

Section 5.5

z-Scores (day 2)

Learning targets:

1. Sketching the **standard normal curve** to solve problems.
2. Using the **z-table** to determine the percent of data
 - a) to the left of a given z-score
 - b) to the right of a given z-score
 - c) between two given z-scores.
3. Using the z-table to determine a z-score for a particular percent (working backwards)

Recall: What is a z-score?

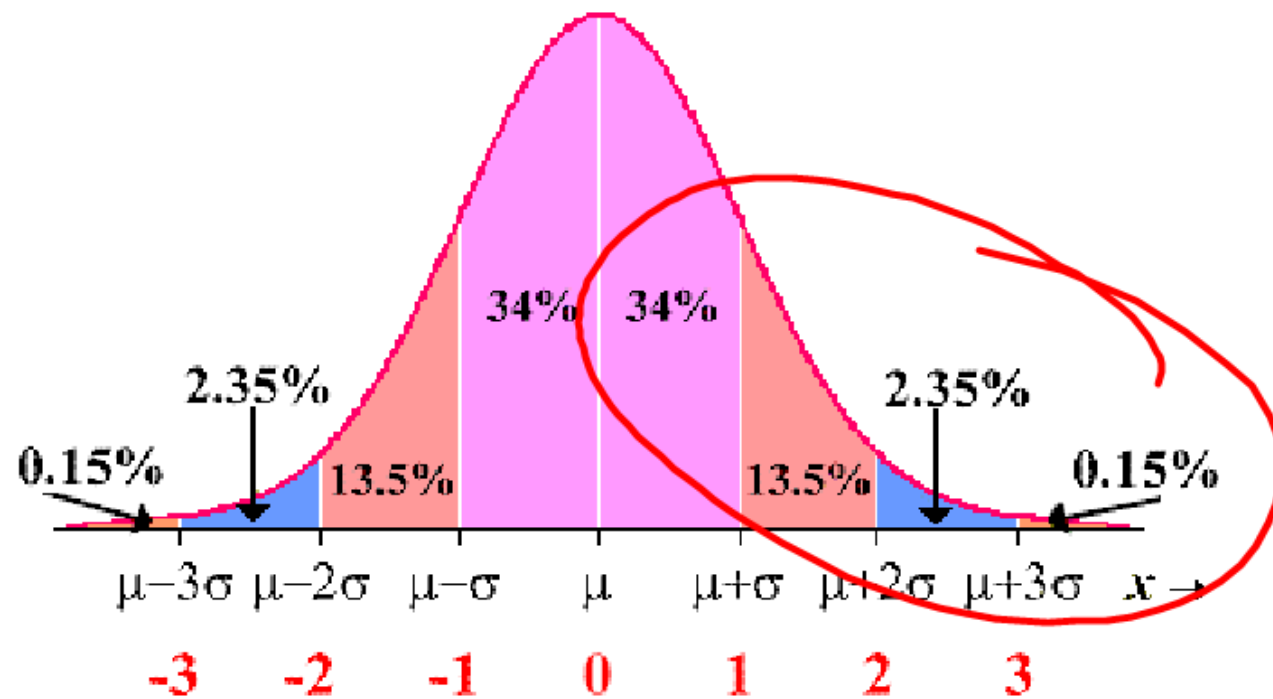
A **z-score** is a numerical value that indicates how far away from the mean a particular data value is, measured in standard deviations:

- A **raw score** of $\mu + \sigma$ has a **z-score** of **+1**
- A **raw score** of $\mu - 2\sigma$ has a **z-score** of **-2**
- A **raw score** equal to μ has a **z-score** of **0**

et cetera...

The *Standard* Normal Curve

Looks like the normal curve, with the same percentages in the strips, but instead of numerical values that are specific to the data being analyzed, the scale along the bottom is noted in **z-scores**:



With the *Standard Normal Curve*, we can solve problems like we solved in Sec. 5.4, but instead of always re-drawing the curve with the different values on the bottom, we can use the same curve over and over – **we just need to convert our data values to z-scores in order to use it.**

Example #2:

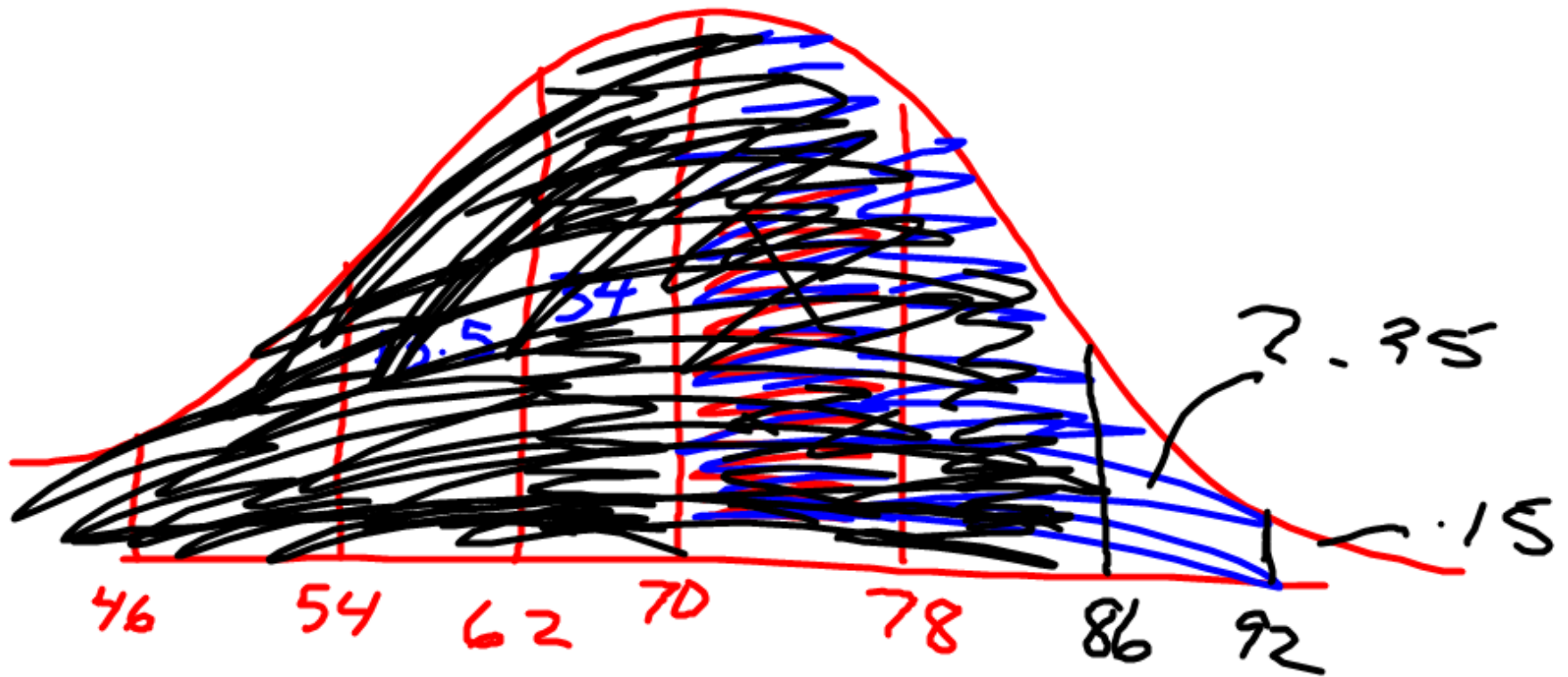
The marks in a university math class were normally distributed with a mean of 70% and a standard deviation of 8%.

- What percentage of students in this class scored less than 78%?

$$50\% + 34\% \\ = 84\%$$

- What percentage of students in this class scored higher than 54%?

$$13.5 + 34 + 50 \\ = 97.5\%$$



- What percentage of students in this class scored between 62% and 86%?

$$68 + 13.5 = 81.5\%$$

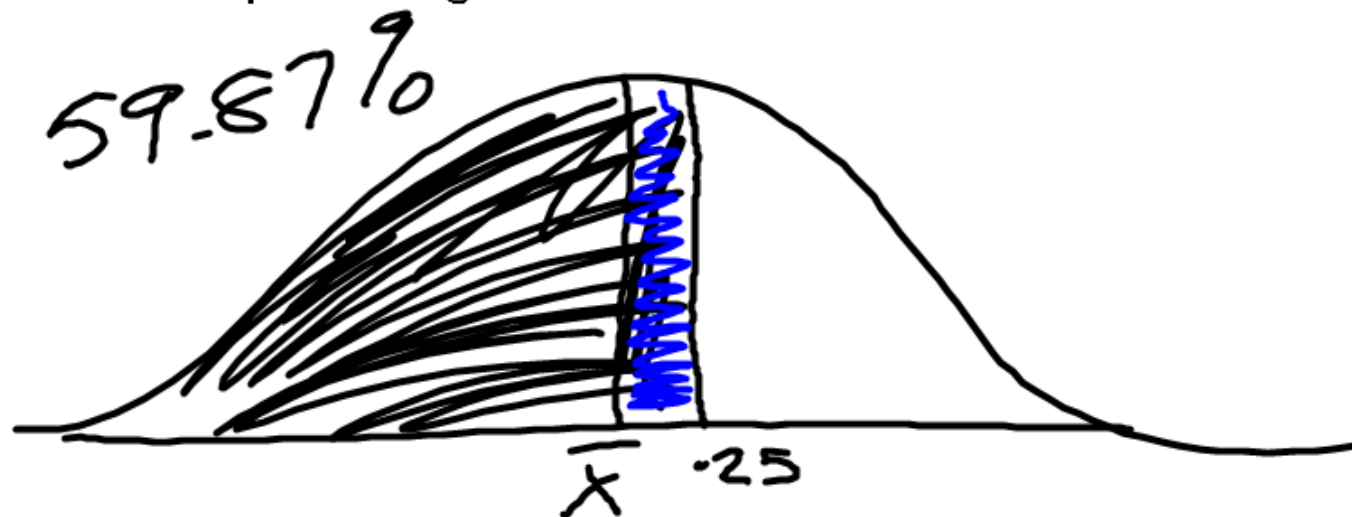
- What percentage of students in this class scored less than 86%?

$$97.5\%$$

/

$$z = \frac{72 - 70}{8} = .25$$

- What percentage of students in this class scored less than 72%?



Since the z-score for 72% is not an integer value we can no longer simply add up percentages from the strips to answer this question.

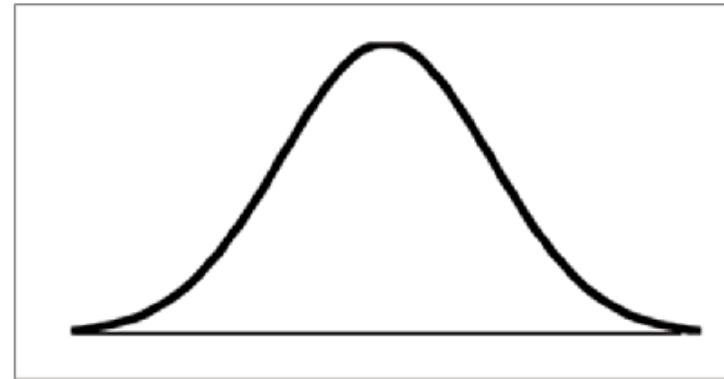
A table of values for this type of problem has been developed (see page 592-593 of text or handout).

Using the z-Table

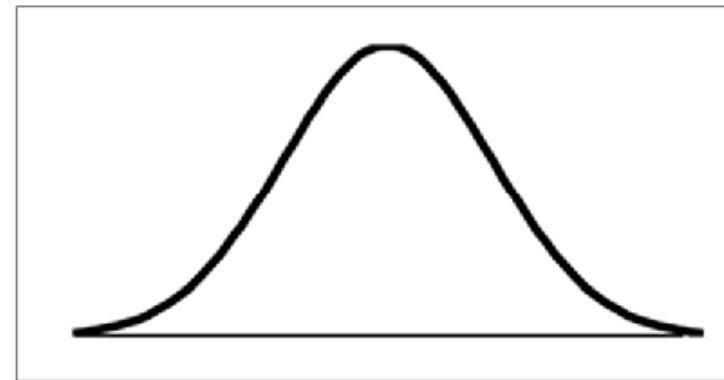
- If we want to find the percentage of the data that is below a particular z-score, the table provides the answer in decimal form to four decimal places.
- If we want to find the percentage of the data that is above a particular z-score, or between two z-scores, the table's values are used, but calculations are required.

z-Pictures:

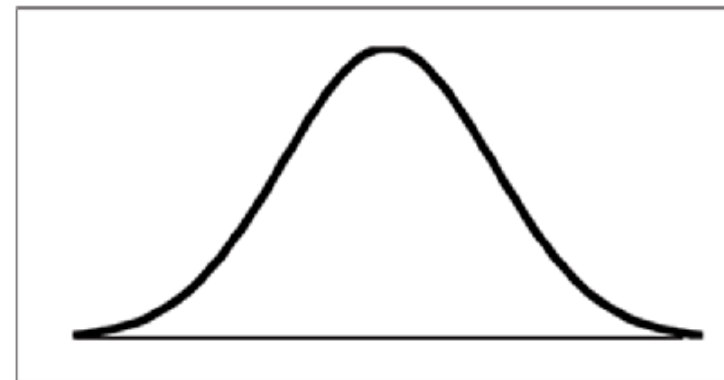
% of data **to the left:**
(below $z = a$)



% of data **to the right:**
(above $z = a$)



% of data **between:**
($a < z < b$)



Finding % of Data to the Left

Use the z-score table to find the percentage of data **to the left of (below)**:

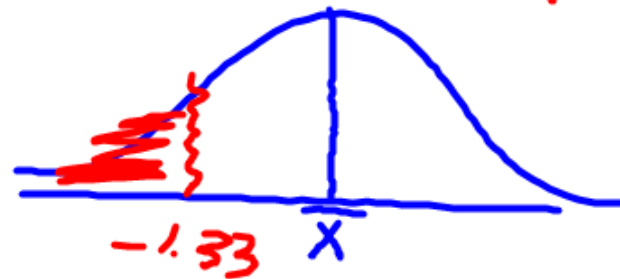
$$z = 2.16$$



98.46%

$$z = -1.33$$

9.18%



You try:

$$z = 0.27$$

6064

$$z = -2.44$$

0073

Finding % of Data to the Right

Use the z-score table to find the percentage of data **to the right of (above)**:

$$z = 1.75$$

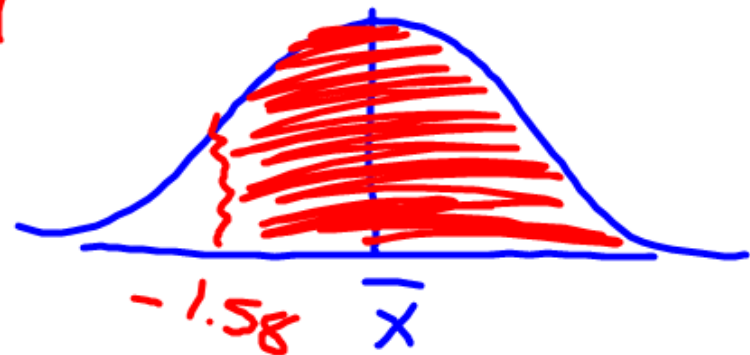


$$1.0 - .9599 \\ = .0401$$

You try: $z = 2.10$

$$z = -0.38$$

$$z = -1.58$$

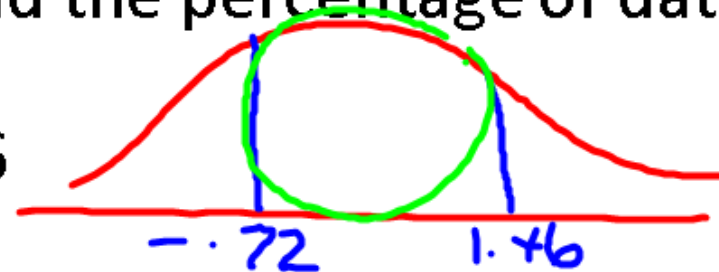


$$1 - .0571 \\ = .9429$$

Finding % of Data Between

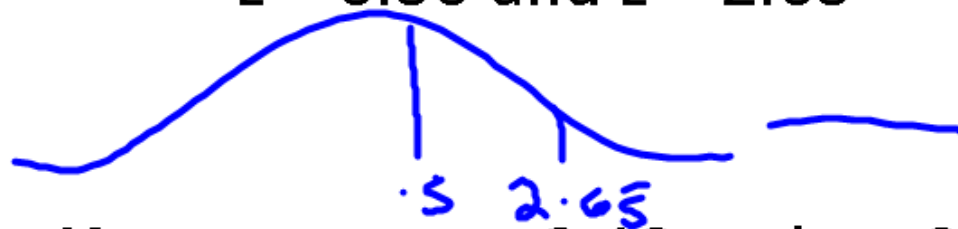
Use the z-score table to find the percentage of data **between:**

$$z = -0.72 \text{ and } z = 1.46$$



$$.9279 - .2358 = .6821$$

$$z = 0.50 \text{ and } z = 2.65$$



$$.9960 - .6915 = .3045$$

You try:

$$z = -2.10 \text{ and } z = 0.04$$

$$z = -1.38 \text{ and } z = -0.21$$



Finding a z-score for a Percentage

Determine the z-score for which you'd find **56.75%** of the data *to the left*.

Determine the z-score for which you'd find **62.55%** of the data *to the right*.

Finding a z-score for a Percentage

Determine the z-score for which you'd find **70%** of the data *to the left*.

Finding a z-score for a Percentage

Determine the z-score for which you'd find **38%** of the data *to the right*.

Find the z-score where 20% of the data is to the left.

ASSIGNMENT:

Page 264 - 265: #2, 3, 4, 6, 7, 8, 9

Handout: 7(a to j), 8(a to d)

(on the handout, area and % of data mean the same thing)