8.6 Scale Factors and 3-D Objects

Learning targets:

1. Demonstrate understanding of new terminology pertaining to scale diagrams and 3-D objects.
2. Calculating a scale factor.
3. Using scale factors to solve problems.

Key Ideas

When two 3-D objects are similar and their proportions are related by a scale factor of k, then:

(1) Their surface areas are related by a factor of k^2

$$k^2 = \frac{\text{surface area new}}{\text{surface area original}}$$

(2) Their volumes are related by a factor of k^3

$$k^3 = \frac{\text{volume new}}{\text{volume original}}$$

<u>Example #1</u>:

A cube is a 3-D shape that is always similar to other cubes.

If one cube has an edge length of 1.5 cm and another cube has an edge length of 6.0 cm,

a) What scale factor is needed to enlarge the small cube to the larger one?

$$k = \frac{\text{enlarged}}{\text{original}} = \frac{6.0\text{ cm}}{1.5\text{ cm}} = 4$$

b) How many times greater will the surface area of the larger cube be?

$$K^2 = (4)^2 = 16 \text{ times greater}$$

b) How many times greater will the volume of the larger cube be?

$$K^3 = (4)^3 = 64 \text{ times greater}$$

You Try:

A stage director needs a large chess bishop for a scene. The bishop in her chess set is 78 mm tall. She wants the enlarged bishop to be 1.56 m tall.

a) What scale factor must she apply to create the enlarged bishop?

$$k = \frac{1560\ \cancel{mm}}{78\ \cancel{mm}} = 20$$

b) How many times greater will the surface area of the larger bishop be?

$k^2 = (20)^2 = 400$ times greater

c) How many times greater will the volume of the larger bishop be?

$k^3 = (20)^3 = 8000$ times greater.

Example #2:

An orange has a diameter of 7 cm. A grapefruit has a diameter of 12 cm. How many times greater is the volume of the grapefruit than that of an orange?

$$k = \frac{12\,\cancel{cm}}{7\,\cancel{cm}} =$$

$$k = \left(\frac{12}{7}\right)$$

$$k^3 = \left(\frac{12}{7}\right)^3 = \frac{1728}{343} = 5.0379$$

Grapefruit is approx 5 times greater volume than orange.

Example #3:

The surface area of an enlarged triangular prism is 6.25 times greater than that of the original prism. How many times greater is the volume of the enlarged prism than the volume of the original?

$$k^2 = \frac{\text{surface area enlarged}}{\text{surface area original}}$$

$$\sqrt{k^2} = \sqrt{6.25}$$

$$K = 2.5$$

$$\frac{\text{Volume}}{(K)^3 = (2.5)^3}$$

$$= 15.625$$

times greater.

You Try:

If the surface area of an enlarged cylinder is 64 times greater than that of the original cylinder, how many times greater is the volume?

$k^2 = 64$

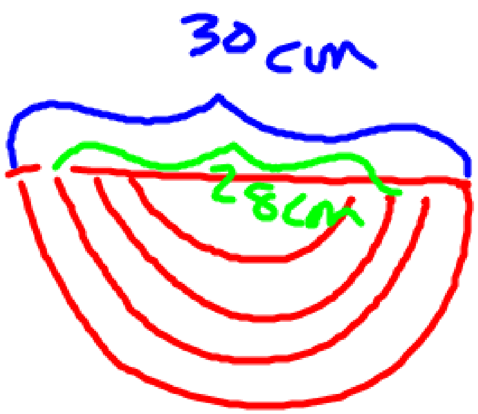
$k = 8$

$K^3 = (8)^3 = 512$ times great

Example #4:

A cook has a set of four mixing bowls. The bowls stack inside each other and are similar. The two largest diameters are 30 cm and 28 cm. The scale factor is the same from each bowl to the next smaller bowl.

What are the diameters of the two smallest bowls to the nearest tenth of a centimetre?



$$k = \frac{28\text{ cm}}{30\text{ cm}}$$

$$k = \frac{14}{15}$$

Bowl 3: $(28\text{ cm})(14/15) = 26.1\text{ cm}$

Bowl 4: $(26.1\text{ cm})(14/15) = 24.4\text{ cm}$

Example 5

The diameter of a Pringles can is 7.5 cm. The height of the can is 23 cm. If the can was to be enlarged by a factor of 4, what would be the surface area of the new can?

$$k^2 = \frac{\text{Surface Area Similar}}{\boxed{\text{Surface Area Original}}}$$

$k = 4$

$r = 3.75\text{ cm}$

$h = 23\text{cm}$

$$SA = 2\pi r^2 + 2\pi r h$$

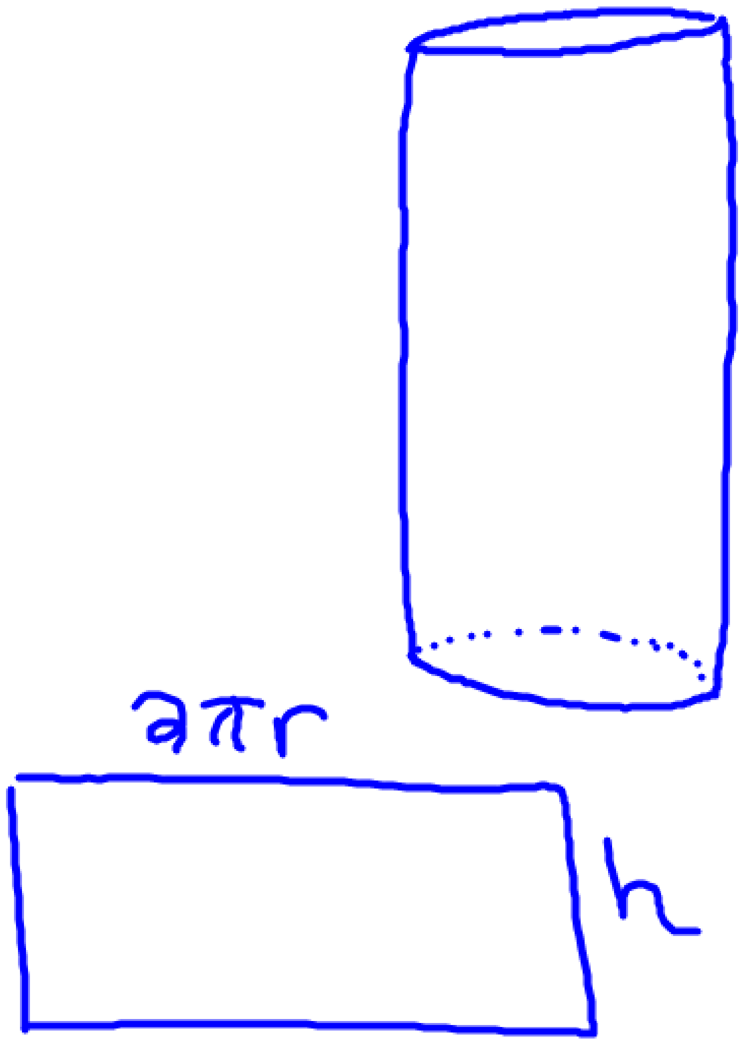
$$SA = 2\pi(3.75\text{cm})^2 + 2\pi(3.75\text{cm})(23\text{cm})$$

$$28.125\pi + 172.5\pi$$

$$SA = 200.625\pi$$

$$(4)^2 = \frac{\text{SA Enlarged}}{200.625\pi\ cm^2}$$

$$3210\pi\ cm^2 = \text{SA enlarged}.$$



$$\underset{\text{(ends)}}{2\pi r^2} + \underset{\text{(walls)}}{2\pi r h}$$

Assignment

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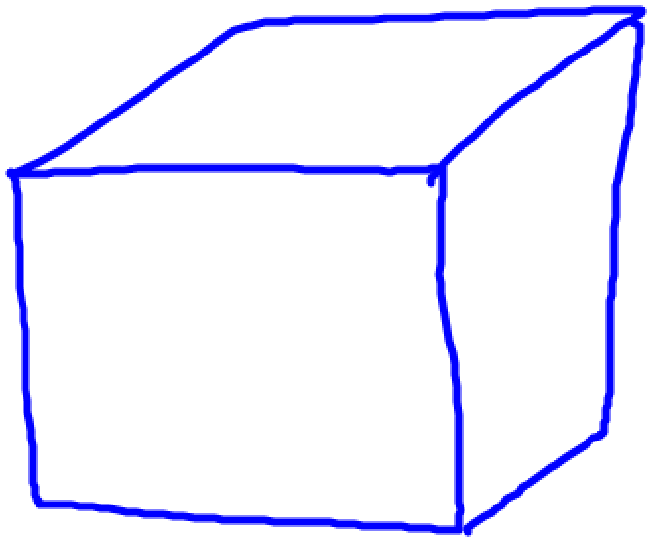
#1, 2, 3, 4, 6, 8, 9, 13



$V = 4500\ cm^3$

SA lid $= 375\ cm^2$

$k = 3$

$k^2 = \dfrac{\text{SA sim}}{\text{SA org.}}$

$(3)^{2} = \dfrac{\text{SA sim}}{375\ cm^2}$

9

9 times larger

$k^3 = \dfrac{VS}{VO}$

$(3)^{3} = \dfrac{VS}{4500\ cm^3}$

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