

Section 2.2

Angles Formed by Parallel Lines

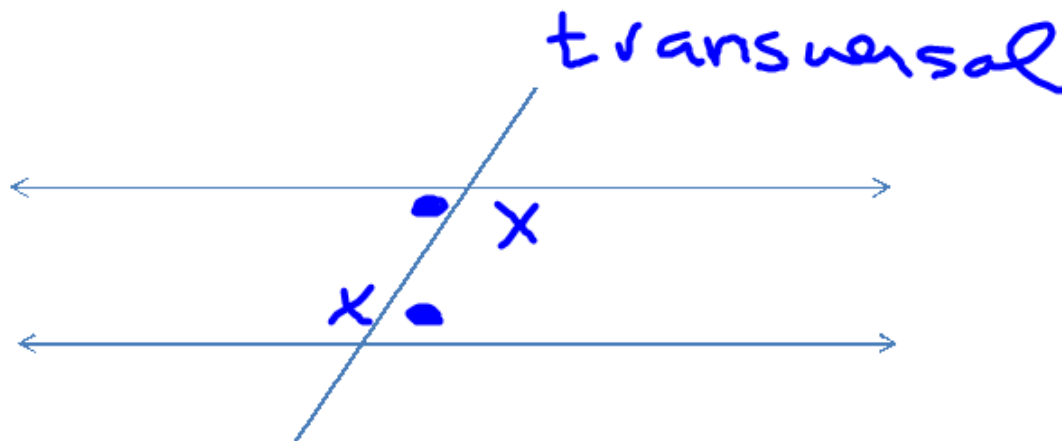
Learning targets:

1. Demonstrate understanding of new terminology (alternate interior angles, alternate exterior angles, vertically opposite angles).
2. Demonstrate understanding of the relationships among pairs of angles when parallel lines are cut by a transversal.
3. Use relationships among pairs of angles when parallel lines are cut by a transversal to solve a problem.
4. Prove relationships deductively in a 2-column proof format.

Key Terms:

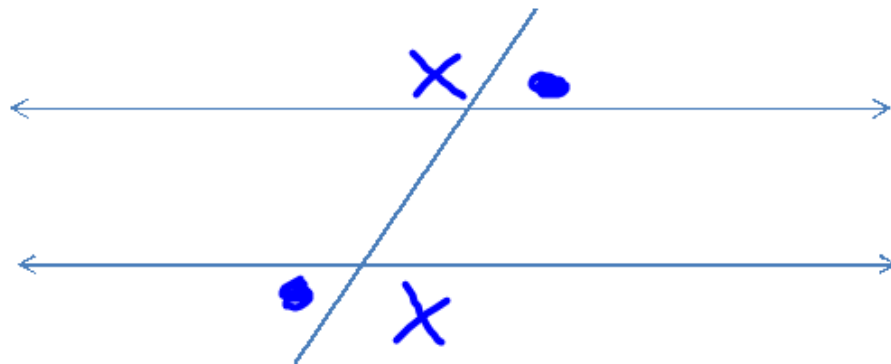
(1) Alternate interior angles:

- two non-adjacent interior angles on opposite sides of the transversal



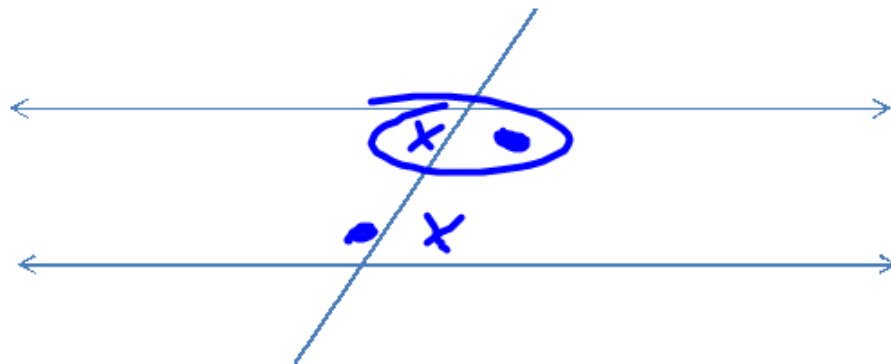
(2) Alternate exterior angles:

- two exterior angles on opposite sides of the transversal



(3) Same-side interior angles:

- two interior angles on the same side of the transversal



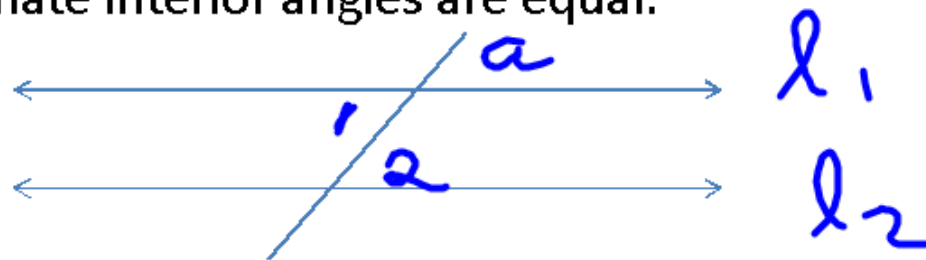
2-Column Deductive Proofs:

- The first column contains statements we believe are true.
- The second column contains the reason or justification for the statement (how do we *know* it's true?)
- These statements and justifications involve known facts about:
 - corresponding angles (they are equal)
 - vertically opposite angles (they are equal)
 - supplementary angles (together they form a straight angle)
- We get to substitute one angle for another once we know they are equal.
- The *transitive* property can be used: if $a = b$ and $b = c$, then $a = c$

Prove the following conjecture:

"When parallel lines are intersected by a transversal, the alternate interior angles are equal."

Create a diagram:



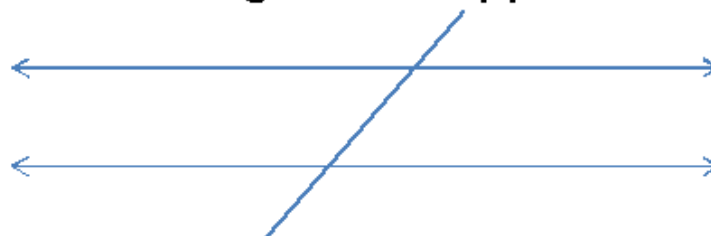
What are you trying to prove?

| <u>Statement</u> | <u>Reason or Justification</u> |
|-----------------------------|--------------------------------|
| ① $l_1 \parallel l_2$ | ① Given |
| ② $\angle 1 \cong \angle a$ | ② V.O. \angle 's are \cong |
| ③ $\angle a = \angle 2$ | ③ Corresponding \angle 's. |
| ④ $\angle 1 = \angle 2$ | ④ Transitive Prop. |

Prove the following conjecture:

“When parallel lines are intersected by a transversal, the same-side interior angles are supplementary.”

Create a diagram:



What are you trying to prove?

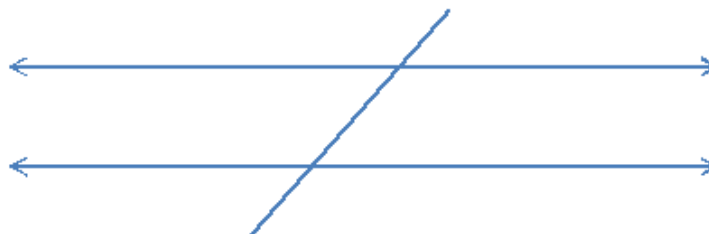
Statement

Reason or Justification

Prove the following conjecture:

“When parallel lines are intersected by a transversal, the alternate exterior angles are equal.”

Create a diagram:



What are you trying to prove?

Statement

Reason or Justification

Results:

Now that we have proved these statements are true, we can use them in other proofs!

(Abbreviations)

1 *Alternate interior angles are equal*

alt int \angle s

2 *Alternate exterior angles are equal*

alt ext \angle s

3 *Same side interior angles are supplementary*

same side \angle s

4 *Corresponding angles are equal*

Corresp \angle s

5 *Vertically opposite angles are equal*

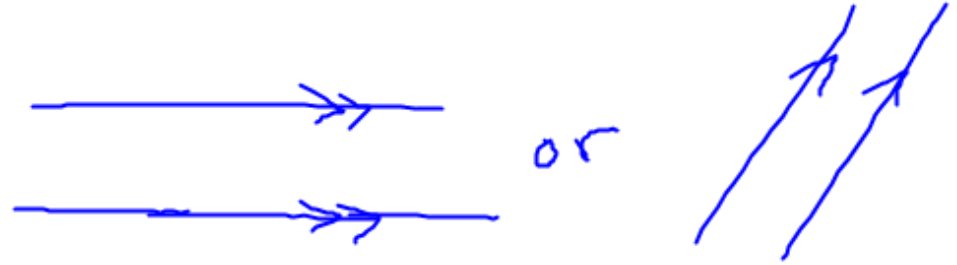
VDA

6 *Angles forming a straight line are supplementary*

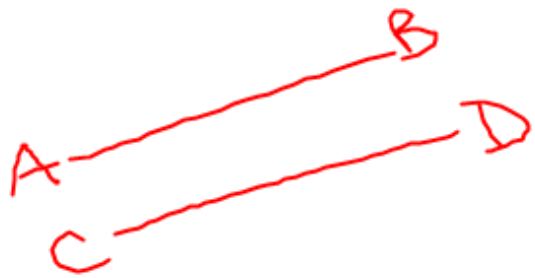
Suppl \angle s

Symbols and Terms:

① The notation used for "parallel" in a diagram:



② The notation for "parallel" in a statement:



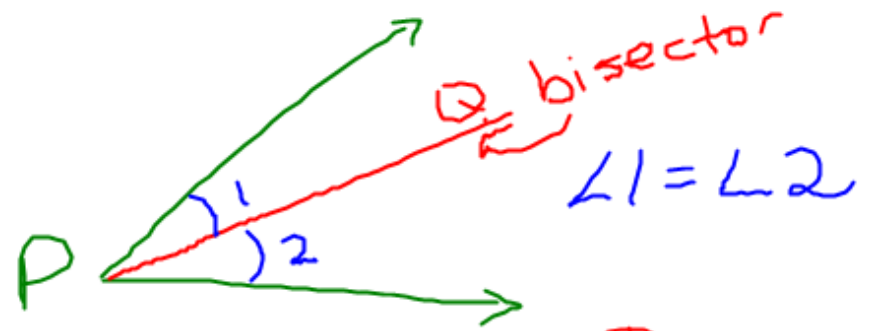
$AB \parallel CD$

↑ "is parallel to"

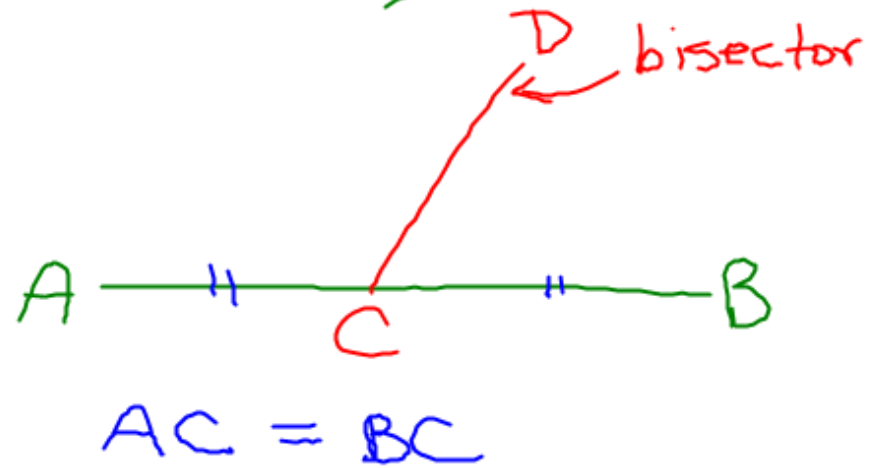


③ "Bisect" means to cut into 2 equal parts:

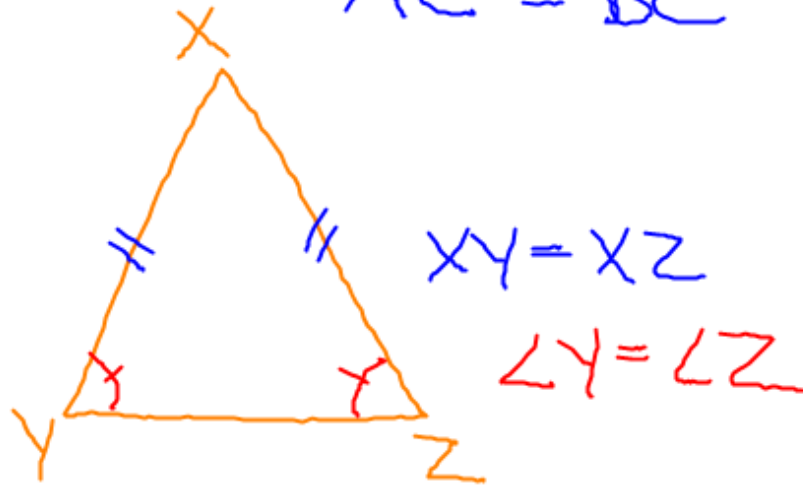
→ angle bisector
PQ bisects $\angle P$



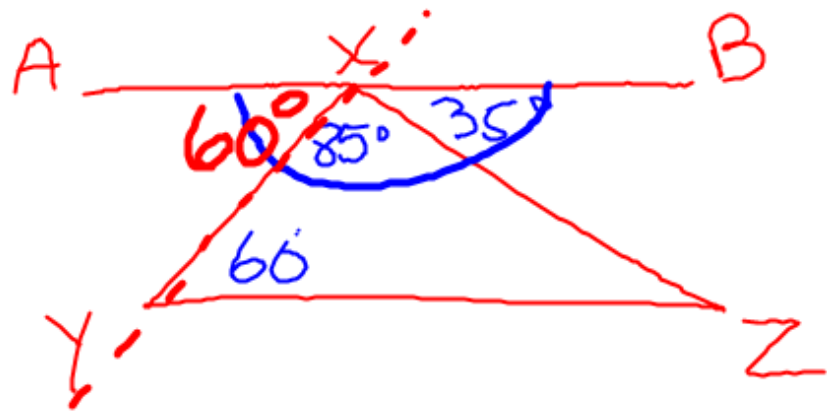
→ segment bisector
CD bisects AB



④ Isosceles \triangle
→ 2 equal sides
→ 2 equal angles



Given:



Prove: $AB \parallel YZ$

S

$$\begin{aligned} \textcircled{1} \quad \angle AXY + \angle YXZ \\ + \angle BXL = 180^\circ \\ \therefore \angle AXY = 60^\circ \end{aligned}$$

$$\textcircled{2} \quad \angle AXY = \angle XYZ$$

$$\textcircled{3} \quad AB \parallel YZ$$

R

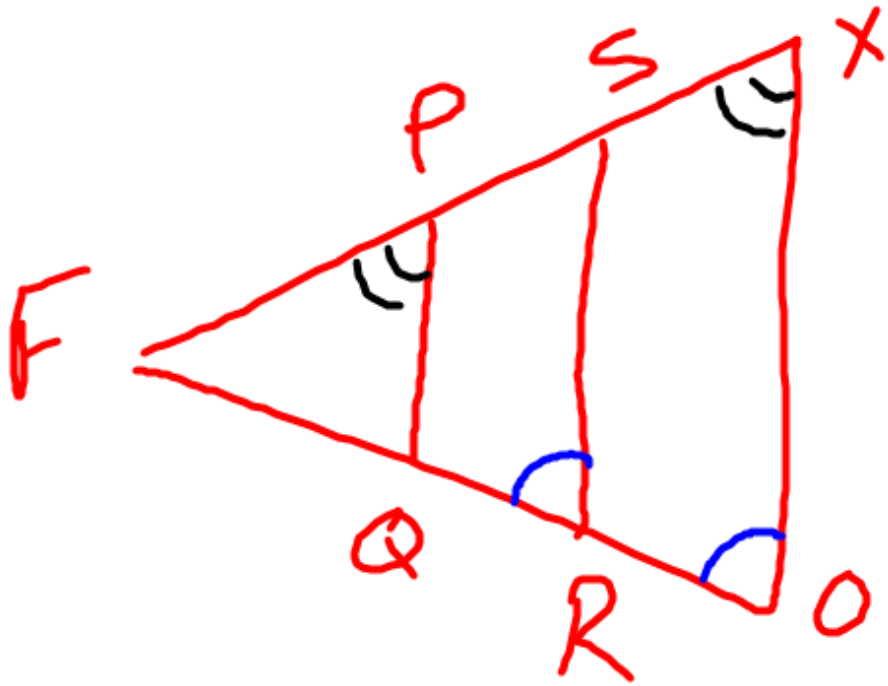
① Supplementary Angles

② alt. int \angle 's

③ alt int \angle 's $\cong \rightarrow \parallel$ lines

ASSIGNMENT:

Text page 79 – 82: #3, 12, 16, 18



Given:

$\triangle FOX$ isosceles

$\angle FOX = \angle FRS$

$\angle FXO = \angle FPQ$

Prove:

$PQ \parallel SR$

$SR \parallel XO$

| S | R |
|---|---|
| ① $LF_{OX} \cong LFRS$ | ① Given |
| ② LF_{OX} and $LFRS$ are corresponding | ② Defn of Correspond. L's. |
| ③ $XO \parallel SR$ | ③ Correspond. L's $\cong \rightarrow \parallel$ lines |
| ④ $LF_{XO} \cong LF_{PQ}$ | ④ Given |
| ⑤ LF_{XO} and LF_{PQ} corresponding | ⑤ see # 2. |
| ⑥ $PQ \parallel XO$ | ⑥ see # 3 |

⑦ PQ || SR

⑦ Transitive Prop.

Relationships we know:

when lines are parallel

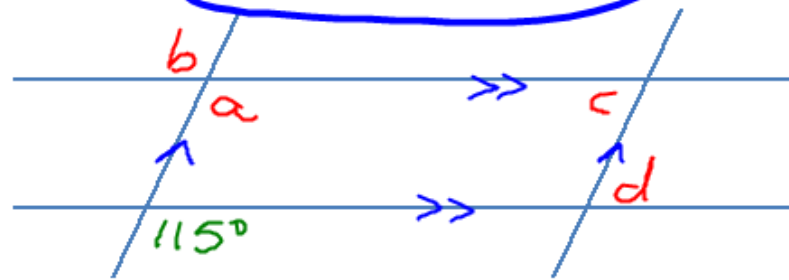
| | | <u>(Abbreviations)</u> | |
|---|---|--|---------------------------------------|
| * | 1 | Corresponding angles are equal | corresp \angles |
| | 2 | Alternate interior angles are equal | alt int \angles |
| | 3 | Alternate exterior angles are equal | alt ext \angles |
| | 4 | Same side interior angles are supplementary | same side \angles |
| | 5 | Vertically opposite angles are equal | VOA |
| | 6 | Angles forming a straight line are supplementary | suppl \angle s |

We can use these relationships to help us solve problems involving missing measurements in geometric diagrams.

Example #1:

Determine the measures of a, b, c and d.
Give justifications and show calculations.

$\angle d = 65^\circ$
alt int \angle 's.



$\angle a = 115^\circ$ corresponding \angle 's

$\angle b = 115^\circ$ \angle 's. (alt ext)

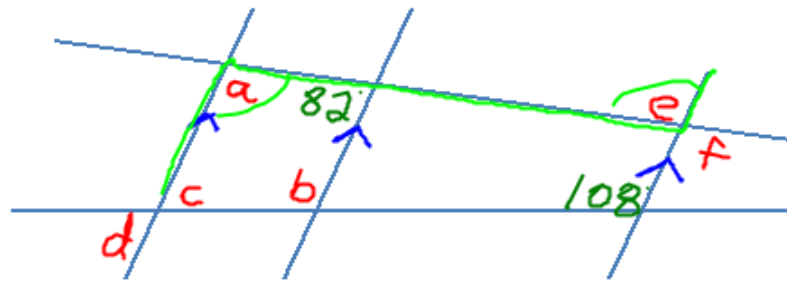
$\angle a + \angle c = 180^\circ$ Supp \angle 's.

$$115^\circ + \angle c = 180^\circ$$

$$\angle c = 65^\circ.$$

Example #2:

Determine the measures of a, b, c, d, e and f.
Give justifications and show calculations.



$$\angle b = 108^\circ$$

corresp. \angle 's.

$$\angle b + \angle c = 180^\circ$$

Supp \angle 's

$$\angle c = 72^\circ$$

$$\angle a + 82^\circ = 180^\circ \quad \text{Supp. } \angle$$
's

$$\angle a = 98^\circ$$

$$\angle e = 98^\circ \quad \text{alt int}$$

$$\angle f = 98^\circ \quad \text{VO } \angle$$
's

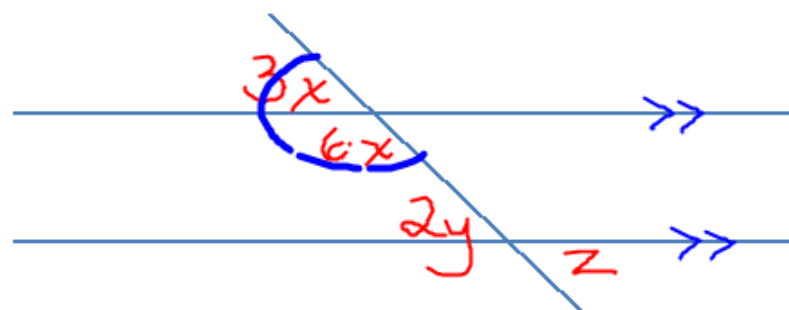
$$\angle d = 72^\circ$$

$$\text{VO } \angle$$
's.

Example #3: $6x + 2y = 150$

Determine the values of x , y , and z .

Give justifications and show calculations.



$$6x + 2y = 150$$

Same Side

int.

$$6(20) + 2y = 180$$

$$2y = 60$$

$$y = 30^\circ$$

$$3x + 6x = 180^\circ$$

$$9x = 180^\circ$$

$$x = 20^\circ$$

Supp \angle 's

$$2y = z$$

VO \angle 's

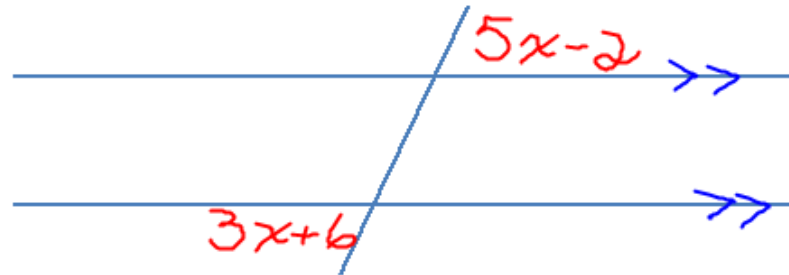
$$2(30^\circ) = z$$

$$60^\circ = z$$

Example #4:

Solve for x .

Give justifications and show calculations.



$$5x - 2 = 3x + 6 \quad \text{alt ext}$$

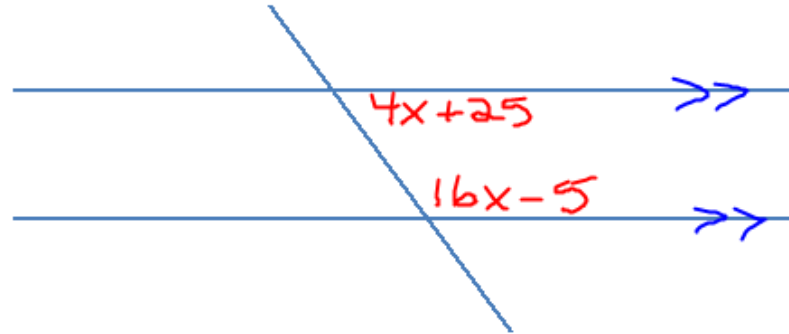
$$2x = 8$$

$$x = 4$$

Example #5:

Solve for x.

Give justifications and show calculations.



$$4x + 25 + 16x - 5 = 180^\circ \quad \text{Supp } \angle\text{'s}$$

$$20x + 20 = 180$$

$$20x = 160$$

$$x = 8$$

ASSIGNMENT:

Text page 78 – 82: #1, 2, 4, 15, 20