Section 5.4
The Normal Distribution

Learning targets:

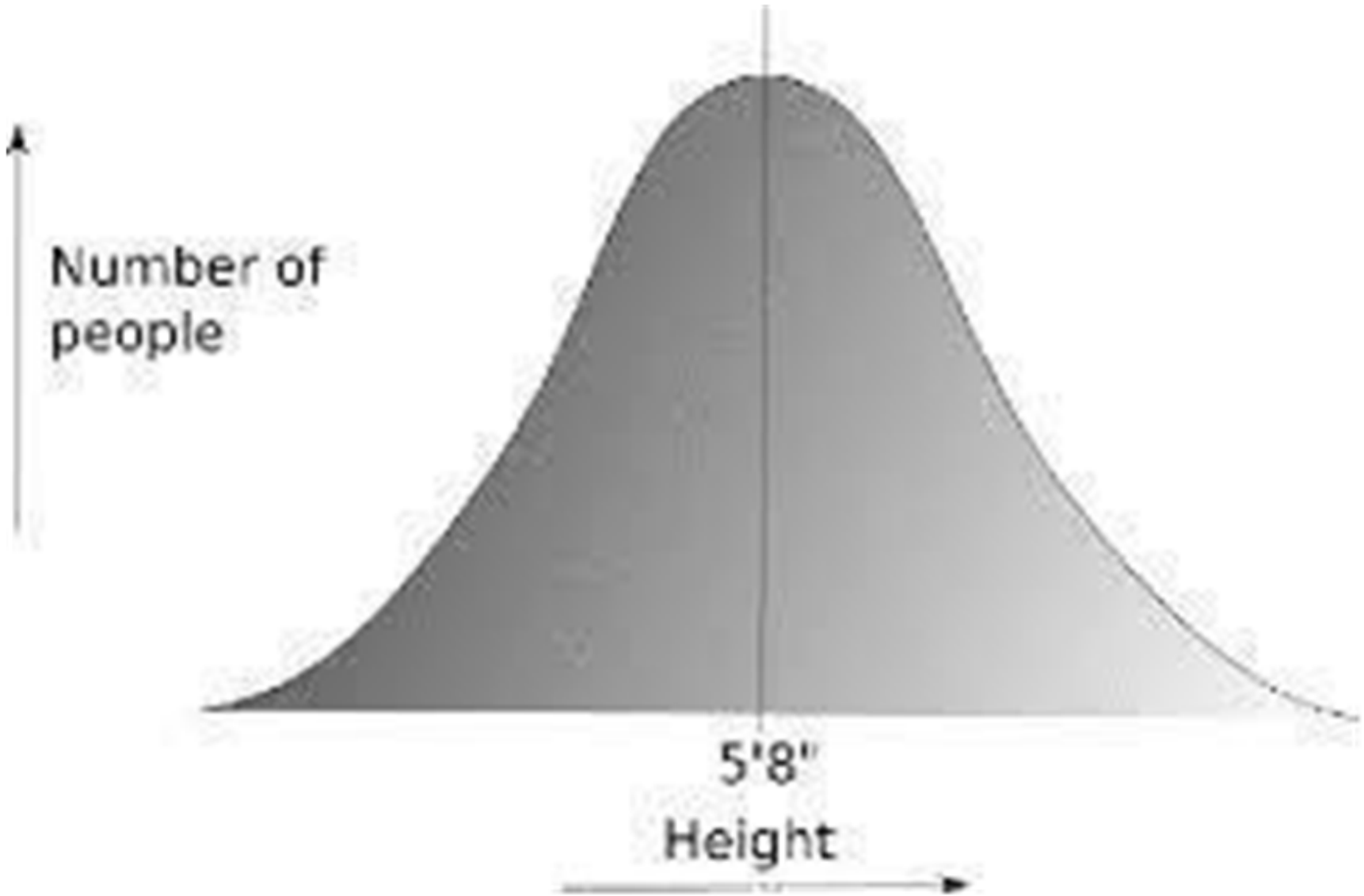
1. Demonstrate understanding of new terminology.
2. Recognize the shape of the normal distribution.
3. The characteristics of the normal curve.
4. Sketching a normal distribution curve for a given situation and using it to answer questions about the population it represents.

Imagine you measured the heights of 10,000 people. You can easily get the average height by adding all the heights together and dividing by 10,000. Say this came out as 5'8".

--> What proportion of the group were exactly 5'8" tall? How many were 5'7" or 6'2"?

So you divide all the heights into a histogram with intervals of 1 inch.

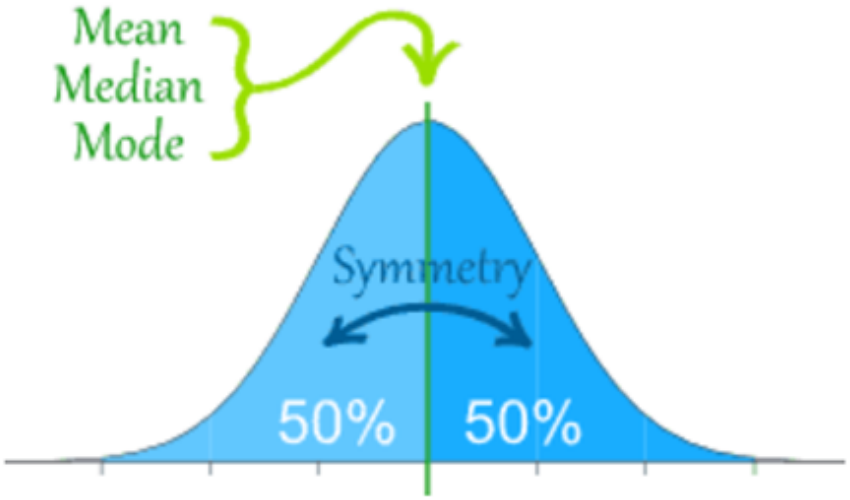
--> The histogram would look something like this:


Number of people
5'8"
Height

This shape is called a **NORMAL DISTRIBUTION**.

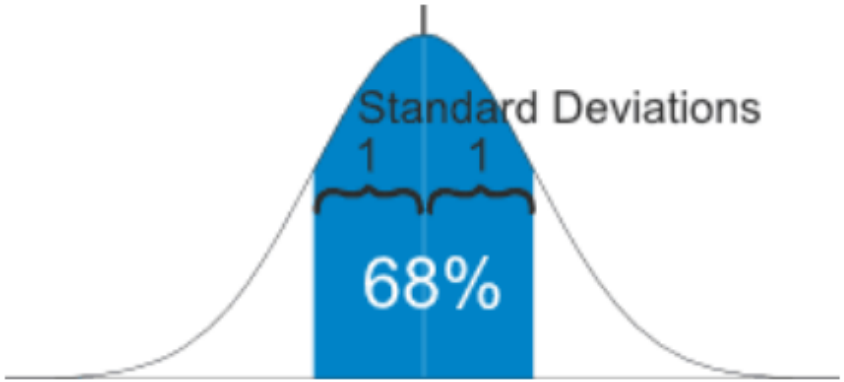
It also gets called the "**Bell Curve**" because it looks like the cross-section of a bell.

We say the data is "normally distributed":



The **Normal Distribution** has:

- mean = median = mode
- symmetry about the center
- 50% of values less than the mean and 50% greater than the mean



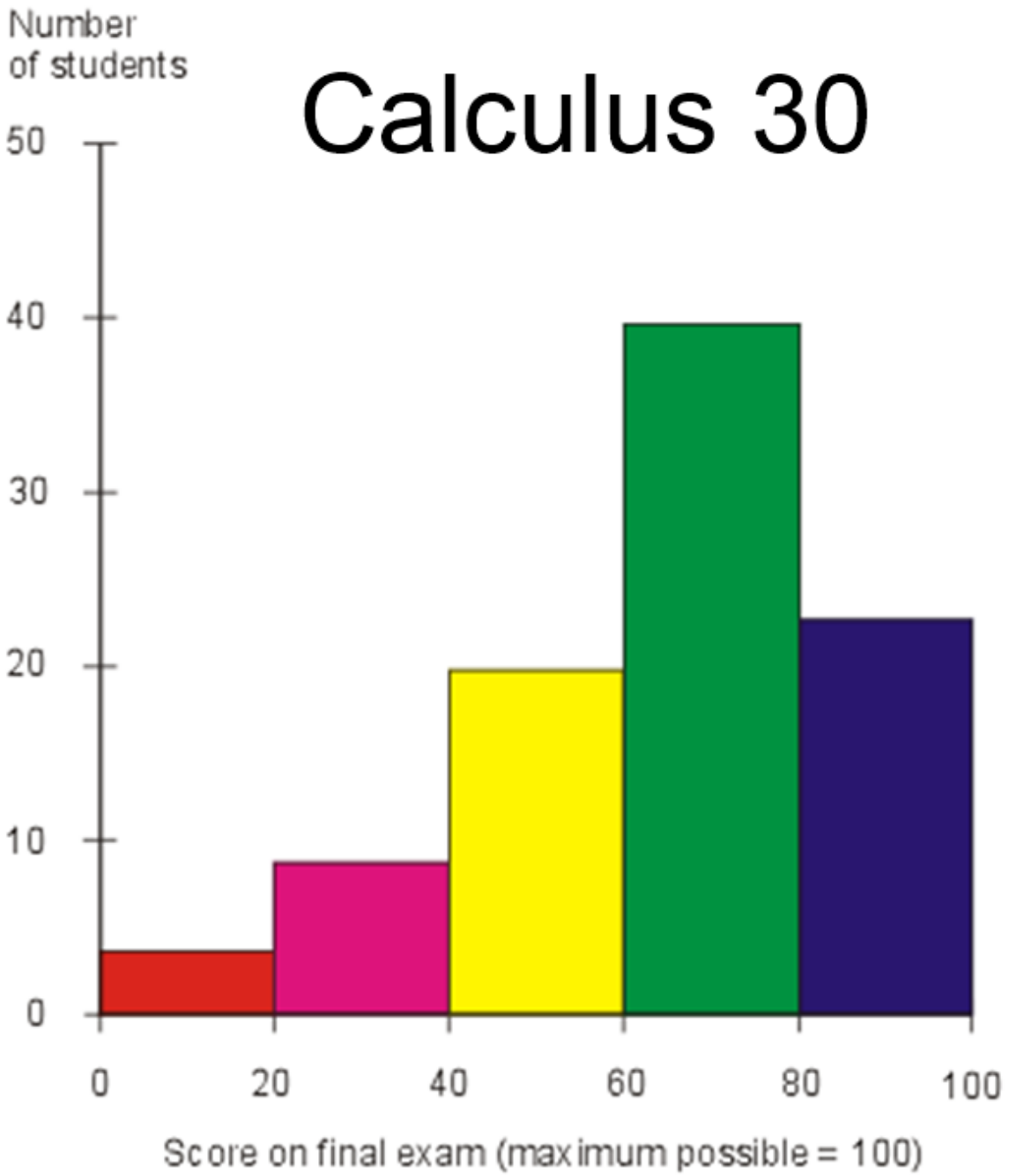
68% of values are within
1 standard deviation of the mean

95% of values are within
2 standard deviations of the mean

99.7% of values are within
3 standard deviations of the mean

Based on this we can now say that any value in a normally distributed set of data is:

- **likely** to be within 1 standard deviation (68 out of 100 should be)
- **very likely** to be within 2 standard deviations (95 out of 100 should be)
- **almost certainly** within 3 standard deviations (997 out of 1000 should be)


Calculus 30
Number of students
50
40
30
20
10
0
0
20
40
60
80
100
Score on final exam (maximum possible = 100)

Analyze the histogram and answer the questions:

1. What height do each of the bars reach?
2. Based on these heights, how many students are represented in this histogram?
3. How many students scored between 40% and 80%?
4. What percentage of students scored between 40% and 80%?
5. How many students scored below 60%?
6. What percentage of students scored below 60%?
7. How many students scored above 80%?
8. What percentage of students scored above 80%?
9. Explain why you can't say for certain how many students scored above 50%?

Mean of a Population:

- In statistics, when an entire population is involved, the symbol μ (read as "**mu**") is used for the mean of the population.

- $\bar{x}$ represents the mean of a smaller portion of a population called a "**sample**".

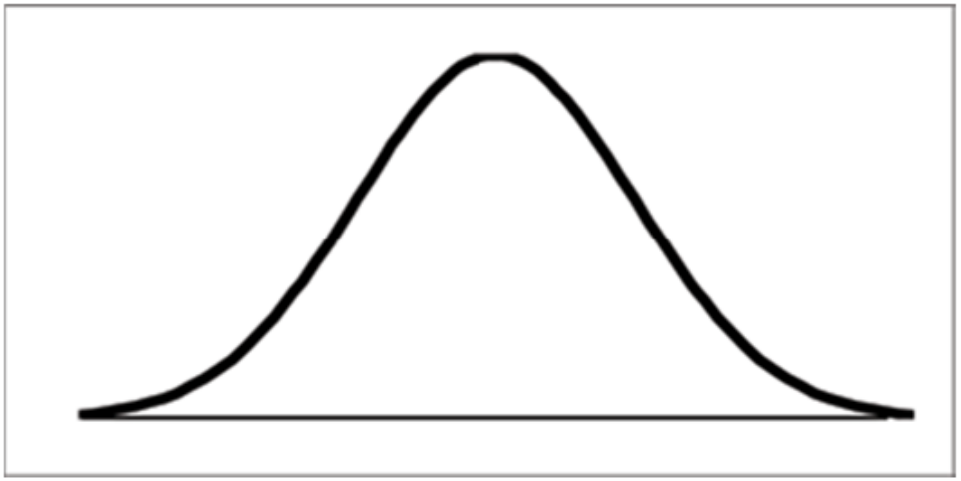
Recall

Definition:

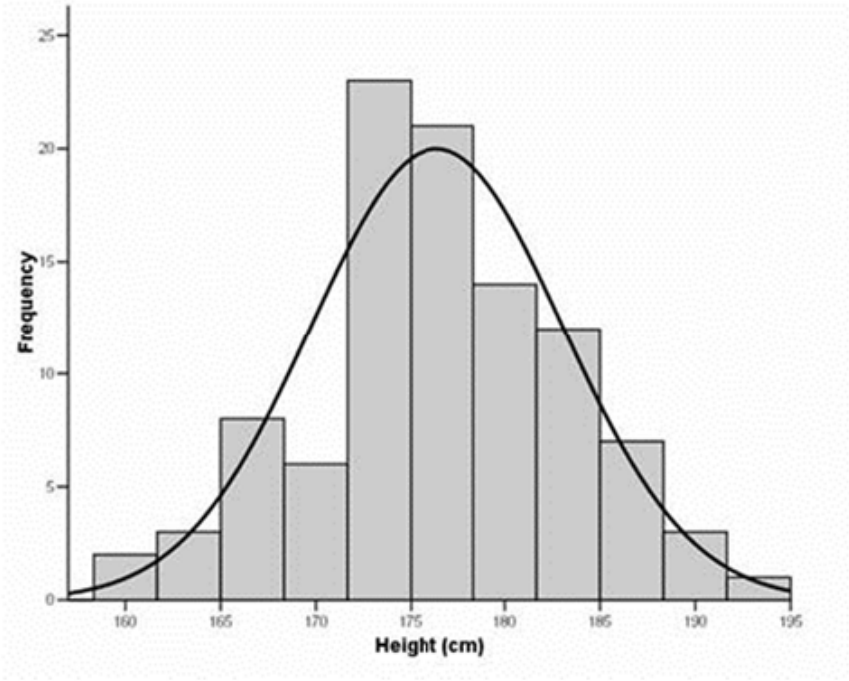
Normal Distribution:

A frequency distribution whose frequency polygon results in a **unimodal** **symmetric** distribution ***about the mean***.

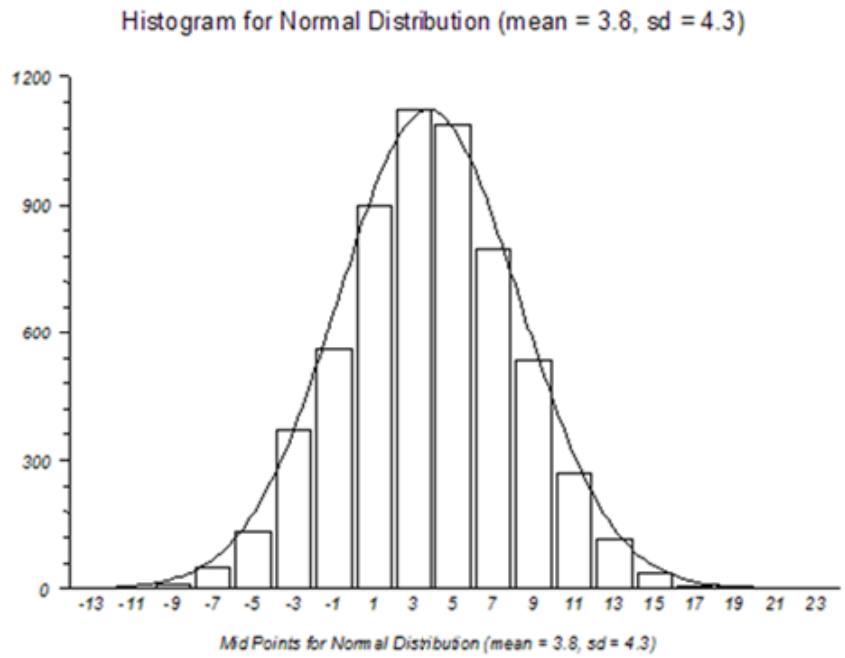
The frequency polygon takes on the shape of a "**bell curve**":



The following graph illustrates a histogram whose shape approximates the normal distribution:



The following graph illustrates a histogram whose shape even more closely approximates the normal distribution:

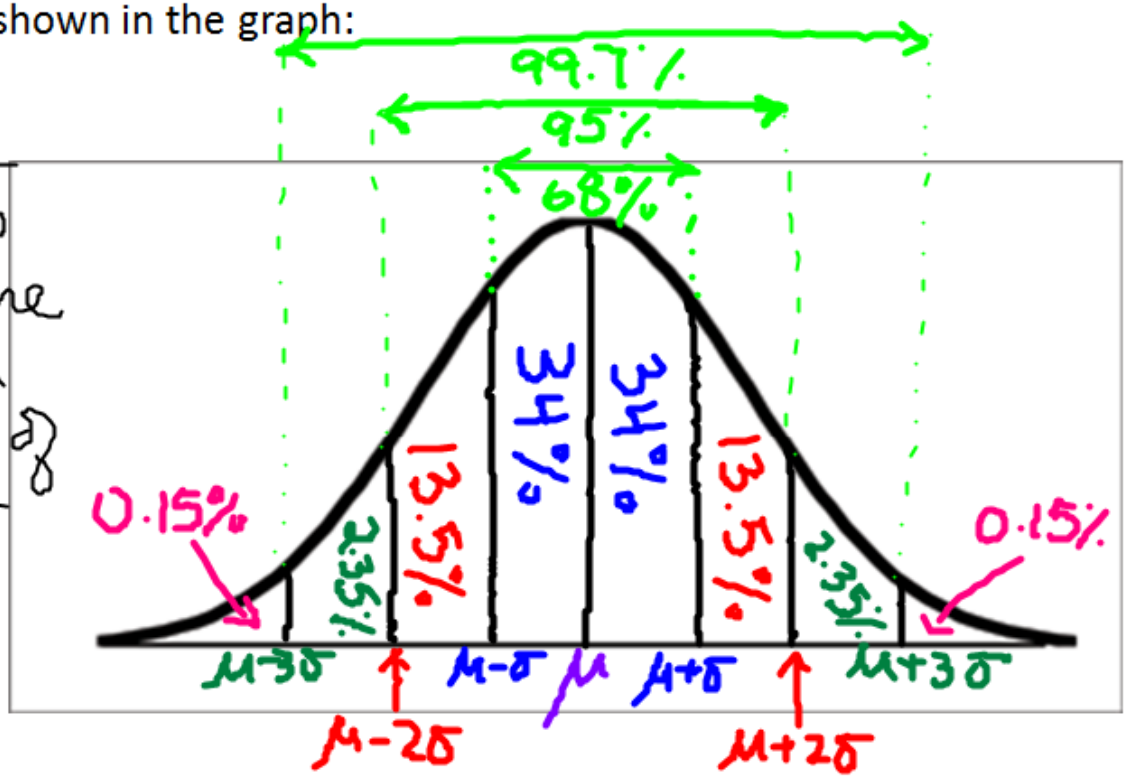


Characteristics of the Normal Distribution Curve

1. The mean, median and mode are all close to equal and fall in the middle of the normal curve, forming the vertical line of symmetry.

2. The amounts of data within one, two and three standard deviations of the mean are shown in the graph:

68% of the data lies within one standard deviation of the mean.



95% of the data lies within two standard deviations of the mean.

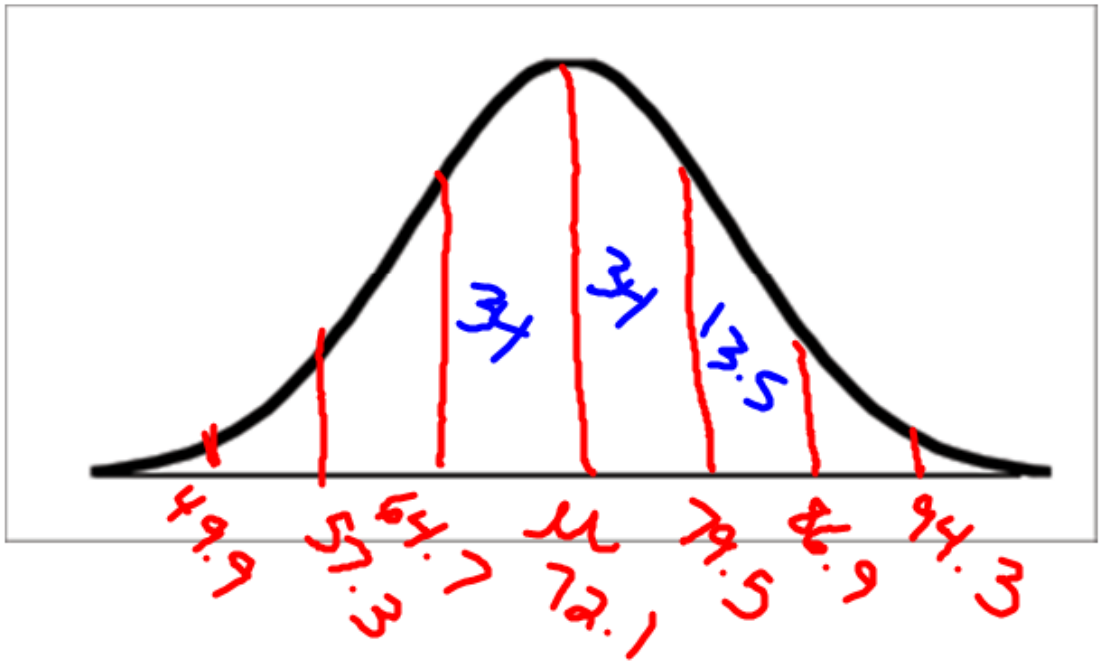
99.7% of the data lies within 3 standard deviations of the mean.

Example #1: Sketch the normal distribution curve for the following:

The ages of the grandparents of students in a math class were collected. The mean of this population and standard deviation were calculated:

$\mu = 72.1$ years $\qquad\qquad$ $\sigma = 7.4$ years

Include the scale along the bottom, and fill in the approximate percentages found between each increment of standard deviation:



Use this graph to answer the following questions about the population:

What percentage of the grandparents are:

(a) older than 72.1 years?

50%

(b) younger than 72.1 years?

50%

(c) between 72.1 and 86.9 years?

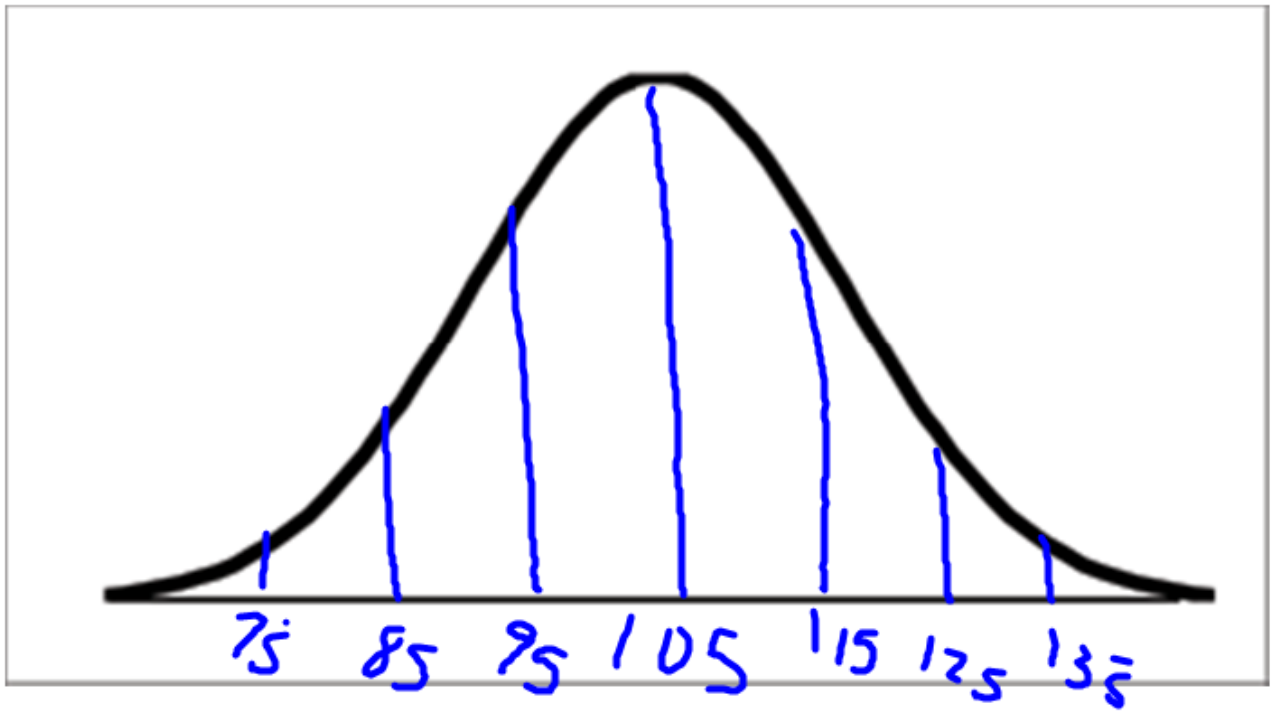
47.5%

(d) older than 64.7 years?

84%

Example #2: Sketch the normal distribution curve for the following:

The speeds of 5000 cars were recorded by photo radar. If the data collected was normally distributed with a **mean of 105 km/h** and a **standard deviation of 10 km/h**, sketch and label the normal curve, including percentages.



Use this graph to answer the following questions about the population:

(a) Determine the **percentage** of cars going between 105 and 115 km/h

34%

(b) Determine the **percentage** of cars going over 125 km/h

2.5%

(c) Determine the **percentage** of cars going less than 95 km/h

16%

(d) Determine the **number** of cars going between 115 and 125 km/h

13.5%

(e) Determine the **number** of cars going between 85 and 105 km/h

47.5%

Suppose tickets for $300 are issued to any car travelling more than 2 standard deviations above the mean. How much money would be charged in total?

2.5% ≥ 2 σ above mean.

$5000\,(.025) = 125$ cars

$(125)(\$300) = \#37500$

ASSIGNMENT:

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Additional problems on handout: #1 - 4