Section 5.5
z-Scores (day 3)
Problem Solving

Learning targets:

Solve problems using z-scores and the z-table.

Example #1:

The mathematics test scores in math class are normally distributed. The mean is 80 with a standard deviation of 5. Callie is in this math class and she scored 83 on the test.

Compare her score with the rest of the class.

What percentage of the class scored lower than Callie on this test?

$= .7257$

$$z = \frac{x - \bar{x}}{\sigma}$$

$$= \frac{83 - 80}{5}$$

$$z = .60$$

$x = 83 \quad \sigma = 5$

You try:

IQ tests are sometimes used to measure a person's intellectual capacity at a particular time. IQ scores are normally distributed, with a **mean of 100** and a **standard deviation of 15**.

If a person scores 117 on an IQ test, how does this score compare with the scores of the general population?

$\bar{x} = 100$

$\sigma = 15$

$x = 117$

$$Z = \frac{117 - 100}{15} = 1.13$$

She scored higher than 87.08% of Pop.

Example #2:

A set of exam scores was recorded and determined to be normally distributed with a **mean of 68%** and a **standard deviation of 9.5%**.

a) Calculate the z-score for a student who scored 80% on the exam.

$$Z = \frac{80 - 68}{9.5} = 1.26$$

b) What percent of the exam scores were lower than 80%?

0.8962 = 89.62%

c) What percent of the exam scores were higher than 80%?

$1 - .8962 = .1038$

d) What percent of the exam scores were between 50% and 80%?

$$z_{50} = \frac{50 - 68}{9.5} = -1.89$$

$.8962 - .0294 = .8668$

e) If 1500 people wrote the exam, how many of them passed (with a mark of 50% or more)?

$1 - .0294 = .9706$

1455

Example #3:

A group of students was given a puzzle to solve and the amount of time it took each to solve the puzzle was normally distributed with a mean of 5 **minutes** and a standard deviation of 30 **seconds**.

(before we begin to ask questions about this situation, we need perform a unit conversion)

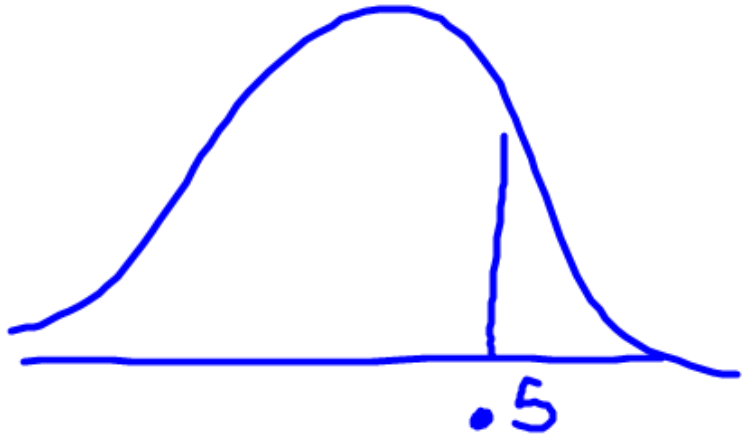
What percentage of the students:

a) Solved the puzzle in less than 5 minutes, 15 seconds?

$x = 315$

$\bar{x} = 300$

$\sigma = 30$



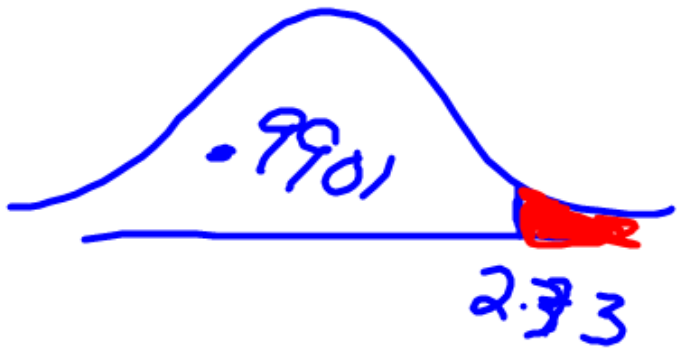
.6915

b) Took longer than 6 minutes, 10 seconds to solve the puzzle?

$X = 370$

$$Z = \frac{370 - 300}{30}$$

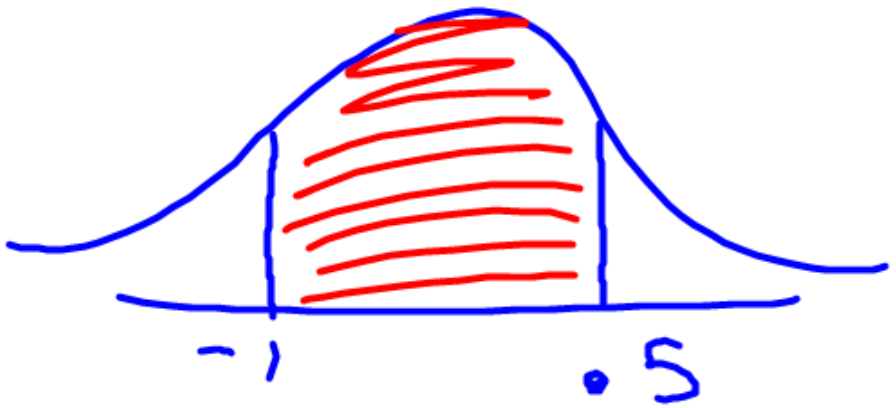
$Z = 2.33$



$1 - .9901$

$= .0099$

c) Solved the puzzle in less than 5 minutes, 15 seconds, but more than 4 minutes, 30 seconds?



.6915 - .1357

= .5558

$$Z = \frac{270 - 300}{30} = -1$$

Z-scores and Probability

The percentage of data to the right, to the left or between z-scores can also be interpreted with respect to probability.

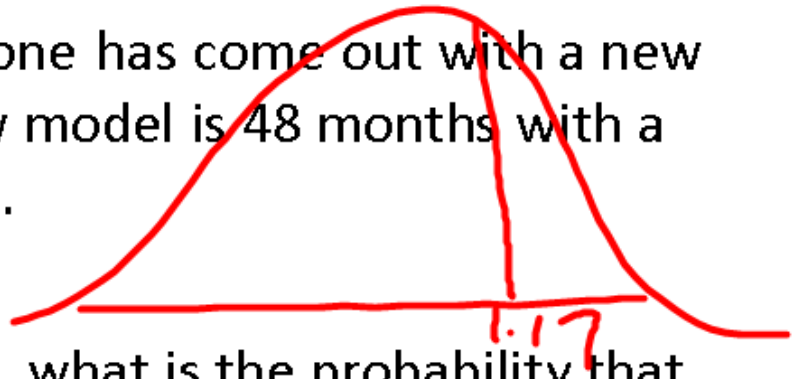
That is, the percentage of data to the left of $z = a$ is the same as the probability of a randomly selected piece of data fitting to the left of that same z-score value.

The percentage of data to the right of $z = a$ is the same as the probability of a randomly selected piece of data fitting to the right of that same z-score value.

The percentage of data between $z = a$ and $z = b$ is the same as the probability of a randomly selected piece of data fitting between those same two z-score values.

Example #4:

The manufacturer of a smart phone has come out with a new model. The mean life of this new model is 48 months with a standard deviation of 12 months.



a) If you bought this new model, what is the probability that your phone would last longer than 5 years and 2 months?

$1 - .8790$

$.1210$

$\bar{X} = 48$ $X = 62$

$\sigma = 12$

$Z = \frac{62 - 48}{12} = 1.17$

b) If you purchased this new model, what is the probability that it would last less than 30 months?

$$Z = \frac{30 - 48}{12}$$

$$Z = -1.5$$

• 0668

$= 6.68\%$

ASSIGNMENT:

Page 264 - 265: #11, 13, 15(a)(b), 16

Handout: #1 – 4