Section 5.5
z-Scores (day 1)

Learning targets:

1. Definition of z-score.
2. The z-score formula.
3. Interpreting z-scores.
4. Using the z-score formula.

What is a z-score?

A **z-score** is a numerical value that indicates how far away from the mean a particular data value is, measured in standard deviations.

- A **positive z-score** indicates that the data value lies **above the mean** of the data set.
- A **negative z-score** indicates that the data value lies **below the mean** of the data set.
- A **z-score of zero** indicates that the data value is **the same as the mean** of the data set.

Example #1:

The marks in a university math class were normally distributed with a mean of 70% and a standard deviation of 8%.

- Would the following marks result in a positive z-score, negative z-score, or z-score of zero?
- Based on the definition of z-scores, what numerical value would each z-score be?

1. Josie scored 86% in this class.

$z = 2.00$

2. Kim scored 70% in this class.

$z = 0$

3. Rick scored 62% in this class.

$z = -1.00$

The z-score formula:

The **z-score** for any data value (x) in a set can be calculated provided we know the mean (μ) and standard deviation (σ) for the population the data value comes from:

$$z = \frac{x - \mu}{\sigma}$$

*A data value is often referred to as a "**raw score**" to differentiate it from its associated "**z-score**".*

In the previous example, the mean, $\mu = 70\%$
and the standard deviation, $\sigma = 8\%$

$x = 72\%$

Calculate Jim's z-score if Jim's mark in the class was 72%. State the answer to 2 decimal places.

$$Z = \frac{x - \mu}{\sigma} = \frac{72 - 70}{8} = 0.25$$

Why do we need z-scores?

Z-scores can be used to compare different sets of data that do not have the same mean and/or standard deviation.

Example #2:

In **Calculus 30**, the class average was 80% with a standard deviation of 5%.

In **Chemistry 30**, the class average was 65% with a standard deviation of 11%.

If Sophia was in both classes and scored 76% in both, in which class did she show more aptitude?

$$Z_{Cal} = \frac{76 - 80}{5}$$

$$= -0.80$$

$$Z_{Chem} = \frac{76 - 65}{11} = 1$$

Example #3:

Use the z-score formula to calculate the z-score for each value of x to two decimal places:

a) $\bar{x} = 200, \sigma = 25, x = 255$

b) $\bar{x} = 18, \sigma = 2.2, x = 17$

You Try:

$$\bar{x} = 3.2, \sigma = 0.3, x = 2.7$$

Example #4:

Use the z-score formula to calculate the raw score for each of the following:

$x = ?$

a) $\bar{x} = 200, \sigma = 25, z = -2.2$

$$Z = \frac{x - \bar{x}}{\sigma}$$

$$-2.2 = \frac{x - 200}{25} \quad (25)$$

$$-55 = x - 200$$

$$x = 145$$

b) $\bar{x} = 18, \sigma = 2.2, z = 0.55$

You Try:

$$\bar{x} = 3.2, \sigma = 0.3, z = -0.65$$

Example #5:

Use the z-score formula to calculate the standard deviation for each of the following:

a) $\bar{x} = 200, x = 250, z = 2.4$

$$z = \frac{x - \bar{x}}{\sigma}$$

$$\sigma = \frac{x - \bar{x}}{z} = \frac{250 - 200}{2.4}$$

$$\sigma = 20.83$$

$$\sigma z = x - \bar{x}$$

$$\bar{x} + \sigma z = x$$

$$Z = \frac{x - \bar{x}(\mu)}{\sigma}$$

b) $\bar{x} = 18, x = 15, z = -0.50$

$$0 \quad Z = \frac{X - \bar{X}}{0}$$

$$\frac{0 \not{Z}}{\not{Z}} = \frac{X - \bar{X}}{Z}$$

$$0 = \frac{15 - 18}{-0.50} = 6$$

You Try:

$$\bar{x} = 3.2, x = 3.3, z = 1.75$$

Example #6:

Use the z-score formula to calculate the mean for each of the following:

a) $\sigma = 20, x = 250, z = -0.40$

$$z = \frac{x - \bar{x}}{\sigma}$$

$$(20)(-0.40) = \frac{250 - \bar{x}}{20}(20)$$

$$-8 = 250 - \bar{x}$$

$$\bar{x} - 8 = 250$$

$$\bar{x} = 258$$

b) $\sigma = 1.6, x = 265, z = 1.35$

You Try:

$$\sigma = 1.2, x = 32, z = 2.35$$

ASSIGNMENT:

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Handout: #1 - 6