AP Calculus Unit 6 Review

1a) $\int (3x^2+2x-1)\,dx$

$= x^3 + x^2 - x + C$

b) $\int \left(\frac{1}{x^3} + 3x + e^{2x}\right) dx$

$= \int (x^{-3} + 3x + e^{2x})\,dx$

$= -\frac{x^{-2}}{2} + \frac{3}{2}x^2 + \frac{e^{2x}}{2} + C$

c) $\int \left(\frac{x-10}{x^3}\right) dx$

$= \int (x^{-2} - 10x^{-3})\,dx$

$= -x^{-1} + 5x^{-2} + C$

d) $\int (\cos 5x + \sin 2x)\,dx$

$= \frac{\sin 5x}{5} - \frac{\cos 2x}{2} + C$

e) $\int \left(3x^2 + \frac{1}{x}\right) dx$

$= x^3 + \ln|x| + C$

f) $\int (\sec^2 x + e^{-3x})\,dx$

$= \tan x - \frac{e^{-3x}}{3} + C$

2. $f(x) = \int (5x^2 - 1)\,dx$

$f(x) = \frac{5}{3}x^3 - x + C$

$3 = \frac{5}{3}(1)^3 - 1 + C$

$3 = \frac{5}{3} - 1 + C$

$4 - \frac{5}{3} = C$

$\frac{7}{3} = C$

$f(x) = \frac{5}{3}x^3 - x + \frac{7}{3}$

3. $v(t) = \int (4t)\,dt$

$v(t) = 2t^2 + C$

$6 = 2(1)^2 + C$

$4 = C$

$v(t) = 2t^2 + 4$

$s(t) = \int (2t^2 + 4)\,dt$

$s(t) = \frac{2t^3}{3} + 4t + C$

$3 = \frac{2(2)^3}{3} + 4(2) + C$

$3 = \frac{16}{3} + 8 + C$

$-31/3 = C \quad \therefore s(t) = \frac{2}{3}t^3 + 4t - \frac{31}{3}$

4. a) $\int x(x^2+2)^3 dx$

let $u = x^2+2$

$du = 2x\, dx$

$\frac{1}{2} du = x\, dx$

$= \frac{1}{2} \int u^3 + C$

$= \frac{1}{2}\left[\frac{u^4}{4} + C\right]$

$= \frac{1}{8} u^4 + C$

$= \frac{1}{8}(x^2+2)^4 + C$

b) $\int (7-2x^3)^{4/3} x^2\, dx$

let $u = 7-2x^3$

$du = -6x^2\, dx$

$-\frac{1}{6} du = x^2\, dx$

$= -\frac{1}{6}\int u^{4/3}\, du$

$= -\frac{1}{6}\left[\frac{3}{7} u^{7/3} + C\right]$

$= -\frac{1}{14} u^{7/3} + C$

$= -\frac{1}{14}(7-2x^3)^{7/3} + C$

c) $\int \cos^3 x \sin x\, dx = \int (\cos x)^3 \sin x\, dx$

let $u = \cos x$

$du = -\sin x\, dx$

$-du = \sin x\, dx$

$= -\int u^3\, du$

$= -\left[\frac{u^4}{4} + C\right]$

$= -\frac{1}{4}(\cos x)^4 + C$

$= -\frac{1}{4}\cos^4 x + C$

d) $\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\, dx$

let $u = \sqrt{x} = x^{1/2}$

$du = \frac{1}{2} x^{-1/2}$

$du = \frac{1}{2\sqrt{x}}$

$2du = \frac{1}{\sqrt{x}}$

$= 2\int e^{u}\, du$

$= 2(e^{u} + C)$

$= 2e^{\sqrt{x}} + C$

e) $\int \frac{3 \ln x}{x}\, dx$

let $u = \ln x$

$du = \frac{1}{x}\, dx$

$= 3\int u\, du$

$= 3\left[\frac{u^2}{2} + C\right]$

$= \frac{3}{2}(\ln x)^2 + C$

f) $\int x \cos(2x^2)\, dx$

let $u = 2x^2$

$du = 4x\, dx$

$\frac{1}{4} du = x\, dx$

$= \frac{1}{4}\int \cos u\, du$

$= \frac{1}{4}[\sin u + C]$

$= \frac{1}{4} \sin(2x^2) + C$

5 a) $\int_{-1}^{0} x\sqrt{1 - x^2}\, dx$

let $u = 1 - x^2$

$du = -2x\, dx$

$-\frac{1}{2} du = x\, dx$

$= -\frac{1}{2}\int_{0}^{1} \sqrt{u}\, du$

$= -\frac{1}{2}\left[\frac{2}{3} u^{3/2} \Big|_0^1\right]$

$= -\frac{1}{2}\left[\frac{2}{3} - 0\right]$

$= -\frac{1}{3}$

b) $\int_0^1 x^3(x^4+2)^3\,dx$

let $u = x^4+2$

$du = 4x^3\,dx$

$\frac{1}{4}\,du = x^3\,dx$

$$= \frac{1}{4}\int_2^3 u^3\,du$$

$$= \frac{1}{4}\left[\frac{u^4}{4}\Big|_2^3\right]$$

$$= \frac{1}{4}\left[\frac{3^4}{4} - \frac{2^4}{4}\right] = \frac{1}{4}\left(\frac{81}{4} - 4\right) = \frac{65}{16}$$

6. Intersection Points

$x^2 = 2x - x^2$

$2x^2 - 2x = 0$

$2x(x-1) = 0$

$x = 0$ or $x = 1$

$$\text{Area} = \int_0^1 (2x - x^2) - (x^2)\,dx = \int_0^1 (2x - 2x^2)\,dx$$

$$= x^2 - \frac{2}{3}x^3\Big|_0^1 = (1)^2 - \frac{2}{3}(1)^3 - \left[0^2 - \frac{2}{3}(0)^3\right]$$

$$= 1 - \frac{2}{3} = \frac{1}{3}$$

⑦ Intersection Points

$x^2 = x^3$

$x^2 - x^3 = 0$

$x^2(x-1) = 0$

$x=0$ OR $x=1$

$$\int_0^1 (x^2 - x^3)\,dx = \frac{1}{12}$$

⑧ Integrate with Respect to y.

$x = y^2 - 5$ $\qquad$ $x = y - 1$

* graph to find intersection points

or $\quad y^2 - 5 = y - 1$

$y^2 - y - 4 = 0$

$y = -1.56155 = a$

$y = 2.56155 = b$

Solve using quad formula to find y coord. of intersection points

$$\int_a^b (y-1) - (y^2-5)\,dy = 11.682$$

9. Three Regions

Region I

$$A = \int_{-1}^{0} x - (5x - x^2)\, dx = \int_{-1}^{0} (x - 5x + x^2)\, dx = \int_{-1}^{0} (x^2 - 4x)\, dx$$

$$= \frac{x^3}{3} - 2x^2 \Big|_{-1}^{0}$$

$$= \left[\frac{(0)^3}{3} - 2(0)^2\right] - \left[\frac{(-1)^3}{3} - 2(-1)^2\right]$$

$$= 0 - \left[-\frac{1}{3} - 2\right]$$

$$= \frac{7}{3}$$

Region II

$$A = \int_{0}^{4} (5x - x^2 - x)\, dx = \int_{0}^{4} (4x - x^2)\, dx$$

$$= 2x^2 - \frac{x^3}{3} \Big|_{0}^{4} = \left[2(4)^2 - \frac{(4)^3}{3}\right] - \left[2(0)^2 - \frac{(0)^3}{3}\right]$$

$$= 32 - \frac{64}{3} = \frac{32}{3}$$

Region III

$$A = \int_{4}^{5} x - (5x - x^2)\, dx = \int_{4}^{5} (x - 5x + x^2)\, dx = \int_{4}^{5} (x^2 - 4x)\, dx$$

$$= \frac{x^3}{3} - 2x^2 \Big|_{4}^{5} = \left[\frac{(5^3)}{3} - 2(5)^2\right] - \left[\frac{(4)^3}{3} - 2(4)^2\right]$$

$$= \left[\frac{125}{3} - 50\right] - \left[\frac{64}{3} - 32\right]$$

$$= \frac{125}{3} - 50 - \frac{64}{3} + 32$$

$$= \frac{61}{3} - 18 \qquad = \frac{7}{3}$$

Total Area

$AR_1 + AR_2 + AR_3$

$$\frac{7}{3} + \frac{32}{3} + \frac{7}{3} = \frac{46}{3}$$