

AP Calculus Unit 6 Review: Indefinite Integrals, U-Substitution, Area Between Curves

1. Integrate the following:

a) $\int (3x^2 + 2x - 1)dx$

b) $\int \left(\frac{1}{x^3} + 3x + e^{2x}\right) dx$

c) $\int \left(\frac{x-10}{x^3}\right) dx$

d) $\int (\cos 5x + \sin 2x)dx$

e) $\int \left(3x^2 + \frac{1}{x}\right) dx$

f) $\int (\sec^2 x + e^{-3x})dx$

2. Find the function $f(x)$ if $f'(x) = 5x^2 - 1$ and $f(1) = 3$.

3. Find the position function of an object, $s(t)$, if the objects acceleration function is $a(t) = 4t$, and $s(2) = 3$ and $v(1) = 6$.

4. Integrate using u-substitution.

a) $\int x(x^2 + 2)^3 dx$

b) $\int (7 - 2x^3)^{\frac{4}{3}} x^2 dx$

c) $\int \cos^3 x \sin x dx$

d) $\int \frac{e^{\sqrt{x}}}{\sqrt{x}} dx$

e) $\int \frac{3 \ln x}{x} dx$

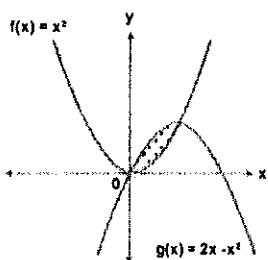
f) $\int x \cos(2x^2) dx$

5. Evaluate the following using u-substitution:

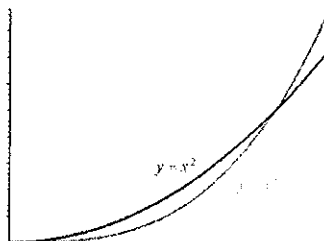
a) $\int_{-1}^0 x \sqrt{1 - x^2} dx$

b) $\int_0^1 x^3 (x^4 + 2)^3 dx$

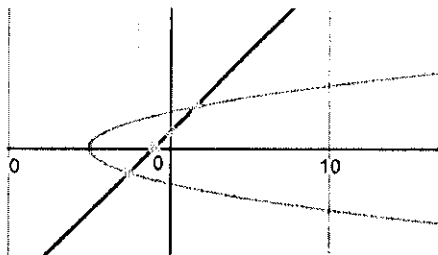
6. Find the area of the region bound by the curves $f(x) = x^2$ and $g(x) = 2x - x^2$.



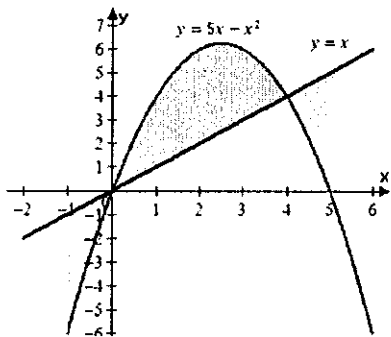
7. Find the area in the first quadrant bound by the curves $y = x^2$ and $y = x^3$.



8. Find the area bound by the curves $y^2 = x + 5$ and $y = x + 1$.



9. Find the area bound the curves $y = 5x - x^2$ and $y = x$ and $x = -1$ and $x = 5$.



Answers

1. a) $x^3 + x^2 - x + c$

c) $-x^{-1} + 5x^{-2} + c$

e) $x^3 + \ln|x| + c$

2. $f(x) = \frac{5}{3}x^3 - x + \frac{7}{3}$

4.a) $\frac{1}{8}(x^2 + 2)^4 + c$

c) $-\frac{1}{4}\cos^4 x + c$

e) $\frac{3}{2}(\ln x)^2 + c$

5.a) $-\frac{1}{3}$

6. $\frac{1}{3}$

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b) $\frac{-x^{-2}}{2} + \frac{3x^2}{2} + \frac{e^{2x}}{2} + c$

d) $\frac{\sin 5x}{5} - \frac{\cos 2x}{2} + c$

f) $\tan x - \frac{e^{-3x}}{3} + c$

3. $s(t) = \frac{2}{3}t^2 + 4t - \frac{31}{3}$

b) $-\frac{1}{14}(7 - 2x^3)^{\frac{7}{3}} + c$

d) $2e^{\sqrt{x}} + c$

f) $\frac{1}{4}\sin(2x^2) + c$

b) $\frac{65}{16}$

7. $\frac{1}{12}$

9. $\frac{46}{3}$