cm, or 0.0254 m
So, to convert 553.3 m to inches, divide:

$553.3\text{ m} = \dfrac{553.3}{0.0254}\text{ in.}$

553.3 m = 21 783.464… in.

Then, convert inches to feet. Use the conversion: 12 in. = 1 ft.
So, to convert 21 783.464… in. to feet, divide:

$21\,783.464\ldots\text{ in.} = \dfrac{21\,783.464\ldots}{12}\text{ ft.}$

21 783.464… in. ≐ 1815.288… ft.
21 783.464… in. ≐ 1815 ft.

To determine the height of the Willis Tower in metres, convert feet to inches, then inches to metres.
1 ft. = 12 in.
1451 ft. = 12(1451) in.
1451 ft. = 17 412 in.

Since 1 in. = 2.54 cm, 1 in. = 0.0254 m
So, 17 412 in. = 17 412(0.0254 m)
17 412 in. = 442.264… m
17 412 in. ≐ 442.3 m

b) Compare the heights in the same units.
Since 553.3 m > 442.3 m, the CN Tower is taller.
Or, since 1815 ft. > 1451 ft., the CN Tower is taller.

c) To determine the difference in heights, subtract.
553.3 m – 442.3 m = 111 m
1815 ft. – 1451 ft. = 364 ft.
So, the difference in heights is approximately 111 m, or 364 ft.

14. Determine the length of a section of casing in metres.
To convert feet to metres, first convert feet to inches, then convert inches to metres.
Use the conversion: 1 ft. = 12 in.
So, 32 ft. = 32(12 in.)
32 ft. = 384 in.
Then, use the conversion: 1 in. = 2.54 cm
So, 384 in. = 384(2.54 cm)
384 in. = 975.36 cm   Divide by 100 to convert centimetres to metres.
$$384 \text{ in.} = \frac{975.36}{100} \text{ m}$$
384 in. = 9.7536 m

To determine how many sections of casings are needed, divide the distance to the oil reserve by the length of one casing.
$$\frac{1400 \text{ m}}{9.7536 \text{ m}} = 143.5367\ldots$$
So, 144 sections of casing are needed.

C

15. Determine the height of the basketball net in metres.
Convert feet to inches, then inches to metres.
Use the conversions: 1 ft. = 12 in. and 1 in. = 2.54 cm
So, 10 ft. = 10(12)(2.54 cm)
10 ft. = 304.8 cm   Divide by 100 to convert centimetres to metres.
$$10 \text{ ft.} = \frac{304.8 \text{ cm}}{100}$$
10 ft. = 3.048 m

Since the player has a maximum reach of 2.5 m, he has to jump 3.048 m – 2.5 m = 0.548 m to reach the rim.
To reach 6 in. above the rim, the player has to jump an extra 6 in.
Convert 0.548 m to inches.
Use the conversion: 1 in. = 2.54 cm, or 0.0254 m
So, to convert 0.548 m to inches, divide:
$$0.548 \text{ m} = \frac{0.548}{0.0254} \text{ in.}$$
0.548 m = 21.5748… in.
So, the player has to jump: 21.5748… + 6 in. = 27.5748…
The player has to jump approximately 28 in. to reach 6 in. above the rim.

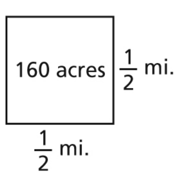
16. The electrician purchased 2(4 m) = 8 m of wire.
The total length needed is: 2(2 ft.) + 2(11 ft.) = 4 ft. + 22 ft.
= 26 ft.
Convert the length of wire needed to metres.
To convert feet to metres, first convert feet to inches, then convert inches to metres.
Use the conversion: 1 ft. = 12 in. and 1 in. = 2.54 cm
So, 26 ft. = 26(12)(2.54 cm)
26 ft. = 792.48 cm
Divide by 100 to convert centimetres to metres.
26 ft. = $\frac{792.48}{100}$ m
26 ft. = 7.9248 m

Since 8 m > 7.9248 m, the electrician will have enough wire.
To determine the amount of wire left over, subtract:
8 m – 7.9248 m = 0.0752 m, or 7.52 cm
So, the electrician will have approximately 8 cm of wire left over.

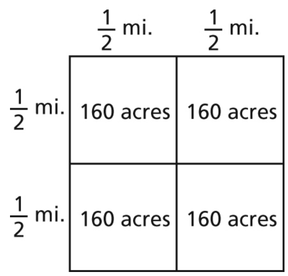
17. Each mobile home requires a width of: 14 ft. + 8 ft. = 22 ft.
Think: How many mobile homes would fit in 50 m?
Convert 50 m to feet. Use the conversion: 1 m $\doteq$ $3\frac{1}{4}$ ft.
So, 50 m $\doteq$ $50(3\frac{1}{4}$ ft.)
50 m $\doteq$ 162.5 ft.
Divide to determine how many mobile homes would fit in 162.5 ft.
$\frac{162.5}{22}$ = 7.386…
So, the maximum number of homes the developer can fit on the land is 7.

18. a) Use the conversion: 1 hectare $\doteq$ 2.471 acres
To convert 160 acres to hectares, divide:
160 acres $\doteq$ $\frac{160}{2.471}$ hectares
160 acres $\doteq$ 64.751… hectares
Each settler received approximately 65 hectares.

b) Sketch a square with side length $\frac{1}{2}$ mi.



Sketch one square mile:



So, one square mile has 4(160 acres) = 640 acres.
Convert 640 acres to hectares. Use the conversion: 1 hectare $\doteq$ 2.471 acres
To convert 640 acres to hectares, divide.

$$640 \text{ acres} \doteq \frac{640}{2.471} \text{ hectares}$$

$$640 \text{ acres} \doteq 259.004\ldots \text{ hectares}$$

So, there are approximately 259 hectares in one square mile.

Checkpoint 1 Assess Your Understanding (page 25)

1.1

1. Answers will vary.

a) Inch; I used my eraser, which is approximately 2 in. long. The distance from my elbow to my wrist is about 5 erasers, or about 10 in.

b) Inch; I used the length of my thumb to the first knuckle, which is approximately 1 in. The length of a cell phone is about 4 thumb lengths, or about 4 in.

c) Foot; I used the length of my slipper, which is approximately 1 ft. long. The width of the bed is about $3\frac{1}{2}$ slippers, or about 3 ft. 6 in.

d) Foot; I used the height of the hockey net as a referent, which is about 3 ft. I estimated that the gymnasium is about 6 hockey nets high, which is about 18 ft.

2. Answers will vary.

a) $9\frac{3}{16}$ in.

b) $4\frac{1}{16}$ in.

3. a) 3 ft. = 1 yd.
To convert feet to yards, divide:

$80 \text{ ft.} = \frac{80}{3} \text{ yd.}$ Write this improper fraction as a mixed number.

$80 \text{ ft.} = 26\frac{2}{3} \text{ yd.}$

80 ft. = 26 yd. 2 ft.

b) 1 mi. = 1760 yd.
So, 3 mi. = 3(1760 yd.)
3 mi. = 5280 yd.

c) First, convert yards to inches. Use the conversion: 1 yd. = 36 in.
So, 2 yd. = 2(36 in.)
2 yd. = 72 in.

Then, convert feet to inches. Use the conversion: 1 ft. = 12 in.
So, 1 ft. = 12 in.

Add. 2 yd. 1 ft. = 72 in. + 12 in.
2 yd. 1 ft. = 84 in.

4. To compare the heights, convert one measurement so the units are the same.
Convert 3 ft. 11 in. to inches.
Use the conversion: 1 ft. = 12 in.

So, 3 ft. = 3(12 in.)
3 ft. = 36 in.
So, 3 ft. 11 in. = 36 in. + 11 in.
3 ft. 11 in. = 47 in.
Since 47 in. < 51 in., Sidney is shorter.

1.2

5. Answers and measurements will vary. For example:

a) 25.4 m; 27 yd. 2 ft. 4 in.

b) 143 cm; 4 ft. 8 in.

c) 77 cm; 30 in.

6. I would use calipers. I would use the unit marked on the calipers.

1.3

7. Answers will vary depending on the conversion ratios used.

a) 1 yd. = 91.44 cm, or 0.9144 m
To convert metres to yards, divide:
$13 \text{ m} = \dfrac{13}{0.9144} \text{ yd.}$
13 m = 14.2169… yd.
13 m ≐ 14.2 yd.

1 yd. = 3 ft.
So, 0.2 yd. = 0.2(3 ft.)
0.2 yd. = 0.6 ft.
So, to the nearest foot, 13 m = 14 yd. 1 ft.

b) 1 ft. = 12 in. and 1 in. = 2.54 cm
4 ft. = 4(12 in.)
4 ft. = 48 in.
48 in. = 48(2.54 cm)
So, 48 in. = 121.92 cm
4 ft. ≐ 122 cm

c) 2 km = 2000 m
Use the conversion: 1 yd. = 91.44 cm, or 0.9144 m
So, $2000 \text{ m} = \dfrac{2000}{0.9144} \text{ yd.}$
2000 m = 2187.226… yd.
1 mi. = 1760 yd.
Subtract: 2187.226… yd. – 1760 yd. = 427.226… yd.
So, 2 km ≐ 1 mi. 427 yd.

d) 91.44 cm = 1 yd.
So, $25\,000 \text{ cm} = \dfrac{25\,000}{91.44} \text{ yd.}$

25 000 cm = 273.403… yd.

1 yd. = 3 ft.
So, 0.403… yd. = (0.403…)(3) ft.
0.403… yd. = 1.209… ft.

1 ft. = 12 in.
0.209… ft. = (0.209…)(12) in.
0.209… ft. = 2.519… in.

So, 25 000 cm = 273 yd. 1 ft. 3 in.

e) 1 in. = 2.54 cm, or 0.0254 m
So, 13 000 in. = (13 000)(0.0254 m)
13 000 in. = (13 000)(0.0254 m)
13 000 in. = 330.2 m

f) 1750 mm = 175 cm
2.54 cm = 1 in.
So, 175 cm = $\frac{175}{2.54}$ in.
175 cm = 68.897… in.

12 in. = 1 ft.
So, 68.897… in. = $\frac{68.897\ldots}{12}$ ft.
68.897… in. = 5.741… ft.
0.741… ft. = 12(0.741…) in.
0.741… ft. = 0.889… ft.
So, 1750 mm ≐ 5 ft. 9 in.

8. The perimeter of the table is:
$P = 2(30\text{ cm}) + 2(115\text{ cm})$
$P = 60\text{ cm} + 230\text{ cm}$
$P = 290\text{ cm}$

2.54 cm = 1 in.
So, 290 cm = $\frac{290}{2.54}$ in.
290 cm = 114.173… in.

1 ft. = 12 in.
114.173… in. = $\frac{114.173\ldots}{12}$ ft.
114.173… in. = 9.514… ft.

0.514… ft. = 12(0.514…) in.
0.514… ft. = 6.168… in.

So, 290 cm $\doteq$ 9 ft. 6 in.
10 ft. of laminate is needed.

Lesson 1.4 **Surface Areas of Right Pyramids and Right Cones** **Exercises (pages 34–35)**

A

4. a) Use the formula for the lateral area, A_L, of a right pyramid with a regular polygon base and slant height, s:

$A_L = \frac{1}{2}s(\text{perimeter of base})$

When the base is a square with side length l:

$A_L = \frac{1}{2}s(4l)$ Substitute: $s = 11$, $l = 6$

$A_L = \frac{1}{2}(11)(4)(6)$

$A_L = (5.5)(24)$

$A_L = 132$

The lateral area of the square pyramid is 132 square inches.

b) Use the formula for the lateral area, A_L, of a right pyramid with a regular polygon base and slant height, s:

$A_L = \frac{1}{2}s(\text{perimeter of base})$

When the base is an equilateral triangle with side length l,

$A_L = \frac{1}{2}s(3l)$ Substitute: $s = 11.3$, $l = 13$

$A_L = \frac{1}{2}(11.3)(3)(13)$

$A_L = (5.65)(39)$

$A_L = 220.35$

The lateral area of the regular tetrahedron is approximately 220 cm^2.

5. a) Use the formula for surface area of a right pyramid with a regular polygon base and slant height s:

$\text{Surface area} = \frac{1}{2}s(\text{perimeter of base}) + (\text{base area})$

When the base is a square with side length l,

$\text{Surface area} = \frac{1}{2}s(4l) + l^2$ Substitute: $s = 11$, $l = 6$

$\text{Surface area} = \frac{1}{2}(11)(4)(6) + (6)^2$

Surface area = 132 + 36

Surface area = 168

The surface area of the square pyramid is 168 square inches.

b) Use the formula for surface area of a right pyramid with a regular polygon base and slant height s:

$\text{Surface area} = \frac{1}{2}s(\text{perimeter of base}) + (\text{base area})$

When the base is an equilateral triangle with side length l, Surface area = $\frac{1}{2}s(3l) + \frac{1}{2}s(l)$ Substitute: $s = 11.3$, $l = 13.0$

Surface area = $\frac{1}{2}(11.3)(3)(13.0) + \frac{1}{2}(13)(11.3)$

Surface area = 220.35 + 73.45

Surface area = 293.8

The surface area of the regular tetrahedron is approximately 294 cm^2.

6. a) Use the formula for the lateral area, A_L, of a right cone with base radius r and slant height s.

$A_L = \pi rs$ Substitute: $r = 4$, $s = 8$

$A_L = \pi(4)(8)$

$A_L = 100.5309\ldots$

The lateral area of the right cone is approximately 101 square inches.

b) Use the formula for the lateral area, A_L, of a right cone with base radius r and slant height s.

$A_L = \pi rs$ Substitute: $r = 15$, $s = 35$

$A_L = \pi(15)(35)$

$A_L = 1649.3361\ldots$

The lateral area of the right cone is approximately 1649 cm^2.

7. a) Use the formula for the surface area of a right cone with base radius r and slant height s.

Surface area = $\pi rs + \pi r^2$ Substitute: $r = 4$, $s = 8$

Surface area = $\pi(4)(8) + \pi(4)^2$

Surface area = $150.7964\ldots$

The surface area of the right cone is approximately 151 square inches.

b) Use the formula for the surface area of a right cone with base radius r and slant height s.

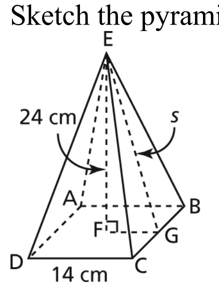
Surface area = $\pi rs + \pi r^2$ Substitute: $r = 15$, $s = 35$

Surface area = $\pi(15)(35) + \pi(15)^2$

Surface area = $2356.1944\ldots$

The surface area of the right cone is approximately 2356 cm^2.

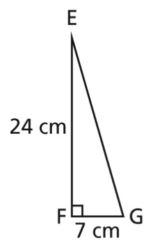
8. a) Sketch the pyramid and label its vertices.



Determine the slant height, s.

In ΔEFG, FG is $\frac{1}{2}$ the length of DC, so FG is 7 cm.

EF is the height of the pyramid, which is 24 cm.



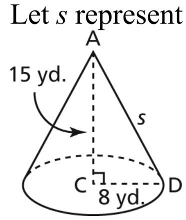
Use the Pythagorean Theorem in right ΔEFG.
$EG^2 = EF^2 + FG^2$ Substitute: EF = 24, FG = 7
$EG^2 = 24^2 + 7^2$
$EG^2 = 625$
$EG = \sqrt{625}$
$EG = 25$
So, the slant height, s, is 25 cm.
Use the formula for the surface area of a right square pyramid with base side length l and slant height s.

Surface area = $\frac{1}{2}s(4l) + l^2$ Substitute: $s = 25$, $l = 14$

Surface area = $\frac{1}{2}(25)(4)(14) + (14)^2$

Surface area = 700 + 196
Surface area = 896
The surface area of the right square pyramid is 896 cm^2.

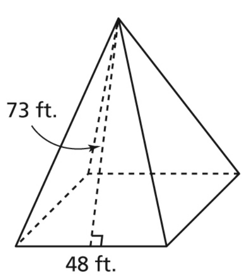
b) Let s represent the slant height. Label the cone.



Use the Pythagorean Theorem in right ΔACD.
$AD^2 = AC^2 + CD^2$
$s^2 = 15^2 + 8^2$
$s^2 = 289$
$s = \sqrt{289}$
$s = 17$

Use the formula for the surface area of a right cone with base radius r and slant height s.
Surface area = $\pi rs + \pi r^2$ Substitute: $r = 8$, $s = 17$
Surface area = $\pi(8)(17) + \pi(8)^2$
Surface area = 628.3185…
The surface area of the right cone is approximately 628 square yards.

B
9. a)



b) Use the formula for the lateral area, A_L, of a right square pyramid with base side length l and slant height s.

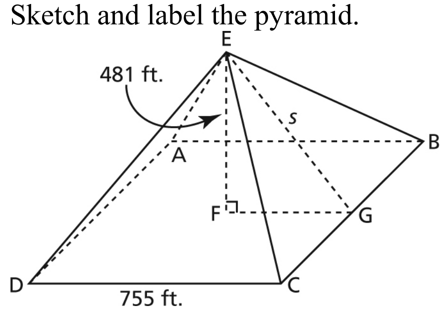
$A_L = \frac{1}{2}s(4l)$ Substitute: $s = 73$, $l = 48$

$A_L = \frac{1}{2}(73)(4)(48)$

$A_L = 7008$

The lateral area of the pyramid is 7008 square feet.

10. Sketch and label the pyramid.



Determine the slant height, s. Use the Pythagorean Theorem in right ΔEFG.

FG is $\frac{1}{2}$ the length of DC. So, FG is $\frac{1}{2}$(755 ft.), or $377\frac{1}{2}$ ft.

$EG^2 = EF^2 + FG^2$

$s^2 = 481^2 + 377.5^2$

$s^2 = 373\ 867.25$

$s = \sqrt{373\ 867.25}$

$s = 611.4468\ldots$

Use the formula for surface area. Do not include the base, as it is not exposed.

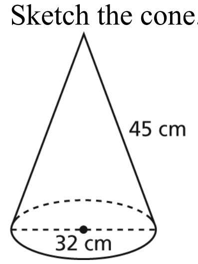
Surface area $= \frac{1}{2}s$(perimeter of base)

Surface area $= \frac{1}{2}(611.4468\ldots)(4)(755)$

Surface area $= 923\ 284.7430\ldots$

The original surface area of the Great Pyramid at Giza was approximately 923 285 square feet.

11. Sketch the cone.



a) Use the formula for the lateral area, A_L, of a right cone with base radius r and slant height s:

$A_L = \pi rs$ Substitute: $r = 16$, $s = 45$

$A_L = \pi(16)(45)$

$A_L = 2261.9467\ldots$

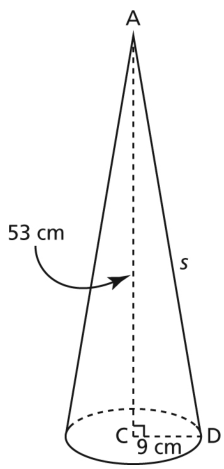
The lateral area of the volcano is approximately 2261.9 cm^2.

b) To determine the number of jars needed, divide the total area by the area covered by one jar:

$$\frac{2261.9\ \text{cm}^2}{400\ \text{cm}^2} \doteq 5.7$$

Since Aiden cannot buy part of a jar of paint, 6 jars are needed.
The total cost is: 6($1.99), or $11.94
The paint will cost $11.94.

12. Sketch and label a diagram. Let s represent the slant height of the cone.



The radius of the cone is 9 cm.

Use the Pythagorean Theorem in right ΔACD to determine the slant height.

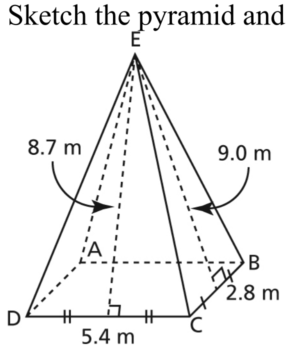
$AD^2 = AC^2 + CD^2$
$s^2 = 53^2 + 9^2$
$s^2 = 2890$
$s = \sqrt{2890}$

Use the formula for the lateral area, A_L, of a right cone with base radius r and slant height s:

$A_L = \pi rs$ Substitute: $r = 9$, $s = \sqrt{2890}$
$A_L = \pi(9)(\sqrt{2890})$
$A_L = 1519.9920\ldots$

The area to be painted is approximately 1520 cm^2.

13. a) Sketch the pyramid and label its vertices.



Area, A, of ΔEDC is:

$A = \frac{1}{2}(5.4)(8.7)$
$A = 23.49$

Since ΔEDC and ΔEAB are congruent, the area of ΔEAB is 23.49 m^2.

Area, A, of ΔEBC is:

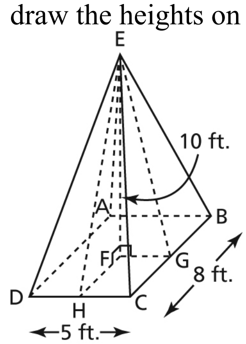
$A = \frac{1}{2}(2.8)(9.0)$
$A = 12.6$

Since ΔEBC and ΔEAD are congruent, the area of ΔEAD is 12.6 m^2.

Area, B, of the base of the pyramid is:
$B = (2.8)(5.4)$
$B = 15.12$

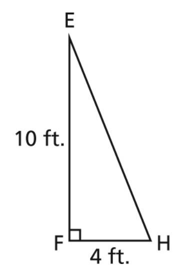
The surface area, SA, of the pyramid is:
$SA = 2(23.49) + 2(12.6) + 15.12$
$SA = 87.3$
The surface area of the right rectangular pyramid is approximately 87 m^2.

b) Sketch the pyramid and label its vertices. Since oposite triangular faces are congruent, draw the heights on two adjacent triangles. Determine these heights.



In ΔEFH, FH is $\frac{1}{2}$ the length of BC, so FH is 4 ft.

Use the Pythagorean Theorem in right ΔEFH.
$EH^2 = EF^2 + FH^2$
$EH^2 = 10^2 + 4^2$
$EH^2 = 116$
$EH = \sqrt{116}$

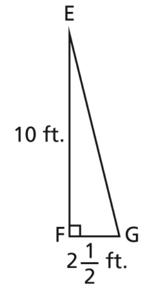


Area, A, of ΔEDC is:
$A = \frac{1}{2}(5)(\sqrt{116})$
$A = 2.5(\sqrt{116})$

Since ΔEDC and ΔEAB are congruent, the area of ΔEAB is $2.5(\sqrt{116})$.

In ΔEFG, FG is $\frac{1}{2}$ the length of DC, so FG is $2\frac{1}{2}$ ft.

Use the Pythagorean Theorem in right ΔEFG.
$GE^2 = EF^2 + FG^2$
$GE^2 = 10^2 + 2.5^2$
$GE^2 = 106.25$
$GE = \sqrt{106.25}$



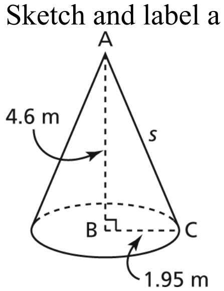
Area, A, of ΔEBC is:
$A = \frac{1}{2}(8)(\sqrt{106.25})$
$A = 4(\sqrt{106.25})$

Since ΔEBC and ΔEAD are congruent, the area of ΔEAD is $4(\sqrt{106.25})$.

Area, B, of the base of the pyramid is:
$B = (8)(5)$
$B = 40$

Surface area, SA, of the pyramid is:
$SA = 2(2.5)(\sqrt{116}) + 2(4)(\sqrt{106.25}) + 40$
$SA = 176.3137\ldots$
The surface area of the right rectangular pyramid is approximately 176 square feet.

14. The tipi approximates a right cone with radius 1.95 m.
Sketch and label a diagram. Let s represent the slant height.



Use the Pythagorean Theorem in right ΔABC.
$AC^2 = AB^2 + BC^2$
$s^2 = 4.6^2 + 1.95^2$
$s^2 = 24.9625$
$s = \sqrt{24.9625}$

Since the base is not covered, use the formula for the lateral area of a right cone with base radius r and slant height s.
$A_L = \pi rs$ Substitute: $r = 1.95$, $s = \sqrt{24.9625}$
$A_L = \pi(1.95)(\sqrt{24.9625})$
$A_L = 30.6075\ldots$

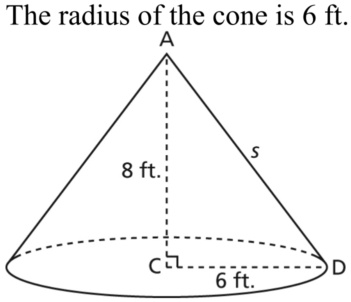
Assume the bison hides have equal areas. Then, Area of 1 hide = $\dfrac{\text{total area}}{\text{number of hides}}$

Area of 1 hide = $\dfrac{30.6075\ldots}{15}$

Area of 1 hide = 2.0405…

Each bison hide covers approximately 2.0 m^2.

15. Sketch and label a diagram. Let s represent the slant height.
The radius of the cone is 6 ft.



Use the Pythagorean Theorem in right ΔACD.
$AD^2 = AC^2 + CD^2$
$s^2 = 8^2 + 6^2$
$s^2 = 64 + 36$
$s^2 = 100$
$s = \sqrt{100}$
$s = 10$

Use the formula for the lateral area, A_L, of a right cone with base radius r and slant height s:
$A_L = \pi rs$ Substitute: $r = 6$, $s = 10$
$A_L = \pi(6)(10)$
$A_L = 188.4955\ldots$
The surface area of the exposed grain is approximately 188 square feet.

16. a) Use the formula for the surface area, SA, of a right cone with base radius r and slant height s.
The radius of the cone is 24 mm.

$SA = \pi rs + \pi r^2$ Substitute: $SA = 7012$ and $r = 24$
$7012 = \pi(24)s + \pi(24)^2$ Solve for s.
$7012 = 24\pi s + 576\pi$ Subtract 576π from each side.
$7012 - 576\pi = 24\pi s$ Divide each side by 24π.
$$\frac{7012 - 576\pi}{24\pi} = \frac{24\pi s}{24\pi}$$
$$s = \frac{7012 - 576\pi}{24\pi}$$
$s = 68.9995\ldots$
s is approximately 69.0 mm.

b) Use the formula for the surface area, SA, of a right square pyramid with base side length l and slant height s.

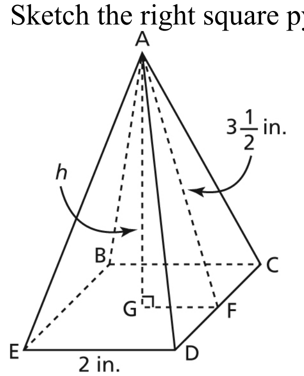
$$SA = \frac{1}{2}s(4l) + l^2 \qquad \text{Substitute: } SA = 65.5,\ l = 3.5$$

$$65.5 = \frac{1}{2}s(4)(3.5) + 3.5^2 \qquad \text{Solve for } s.$$

$$65.5 = 7s + 12.25$$
$$65.5 - 12.25 = 7s$$
$$53.25 = 7s$$
$$\frac{53.25}{7} = \frac{7s}{7}$$
$$s = \frac{53.25}{7}$$
$$s = 7.6071\ldots$$

s is approximately 7.6 m.

17. a) Determine the height, h, of each block.
Sketch the right square pyramid and label its vertices.



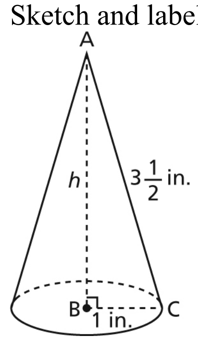
In ΔAGF, GF is $\frac{1}{2}$ the length of ED, so GF is 1 in.

Use the Pythagorean Theorem in right ΔAGF.

$AG^2 = AF^2 - GF^2$
$h^2 = 3.5^2 - 1^2$
$h^2 = 11.25$
$h = \sqrt{11.25}$
$h = 3.3541\ldots$

The height of the right square pyramid is approximately $3\frac{4}{10}$ in., or $3\frac{2}{5}$ in.

Sketch and label the right cone.



Use the Pythagorean Theorem in right ΔABC.
$AB^2 = AC^2 - BC^2$
$h^2 = 3.5^2 - 1^2$
$h^2 = 11.25$
$h = \sqrt{11.25}$
$h = 3.3541\ldots$
The height of the right cone is the same as the height of the right square pyramid, approximately $3\frac{4}{10}$ in., or $3\frac{2}{5}$ in.

Compare the heights.
The right square pyramid and right cone are the tallest blocks.

b) Determine the surface area of each block.
Use the formula for the surface area, *SA*, of a right square pyramid with base side length *l* and slant height *s*:

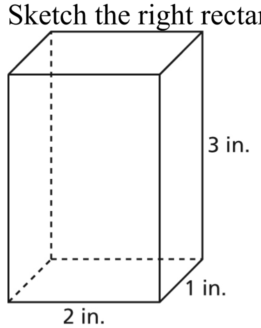
$SA = \frac{1}{2}s(4l) + l^2$ Substitute: $s = 3.5$, $l = 2$

$SA = \frac{1}{2}(3.5)(4)(2) + 2^2$

$SA = 14 + 4$
$SA = 18$
The surface area of the right square pyramid is 18 square inches.

Use the formula for the surface area, *SA*, of a right cone with base radius *r* and slant height *s*:
$SA = \pi rs + \pi r^2$ Substitute: $r = 1$, $s = 3.5$
$SA = \pi(1)(3.5) + \pi(1)^2$
$SA = 14.1371\ldots$
The surface area of the right cone is approximately 14 square inches.

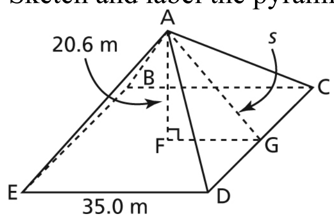
Sketch the right rectangular prism:



Its surface area, *SA*, is:
$SA = 2 \times$ area of top face $+ 2 \times$ area of front face $+ 2 \times$ area of side face
$SA = 2(2 \times 1) + 2(2 \times 3) + 2(3 \times 1)$
$SA = 4 + 12 + 6$
$SA = 22$
The surface area of the right rectangular prism is 22 square inches.

Compare the surface areas. The right rectangular prism has the greatest surface area. So, it requires the most paint.

18. Determine the lateral area of each pyramid.
Sketch and label the pyramid at the Louvre.



In ΔAFG, FG is $\frac{1}{2}$ the length of ED, so FG is 17.5 m.

Use the Pythagorean Theorem in right ΔAFG.
$AG^2 = AF^2 + FG^2$
$s^2 = 20.6^2 + 17.5^2$
$s^2 = 730.61$
$s = \sqrt{730.61}$

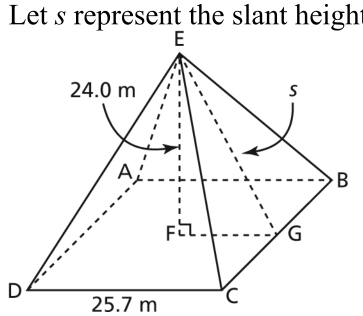
Use the formula for the lateral area, A_L, of a right square pyramid with base side length l and slant height s. The base is not included, because it is not a glass face.

$A_L = \frac{1}{2}s(4l)$ Substitute: $s = \sqrt{730.61}$, $l = 35.0$

$A_L = \frac{1}{2}(\sqrt{730.61})(4)(35.0)$

$A_L = 1892.0858\ldots$
The lateral area of the pyramid at the Louvre is approximately 1892 m^2.

Sketch the pyramid at the Muttart Conservatory.
Let s represent the slant height.



In ΔEFG, FG is $\frac{1}{2}$ the length of DC, so FG is 12.85 m.

Use the Pythagorean Theorem in right ΔEFG.
$s^2 = 24.0^2 + 12.85^2$
$s^2 = 741.1225$
$s = \sqrt{741.1225}$

Use the formula for the lateral area, A_L, of a right square pyramid with base side length l and slant height s.

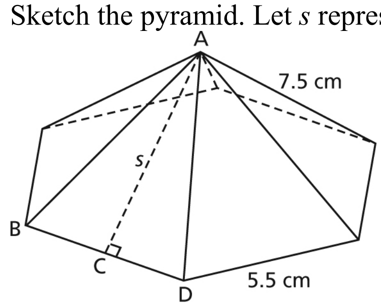
$A_L = \frac{1}{2}s(4l)$ Substitute: $s = \sqrt{741.1225}$, $l = 25.7$

$A_L = \frac{1}{2}(\sqrt{741.1225})(4)(25.7)$
$A_L = 1399.2912\ldots$
The lateral area of the pyramid at the Muttart Conservatory is approximately 1399 m^2.

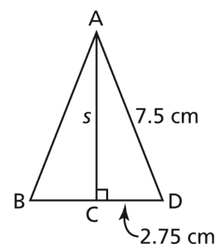
Compare the lateral areas. The pyramid at the Louvre requires more glass to enclose its space.

C

19. a) Sketch the pyramid. Let *s* represent the slant height.

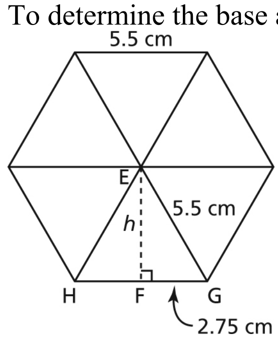


Sketch and label one triangular face.



In ΔACD, CD is $\frac{1}{2}$ the length of BD, so CD is 2.75 cm.
Use the Pythagorean Theorem in right ΔACD.
$AC^2 = AD^2 - CD^2$
$s^2 = 7.5^2 - 2.75^2$
$s^2 = 48.6875$
$s = \sqrt{48.6875}$

To determine the base area, divide the hexagon into 6 equilateral triangles.



In ΔEFG, FG is $\frac{1}{2}$ the length of HG, so FG is 2.75 cm.
Determine the area of one equilateral triangle. Use the Pythagorean Theorem in right ΔEFG to determine the height, *h*.
$EF^2 = EG^2 - FG^2$
$h^2 = 5.5^2 - 2.75^2$
$h^2 = 22.6875$
$h = \sqrt{22.6875}$

So, the area, *A*, of one triangle:
$A = \frac{1}{2}$(base)(height)

$A = \frac{1}{2}(5.5)(\sqrt{22.6875})$

Multiply the area of one triangle by 6 to get the total area:

$6 \times \frac{1}{2}(5.5)(\sqrt{22.6875}) = 3(5.5)(\sqrt{22.6875})$

Use the formula for the surface area, *SA*, of a right regular pyramid:

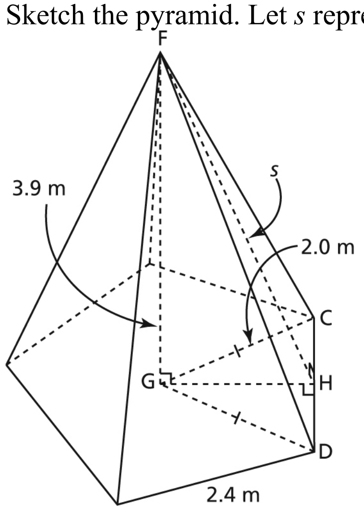
$SA = \frac{1}{2}s(\text{perimeter of base}) + (\text{base area})$

$SA = \frac{1}{2}(\sqrt{48.6875})(6)(5.5) + 3(5.5)(\sqrt{22.6875})$

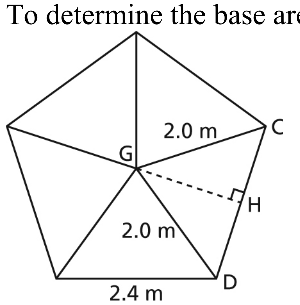
$SA = 193.7229\ldots$

The surface area of the right pyramid is approximately 193.7 cm^2.

b) Sketch the pyramid. Let *s* represent the slant height.



To determine the base area, divide the pentagon into 5 isosceles triangles.



To determine the area of one of these triangles, we need to know its height GH.

In ΔGHC, HC is $\frac{1}{2}$ the length of DC, so HC is 1.2 m.

Use the Pythagorean Theorem in ΔGHC.

$GH^2 = GC^2 - HC^2$

$GH^2 = 2.0^2 - 1.2^2$

$GH^2 = 2.56$

$GH = \sqrt{256}$

$GH = 1.6$

The area, A, of one triangle is:

$A = \frac{1}{2}$(base)(height)

$A = \frac{1}{2}(2.4)(1.6)$

$A = 1.92$

So, the area of the base is: $5(1.92) = 9.6$

To determine the slant height, use the Pythagorean Theorem in right ΔFGH in the diagram of the pyramid.

$FH^2 = FG^2 + GH^2$

$s^2 = 3.9^2 + 1.6^2$

$s^2 = 17.77$

$s = \sqrt{17.77}$

Use the formula for the surface area of a right regular pyramid:

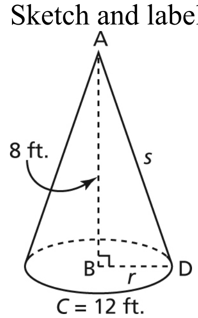
$SA = \frac{1}{2}s(\text{perimeter of base}) + (\text{base area})$

$SA = \frac{1}{2}(\sqrt{17.77})(5)(2.4) + 9.6$

$SA = 34.8926\ldots$

The surface area of the right pyramid is approximately 34.9 m^2.

20. Sketch and label the cone. Let r represent the base radius.



Use the formula for the circumference, C, of a circle.

$C = 2\pi r$ Substitute: $C = 12$

$12 = 2\pi r$ Solve for r.

$\frac{12}{2\pi} = \frac{2\pi r}{2\pi}$

$r = \frac{12}{2\pi}$

$r = 1.9098\ldots$

Use the Pythagorean Theorem in right ΔABD.

$AD^2 = AB^2 + BD^2$

$s^2 = 8^2 + (1.9098\ldots)^2$

$s^2 = 67.6475\ldots$

$s = \sqrt{67.6475\ldots}$

$s = 8.2248\ldots$

Use the formula for the surface area of a right cone with base radius r and slant height s:

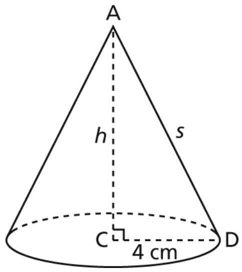
$SA = \pi rs + \pi r^2$ Substitute: $r = 1.9098\ldots$, $s = 8.2248\ldots$

$SA = \pi(1.9098\ldots)(8.2248\ldots) + \pi(1.9098\ldots)^2$

$SA = 60.8080\ldots$

The surface area of the cone is approximately 61 square feet.

21. Sketch and label the cone. Let h represent the height.



Use the formula for the surface area of a right cone with base radius r and slant height s.

$SA = \pi rs + \pi r^2$ Substitute: $SA = 258$, $r = 4$

$258 = \pi(4)s + \pi(4)^2$ Solve for s.

$258 = 4\pi s + 16\pi$

$258 - 16\pi = 4\pi s$

$$\frac{258 - 16\pi}{4\pi} = \frac{4\pi s}{4\pi}$$

$$s = \frac{258 - 16\pi}{4\pi}$$

$s = 16.5309\ldots$

Use the Pythagorean Theorem in right ΔACD to determine the height, h, of the cone.

$AC^2 = AD^2 - CD^2$

$h^2 = (16.5309\ldots)^2 - 4^2$

$h^2 = 257.2735\ldots$

$h = \sqrt{257.2735...}$

$h = 16.0397\ldots$

The height of the cone is approximately 16.0 cm.

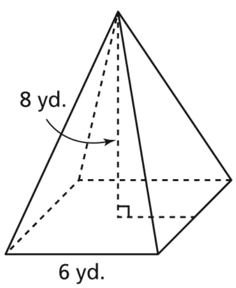
Lesson 1.5	**Volumes of Right Pyramids and Right Cones**	**Exercises (pages 42–44)**

A

4. a) Use the formula for the volume of a right rectangular prism.
$V = lwh$ Substitute: $l = 6$, $w = 6$, $h = 8$
$V = (6)(6)(8)$
$V = 288$
The volume of the prism is 288 cubic yards.

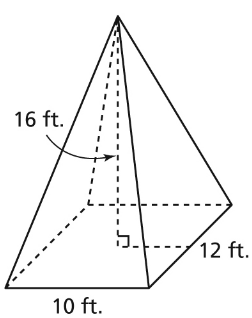
b) Use the formula for the volume of a right rectangular prism.
$V = lwh$ Substitute: $l = 12$, $w = 10$, $h = 16$
$V = (12)(10)(16)$
$V = 1920$
The volume of the prism is 1920 cubic feet.

5. a)



Use the formula for the volume of a right rectangular pyramid.
$V = \frac{1}{3}lwh$ Substitute: $l = 6$, $w = 6$, $h = 8$
$V = \frac{1}{3}(6)(6)(8)$
$V = 96$
The volume of the pyramid is 96 cubic yards.

b)



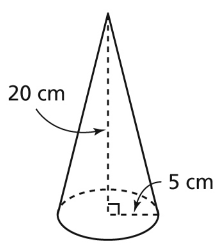
Use the formula for the volume of a right rectangular pyramid.
$V = \frac{1}{3}lwh$ Substitute: $l = 12$, $w = 10$, $h = 16$
$V = \frac{1}{3}(12)(10)(16)$

$V = 640$
The volume of the pyramid is 640 cubic feet.

6. a) Use the formula for the volume of a right cylinder.
$V = \pi r^2 h$ Substitute: $r = 5$, $h = 20$
$V = \pi(5)^2(20)$
$V = 1570.7963\ldots$
The volume of the cylinder is approximately 1571 cm^3.

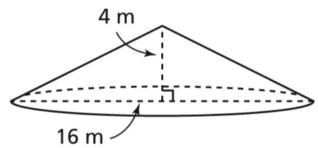
b) The radius, r, of the base of the cylinder is $\frac{1}{2}$ the diameter; so, r is 8 m.
Use the formula for the volume of a right cylinder.
$V = \pi r^2 h$ Substitute: $r = 8$, $h = 4$
$V = \pi(8)^2(4)$
$V = 804.2477\ldots$
The volume of the cylinder is approximately 804 m^3.

7. a)



Use the formula for the volume of a right cone.
$V = \frac{1}{3}\pi r^2 h$ Substitute: $r = 5$, $h = 20$
$V = \frac{1}{3}\pi(5)^2(20)$
$V = 523.5987\ldots$
The volume of the cone is approximately 524 cm^3.

b)



The radius, r, of the base of the cone is $\frac{1}{2}$ the diameter; so, r is 8 m.

Use the formula for the volume of a right cone.

$V = \frac{1}{3}\pi r^2 h$ Substitute: $r = 8$, $h = 4$

$V = \frac{1}{3}\pi (8)^2(4)$

$V = 268.0825\ldots$

The volume of the cone is approximately 268 m^3.

8. a) Use the formula for the volume of a right rectangular pyramid.

$V = \frac{1}{3}lwh$ Substitute: $l = 3$, $w = 3$, $h = 6$

$V = \frac{1}{3}(3)(3)(6)$

$V = 18$

The volume of the pyramid is 18 m^3.

b) Use the formula for the volume of a right rectangular pyramid.

$V = \frac{1}{3}lwh$ Substitute: $l = 7$, $w = 6$, $h = 12$

$V = \frac{1}{3}(7)(6)(12)$

$V = 168$

The volume of the pyramid is 168 cubic yards.

9. a) Use the formula for the volume of a right cone.

$V = \frac{1}{3}\pi r^2 h$ Substitute: $r = 2.0$, $h = 9.0$

$V = \frac{1}{3}\pi (2.0)^2(9.0)$

$V = 37.6991\ldots$

The volume of the cone is approximately 37.7 m^3.

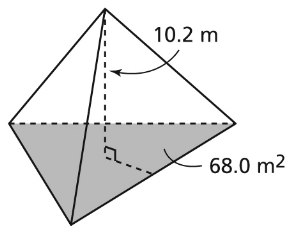
b) The radius, r, of the base of the cone is $\frac{1}{2}$ the diameter; so, r is 16 cm.

Use the formula for the volume of a right cone.

$V = \frac{1}{3}\pi r^2 h$ Substitute: $r = 16$, $h = 11$

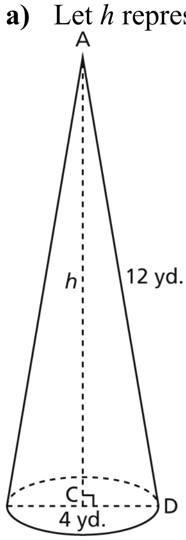
$V = \frac{1}{3}\pi(16)^2(11)$
$V = 2948.9083\ldots$
The volume of the cone is approximately 2948.9 cm^3.

B
10. a)



b) Use the formula for the volume of a right pyramid.
$V = \frac{1}{3}$(base area)(height)
$V = \frac{1}{3}(68)(10.2)$
$V = 231.2$
The volume of the tetrahedron is 231.2 m^3.

11. a) Let h represent the height.



b) The radius, r, of the cone is $\frac{1}{2}$ the diameter; so, r is 2 yd.
Use the Pythagorean Theorem in right ΔACD to determine the height, h, of the cone.
$AC^2 = AD^2 - CD^2$
$h^2 = 12^2 - 2^2$
$h^2 = 140$
$h = \sqrt{140}$

Use the formula for the volume of a right cone.

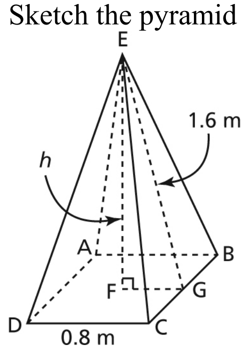
$V = \frac{1}{3}\pi r^2 h$ Substitute: $r = 2$, $h = \sqrt{140}$

$V = \frac{1}{3}\pi(2)^2(\sqrt{140})$

$V = 49.5624\ldots$

The volume of the cone is approximately 50 cubic yards.

12. Sketch the pyramid and label its vertices. Let h represent the height.



In ΔEFG, FG is $\frac{1}{2}$ the length of DC, so FG is 0.4 m.

Use the Pythagorean Theorem in right ΔEFG.

$EF^2 = EG^2 - FG^2$

$h^2 = 1.6^2 - 0.4^2$

$h^2 = 2.4$

$h = \sqrt{2.4}$

Use the formula for the volume of a right rectangular pyramid.

$V = \frac{1}{3}lwh$ Substitute: $l = 0.8$, $w = 0.8$, $h = \sqrt{2.4}$

$V = \frac{1}{3}(0.8)(0.8)(\sqrt{2.4})$

$V = 0.3304\ldots$

The volume of the monument is approximately 0.3 m^3.

13. a) Answers may vary. For example, Annika can use the formula for the volume of a right rectangular pyramid. She can multiply the three measurements, then divide by 3.

b) Use the formula for the volume of a right rectangular pyramid.

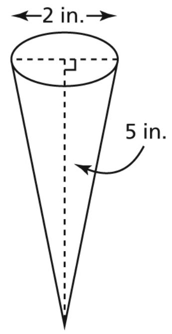
$V = \frac{1}{3}lwh$ Substitute: $l = 10.4$, $w = 8.6$, $h = 14.8$

$V = \frac{1}{3}(10.4)(8.6)(14.8)$

$V = 441.2373\ldots$

The volume of the pyramid is approximately 441.2 cm^3.

14. a) Sketch the cone.



The radius, r, is $\frac{1}{2}$ the diameter; so, $r = 1$ in.

Use the formula for the volume of a right cone.

$V = \frac{1}{3}\pi r^2 h$ Substitute: $r = 1$, $h = 5$

$V = \frac{1}{3}\pi(1)^2(5)$

$V = 5.2359\ldots$

The cone can hold approximately 5 cubic inches of ice cream.

b) Determine the total cost, C dollars, of an ice cream cone with whipped topping and sprinkles:

$C = 0.55(5.2359\ldots) + 0.35 + 0.10$

$C = 3.3297\ldots$

This dessert will cost \$3.33 to produce.

c) Use the formula for the volume of a right rectangular pyramid.

$V = \frac{1}{3}lwh$ Substitute: $l = 2$, $w = 2$, $h = 5$

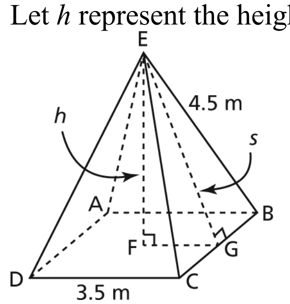
$V = \frac{1}{3}(2)(2)(5)$

$V = 6.6666\ldots$

The pyramid would hold approximately 7 cubic inches of ice cream.

15. a) Sketch the pyramid and label its vertices.

Let h represent the height and let s represent the slant height.



b) First determine the slant height, s.

In ΔEGB, GB is $\frac{1}{2}$ the length of CB, so GB = 1.75 m.

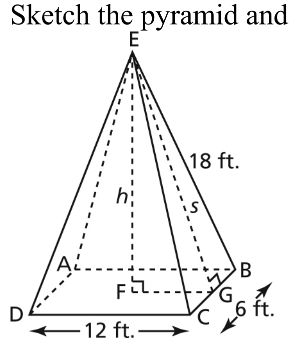
Use the Pythagorean Theorem in right ΔEGB.
$EG^2 = EB^2 - GB^2$
$s^2 = 4.5^2 - 1.75^2$
$s^2 = 17.1875$
$s = \sqrt{17.1875}$

In ΔEFG, FG is $\frac{1}{2}$ the length of DC, so FG = 1.75 m.
To calculate the height, h, use the Pythagorean Theorem in right ΔEFG.
$EF^2 = EG^2 - FG^2$
$h^2 = (\sqrt{17.1875})^2 - 1.75^2$
$h^2 = 14.125$
$h = 3.7583\ldots$
The height of the pyramid is approximately 3.8 m.

c) Use the formula for the volume of a right rectangular pyramid.
$V = \frac{1}{3}lwh$ Substitute: $l = 3.5$, $w = 3.5$, $h = 3.7583\ldots$
$V = \frac{1}{3}(3.5)(3.5)(3.7583\ldots)$
$V = 15.3464\ldots$
The volume of the pyramid is approximately 15.3 m^3.

16. Convert the equal side length of each triangular face from yards to feet, so all measurements are in the same unit. Use the conversion: 1 yd. = 3 ft.
So, 6 yd. = 6(3 ft.)
6 yd. = 18 ft.

Sketch the pyramid and label its vertices. Let h represent its height.



First determine the slant height, s. In ΔEGB, GB is $\frac{1}{2}$ the length of CB, so GB = 3 ft.
Use the Pythagorean Theorem in right ΔEGB.
$EG^2 = EB^2 - GB^2$
$s^2 = 18^2 - 3^2$
$s^2 = 315$
$s = \sqrt{315}$

Now use the Pythagorean Theorem in right ΔEFG to determine h.

Side FG is $\frac{1}{2}$ the length of DC, so FG = 6 ft.

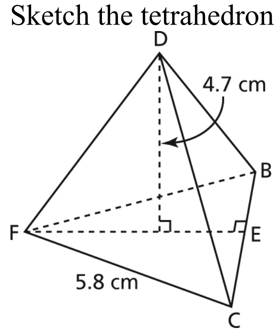
$EF^2 = EG^2 - FG^2$
$h^2 = (\sqrt{315})^2 - 6^2$
$h^2 = 279$
$h = \sqrt{279}$

Use the formula for the volume of a right rectangular pyramid.

$V = \frac{1}{3}lwh$ Substitute: $l = 12$, $w = 6$, $h = \sqrt{279}$

$V = \frac{1}{3}(12)(6)(\sqrt{279})$
$V = 400.8790\ldots$
The volume of the pyramid is approximately 401 cubic feet.

17. a) Sketch the tetrahedron and label its vertices.



In equilateral ΔFBC, EB is $\frac{1}{2}$ the length of CB, so EB = 2.9 cm.
Use the Pythagorean Theorem in right ΔFEB to determine the height of the base, FE.
$FE^2 = 5.8^2 - 2.9^2$
$FE^2 = 25.23$
$FE = \sqrt{25.23}$

So, the area, A, of the base is:

$A = \frac{1}{2}(5.8)(\sqrt{25.23})$
$A = 14.5665\ldots$
The area of the base is approximately 15 cm^2.

b) Use the formula for the volume of a right pyramid.

$V = \frac{1}{3}$(base area)(height)

$V = \frac{1}{3}(14.5665\ldots)(4.7)$
$V = 22.8209\ldots$
The volume of the tetrahedron is approximately 23 cm^3.

c) No, the volume of tea in the bag will be less than 23 cm^3. There is some air inside the tea bag to allow the tea to expand when boiling water is poured on it.

18. a) Use the formula for the volume of a rectangular prism.

$V = lwh$ Substitute: $V = 88.8$, $l = 5.9$, $w = 3.2$

$88.8 = (5.9)(3.2)h$ Solve for h.

$$88.8 = 18.88h$$
$$\frac{88.8}{18.88} = \frac{18.88h}{18.88}$$
$$h = \frac{88.8}{18.88}$$
$$h = 4.7033\ldots$$

h is approximately 4.7 cm.

b) Use the formula for the volume of a right rectangular pyramid.

$V = \frac{1}{3}lwh$ Substitute: $V = 554.9$, $l = x$, $w = x$, $h = 15.1$

$554.9 = \frac{1}{3}(x)(x)(15.1)$ Multiply both sides by 3.

$$(3)(554.9) = (x)(x)(15.1)$$

$1664.7 = 15.1x^2$ Solve for x.

$$\frac{1664.7}{15.1} = \frac{15.1x^2}{15.1}$$
$$x^2 = \frac{1664.7}{15.1}$$
$$x = \sqrt{\frac{1664.7}{15.1}}$$
$$x = 10.4997\ldots$$

x is approximately 10.5 m.

c) Use the formula for the volume of a right cylinder.

$V = \pi r^2 h$ Substitute: $V = 219.0$, $h = 6.4$

$219.0 = \pi r^2(6.4)$ Divide both sides by 6.4π .

$\frac{219.0}{6.4\pi} = \frac{6.4\pi r^2}{6.4\pi}$ Solve for r.

$$r^2 = \frac{219.0}{6.4\pi}$$
$$r = \sqrt{\frac{219.0}{6.4\pi}}$$
$$r = 3.300\ldots$$

r is approximately 3.3 m.

d) Use the formula for the volume of a right cone.

$V = \frac{1}{3}\pi r^2 h$ Substitute: $V = 164.9$, $h = 11.5$

$164.9 = \frac{1}{3}\pi r^2(11.5)$	Multiply both sides by 3.
$494.7 = 11.5\pi r^2$	Divide both sides by 11.5π.
$\frac{494.7}{11.5\pi} = \frac{11.5\pi r^2}{11.5\pi}$	Solve for r.
$r^2 = \frac{494.7}{11.5\pi}$	
$r = \sqrt{\frac{494.7}{11.5\pi}}$	

The diameter, d, of the cone is twice the radius.

$d = 2 \times \sqrt{\frac{494.7}{11.5\pi}}$

$d = 7.400\ldots$

d is approximately 7.4 cm.

19. a) Answers may vary. For example, Sunil could use the diameter to determine the radius, substitute the known data in the formula for the volume of a right cone, then solve for h.

b) The radius, r, is $\frac{1}{2}$ the diameter; so, $r = 2.0$ cm

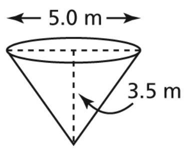
Use the formula for the volume of a right cone.

$V = \frac{1}{3}\pi r^2 h$	Substitute: $V = 33.5$, $r = 2.0$
$33.5 = \frac{1}{3}\pi(2.0)^2 h$	Multiply both sides by 3.
$100.5 = 4\pi h$	Divide both sides by 4π.
$\frac{100.5}{4\pi} = \frac{4\pi h}{4\pi}$	Solve for h.
$h = \frac{100.5}{4\pi}$	
$h = 7.9975\ldots$	

The height of the cone is approximately 8.0 cm.

C

20. a) The radius, r, is $\frac{1}{2}$ the diameter of the cone; so, $r = 2.5$ m



Use the formula for the volume of a right cone.

$V = \frac{1}{3}\pi r^2 h$ Substitute: $r = 2.5$, $h = 3.5$

$V = \frac{1}{3}\pi (2.5)^2(3.5)$

$V = 22.9074\ldots$

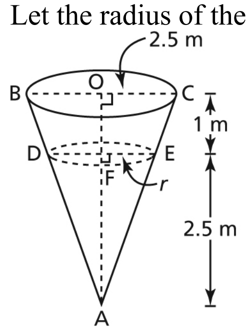
The volume is approximately 22.9 m^3.

Since 1 m^3 = 1 kL, the capacity of the tank is approximately 22.9 kL.

b) Sketch and label a diagram.

When the water level is below the top of the tank, the water is 2.5 m deep.

Let the radius of the water surface be r.



Use similar triangles to determine r.

Right ΔAFE and ΔAOC are similar. So, the ratio of corresponding side lengths is equal.

$\frac{AF}{AO} = \frac{EF}{CO}$ Substitute: AF = 2.5, AO = 3.5, CO = 2.5, EF = r

$\frac{2.5}{3.5} = \frac{r}{2.5}$ Solve for r.

$r = \frac{(2.5)(2.5)}{3.5}$

$r = 1.7857\ldots$

Use the formula for the volume of a right cone.

$V = \frac{1}{3}\pi r^2 h$ Substitute: $r = 1.7857\ldots$, $h = 2.5$

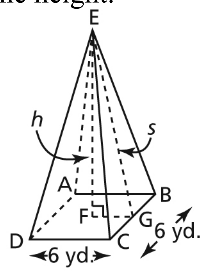
$V = \frac{1}{3}\pi(1.7857...)^2(2.5)$

$V = 8.3481\ldots$

The volume is approximately 8.3 m^3.

Since 1 m^3 = 1 kL, there is approximately 8.3 kL of water in the tank.

21. Sketch the pyramid and label its vertices. Let s represent the slant height and let h represent the height.



V = 111 cubic yards

First determine the height, h. Use the formula for the volume of a right rectangular pyramid.

$V = \frac{1}{3}lwh$ Substitute: $V = 111$, $l = w = 6$

$111 = \frac{1}{3}(6)(6)h$ Solve for h.

$111 = 12h$

$\frac{111}{12} = \frac{12h}{12}$

$h = \frac{111}{12}$

$h = 9.25$

To determine s, use the Pythagorean Theorem in right ΔEFG. Side length FG is $\frac{1}{2}$ the length of DC, so FG = 3 yd.

$EG^2 = EF^2 + FG^2$

$s^2 = 9.25^2 + 3^2$

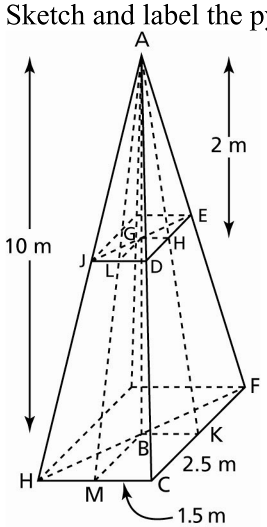
$s^2 = 94.5625$

$s = \sqrt{94.5625}$

$s = 9.7243\ldots$

The slant height of the pyramid is approximately 10 yd.

22. Sketch and label the pyramid.



Use similar triangles to determine the dimensions of the base of the pyramid that is removed.

ΔAGH and ΔABK are similar.

So, $\frac{GH}{BK} = \frac{AG}{AB}$

$\frac{GH}{1.5} = \frac{2}{10}$

$GH = \frac{2(1.5)}{10}$

$GH = \frac{3}{10}$, or 0.3

ΔAGL and ΔABM are similar.

So, $\frac{AG}{AB} = \frac{GL}{BM}$

$\frac{2}{10} = \frac{GL}{2.5}$

$GL = \frac{2(2.5)}{10}$

$GL = \frac{5}{10}$, or 0.5

The dimensions of the base of the smaller pyramid are 2(0.3) m by 2(0.5) m, or 0.6 m by 1.0 m

So, the volume of the smaller pyramid is: $\frac{1}{3}(0.6)(1.0)(2) = 0.4$

The volume of the larger pyramid is: $\frac{1}{3}(5)(3)(10) = 50$

So, the remaining piece has volume: 50 – 0.4 = 49.6
The volume of the remaining piece is 49.6 m^3.

Lesson 1.6 Surface Area and Volume of a Sphere Exercises (pages 51–52)

A

3. a) Use the formula for the surface area of a sphere.
$SA = 4\pi r^2$ Substitute: $r = 5$
$SA = 4\pi(5)^2$
$SA = 314.1592\ldots$
The surface area of the sphere is approximately 314 cm^2.

b) Use the formula for the surface area of a sphere.
$SA = 4\pi r^2$ Substitute: $r = 1.6$
$SA = 4\pi(1.6)^2$
$SA = 32.1699\ldots$
The surface area of the sphere is approximately 32 m^2.

c) Use the formula for the surface area of a sphere.
The radius, r, is $\frac{1}{2}(8 \text{ ft.}) = 4$ ft.
$SA = 4\pi r^2$ Substitute: $r = 4$
$SA = 4\pi(4)^2$
$SA = 201.0619\ldots$
The surface area of the sphere is approximately 201 square feet.

d) Use the formula for the surface area of a sphere.
The radius, r, is $\frac{1}{2}(5.6 \text{ cm}) = 2.8$ cm
$SA = 4\pi r^2$ Substitute: $r = 2.8$
$SA = 4\pi(2.8)^2$
$SA = 98.5203\ldots$
The surface area of the sphere is approximately 99 cm^2.

4. a) Use the formula for the volume of a sphere.
$V = \frac{4}{3}\pi r^3$ Substitute: $r = 5$
$V = \frac{4}{3}\pi(5)^3$
$V = 523.5987\ldots$
The volume of the sphere is approximately 524 cm^3.

b) Use the formula for the volume of a sphere.
$V = \frac{4}{3}\pi r^3$ Substitute: $r = 1.6$
$V = \frac{4}{3}\pi(1.6)^3$
$V = 17.1572\ldots$
The volume of the sphere is approximately 17 m^3.

c) Use the formula for the volume of a sphere.

The radius, r, is $\frac{1}{2}$(8 ft.) = 4 ft.

$V = \frac{4}{3}\pi r^3$ Substitute: $r = 4$

$V = \frac{4}{3}\pi(4)^3$

$V = 268.0825\ldots$

The volume of the sphere is approximately 268 cubic feet.

d) Use the formula for the volume of a sphere.

The radius, r, is $\frac{1}{2}$(5.6 cm) = 2.8 cm

$V = \frac{4}{3}\pi r^3$ Substitute: $r = 2.8$

$V = \frac{4}{3}\pi(2.8)^3$

$V = 91.9523\ldots$

The volume of the sphere is approximately 92 cm^3.

5. a) SA of a hemisphere = SA of one-half a sphere + area of circle

$SA = \frac{1}{2}(4\pi r^2) + \pi r^2$

$SA = 2\pi r^2 + \pi r^2$

$SA = 3\pi r^2$ Substitute: $r = 6$

$SA = 3\pi(6)^2$

$SA = 339.2920\ldots$

The surface area of the hemisphere is approximately 339 m^2.

Volume of a hemisphere = volume of one-half a sphere

$V = \frac{1}{2}\left(\frac{4}{3}\pi r^3\right)$

$V = \frac{2}{3}\pi r^3$ Substitute: $r = 6$

$V = \frac{2}{3}\pi(6)^3$

$V = 452.3893\ldots$

The volume of the hemisphere is approximately 452 m^3.

b) The radius, r, is $\frac{1}{2}$(9 yd.) = $4\frac{1}{2}$ yd.

SA of a hemisphere = SA of one-half a sphere + area of circle

$SA = \frac{1}{2}(4\pi r^2) + \pi r^2$

$SA = 2\pi r^2 + \pi r^2$

$SA = 3\pi r^2$ Substitute: $r = 4.5$

$SA = 3\pi(4.5)^2$
$SA = 190.8517\ldots$
The surface area of the hemisphere is approximately 191 square yards.

Volume of a hemisphere = volume of one-half a sphere

$V = \frac{1}{2}\left(\frac{4}{3}\pi r^3\right)$

$V = \frac{2}{3}\pi r^3$ Substitute: $r = 4.5$

$V = \frac{2}{3}\pi(4.5)^3$

$V = 190.8517\ldots$
The volume of the hemisphere is approximately 191 cubic yards.

B

6. Answers will vary depending on object chosen.
For example, I used a baseball. I measured its diameter to be 3 in.

The radius, r, of the baseball is:

$r = \frac{1}{2}(3 \text{ in.})$

$r = 1\frac{1}{2}$ in.

Use the formula for the surface area of a sphere.
$SA = 4\pi r^2$ Substitute: $r = 1.5$
$SA = 4\pi(1.5)^2$
$SA = 28.2743\ldots$
The surface area of the baseball is approximately 28 square inches.

Use the formula for the volume of a sphere.

$V = \frac{4}{3}\pi r^3$ Substitute: $r = 1.5$

$V = \frac{4}{3}\pi(1.5)^3$

$V = 14.1371\ldots$
The volume of the baseball is approximately 14 cubic inches.

7. Use the formula for the surface area of a sphere.
$SA = 4\pi r^2$ Substitute: $r = 8.4$
$SA = 4\pi(8.4)^2$
$SA = 886.6831\ldots$
The surface area of the sphere is approximately 886.7 m^2.

Use the formula for the volume of a sphere.

$V = \frac{4}{3}\pi r^3$ Substitute: $r = 8.4$

$V = \frac{4}{3}\pi(8.4)^3$
$V = 2482.7127\ldots$
The volume of the sphere is approximately 2482.7 m^3.

8. Use the formula for the surface area of a sphere.

$SA = 4\pi r^2$ Substitute: $SA = 127$
$127 = 4\pi r^2$ Solve for r.

$$\frac{127}{4\pi} = \frac{4\pi r^2}{4\pi}$$
$$r^2 = \frac{127}{4\pi}$$
$$r = \sqrt{\frac{127}{4\pi}}$$
$$r = 3.1790\ldots$$

The radius of the tennis ball is approximately 3.2 cm.

9. Use the formula for the surface area of a sphere and determine r.

$SA = 4\pi r^2$ Substitute: $SA = 452$
$452 = 4\pi r^2$ Solve for r.

$$\frac{452}{4\pi} = \frac{4\pi r^2}{4\pi}$$
$$r^2 = \frac{452}{4\pi}$$
$$r = \sqrt{\frac{452}{4\pi}}$$

The diameter, d, is:

$$d = 2\left(\sqrt{\frac{452}{4\pi}}\right)$$
$$d = 11.9948\ldots$$

The diameter of the sphere is approximately 12 in.

10. a) The radius, r, is $\frac{1}{2}(20 \text{ cm}) = 10 \text{ cm}$

Volume of a hemisphere = volume of one-half a sphere

$$V = \frac{1}{2}\left(\frac{4}{3}\pi r^3\right)$$
$V = \frac{2}{3}\pi r^3$ Substitute: $r = 10$
$$V = \frac{2}{3}\pi(10)^3$$
$$V = 2094.3951\ldots$$

Since $1000\text{ cm}^3 = 1\text{ L}$, the capacity of the bowl is: $\dfrac{2094.3951\ldots}{1000} = 2.0943\ldots$

The capacity of the bowl is approximately 2.1 L.

b) 1 cup = 250 mL, or 0.250 L

The number of cups is: $\dfrac{2.0943\ldots}{0.250} = 8.3775\ldots$

The bowl can hold approximately 8 cups of punch.

11. a) Determine the surface areas of the sphere and hemisphere, then compare.
Use the formula for the surface area of a sphere.

The radius, r, of the sphere is: $\frac{1}{2}(12\text{ cm}) = 6\text{ cm}$

$SA = 4\pi r^2$ Substitute: $r = 6$
$SA = 4\pi(6)^2$
$SA = 452.3893\ldots$
The surface area of the sphere is approximately 452.4 cm^2.

SA of a hemisphere = SA of one-half a sphere + area of circle

$SA = \frac{1}{2}(4\pi r^2) + \pi r^2$
$SA = 2\pi r^2 + \pi r^2$
$SA = 3\pi r^2$ Substitute: $r = 8$
$SA = 3\pi(8)^2$
$SA = 603.1857\ldots$
The surface area of the hemisphere is approximately 603.2 cm^2.

Since 603.2 > 452.4, the hemisphere has the greater surface area.

b) Determine the volumes of the sphere and hemisphere, then compare.
Use the formula for the volume of a sphere.
The radius, r, of the sphere is: 6 cm

$V = \frac{4}{3}\pi r^3$ Substitute: $r = 6$
$V = \frac{4}{3}\pi(6)^3$
$V = 904.7786\ldots$
The volume of the sphere is approximately 904.8 cm^3.

Volume of a hemisphere = volume of one-half a sphere

$V = \frac{1}{2}\left(\frac{4}{3}\pi r^3\right)$
$V = \frac{2}{3}\pi r^3$ Substitute: $r = 8$

$V = \frac{2}{3}\pi(8)^3$
$V = 1072.3302\ldots$
The volume of the hemisphere is approximately 1072.3 cm^3.

Since 1072.3 > 904.8, the hemisphere has the greater volume.

12. a) Use the formula for the surface area of a sphere.
The radius, r, of the sphere is: $\frac{1}{2}(15.8 \text{ m}) = 7.9 \text{ m}$

$SA = 4\pi r^2$ Substitute: $r = 7.9$
$SA = 4\pi(7.9)^2$
$SA = 784.2671\ldots$
The surface area of the sphere is approximately 784 m^2.

b) Use the formula for the volume of a sphere.
$V = \frac{4}{3}\pi r^3$ Substitute: $r = 7.9$
$V = \frac{4}{3}\pi(7.9)^3$
$V = 2065.2369\ldots$
The volume of the sphere is approximately 2065 m^3.
Since 1 kL = 1 m^3, the capacity of the sphere is approximately 2065 kL.

13. a) Use the formula for the surface area of a sphere.
The radius, r, of Earth is: $\frac{1}{2}(12\ 756 \text{ km}) = 6378 \text{ km}$

$SA = 4\pi r^2$ Substitute: $r = 6378$
$SA = 4\pi(6378)^2$
$SA = 511\ 185\ 932.5\ldots$
The surface area of Earth is approximately 511 185 933 km^2.

b) Determine 70% of 511 185 933.5…
$0.70 \times 511\ 185\ 933.5\ldots = 357\ 830\ 152.8\ldots$
Approximately 357 830 153 km^2 of Earth's surface is covered in water.

c) Use the formula for the volume of a sphere.
$V = \frac{4}{3}\pi r^3$ Substitute: $r = 6378$
$V = \frac{4}{3}\pi(6378)^3$
$V = 1\ 086\ 781\ 293\ 000\ldots$
The volume of Earth is approximately 1 086 781 293 000 km^3.

d) Determine the volume of the inner core of Earth.
Use the formula for the volume of a sphere.

$V = \frac{4}{3}\pi r^3$ Substitute: $r = 1278$

$V = \frac{4}{3}\pi(1278)^3$

$V = 8\ 743\ 416\ 579\ldots$

So, the volume of Earth that is not part of the inner core, in cubic kilometres, is:
volume of Earth – volume of inner core
= 1 086 781 293 000… – 8 743 416 579…
= 1 078 037 876 000…
The volume of Earth that is not part of the inner core is approximately 1 078 037 876 000 km^3.

14. The diameter of Earth through the poles is: 12 756 km – 16 km = 12 740 km

The radius, r, at the poles is: $\frac{1}{2}$(12 740 km) = 6370 km

Use the formula for the volume of a sphere.

$V = \frac{4}{3}\pi r^3$ Substitute: $r = 6370$

$V = \frac{4}{3}\pi(6370)^3$

$V = 1\ 082\ 696\ 932\ 000\ldots$

The volume of Earth, using the polar radius, is approximately 1 082 696 932 000 km^3.

The diameter of Earth at the equator is: 12 756 km + 26 km = 12 782 km

The radius, r, at the equator is: $\frac{1}{2}$(12 782 km) = 6391 km

Use the formula for the volume of a sphere.

$V = \frac{4}{3}\pi r^3$ Substitute: $r = 6391$

$V = \frac{4}{3}\pi(6391)^3$

$V = 1\ 093\ 440\ 264\ 000\ldots$

The volume of Earth, using the equatorial radius, is approximately 1 093 440 264 000 km^3.

15. Determine the surface area of one sphere of dough.
Use the formula for the surface area of a sphere.

The radius, r, is: $\frac{1}{2}$(2.5 cm) = 1.25 cm

$SA = 4\pi r^2$ Substitute: $r = 1.25$

$SA = 4\pi(1.25)^2$

$SA = 19.6349\ldots$

The number of spheres that can be glazed is:

$$\frac{\text{total surface area}}{\text{area of one sphere}} = \frac{4710}{19.6349\ldots}$$
$$= 239.8783\ldots$$

So, 239 doughnut spheres can be glazed.

16. a) Use the formula for the circumference, C, of a circle to determine r.

For the volleyball:

$C = 2\pi r$ Substitute: $C = 66$

$66 = 2\pi r$ Solve for r. Divide both sides by 2π.

$$\frac{66}{2\pi} = \frac{2\pi r}{2\pi}$$
$$r = \frac{66}{2\pi}$$
$$r = 10.5042\ldots$$

The radius of a volleyball is approximately 11 cm.

For the basketball:

$C = 2\pi r$ Substitute: $C = 29.5$

$29.5 = 2\pi r$ Solve for r. Divide both sides by 2π.

$$\frac{29.5}{2\pi} = \frac{2\pi r}{2\pi}$$
$$r = \frac{29.5}{2\pi}$$
$$r = 4.6950\ldots$$

The radius of a basketball is approximately 5 in.

b) Use the formula for the surface area of a sphere.

For the volleyball:

$SA = 4\pi r^2$ Substitute: $r = 10.5042\ldots$

$SA = 4\pi(10.5042\ldots)^2$

$SA = 1386.5578\ldots$

The surface area of a volleyball is approximately 1387 cm^2.

For the basketball:

$SA = 4\pi r^2$ Substitute: $r = 4.6950\ldots$

$SA = 4\pi(4.6950\ldots)^2$

$SA = 277.0091\ldots$

The surface area of a basketball is approximately 277 square inches.

c) Use the formula for the volume of a sphere.

For the volleyball:

$V = \frac{4}{3}\pi r^3$ Substitute: $r = 10.5042\ldots$

$$V = \frac{4}{3}\pi(10.5042\ldots)^3$$
$$V = 4854.9058\ldots$$

The volume of a volleyball is approximately 4855 cm^3.

For the basketball:

$V = \frac{4}{3}\pi r^3$ Substitute: $r = 4.6950\ldots$

$V = \frac{4}{3}\pi (4.6950\ldots)^3$

$V = 433.5259\ldots$

The volume of a basketball is approximately 434 cubic inches.

d) To determine which ball is larger, compare their radii. Convert the radius of the basketball to centimetres. Use the exact conversion: 1 in. = 2.54 cm
So, 29.5 in. = 29.5(2.54 cm)
29.5 in. = 74.93 cm
Since 74.93 > 66, the basketball is larger.

17. a) Use the formula for the volume of a sphere.

The radius, r, is: $\frac{1}{2}(3.15\text{ m}) = 1.575\text{ m}$

$V = \frac{4}{3}\pi r^3$ Substitute: $r = 1.575$

$V = \frac{4}{3}\pi (1.575)^3$

$V = 16.3655\ldots$

The volume of the inside of the shell is approximately 16.4 m^3.

b) Determine the surface areas of the inside and outside shells. Use the formula for the surface area of a sphere.
For the outside shell:

The radius, r, is: $\frac{1}{2}(3.20\text{ m}) = 1.6\text{ m}$

$SA = 4\pi r^2$ Substitute: $r = 1.6$

$SA = 4\pi (1.6)^2$

$SA = 32.1699\ldots$

For the inside shell:

$SA = 4\pi r^2$ Substitute: $r = 1.575$

$SA = 4\pi (1.575)^2$

$SA = 31.1724\ldots$

Subtract the two surface areas to determine the difference:
$32.1699\ldots - 31.1724\ldots = 0.9974\ldots$

So, the difference between the outside and inside surface areas is approximately 1.0 m^2.

18. Use the formula for the circumference, C, of a circle to determine r.

$C = 2\pi r$ Substitute: $C = 47.1$

$47.1 = 2\pi r$ Solve for r. Divide both sides by 2π.

$\frac{47.1}{2\pi} = \frac{2\pi r}{2\pi}$

$r = \frac{47.1}{2\pi}$

$r = 7.4961...$

SA of a hemisphere = SA of one-half a sphere + area of circle

$SA = \frac{1}{2}(4\pi r^2) + \pi r^2$

$SA = 2\pi r^2 + \pi r^2$

$SA = 3\pi r^2$ Substitute: $r = 7.4961...$

$SA = 3\pi(7.4961...)^2$

$SA = 529.6063...$

The surface area of the hemisphere is approximately 529.6 m^2.

Volume of a hemisphere = volume of one-half a sphere

$V = \frac{1}{2}\left(\frac{4}{3}\pi r^3\right)$

$V = \frac{2}{3}\pi r^3$ Substitute: $r = 7.4961...$

$V = \frac{2}{3}\pi(7.4961...)^3$

$V = 882.2298...$

The volume of the sphere is approximately 882.2 m^3.

19. Determine the volume of the ball. Use the formula for the volume of a sphere.

The radius, r, is: $\frac{1}{2}(28 \text{ cm}) = 14 \text{ cm}$

$V = \frac{4}{3}\pi r^3$ Substitute: $r = 14$

$V = \frac{4}{3}\pi(14)^3$

$V = 11\,494.0403...$

Divide the total volume by the volume per pump to determine the number of pumps needed:

$\frac{11\,494.0403...}{280} = 41.0501...$

So, 42 pumps are needed to inflate the ball.

20. Determine the volume of the pail of cookie dough and the volume of a scoop of dough.

For the pail, use the formula for the volume of a cylinder.

The radius, r, is: $\frac{1}{2}(17 \text{ cm}) = 8.5 \text{ cm}$

$V = \pi r^2 h$ Substitute: $r = 8.5$, $h = 13$
$V = \pi (8.5)^2(13)$
$V \doteq 2950.7409\ldots$

For the scoop, use the formula for the volume of a sphere.

The radius, r, is: $\frac{1}{2}(5 \text{ cm}) = 2.5 \text{ cm}$

$V = \frac{4}{3}\pi r^3$ Substitute: $r = 2.5$
$V = \frac{4}{3}\pi (2.5)^3$
$V = 65.4498\ldots$

To determine how many cookies can be made, divide the volume of the pail by the volume of a scoop:

$\frac{2950.7408\ldots}{65.4498\ldots} = 45.0840\ldots$

So, 45 cookies can be made from the pail of dough.

C

21. a) Determine the volume of the block of wood.
Use the formula for the volume of a right rectangular prism.
$V = lwh$ Substitute: $l = 14$, $w = 12$, $h = 10$
$V = (14)(12)(10)$
$V = 1680$
The volume of the block of wood is 1680 cm^3.

Determine the volume of the sphere. Use the formula for the volume of a sphere.
The diameter of the sphere is the least measurement of the block, or 10 cm.

The radius, r, is: $\frac{1}{2}(10 \text{ cm}) = 5 \text{ cm}$

$V = \frac{4}{3}\pi r^3$ Substitute: $r = 5$
$V = \frac{4}{3}\pi (5)^3$
$V = 523.5987\ldots$
The volume of the sphere is 523.5987… cm^3.

The volume of wood wasted is: 1680 cm^3 – 523.5987… cm^3 = 1156.4012… cm^3
Write the volume of wood wasted as a fraction of the total amount of wood:

$\frac{1156.4012\ldots}{1680} = 0.6883\ldots$

Write this decimal as a percent: $0.6883\ldots \times 100\% = 68.83\ldots\%$
So, approximately 69% of the wood is wasted.

b) I assume that the ball is created from one solid piece of wood and that it has the greatest possible diameter.

22. Use the formula for the surface area of a sphere.

The radius r, is one-half the diameter, d: $r = \frac{1}{2}d$

$SA = 4\pi r^2$ Substitute: $r = \frac{1}{2}d$

$SA = 4\pi\left(\frac{1}{2}d\right)^2$

$SA = 4\pi\left(\frac{1}{4}d^2\right)$

$SA = \pi d^2$

To check, calculate the surface area of the sphere in question 3c.

$SA = \pi d^2$ Substitute: $d = 8$

$SA = \pi(8)^2$

$SA = 201.0619\ldots$

The surface area of the sphere is approximately 201 square feet. This is the same as the answer for question 3c; so, my formula is correct.

Use the formula for the volume of a sphere.

$V = \frac{4}{3}\pi r^3$ Substitute: $r = \frac{1}{2}d$

$V = \frac{4}{3}\pi\left(\frac{1}{2}d\right)^3$

$V = \frac{4}{3}\pi\left(\frac{1}{8}d^3\right)$

$V = \frac{4}{24}\pi d^3$

$V = \frac{1}{6}\pi d^3$

To check, calculate the volume of the sphere from question 3c.

$V = \frac{1}{6}\pi d^3$ Substitute: $d = 8$

$V = \frac{1}{6}\pi(8)^3$

$V = 268.0825\ldots$

The volume of the sphere is approximately 268 cubic feet. This is the same as the answer for question 4c; so, my formula is correct.

23. 70% of the volume is: 420 cubic inches

So, 1% of the volume is: $\frac{420}{70}$ cubic inches

And, 100% of the volume is: $\frac{420(100)}{70}$ cubic inches = 600 cubic inches

The maximum volume of the beach ball is 600 cubic inches.

Use the formula for the volume of a sphere to determine r.

$V = \frac{4}{3}\pi r^3$	Substitute: $V = 600$
$600 = \frac{4}{3}\pi r^3$	Solve for r. Multiply both sides by 3.
$1800 = 4\pi r^3$	Divide both sides by 4π.
$\frac{1800}{4\pi} = \frac{4\pi r^3}{4\pi}$	
$r^3 = \frac{1800}{4\pi}$	
$r = \sqrt[3]{\frac{1800}{4\pi}}$	
$r = 5.2322\ldots$	

At its maximum volume, the radius of the beach ball is approximately 5 in.

24. a) Determine the circumference, C, of the original balloon. Use the formula for the circumference of a circle.

$C = 2\pi r$	Substitute: $r = 10$
$C = 2\pi(10)$	
$C = 20\pi$	

Determine the circumference, C, of the inflated balloon. Use the formula for the circumference of a circle.

The radius, r, is three times the original radius:

$r = 3(10 \text{ cm}) = 30 \text{ cm}$

$C = 2\pi r$	Substitute: $r = 30$
$C = 2\pi(30)$	
$C = 60\pi$	

To compare the circumferences, divide: $\frac{20\pi}{60\pi} = 3$

The circumference of the inflated balloon is 3 times as great as the circumference of the original balloon.

b) Determine the surface area of the original balloon. Use the formula for the surface area of a sphere.

$SA = 4\pi r^2$	Substitute: $r = 10$
$SA = 4\pi(10)^2$	
$SA = 400\pi$	

Determine the surface area of the inflated balloon. Use the formula for the surface area of a sphere.

$SA = 4\pi r^2$ Substitute: $r = 30$
$SA = 4\pi(30)^2$
$SA = 3600\pi$

To compare the surface areas, divide: $\dfrac{3600\pi}{400\pi} = 9$
The surface area of the inflated balloon is 9 times as great as the surface area of the original balloon.

c) Determine the volume of the original balloon. Use the formula for the volume of a sphere.

$V = \dfrac{4}{3}\pi r^3$ Substitute: $r = 10$
$V = \dfrac{4}{3}\pi(10)^3$
$V = \dfrac{4000\pi}{3}$

Determine the volume of the inflated balloon. Use the formula for the volume of a sphere.

$V = \dfrac{4}{3}\pi r^3$ Substitute: $r = 30$
$V = \dfrac{4}{3}\pi(30)^3$
$V = 36\,000\pi$

To compare the volumes, divide: $\dfrac{36\,000\pi}{\dfrac{4000\pi}{3}} = \dfrac{108}{4} = 27$
The volume of the inflated balloon is 27 times as great as the volume of the original balloon.

Checkpoint 2 Assess Your Understanding (page 54)

1.4

1. a) Use the formula for surface area of a right square pyramid with slant height s and base side length l:

Surface area = $\frac{1}{2}s(4l) + l^2$ Substitute: $s = 8$, $l = 4$

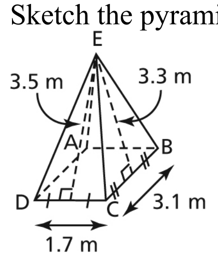
Surface area = $\frac{1}{2}(8)(4)(4) + (4)^2$

Surface area = $64 + 16$

Surface area = 80

The surface area of the pyramid is 80 square feet.

b) Sketch the pyramid and label its vertices.



Area, A, of ΔEDC is:

$A = \frac{1}{2}(1.7)(3.5)$

$A = 2.975$

Since ΔEDC and ΔEAB are congruent, the area of ΔEAB is 2.975 m^2.

Area, A, of ΔEBC is:

$A = \frac{1}{2}(3.1)(3.3)$

$A = 5.115$

Since ΔEBC and ΔEAD are congruent, the area of ΔEAD is 5.115 m^2.

Area, B, of the base of the pyramid is:

$B = (1.7)(3.1)$

$B = 5.27$

Each of two triangles has area 2.975 m^2, and each of the other two triangles has area 5.115 m^2.

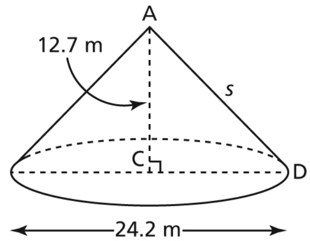
The surface area, SA, of the pyramid is:

$SA = 2(2.975) + 2(5.115) + 5.27$

$SA = 21.45$

The surface area of the right rectangular pyramid is approximately 21 m^2.

c) Sketch the cone and label its vertices. Let s represent the slant height.



The radius, CD, of the cone is one-half the diameter: $\frac{1}{2}(24.2 \text{ m}) = 12.1$ m

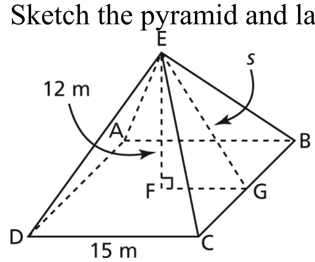
Use the Pythagorean Theorem in right ΔACD to determine s.

$AD^2 = AC^2 + CD^2$
$s^2 = 12.7^2 + 12.1^2$
$s^2 = 307.7$
$s = \sqrt{307.7}$

Use the formula for the surface area of a right cone.

Surface area = $\pi rs + \pi r^2$ Substitute: $r = 12.1$, $s = \sqrt{307.7}$
Surface area = $\pi(12.1)(\sqrt{307.7}) + \pi(12.1)^2$
Surface area = 1126.7658…
The surface area of the right cone is approximately 1127 m^2.

2. Sketch the pyramid and label its vertices. Let s represent the slant height.



FG is $\frac{1}{2}$ the length of DC, so FG is 7.5 m.

Use the Pythagorean Theorem in right ΔEFG to determine s.

$EG^2 = EF^2 + FG^2$
$s^2 = 12^2 + 7.5^2$
$s^2 = 200.25$
$s = \sqrt{200.25}$

Determine the lateral area, A_L, of the right square pyramid with slant height s and base side length l.

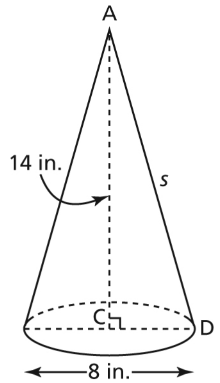
$A_L = \frac{1}{2}s(4l)$ Substitute: $s = \sqrt{200.25}$, $l = 15$

$A_L = \frac{1}{2}(\sqrt{200.25})(4)(15)$

$A_L = 424.5291\ldots$

The area that needs to be re-covered is approximately 425 m^2.

3. Sketch the cone. Let s represent the slant height.



CD is $\frac{1}{2}$ the diameter, so CD is 4 in.

Use the Pythagorean Theorem in right ΔACD to determine s.

$AD^2 = AC^2 + CD^2$
$s^2 = 14^2 + 4^2$
$s^2 = 212$
$s = \sqrt{212}$

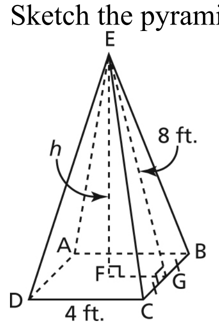
Use the formula for the lateral area, A_L, of a right cone with base radius r and slant height s.

$A_L = \pi rs$ Substitute: $r = 4$, $s = \sqrt{212}$
$A_L = \pi(4)(\sqrt{212})$
$A_L = 182.9691…$

The lateral area of the right cone is approximately 183 square inches.

1.5

4. a) Sketch the pyramid and label its vertices. Let h represent the height.



FG is $\frac{1}{2}$ the length of DC, so FG is 2 ft.

Use the Pythagorean Theorem in right ΔEFG to determine h.

$EF^2 = EG^2 - FG^2$
$h^2 = 8^2 - 2^2$
$h^2 = 60$
$h = \sqrt{60}$

Use the formula for the volume of a right rectangular pyramid.

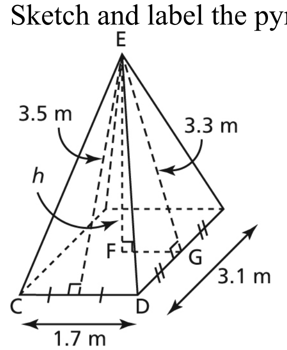
$V = \frac{1}{3}lwh$ Substitute: $l = 4$, $w = 4$, $h = \sqrt{60}$

$V = \frac{1}{3}(4)(4)(\sqrt{60})$

$V = 41.3118\ldots$

The volume of the pyramid is approximately 41 cubic feet.

b) Sketch and label the pyramid. Let h represent the height.



FG is $\frac{1}{2}$ the length of DC, so FG is 0.85 m.

Use the Pythagorean Theorem in right ΔEFG to determine h.

$EF^2 = EG^2 - FG^2$

$h^2 = 3.3^2 - 0.85^2$

$h^2 = 10.1675$

$h = \sqrt{10.1675}$

Use the formula for the volume of a right rectangular pyramid.

$V = \frac{1}{3}lwh$ Substitute: $l = 3.1$, $w = 1.7$, $h = \sqrt{10.1675}$

$V = \frac{1}{3}(3.1)(1.7)(\sqrt{10.1675})$

$V = 5.6013\ldots$

The volume of the pyramid is approximately 6 m^3.

c) The radius, r, of the base of the cone is $\frac{1}{2}$ the diameter: $\frac{1}{2}(24.2\text{ m}) = 12.1$ m

Use the formula for the volume of a right cone.

$V = \frac{1}{3}\pi r^2 h$ Substitute: $r = 12.1$, $h = 12.7$

$V = \frac{1}{3}\pi(12.1)^2(12.7)$

$V = 1947.1664\ldots$

The volume of the cone is approximately 1947 m^3.

5. a) Use the formula for the volume of a right rectangular pyramid to determine h.

$V = \frac{1}{3}lwh$ Substitute: $l = w = 7.3$, $V = 168.8$

$168.8 = \frac{1}{3}(7.3)(7.3)h$ Multiply both sides by 3.

$506.4 = 53.29h$ Solve for h. Divide both sides by 53.29.

$h = \dfrac{506.4}{53.29}$

$h = 9.5027\ldots$

h is approximately 9.5 cm.

b) Use the formula for the volume of a right cone to determine h.

$V = \dfrac{1}{3}\pi r^2 h$ Substitute: $r = 1.7$, $V = 8.2$

$8.2 = \dfrac{1}{3}\pi(1.7)^2 h$ Multiply both sides by 3.

$24.6 = 2.89\pi h$ Solve for h. Divide both sides by 2.89π.

$\dfrac{24.6}{2.89\pi} = \dfrac{2.89\pi h}{2.89\pi}$

$h = 2.7094\ldots$

h is approximately 2.7 m.

c) Use the formula for the volume of a right rectangular pyramid to determine x.

$V = \dfrac{1}{3}lwh$ Substitute: $V = 1594.5$, $l = w = x$, $h = 15.8$

$1594.5 = \dfrac{1}{3}(x)(x)(15.8)$ Multiply both sides by 3.

$4783.5 = 15.8x^2$ Solve for x. Divide both sides by 15.8.

$\dfrac{4783.5}{15.8} = \dfrac{15.8x^2}{15.8}$

$x^2 = \dfrac{4783.5}{15.8}$

$x = \sqrt{\dfrac{4783.5}{15.8}}$

$x = 17.3998\ldots$

x is approximately 17.4 cm.

1.6

6. **a)** Use the formula for the surface area of a sphere.

$SA = 4\pi r^2$ Substitute: $r = 8.8$

$SA = 4\pi(8.8)^2$

$SA = 973.1397\ldots$

The surface area of the sphere is approximately 973.1 km^2.

Use the formula for the volume of a sphere.

$V = \dfrac{4}{3}\pi r^3$ Substitute: $r = 8.8$

$V = \dfrac{4}{3}\pi(8.8)^3$

$V = 2854.5432\ldots$

The volume of the sphere is approximately 2854.5 km^3.

b) The radius, r, is: $\frac{1}{2}$(6.8 cm) = 3.4 cm

SA of a hemisphere = SA of one-half a sphere + area of circle

$SA = \frac{1}{2}(4\pi r^2) + \pi r^2$

$SA = 2\pi r^2 + \pi r^2$

$SA = 3\pi r^2$ Substitute: $r = 3.4$

$SA = 3\pi(3.4)^2$

$SA = 108.9504\ldots$

The surface area of the hemisphere is approximately 109.0 cm^2.

Volume of a hemisphere = volume of one-half a sphere

$V = \frac{1}{2}\left(\frac{4}{3}\pi r^3\right)$

$V = \frac{2}{3}\pi r^3$ Substitute: $r = 3.4$

$V = \frac{2}{3}\pi(3.4)^3$

$V = 82.3181\ldots$

The volume of the hemisphere is approximately 82.3 cm^3.

7. The circumference is the length of the longest circle that can be drawn on the surface of the spherical globe.
Use the formula for the circumference, C, of a circle to determine r.

$C = 2\pi r$ Substitute: $C = 158$

$158 = 2\pi r$ Solve for r. Divide both sides by 2π.

$\frac{158}{2\pi} = \frac{2\pi r}{2\pi}$

$r = \frac{158}{2\pi}$

$r = 25.1464\ldots$

Use the formula for the surface area of a sphere.

$SA = 4\pi r^2$ Substitute: $r = 25.1464\ldots$

$SA = 4\pi(25.1464\ldots)^2$

$SA = 7946.2879\ldots$

The area to be painted is approximately 7946 cm^2.

Lesson 1.7 **Solving Problems Involving Objects** **Exercises (pages 59–61)**

A

3. a) The surface area comprises the lateral area of the cone, one base of the cylinder, and the curved surface of the cylinder.

The cone and cylinder have equal radii.

Surface area of composite object is:
Lateral area of cone + area of base of cylinder + area of curved surface of cylinder
Use the algebraic formulas for surface area.

$SA = \pi rs + \pi r^2 + 2\pi rh$ Substitute: $r = 3$, $s = 5$, $h = 5$
$SA = \pi(3)(5) + \pi(3)^2 + 2\pi(3)(5)$
$SA = 15\pi + 9\pi + 30\pi$
$SA = 54\pi$
$SA = 169.6460\ldots$
The surface area of the composite object is approximately 170 cm^2.

b) The surface area comprises the lateral area of the right square pyramid, plus the areas of the four rectangular faces and base of the right square prism.

The base of the prism is a square.
Surface area of composite object is:
lateral area of right square pyramid + area of 4 rectangular faces + area of base of prism

Use the algebraic formulas for surface area.
$SA = \frac{1}{2}s(4l) + 4hw + lw$ Substitute $s = 13$, $l = w = 10$, $h = 17$
$SA = \frac{1}{2}(13)(4)(10) + 4(17)(10) + (10)(10)$
$SA = 1040$
The surface area of the composite object is 1040 square feet.

c) The surface area comprises the curved surface of the cylinder, plus the surface area of the right square prism.
The top face of the cylinder is equal to the area of the square prism that is covered by the bottom face of the cylinder. So, do not include the area of the top face of the cylinder and do include the top face of the prism.

Surface area of composite object is:
Curved surface of cylinder + 2(area of top face of right square prism) + 2(area of front face of right square prism) + 2(area of side face of right square prism)

Use the algebraic formulas for surface area.
$SA = 2\pi rh + 2lw + 2hw + 2hl$ Substitute: $r = 1$, $l = w = 5$, $h = 1$
$SA = 2\pi(1)(4) + 2(5)(5) + 2(1)(5) + 2(1)(5)$
$SA = 95.1327\ldots$
The surface area of the composite object is approximately 95 square inches.

d) The surface area comprises the curved surface of the hemisphere, plus the lateral area of the right cone.

The hemisphere and cone have equal radii.

Surface area of composite object is:
Area of curved surface of hemisphere + lateral area of cone

Use the algebraic formulas for surface area.

$SA = 2\pi r^2 + \pi rs$ Substitute: $r = 4$, $s = 17$

$SA = 2\pi(4)^2 + \pi(4)(17)$

$SA = 314.1592\ldots$

The surface area of the composite object is approximately 314 square inches.

4. a) I could calculate the volume of the object in part c without determining any further dimensions.

b) Use the formula for the volume of a right cylinder and the formula for the volume of a right rectangular prism.

$V = \pi r^2 H + lwh$ Substitute: $r = 1$, $H = 4$, $l = w = 5$, $h = 1$

$V = \pi(1)^2(4) + (5)(5)(1)$

$V = 37.5663\ldots$

The volume of the composite object is approximately 38 cubic inches.

B

5. a) The surface area comprises the curved surfaces of the two hemispheres, plus the curved surface of the cylinder.

The hemispheres and cylinder have equal radii.

The radius, r, is: $\frac{1}{2}(6.0 \text{ cm}) = 3.0 \text{ cm}$

Surface area of composite object is:
Area of curved surface of hemisphere + area of curved surface of cylinder + area of curved surface of hemisphere

Use the algebraic formulas for surface area.

$SA = 2\pi r^2 + 2\pi rh + 2\pi r^2$

$SA = 4\pi r^2 + 2\pi rh$ Substitute: $r = 3.0$, $h = 8.5$

$SA = 4\pi(3.0)^2 + 2\pi(3.0)(8.5)$

$SA = 273.3185\ldots$

The surface area of the composite object is approximately 273.3 cm^2.

Volume of composite object:
Volume of hemisphere + volume of cylinder + volume of hemisphere
= volume of sphere + volume of cylinder

Use the algebraic formulas for volume.

$V = \frac{4}{3}\pi r^3 + \pi r^2 h$ Substitute: $r = 3.0$, $h = 8.5$

$V = \frac{4}{3}\pi (3.0)^3 + \pi (3.0)^2(8.5)$

$V = 353.4291\ldots$

The volume of the composite object is approximately 353.4 cm^3.

b) The surface area comprises the surface area of the right square prism, the lateral area of the right square pyramid, minus the overlap.

Surface area of composite object is:
surface area of right square prism + lateral area of right square pyramid – overlap

The overlap is the base of the right square pyramid.

Use the algebraic formulas for surface area.

$SA = 4lH + 2lw + \frac{1}{2}s(4l) - lw$ Substitute: $l = 1.0$, $H = 2.0$, $w = 1.0$, $s = 1.5$

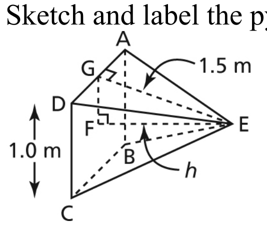
$SA = 4(1.0)(2.0) + 2(1.0)(1.0) + \frac{1}{2}(1.5)(4)(1.0) - (1.0)(1.0)$

$SA = 12.0$

The surface area of the composite object is 12.0 m^2.

The volume of the composite object is:
Volume of right square prism + volume of right square pyramid

Sketch and label the pyramid. Let h represent its height.



FG is $\frac{1}{2}$ the length of CD, so FG is 0.5 m.

Use the Pythagorean Theorem in right ΔEFG.

$FE^2 = GE^2 - FG^2$

$h^2 = 1.5^2 - 0.5^2$

$h^2 = 2$

$h = \sqrt{2}$

Use the algebraic formulas for volume.

$V = lwH + \frac{1}{3}lwh$ Substitute: $l = w = 1.0$, $H = 2.0$, $h = \sqrt{2}$

$V = (1.0)(1.0)(2.0) + \frac{1}{3}(1.0)(1.0)(\sqrt{2})$

$V = 2.4714\ldots$

The volume of the composite object is approximately 2.5 m^3.

6. a) Use the formula for the curved surface area of a right cylinder to determine r.

$$SA = 2\pi rh \qquad \text{Substitute: } SA = 219,\ h = 12$$
$$219 = 2\pi r(12)$$
$$219 = 24\pi r \qquad \text{Solve for } r\text{. Divide both sides by } 24\pi\text{.}$$
$$\frac{219}{24\pi} = \frac{24\pi r}{24\pi}$$
$$r = \frac{219}{24\pi}$$

The diameter, d, is:

$$d = 2r \qquad \text{Substitute: } r = \frac{219}{24\pi}$$
$$d = 2\left(\frac{219}{24\pi}\right)$$
$$d = 5.809\ldots$$

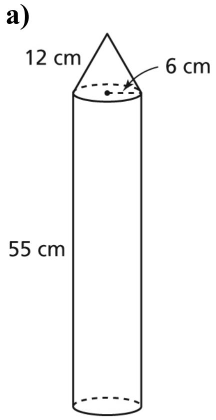
d is approximately $5\frac{4}{5}$ in.

b) Use the formula for the surface area of a right cylinder to determine h.

$$SA = 2\pi r^2 + 2\pi rh \qquad \text{Substitute: } SA = 137.2,\ r = 2.4$$
$$137.2 = 2\pi(2.4)^2 + 2\pi(2.4)h$$
$$137.2 = 11.52\pi + 4.8\pi h$$
$$137.2 - 11.52\pi = 4.8\pi h \qquad \text{Solve for } h\text{. Divide both sides by } 4.8\pi\text{.}$$
$$\frac{137.2 - 11.52\pi}{4.8\pi} = \frac{4.8\pi h}{4.8\pi}$$
$$h = \frac{137.2 - 11.52\pi}{4.8\pi}$$
$$h = 6.6983\ldots$$

h is approximately 6.7 cm.

7. a)



b) The surface area of the composite object comprises the curved surface area of the cylinder plus the area of one base of the cylinder, plus the lateral surface area of the cone.

The cone and cylinder have equal radii.

Since the area of the bottom base of the cylinder is equal to the area of the base of the cone, the surface area of composite object:
Curved surface area of cylinder + surface area of cone

Use the algebraic formulas for surface area.

$SA = 2\pi rH + \pi rs + \pi r^2$ Substitute: $r = 6$, $H = 55$, $s = 12$

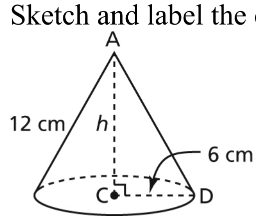
$SA = 2\pi(6)(55) + \pi(6)(12) + \pi(6)^2$

$SA = 2412.7431\ldots$

The surface area of the rocket is approximately 2413 cm^2.

c) The volume of the rocket is the volume of the cone, plus the volume of the cylinder.

Sketch and label the cone. Let h represent its height.



Use the Pythagorean Theorem in right ΔACD to determine h.

$AC^2 = AD^2 - CD^2$

$h^2 = 12^2 - 6^2$

$h^2 = 108$

$h = \sqrt{108}$

Use the algebraic formulas for volume.

$V = \frac{1}{3}\pi r^2 h + \pi r^2 H$ Substitute: $r = 6$, $h = \sqrt{108}$, $H = 55$

$V = \frac{1}{3}\pi(6)^2(\sqrt{108}) + \pi(6)^2(55)$

$V = 6612.1341\ldots$

The volume of the rocket is approximately 6612 cm^3.

d) One-third of the interior space is one-third of the volume of the rocket.

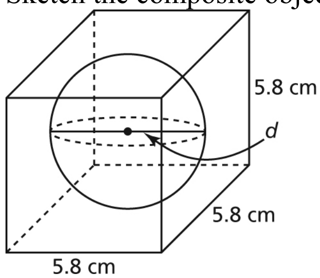
$V_{interior} = \frac{1}{3}V_{rocket}$ Substitute: $V_{rocket} = 6612.1341\ldots$

$V_{interior} = \frac{1}{3}(6612.1341\ldots)$

$V_{interior} = 2204.0447\ldots$

The rocket can hold approximately 2204 cm^3, or 2204 mL of fuel.

8. Sketch the composite object.



Determine the volume of the sphere and the volume of the cube.

The radius, r, of the sphere is: $\frac{1}{2}$(5.8 cm) = 2.9 cm

Use the formula for the volume of a sphere.

$V = \frac{4}{3}\pi r^3$ Substitute: $r = 2.9$

$V = \frac{4}{3}\pi(2.9)^3$

$V = 102.1604\ldots$

Use the formula for the volume of a cube.

$V = lwh$ Substitute: $l = w = h = 5.8$

$V = (5.8)(5.8)(5.8)$

$V = 195.112$

The volume of air in the cube is:

Volume of cube – volume of sphere = 195.112 – 102.1604…

= 92.9515…

The volume of air in the cube is approximately 93 cm^3.

9. **a)** Determine the volumes of the bins, then compare them.

The first bin comprises a right square prism and a right square pyramid.
Use the algebraic formulas for volume.

$V = lwh + \frac{1}{3}lwH$ Substitute: $l = w = 15$, $h = 10$, $H = 4$

$V = (15)(15)(10) + \frac{1}{3}(15)(15)(4)$

$V = 2250 + 300$

$V = 2550$

The volume of the first bin is 2550 cubic feet.

The second bin comprises a right cylinder and a right cone.
Use the algebraic formulas for volume.

$V = \pi r^2 h + \frac{1}{3}\pi r^2 H$ Substitute: $r = 12$, $h = 8$, $H = 3$

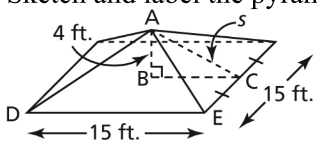
$V = \pi(12)^2(8) + \frac{1}{3}\pi(12)^2(3)$

$V = 3619.1147\ldots + 452.3893\ldots$
$V = 4071.5040\ldots$
The volume of the second bin is approximately 4072 cubic feet.

Since 4072 > 2550, the second bin holds more grain.

b) Determine the surface areas of the bins, then compare them.

The surface area of the first bin is the sum of the areas of the 4 walls, plus the lateral area of the pyramid. First determine the slant height, *s,* of the right square pyramid.
Sketch and label the pyramid.



In ΔABC, BC is $\frac{1}{2}$ the length of DE, so BC is $7\frac{1}{2}$ ft.
Use the Pythagorean Theorem in ΔABC to determine *s*.
The slant height, *s*, of the pyramid is:
$AC^2 = AB^2 + BC^2$
$s^2 = 4^2 + 7.5^2$
$s^2 = 72.25$
$s = \sqrt{72.25}$

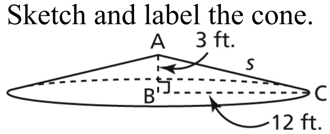
Use the algebraic formulas for surface area.

$SA = 4lh + \frac{1}{2}s(4l)$ Substitute: $l = 15$, $h = 10$, $s = \sqrt{72.25}$

$SA = 4(15)(10) + \frac{1}{2}(\sqrt{72.25})(4)(15)$

$SA = 855$
The first bin has a surface area of 855 square feet.
The cost to build this bin is:
855($10.49) = $8968.95
The cost to build the first bin is approximately $8970.

The surface area of the second bin is the area of the curved surface of the cylinder, plus the lateral area of the cone. First determine the slant height, *s*, of the cone.
Sketch and label the cone.



Use the Pythagorean Theorem in ΔABC to determine *s*.
$AC^2 = AB^2 + BC^2$
$s^2 = 3^2 + 12^2$
$s^2 = 153$
$s = \sqrt{153}$

Use the algebraic formulas for surface area.
$SA = 2\pi rh + \pi rs$ Substitute: $r = 12$, $h = 8$, $s = \sqrt{153}$

$SA = 2\pi(12)(8) + \pi(12)(\sqrt{153})$
$SA = 1069.4980\ldots$
The second bin has a surface area of approximately 1069 square feet.
The cost to build this bin is:
$(1069.4980\ldots)(\$9.25) = \$9892.8569\ldots$
The cost to build the second bin is approximately \$9890.

Since \$8970 < \$9890, it is cheaper to build the first bin.

10. a) The volume of the object is:
Volume of right square prism – volume of right square pyramid

Use the algebraic formulas for volume.

$V = lwH - \frac{1}{3}lwh$ Substitute: $l = w = 10$, $H = 15$, $h = 6$

$V = (10)(10)(15) - \frac{1}{3}(10)(10)(6)$

$V = 1300$
The volume of the object is 1300.0 cm^3.

b) The volume of the object is:
Volume of right cylinder – volume of hemisphere

The cylinder and hemisphere have equal radii.

The radius, r, is: $\frac{1}{2}(1.5 \text{ m}) = 0.75$ m

Use the algebraic formulas for volume.

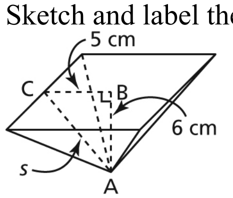
$V = \pi r^2 h - \frac{2}{3}\pi r^3$ Substitute: $r = 0.75$, $h = 4.0$

$V = \pi(0.75)^2(4.0) - \frac{2}{3}\pi(0.75)^3$

$V = 6.1850\ldots$
The volume of the object is approximately 6.2 m^3.

11. a) The surface area of the object is:
Area of 4 rectangular faces of prism + area of base of prism + lateral area of pyramid

First determine the slant height, s, of the pyramid.
Sketch and label the pyramid.



Use the Pythagorean Theorem in ΔABC.
$AC^2 = AB^2 + BC^2$
$s^2 = 6^2 + 5^2$
$s^2 = 61$

$s = \sqrt{61}$

Use the algebraic formulas for surface area.

$SA = 4lh + lw + \frac{1}{2}s(4l)$ Substitute: $l = 10$, $h = 15$, $w = 10$, $s = \sqrt{61}$

$SA = 4(10)(15) + (10)(10) + \frac{1}{2}(\sqrt{61})(4)(10)$

$SA = 856.2049\ldots$

The surface area of the object is approximately 856.2 cm^2.

b) The surface area of the object is:
Area of the curved surface of the cylinder + area of base of cylinder + area of curved surface of hemisphere

Use the algebraic formulas for surface area.

$SA = 2\pi rh + \pi r^2 + 2\pi r^2$

$SA = 2\pi rh + 3\pi r^2$ Substitute: $r = 0.75$, $h = 4$

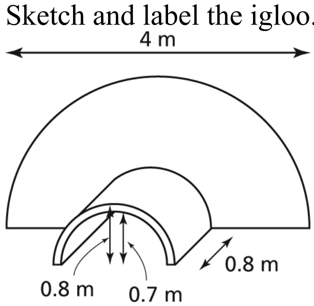
$SA = 2\pi(0.75)(4) + 3\pi(0.75)^2$

$SA = 24.1509\ldots$

The surface area of the object is approximately 24.2 m^2.

C

12. Sketch and label the igloo.



The surface area is:

curved surface area of hemisphere – $\frac{1}{2}$ area of base of cylinder + $\frac{1}{2}$ curved surface area of cylinder + surface area of the entrance

The curved surface area of hemisphere is: $2\pi r^2$, where $r = 2.0$

So, the curved surface area of hemisphere is: $2\pi(2)^2 = 8\pi$

$\frac{1}{2}$ area of base of cylinder is: $\frac{1}{2}\pi r^2$, where $r = 0.8$

So, $\frac{1}{2}$ area of base of cylinder is: $\frac{1}{2}\pi(0.8)^2$

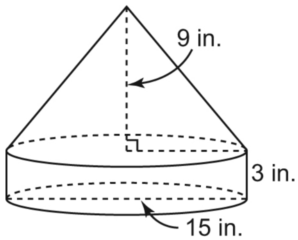
$\frac{1}{2}$ curved surface area of cylinder is: πrh, where $r = 0.8$ and $h = 0.8$

So, $\frac{1}{2}$ curved surface area of cylinder is: $\pi(0.8)(0.8) = \pi(0.8)^2$

Surface area of the entrance is the difference in areas of those semicircles with radii 0.8 m and 0.7 m; that is, $\frac{1}{2}\pi(0.8)^2 - \frac{1}{2}\pi(0.7)^2$

So, the total surface area *SA* is:
$SA = 8\pi - \frac{1}{2}\pi(0.8)^2 + \pi(0.8)^2 + \frac{1}{2}\pi(0.8)^2 - \frac{1}{2}\pi(0.7)^2$
$SA = 26.3736\ldots$
The surface area of the outside of the igloo and tunnel is approximately 26.4 m^2.

13. a) Sketch and label the composite object.



The volume of the composite object is:
Volume of right cylinder + volume of right cone

The radius, *r*, is: $\frac{1}{2}(15 \text{ in.}) = 7\frac{1}{2}$ in.

Use the algebraic formulas for volume.
$V = \pi r^2 H + \frac{1}{3}\pi r^2 h$ Substitute: $r = 7.5$, $H = 3$, $h = 9$
$V = \pi(7.5)^2(3) + \frac{1}{3}\pi(7.5)^2(9)$
$V = 1060.2875\ldots$
The volume of the sculpture is approximately 1060 cubic inches.

b) The sculpture has diameter 15 in. and total height: 9 in. + 3 in. = 12 in. So, the least possible dimensions for the right square prism are 15 in. by 15 in. by 12 in.

c) Determine the volume of the right rectangular prism.
Use the algebraic formula for volume.
$V = lwh$ Substitute: $l = 16$, $w = 15$, $h = 12$
$V = (16)(15)(12)$
$V = 2880$

The volume of ice remaining is:
Volume of prism – volume of sculpture = 2880 – 1060.2875…
= 1819.7124…
The volume of ice remaining is approximately 1820 cubic inches.

Review (pages 64–66)

1.1

1. Answers may vary. For example:

a) Inch; my arm is between 2 ft. and 3 ft. long, but for the most accurate measure I'd use inches.

b) Foot; the measurement would be more accurate in feet than yards.

c) Yard; the length of the gym is measured in yards, and since I ran the length of the gym many times, I'd measure my distance in yards.

2. Answers will vary. For example:

a) The length of my arm: The referent was the length of my thumb to the first knuckle, which is approximately 1 inch. I estimated that the length of my arm from shoulder to fingertip is 27 in.
The width of my classroom: My referent was my foot length from heel to toe, which is a little less than 1 ft. I estimated that the width of my classroom is approximately 25 ft.

b) I used a ruler to measure the width of the length of my arm: 26 in.
My estimate was reasonable.
I used a tape measure to measure the width of the classroom: 24 ft.
My estimate was reasonable.

3. a) Since 1 yd. = 3 ft., to convert yards to feet, multiply by 3.
14 yd. = 14(3 ft.)
14 yd. = 42 ft.

b) Since 1 mi. = 1760 yd., to convert miles to yards, multiply by 1760.
5 mi. = 5(1760 yd.)
5 mi. = 8800 yd.

c) Convert 6 ft. to inches.
Since 1 ft. = 12 in., to convert feet to inches, multiply by 12.
6 ft. = 6(12 in.)
6 ft. = 72 in.
So, 6 ft. 3 in. = 72 in. + 3 in.
= 75 in.

d) First convert inches to feet.
Since 12 in. = 1 ft., divide by 12.

$123 \text{ in.} = \dfrac{123}{12} \text{ ft.}$

$123 \text{ in.} = 10\dfrac{3}{12} \text{ ft.}$

123 in. = 10 ft. 3 in.
Now convert feet to yards.
Since 3 ft. = 1 yd., divide by 3.

$10 \text{ ft.} = \dfrac{10}{3} \text{ yd.}$

$10 \text{ ft.} = 3\frac{1}{3} \text{ yd.}$
10 ft. = 3 yd. 1 ft.
So, 123 in. = 3 yd. 1 ft. 3 in.

4. The scale of the model is 1 in. represents 40 in.
So, 8 in. represents 8(40 in.) = 320 in.

First convert inches to feet.
Since 12 in. = 1 ft., divide by 12.

$320 \text{ in.} = \frac{320}{12} \text{ ft.}$

$320 \text{ in.} = 26\frac{8}{12} \text{ ft.}$

320 in. = 26 ft. 8 in.
Now convert feet to yards.
Since 3 ft. = 1 yd., divide by 3.

$26 \text{ ft.} = \frac{26}{3} \text{ yd.}$

$26 \text{ ft.} = 8\frac{2}{3} \text{ yd.}$

26 ft. = 8 yd. 2 ft.

So, the actual plane is 320 in., or 8 yd. 2 ft. 8 in.

1.2

5. Strategies may vary. For example:

a) I would use the width of my hand, which is about 4 in. or 10 cm, to estimate the diameter. I would place one hand next to the other, beginning where the tire touches the ground, and measuring to the highest point on the tire.
I would use a tape measure to measure the diameter in imperial units or SI units.

b) I would use the length of my stride, which is about 0.6 m or about 2 ft., to estimate the length of a car. I would use a tape measure marked in imperial or SI units to measure the length.

c) I would use the width of my thumb as a referent, which is approximately 1.5 cm or $\frac{1}{2}$ in.
I would use calipers to measure the diameter in inches or centimetres, then divide by 2 to determine the radius.

1.3

6. Answers will vary depending on the conversion ratios used.

a) Use the exact conversion: 2.54 cm = 1 in.

So, $261 \text{ cm} = \frac{261}{2.54} \text{ in.}$

261 cm = 102.755… in.
$261 \text{ cm} \doteq 103 \text{ in.}$

12 in. = 1 ft.

So, 103 in. = $\frac{103}{12}$ ft.

103 in. = $8\frac{7}{12}$ ft.

103 in. = 8 ft. 7 in.

So, 261 cm ≐ 8 ft. 7 in.

b) Use the exact conversion:
1 yd. = 91.44 cm
1 yd. = 0.9144 m
Divide to convert metres to yards.

125 m = $\frac{125}{0.9144}$ yd.

125 m = 136.7016… yd.

1 yd. = 3 ft.
So, 0.7016… yd. = 0.7016…(3 ft.)
0.7016… yd. = 2.1049… ft.

1 ft. = 12 in.
So, 0.1049… ft. = 0.1049…(12 in.)
0.1049… ft. = 1.2598… in.

So, 125 m ≐ 136 yd. 2 ft. 1 in.

c) Use: 1 km ≐ $\frac{6}{10}$ mi.

So, 6 km = $6\left(\frac{6}{10}\right)$ mi.

6 km = 3.6 mi.

Use: 1 mi. = 1760 yd.
So, 0.6 mi. = 0.6(1760) yd.
0.6 mi. = 1056 yd.

So, 6 km ≐ 3 mi. 1056 yd.

d) Use: 10 mm = 1 cm
So, 350 mm = 35 cm

Use: 2.54 cm = 1 in.

So, 35 cm = $\frac{35}{2.54}$ in.

35 cm = 13.7795… in.
35 cm ≐ 14 in., or 1 ft. 2 in.

So, 350 mm ≐ 1 ft. 2 in.

7. **a)** First, convert 13 yd. 2 ft. to feet.
Use the conversion: 1 yd. = 3 ft.
So, 13 yd. = 13(3 ft.)
13 yd. = 39 ft.
So, 13 yd. 2 ft. = 39 ft. + 2 ft.
13 yd. 2 ft. = 41 ft.
Since 1 in. = 2.54 cm and 12 in. = 1 ft.,
1 ft. = 12(2.54 cm)
1 ft. = 30.48 cm
So, 41 ft. = 41(30.48 cm)
41 ft. = 1249.68 cm
41 ft. = 12.49… m
41 ft. ≐ 12.5 m

So, 13 yd. 2 ft. ≐ 12.5 m

b) First, convert 4 mi. 350 yd. to yards. Use the conversion 1 mi. = 1760 yd.
Then, 4 mi. = 4(1760 yd.)
4 mi. = 7040 yd.
So, 4 mi. 350 yd. = 7040 yd. + 350 yd.
4 mi. 350 yd. = 7390 yd.

Then, use the conversion: 1 yd. = 91.44 cm
7390 yd. = 7390(91.44 cm)
7390 yd. = 675 741.6 cm
7390 yd. = 6.7574… km
7390 yd. ≐ 6.8 km

So, 4 mi. 350 yd. ≐ 6.8 km

c) First, convert 1 ft. 7 in. to inches. Use the conversion: 1 ft. = 12 in.
So, 1 ft. 7 in. = 12 in. + 7 in.
1 ft. 7 in. ≐ 19 in.

Then, use the conversion: 1 in. = 2.54 cm
So, 19 in. = 19(2.54 cm)
19 in. = 48.26 cm
19 in. ≐ 48.3 cm

So, 1 ft. 7 in. ≐ 48.3 cm

d) Use the conversion: 1 in. = 2.54 cm
$8\frac{1}{2}$ in. = 8.5 in.

So, $8\frac{1}{2}$ in. = 8.5(2.54 cm)

$8\frac{1}{2}$ in. = 21.59 cm

So, $8\frac{1}{2}$ in. = 215.9 mm

8. First convert 460 km to centimetres.
460 km = 460(1000) m
460 km = 460(1000)(100) cm
460 km = 46 000 000 cm

Now convert 46 000 000 cm to inches.
Use: 2.54 cm = 1 in.
So, 46 000 000 cm = $\frac{46\ 000\ 000}{2.54}$ in.
46 000 000 cm = 18 110 236.22… in.

The length of Vancouver Island is 18 110 236.22… in. If Sarah's average stride is 27 in., divide to determine how many strides she takes:
$\frac{18\ 110\ 236.22\text{... in.}}{27\text{ in.}} = 670\ 749.4896\ldots$
Sarah takes approximately 670 750 strides.

1.4
9. **a)** Use the formula for the surface area of a right cone.
$SA = \pi rs + \pi r^2$ Substitute: $r = 3$, $s = 5$
$SA = \pi(3)(5) + \pi(3)^2$
$SA = 75.3982\ldots$
The surface area of the cone is approximately 75 square feet.

b) Use the formula for the surface area of a right pyramid with a regular triangle base.
$SA = \frac{1}{2}s(3l) + \frac{1}{2}sl$ Substitute: $s = 6.1$, $l = 7.0$
$SA = \frac{1}{2}(6.1)(3)(7.0) + \frac{1}{2}(7.0)(6.1)$
$SA = 85.4$
The surface area of the regular tetrahedron is approximately 85 cm^2.

c) The radius, r, is: $\frac{1}{2}(22\text{ mm}) = 11\text{ mm}$

Use the formula for the surface area of a right cone.
$SA = \pi rs + \pi r^2$ Substitute: $r = 11$, $s = 15$
$SA = \pi(11)(15) + \pi(11)^2$
$SA = 898.4954\ldots$
The surface area of the cone is approximately 898 mm^2.

d) The surface area is the area of the 4 triangular faces, plus the area of the base. Opposite triangular faces are congruent.

The area, A, of the front triangular face is:
$A = \frac{1}{2}(7.8)(11.2)$
$A = 43.68$
Since the front and back triangular faces are congruent, the area of the back triangular face is 43.68.

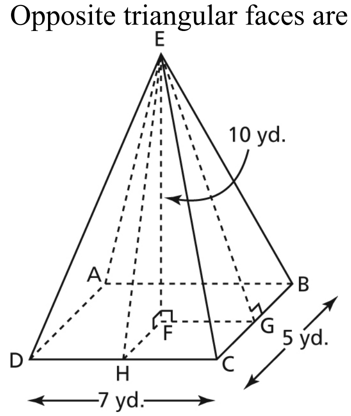
The area, A, of the triangular face at the right is:
$A = \frac{1}{2}(5.4)(11.6)$
$A = 31.32$
Since the triangular faces at the right and left are congruent, the area of the triangular face at the left is 31.32.

The area, B, of the base of the pyramid is:
$B = (7.8)(5.4)$
$B = 42.12$

Surface area, SA, of the pyramid is:
$SA = 2(43.68) + 2(31.32) + 42.12$
$SA = 192.12$

The surface area of the pyramid is approximately 192 m^2.

10. There are 4 triangular faces and a rectangular base. Sketch the pyramid and label its vertices. Opposite triangular faces are congruent. Draw the heights on two adjacent triangles.



In ΔEFH, FH is $\frac{1}{2}$ the length of BC, so FH is $2\frac{1}{2}$ yd.
EF is the height of the pyramid, which is 10 yd.
Use the Pythagorean Theorem in right ΔEFH to determine the length of EH.
$EH^2 = EF^2 + FH^2$
$EH^2 = 10^2 + 2.5^2$
$EH^2 = 106.25$
$EH = \sqrt{106.25}$

Area, A, of ΔEDC is:

$A = \frac{1}{2}(7)(\sqrt{106.25})$

$A = 3.5(\sqrt{106.25})$

Since ΔEDC and ΔEAB are congruent, the area of ΔEAB is $3.5(\sqrt{106.25})$.

In ΔEFG, FG is $\frac{1}{2}$ the length of DC, so FG is $3\frac{1}{2}$ yd.

Use the Pythagorean Theorem in right ΔEFG to determine the length of GE.

$GE^2 = EF^2 + FG^2$

$GE^2 = 10^2 + 3.5^2$

$GE^2 = 112.25$

$GE = \sqrt{112.25}$

Area, A, of ΔEBC is:

$A = \frac{1}{2}(5)(\sqrt{112.25})$

$A = 2.5(\sqrt{112.25})$

Since ΔEBC and ΔEAD are congruent, the area of ΔEAD is $2.5(\sqrt{112.25})$.

Area, B, of the base of the pyramid is:

$B = (7)(5)$

$B = 35$

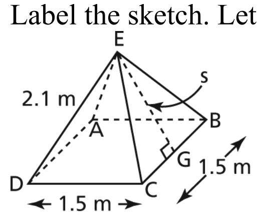
Surface area, SA, of the pyramid is:

$SA = 2(3.5)(\sqrt{106.25}) + 2(2.5)(\sqrt{112.25}) + 35$

$SA = 160.1283\ldots$

The surface area of the pyramid is approximately 160 square yards.

11. a) Label the sketch. Let s represent the slant height of the tent.



b) In ΔEGB, GB is $\frac{1}{2}$ the length of CB, so GB is 0.75 m.

Use the Pythagorean Theorem in right ΔEGB.

$EG^2 = EB^2 - GB^2$

$s^2 = 2.1^2 - 0.75^2$

$s^2 = 3.8475$

$s = \sqrt{3.8475}$

$s = 1.9615\ldots$

The slant height of the tent is approximately 2.0 m.

c) The lateral area, A_L, of the tent is:

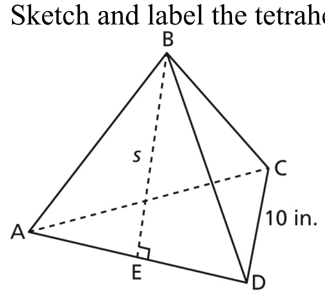
$A_L = \left(\frac{1}{2}s\right)(4l)$ Substitute: $s = \sqrt{3.8475}$, $l = 1.5$

$A_L = \frac{1}{2}(\sqrt{3.8475})(4)(1.5)$

$A_L = 5.8845\ldots$

The lateral surface area of the tent is approximately 6 m^2.

12. a) Sketch and label the tetrahedron. Let s represent the slant height.



In ΔBED, ED is $\frac{1}{2}$ the length of AD, so ED is 5 in.

Use the Pythagorean Theorem in right ΔBED.

$BE^2 = BD^2 - ED^2$

$s^2 = 10^2 - 5^2$

$s^2 = 75$

$s = \sqrt{75}$

$s = 8.6602\ldots$

The slant height of the tetrahedron is approximately $8\frac{7}{10}$ in.

b) The area, A, of each face of the tetrahedron is:

$A = \frac{1}{2}(10)(\sqrt{75})$

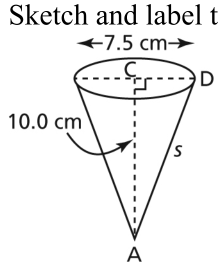
The surface area, SA, is:

$SA = 4\left(\frac{1}{2}\right)(10)(\sqrt{75})$

$SA = 173.2050\ldots$

The surface area of the tetrahedron is approximately 173 square inches.

13. Sketch and label the cone. Let s represent the slant height.



The radius, r, is: $\frac{1}{2}$(7.5 cm) = 3.75 cm

Use the Pythagorean Theorem in right ΔACD to determine s.

$AD^2 = AC^2 + CD^2$

$s^2 = 10^2 + 3.75^2$

$s^2 = 114.0625$

$s = \sqrt{114.0625}$

Use the formula for the lateral area of a right cone.

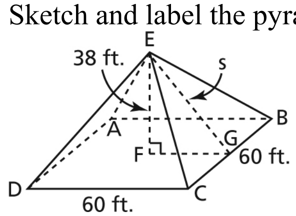
$SA = \pi rs$ Substitute: $r = 3.75$, $s = \sqrt{114.0625}$

$SA = \pi(3.75)(\sqrt{114.0625})$

$SA = 125.8208\ldots$

The area to be coated is approximately 125.8 cm^2.

14. Sketch and label the pyramid. Let s represent the slant height.



In ΔEFG, FG is $\frac{1}{2}$ the length of DC, so FG is 30 ft.

Use the Pythagorean Theorem in right ΔEFG to determine s.

$EG^2 = EF^2 + FG^2$

$s^2 = 38^2 + 30^2$

$s^2 = 2344$

$s = \sqrt{2344}$

Determine the lateral area, A_L, of the pyramid.

$A_L = \left(\frac{1}{2}s\right)(4l)$ Substitute: $s = \sqrt{2344}$, $l = 60$

$A_L = \frac{1}{2}(\sqrt{2344})(4)(60)$

$A_L = 5809.7848\ldots$

The area of limestone needed is approximately 5810 square feet.

1.5

15. a) Use the formula for the volume of a right rectangular pyramid.

$V = \frac{1}{3}lwh$ Substitute: $l = 3.5$, $w = 2.0$, $h = 4.5$

$V = \frac{1}{3}(3.5)(2.0)(4.5)$

$V = 10.5$

The volume of the pyramid is approximately 11 m^3.

b) Use the formula for the volume of a right cone.

$V = \frac{1}{3}\pi r^2 h$ Substitute: $r = 18$, $h = 26$

$V = \frac{1}{3}\pi(18)^2(26)$
$V = 8821.5921\ldots$
The volume of the cone is approximately 8822 cubic inches.

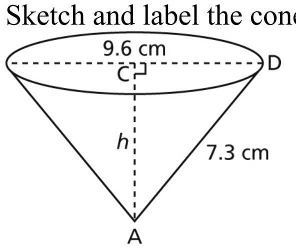
c) Use the formula for the volume of a right rectangular pyramid.
$V = \frac{1}{3}lwh$ Substitute: $l = 2$, $w = 2$, $h = 5$
$V = \frac{1}{3}(2)(2)(5)$
$V = 6.6666\ldots$
The volume of the pyramid is approximately 7 cubic feet.

d) The radius, r, is: $\frac{1}{2}(13 \text{ mm}) = 6.5 \text{ mm}$

Use the formula for the volume of a right cone.
$V = \frac{1}{3}\pi r^2 h$ Substitute: $r = 6.5$, $h = 5$
$V = \frac{1}{3}\pi(6.5)^2(5)$
$V = 221.2204\ldots$
The volume of the cone is approximately 221 mm^3.

16. No, Owen needs to determine the height of the cone to determine its volume.

Sketch and label the cone. Let h represent the height of the cone.



The radius, r, is: $\frac{1}{2}(9.6 \text{ cm}) = 4.8 \text{ cm}$

Use the Pythagorean Theorem in right ΔACD to determine h.
$AC^2 = AD^2 - CD^2$
$h^2 = 7.3^2 - 4.8^2$
$h^2 = 30.25$
$h = \sqrt{30.25}$
$h = 5.5$

Use the formula for the volume of a right cone.
$V = \frac{1}{3}\pi r^2 h$ Substitute: $r = 4.8$, $h = 5.5$

$V = \frac{1}{3}\pi(4.8)^2(5.5)$
$V = 132.7008\ldots$
The volume of the cone is approximately 132.7 cm^3.

17. Use the formula for the volume of a right rectangular pyramid to determine h.

$V = \frac{1}{3}lwh$ Substitute: $V = 400$, $l = w = 10$

$400 = \frac{1}{3}(10)(10)h$

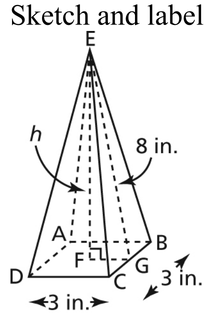
$1200 = 100h$ Solve for h.

$\frac{1200}{100} = \frac{100h}{100}$

$h = 12$

The height of the pyramid was 12 cm.

18. a) Sketch and label the pyramid. Let h represent its height.



In ΔEFG, FG is $\frac{1}{2}$ the length of DC, so FG is $1\frac{1}{2}$ in.

Use the Pythagorean Theorem in right ΔEFG to determine h.

$EF^2 = EG^2 - FG^2$
$h^2 = 8^2 - 1.5^2$
$h^2 = 61.75$
$h = \sqrt{61.75}$

Use the formula for the volume of a right rectangular pyramid.

$V = \frac{1}{3}lwh$ Substitute: $l = 3$, $w = 3$, $h = \sqrt{61.75}$

$V = \frac{1}{3}(3)(3)(\sqrt{61.75})$

$V = 23.5743\ldots$

The volume of the ornament is approximately 24 cubic inches.

b) Let x inches represent the side length of the base.
Use the formula for the volume of a right rectangular pyramid to determine x.

$V = \frac{1}{3}lwh$ Substitute: $V = 96$, $l = w = x$, $h = \sqrt{61.75}$

$96 = \frac{1}{3}(x)(x)(\sqrt{61.75})$ Multiply both sides by 3.

$$288 = \sqrt{61.75}\,x^2 \qquad \text{Divide both sides by } \sqrt{61.75}.$$
$$\frac{288}{\sqrt{61.75}} = \frac{\sqrt{61.75}x^2}{\sqrt{61.75}}$$
$$x^2 = \frac{288}{\sqrt{61.75}}$$
$$x = \sqrt{\frac{288}{\sqrt{61.75}}}$$
$$x = 6.0539\ldots$$

The side length of the base is approximately 6 in.

19. a) Use the formula for the volume of a right cone to determine r.

$$V = \frac{1}{3}\pi r^2 h \qquad \text{Substitute: } V = 41.6,\ h = 9.0$$
$$41.6 = \frac{1}{3}\pi r^2(9.0) \qquad \text{Multiply both sides by 3.}$$
$$124.8 = 9.0\pi r^2 \qquad \text{Solve for } r\text{. Divide both sides by } 9.0\pi.$$
$$\frac{124.8}{9.0\pi} = \frac{9.0\pi r^2}{9.0\pi}$$
$$r^2 = \frac{124.8}{9.0\pi}$$
$$r = \sqrt{\frac{124.8}{9.0\pi}}$$
$$r = 2.1009\ldots$$

r is approximately 2.1 m.

b) Use the formula for the volume of a right rectangular pyramid to determine x.

$$V = \frac{1}{3}lwh \qquad \text{Substitute: } V = 68.4,\ l = 9.0,\ w = x,\ h = 10.0$$
$$68.4 = \frac{1}{3}(9.0)(x)(10.0) \qquad \text{Multiply both sides by 3.}$$
$$205.2 = 90.0x \qquad \text{Solve for } x\text{. Divide both sides by 3.}$$
$$\frac{205.2}{90.0} = \frac{90.0x}{90.0}$$
$$x = \frac{205.2}{90.0}$$
$$x = 2.28$$

x is approximately 2.3 cm.

1.6

20. a) Use the formula for the surface area of a sphere.

$SA = 4\pi r^2$ Substitute: $r = 4.5$

$SA = 4\pi(4.5)^2$

$SA = 254.4690\ldots$

The surface area of the sphere is approximately 254 square inches.

Use the formula for the volume of a sphere.

$V = \frac{4}{3}\pi r^3$ Substitute: $r = 4.5$

$V = \frac{4}{3}\pi (4.5)^3$

$V = 381.7035\ldots$

The volume of the sphere is approximately 382 cubic inches.

b) The radius, r, is: $\frac{1}{2}(6.5 \text{ m}) = 3.25 \text{ m}$

Use the formula for the surface area of a sphere.

$SA = 4\pi r^2$ Substitute: $r = 3.25$

$SA = 4\pi (3.25)^2$

$SA = 132.7322\ldots$

The surface area of the sphere is approximately 133 m^2.

Use the formula for the volume of a sphere.

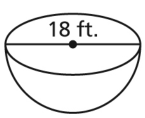
$V = \frac{4}{3}\pi r^3$ Substitute: $r = 3.25$

$V = \frac{4}{3}\pi (3.25)^3$

$V = 143.7933\ldots$

The volume of the sphere is approximately 144 m^3.

21.



a) The radius, r, of the hemisphere is: $\frac{1}{2}(18 \text{ ft.}) = 9 \text{ ft.}$

SA of a hemisphere = *SA* of one-half a sphere + area of circle

$SA = \frac{1}{2}(4\pi r^2) + \pi r^2$

$SA = 2\pi r^2 + \pi r^2$

$SA = 3\pi r^2$ Substitute: $r = 9$

$SA = 3\pi (9)^2$

$SA = 763.4070\ldots$

The surface area of the hemisphere is approximately 763 square feet.

b) Volume of a hemisphere = volume of one-half a sphere

$V = \frac{1}{2}\left(\frac{4}{3}\pi r^3\right)$

$V = \frac{2}{3}\pi r^3$ Substitute: $r = 9$

$V = \frac{2}{3}\pi(9)^3$
$V = 1526.8140\ldots$
The volume of the hemisphere is approximately 1527 cubic feet.

22. Use the formula for the surface area of a sphere to determine r.

$SA = 4\pi r^2$	Substitute: $SA = 66$
$66 = 4\pi r^2$	Solve for r. Divide both sides by 4π.
$\frac{66}{4\pi} = \frac{4\pi r^2}{4\pi}$	
$r^2 = \frac{66}{4\pi}$	
$r = \sqrt{\frac{66}{4\pi}}$	

The diameter, d, is:
$d = 2r$
$d = 2\left(\sqrt{\frac{66}{4\pi}}\right)$
$d = 4.5834\ldots$
$d \doteq 4.6$
The diameter of the sphere is approximately $4\frac{6}{10}$ in., or $4\frac{3}{5}$ in.

23. Use the formula for circumference, C, to determine r.

$C = 2\pi r$	Substitute: $C = 18$
$18 = 2\pi r$	Solve for r. Divide both sides by 2π.
$\frac{18}{2\pi} = \frac{2\pi r}{2\pi}$	
$r = \frac{18}{2\pi}$	
$r = 2.8647\ldots$	

Use the formula for the volume of a sphere.

$V = \frac{4}{3}\pi r^3$	Substitute: $r = 2.8647\ldots$
$V = \frac{4}{3}\pi(2.8647\ldots)^3$	
$V = 98.4841\ldots$	

The volume of the sphere is approximately 98 cm^3.

24. Use the formula for the surface area of a sphere to determine r.

$SA = 4\pi r^2$	Substitute: $SA = 314$
$314 = 4\pi r^2$	Solve for r. Divide both sides by 4π.
$\frac{314}{4\pi} = \frac{4\pi r^2}{4\pi}$	

$r^2 = \frac{314}{4\pi}$

$r = \sqrt{\frac{314}{4\pi}}$

$r = 4.9987\ldots$

Use the formula for the volume of a sphere.

$V = \frac{4}{3}\pi r^3$ Substitute: $r = 4.9987\ldots$

$V = \frac{4}{3}\pi(4.9987\ldots)^3$

$V = 523.2006\ldots$

The volume of the sphere is approximately 523 cubic inches.

1.7

25. a) The surface area comprises the lateral area of the right square pyramid, plus the areas of the four rectangular faces and base of the right square prism.

Surface area of composite object is:
Lateral area of pyramid + area of 4 rectangular faces + area of base

Use the algebraic formulas for surface area.

$SA = \frac{1}{2}s(4l) + 4Hw + lw$ Substitute: $s = 11$, $l = w = 6$, $H = 13$

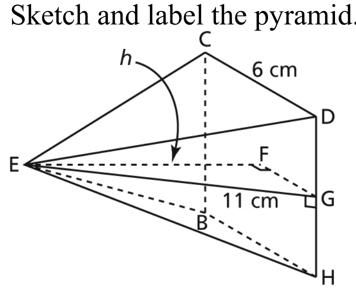
$SA = \frac{1}{2}(11)(4)(6) + 4(13)(6) + (6)(6)$

$SA = 480$

The surface area of the composite object is 480 cm^2.

The volume of the composite object is:
Volume of right square pyramid + volume of right square prism

Sketch and label the pyramid. Let h represent its height.



In ΔEFG, FG is $\frac{1}{2}$ the length of CD, so FG is 3 m.

Use the Pythagorean Theorem in right ΔEFG to determine h.

$EF^2 = EG^2 - FG^2$

$h^2 = 11^2 - 3^2$

$h^2 = 112$

$h = \sqrt{112}$

Use the algebraic formulas for volume.

$V = \frac{1}{3}lwh + lwH$ Substitute: $l = w = 6$, $h = \sqrt{112}$, $H = 13$

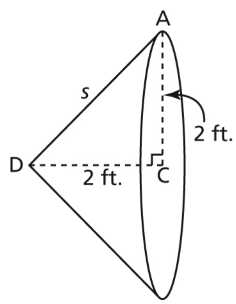
$V = \frac{1}{3}(6)(6)(\sqrt{112}) + (6)(6)(13)$

$V = 594.9960\ldots$

The volume of the composite object is approximately 595 cm^3.

b) The surface area comprises the curved surface area of the cylinder, plus the lateral areas of the cones.

Surface area of composite object:
Lateral area of cone at the left + curved surface area of cylinder + lateral area of cone at the right

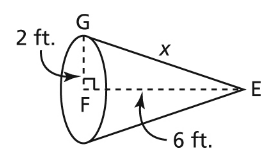


Determine the slant heights of the cones.
For the cone at the left, let s represent the slant height.
Use the Pythagorean Theorem in right ΔACD.

$AD^2 = AC^2 + CD^2$
$s^2 = 2^2 + 2^2$
$s^2 = 8$
$s = \sqrt{8}$

For the cone at the right, let x represent the slant height.
Use the Pythagorean Theorem in right ΔEFG.

$EG^2 = EF^2 + FG^2$
$x^2 = 6^2 + 2^2$
$x^2 = 40$
$x = \sqrt{40}$



Use the algebraic formulas for surface area.

$SA = \pi rs + 2\pi rh + \pi rx$ Substitute: $r = 2$, $s = \sqrt{8}$, $h = 4$, $x = \sqrt{40}$

$SA = \pi(2)(\sqrt{8}) + 2\pi(2)(4) + \pi(2)(\sqrt{40})$

$SA = 107.7753\ldots$

The surface area of the composite object is approximately 108 square feet.

The volume of the composite object is:
Volume of cone at the left + volume of right cylinder + volume of cone at the right

Use the algebraic formulas for volume.

$V = \frac{1}{3}\pi r^2 h_1 + \pi r^2 h_2 + \frac{1}{3}\pi r^2 h_3$ Substitute: $r = 2$, $h_1 = 2$, $h_2 = 4$, $h_3 = 6$

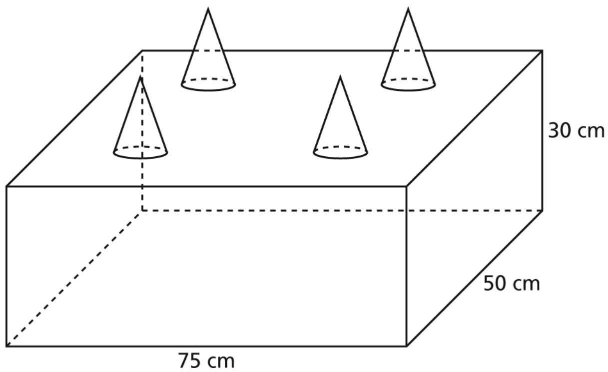
$V = \frac{1}{3}\pi(2)^2(2) + \pi(2)^2(4) + \frac{1}{3}\pi(2)^2(6)$

$V = 83.7758\ldots$

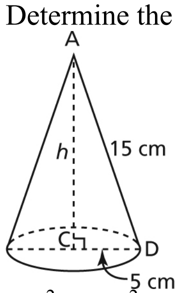
The volume of the composite object is approximately 84 cubic feet.

26. a) The volume of the sandcastle is:
Volume of right rectangular prism + volume of 4 right cones

The radius, r, of each cone is: $\frac{1}{2}$(10 cm) = 5 cm



Determine the height, h, of a cone. Use the Pythagorean Theorem in right ΔACD.



$AC^2 = AD^2 - CD^2$
$h^2 = 15^2 - 5^2$
$h^2 = 200$
$h = \sqrt{200}$

Use the algebraic formulas for volume.

$V = lwH + 4(\frac{1}{3}\pi r^2 h)$ Substitute: $l = 75$, $w = 50$, $H = 30$, $r = 5$, $h = \sqrt{200}$

$V = (75)(50)(30) + 4(\frac{1}{3}\pi (5)^2(\sqrt{200}))$

$V = 113\ 980.961$
The volume of sand required to construct the castle is approximately 113 981 cm^3.

b) The surface area of the castle is:
Surface area of four side faces and top face of right rectangular prism + lateral area of 4 right cones – area of the bases of 4 cones

Use the algebraic formulas for surface area.
SA = 2(area of front face) + 2(area of side face) + area of top face + $4\pi rs - 4\pi r^2$
$SA = 2(75)(30) + 2(50)(30) + (50)(75) + 4\pi(5)(15) - 4\pi(5)^2$
$SA = 11\ 878.3185\ldots$
The surface area of the sandcastle is approximately 11 878 cm^2.

27. a) Use the formula for the surface area of a right square pyramid to determine s.

$$SA = \frac{1}{2}s(4l) + l^2 \qquad \text{Substitute: } SA = 132,\ l = 6$$

$$132 = \frac{1}{2}s(4)(6) + (6)(6)$$

$$132 = 12s + 36$$

$$96 = 12s$$

$$\frac{96}{12} = \frac{12s}{12}$$

$$s = \frac{96}{12}$$

$$s = 8$$

s is 8 cm.

b) Use the formula for the surface area of a right cone to determine s.

$$SA = \pi rs + \pi r^2 \qquad \text{Substitute: } SA = 176,\ r = 4$$

$$176 = \pi(4)s + \pi(4)^2$$

$$176 = 4\pi s + 16\pi$$

$$176 - 16\pi = 4\pi s$$

$$\frac{176 - 16\pi}{4\pi} = s$$

$$s = 10.0056\ldots$$

s is approximately 10 mm.

Practice Test (page 67)

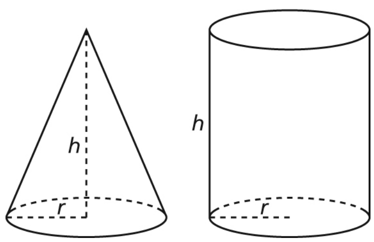
1. Use the conversion: 1 cm $\doteq \dfrac{4}{10}$ in.

To convert centimetres to inches, multiply by $\dfrac{4}{10}$.

So, 3 cm $\doteq 3(\dfrac{4}{10}$ in.)
3 cm $\doteq$ 3(0.4 in.)
So, the correct answer is B.

2. The formula for the volume of a sphere is $V = \dfrac{4}{3}\pi r^3$, so the correct answer is C.

3.



Use the formula for the volume of a cylinder.
$V_{cylinder} = \pi r^2 h$
Use the formula for the volume of a cone.
$V_{cylinder} = \dfrac{1}{3}\pi r^2 h$
To compare the volumes, divide:
$$\frac{V_{cylinder}}{V_{cone}} = \frac{\cancel{\pi}\cancel{r}^2\cancel{h}}{\dfrac{\cancel{\pi}\cancel{r}^2\cancel{h}}{3}} = 3$$
So, the volume of a right cylinder is 3 times the volume of a right cone with the same base radius and the same height.

4. a) The volume of the composite object is:
Volume of rectangular prism + volume of rectangular pyramid

Use the algebraic formulas for volume.

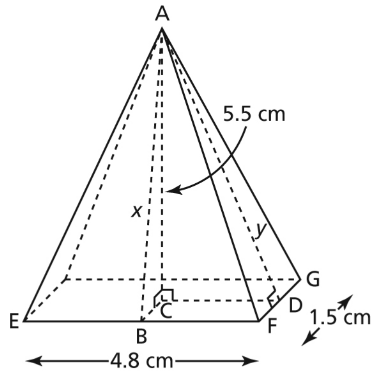
$V = lwH + \dfrac{1}{3}(lwh)$ Substitute: $l = 4.8$, $w = 1.5$, $H = 1.5$, $h = 5.5$

$V = (4.8)(1.5)(2.1) + \dfrac{1}{3}(4.8)(1.5)(5.5)$

$V = 28.32$

The volume of the composite object is approximately 28.32 cm^3.

For the surface area, first determine the heights of adjacent triangular faces of the pyramid.



BC is $\frac{1}{2}$(1.5 cm), or 0.75 cm.
Let x represent the height of ΔAEF.
Use the Pythagorean Theorem in right ΔABC.
$x^2 = 5.5^2 + 0.75^2$
$x^2 = 30.8125$
$x = \sqrt{30.8125}$
Let y represent the height of ΔAFG.
CD is $\frac{1}{2}$(4.8 cm), or 2.4 cm.
Use the Pythagorean Theorem in right ΔACD.
$y^2 = 5.5^2 + 2.4^2$
$y^2 = 36.01$
$y = \sqrt{36.01}$

Surface area of the composite object is:
Area of 4 triangular faces + area of 4 rectangular faces + area of base

Use the algebraic formulas for surface area.

$$SA = 2\left(\frac{1}{2}xl\right) + 2\left(\frac{1}{2}yw\right) + 2(lh) + 2(wh) + lw$$

Substitute: $x = \sqrt{30.8125}$, $l = 4.8$, $y = \sqrt{36.01}$, $w = 1.5$, $h = 2.1$

$SA = \left(\sqrt{30.8125}\right)(4.8) + \left(\sqrt{36.01}\right)(1.5) + 2(4.8)(2.1) + 2(1.5)(2.1) + (4.8)(1.5)$

$SA = 69.3055\ldots$

The surface area of the composite object is approximately 69.3 cm^2.

b) The radius of the cylinder is r: $\frac{1}{2}(10 \text{ m}) = 5$ m

The radius of the cone is R: $\frac{1}{2}(6 \text{ m}) = 3$ m

The volume of the composite object is:
Volume of cylinder + volume of cone

Use the algebraic formulas for volume:

$V = \pi r^2 h + \frac{1}{3}\pi R^2 h$ Substitute: $r = 5$, $h = 15$, $R = 3$, $H = 4$

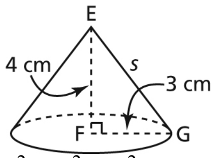
$V = \pi(5)^2(15) + \frac{1}{3}\pi(3)^2(4)$

$V = 1215.7963\ldots$

The volume of the composite object is approximately 1215.8 m^3.

The surface area of the composite object is:
Surface area of cylinder + lateral area of cone – overlap

To determine the slant height, s, of the cone, use the Pythagorean Theorem in ΔEGF.



$s^2 = 3^2 + 4^2$
$s^2 = 25$
$s = 5$

Use the algebraic formulas for surface area.
$SA = 2\pi r^2 + 2\pi rh + \pi Rs - \pi R^2$
Substitute: $r = 5$, $h = 15$, $s = 5$, $R = 3$
$SA = 2\pi(5)^2 + 2\pi(5)(15) + \pi(3)(5) - \pi(3)^2$
$SA = 647.1680\ldots$

The surface area of the composite object is approximately 647.2 m^2.

5. Answers may vary.
a) The student may have used a ruler marked in inches, because she recorded her measurements in inches.

b) To calculate the volume, the student could determine the area of the square base, then use the formula for the volume of a right rectangular pyramid.
To calculate the surface area, the student could use the Pythagorean Theorem to determine the slant height of the pyramid, then use the formula for the surface area of a right square pyramid.

6. Use the formula for the surface area of a sphere, radius r.
$SA = 4\pi r^2$ Substitute: $r = 5.0$
$SA = 4\pi(5.0)^2$
$SA = 100\pi$

Use the formula for the surface area of a hemisphere, radius R.
$SA = 2\pi R^2 + \pi R^2$
$SA = 3\pi R^2$ Substitute: $SA = 100\pi$
$100\pi = 3\pi R^2$ Solve for R. Divide both sides by 3π.
$R^2 = \dfrac{100\pi}{3\pi}$
$R = \sqrt{\dfrac{100}{3}}$
$R = 5.7735\ldots$
The radius of the hemisphere is approximately 5.8 cm.