Explore:

$k^2 = \frac{\text{area new}}{\text{area original}}$

A 2 cm x 5 cm rectangle has been enlarged by a scale factor of 3 to create a similar rectangle.

How are the *areas* of these rectangles related?

Original Rectangle:

Dimensions:

2cm x 5cm

Area:

$10\ cm^2$

Enlarged Rectangle:

Dimensions:

6cm x 15cm

Area:

$90\ cm^2$

Conclusion:

$k = \sqrt{\frac{\text{area new}}{\text{area original}}}$

8.4 Scale Factors and Areas of 2-D Shapes

Learning targets:

1. Demonstrate understanding of how the areas of similar shapes are related by their scale factor.
2. Calculating a scale factor using measurements from similar shapes.
3. Using scale factors to solve problems.

Key Ideas

When a 2-D shape undergoes an enlargement or reduction by a scale factor, k, the resulting shape is **SIMILAR** to the original shape.

If two 2-D shapes are similar and their dimensions are related by a scale factor, k, then the relationship between the **area** of the similar shape and the **area** of the original shape can be expressed as:

Area of similar 2-D shape = k^2(Area of original shape)

If the area of a similar 2-D shape and the area of the original shape are known, then the scale factor, k, can be determined using the ratios of their areas:

k^2(area original) = area of similar

$$k^2 = \frac{\textbf{Area of similar 2 - D shape}}{\textbf{Area of original shape}}$$

$$k = \sqrt{\frac{\textbf{Area of similar 2 - D shape}}{\textbf{Area of original shape}}}$$

Example #1:

Finding the area of a similar shape

A triangle whose area is 76 cm^2 will be reduced by a scale factor of $\frac{5}{8}$. Determine the area of the reduced triangle to the nearest square centimetre.

$k = \frac{5}{8}$ area of reduced $\triangle$?

area reduced = k^2(area original)

area reduced = $\left(\frac{5}{8}\right)^2(76\,cm^2)$

$= \left(\frac{25}{64}\right)(76\,cm^2) \approx 29.7\,cm^2$

Example #1:

Finding the area of a similar shape

A triangle whose area is 76 cm^2 will be reduced by a scale factor of $\frac{5}{8}$. Determine the area of the reduced triangle to the nearest square centimetre.

Area of reduced triangle = $k^2 \times$ area of original triangle

$$= \left(\frac{5}{8}\right)^2 \times (76\ \text{cm}^2)$$

$$= \frac{25}{64} \times 76\ \text{cm}^2$$

$$= 29.6875\ \text{cm}^2$$

$\therefore$ The reduced triangle will have an area of approximately 30 cm^2

You Try:

A circle of area 10 cm^2 will be enlarged by a scale factor of 5.5
Determine the area of the enlarged circle.

area enlarged $= k^2$ (area of original)

area enlarged $= (5.5)^2 (10\ cm^2)$

area enlarged $= 302.5\ cm^2$

Example #2:

Finding a scale factor from measurements

The area of a parallelogram is $900\ \text{mm}^2$

The area of a smaller similar parallelogram is $100\ \text{mm}^2$

What scale factor was used to make this reduction?

$$k^2 = \frac{\text{area similar}}{\text{area original}}$$

$$k^2 = \frac{100\ \text{mm}^2}{900\ \text{mm}^2}$$

$$k^2 = \frac{1}{9}$$

$$k = \pm\sqrt{\frac{1}{9}}$$

$$k = \frac{1}{3}$$

Example #2:

Finding a scale factor from measurements

The area of a parallelogram is $900\ \text{mm}^2$

The area of a smaller similar parallelogram is $100\ \text{mm}^2$

What scale factor was used to make this reduction?

$$k^2 = \frac{Area\ of\ similar\ 2\text{–}D\ shape}{Area\ of\ original\ shape}$$

$$k^2 = \frac{100\ \text{mm}^2}{900\ \text{mm}^2}$$

$$k^2 = \frac{1}{9}$$

$$\therefore k = \frac{\sqrt{1}}{\sqrt{9}} = \frac{1}{3}$$

The scale factor used was $\frac{1}{3}$

You Try:

A trapezoid was reduced from an area of 750 mm^2 to 30 mm^2.

What scale factor was used to make this reduction?

$$k^2 = \frac{\text{area similar}}{\text{area original}}$$

$$k^2 = \frac{30\ \text{mm}^2}{750\ \text{mm}^2}$$

$$k^2 = \frac{3}{75}$$

$$k^2 = \frac{1}{25}$$

$$k = \sqrt{\frac{1}{25}}$$

$$k = \frac{1}{5}$$

Example #3: Application

Jasmine is making a kite from a 2 : 25 scale diagram. The **area** of the scale diagram is $20\ \text{cm}^2$. How much fabric will she need for her kite?

$k = \frac{2}{25}$

$k^2 = \frac{as}{ao}$

area similar $= 20\text{cm}^2$

$\left(\frac{2}{25}\right)^2 = \frac{20\ \text{cm}^2}{\text{area } o.}$

$\frac{4}{625} = \frac{20\ \text{cm}^2}{\text{area } o.}$

$$\frac{4(\text{area } 0)}{4} = \frac{(625)(20\ \text{cm}^2)}{4}$$

$$\text{area } 0 = 3125\ \text{cm}^2$$

Example #3: Application

Jasmine is making a kite from a 2 : 25 scale diagram. The **area** of the scale diagram is $20\ \text{cm}^2$. How much fabric will she need for her kite?

Normally the diagram measurement is the top part of our scale factor. Let's "flip" our thinking to find the scale factor needed to produce the kite from the drawing, rather than the scale factor needed to produce the drawing based on the kite.

Scale factor to make the kite: $\frac{25}{2} = 12.5$

Making the kite from the drawing is an enlargement.

Example #3: Application

Jasmine is making a kite from a 2 : 25 scale diagram. The **area** of the scale diagram is $\mathbf{20\ cm^2}$. How much fabric will she need for her kite?

Therefore, the area of the kite is $k^2 \times$ **(the area of the diagram)**:

Area of kite: $(12.5)^2(20\text{cm}^2) = 3125\text{cm}^2$

$\therefore$ Jasmine would need $3125\ \text{cm}^2$ of fabric to make the kite.

You Try:

Suppose Jasmine's scale diagram for the kite had been drawn using a ratio of 1 : 20, and the area of the scale diagram had been $30\ \text{cm}^2$.

How much fabric would Jasmine have needed for her kite?

Example #4: Application

A computer screen measures 35 cm by 55 cm. An image on the computer is projected onto a whiteboard with a screen area of 7238 cm^2. Determine the length and width of the whiteboard to the nearest cm.

Area of comp. screen

$ACS = (35cm)(55cm)$

$ACS = 1925cm^2$

$k^2 = \frac{\text{area sim.}}{\text{area org.}}$

$k^2 = \frac{7238\ cm^2}{1925\ cm^2}$

$k^2 = 3.76$

$k = 1.94$

Length Whiteboard

$(55\text{cm})(1.94) = 106.7\text{cm}$

Width

$(35\text{cm})(1.94) = 67.9\text{cm}$

Example #4: Application

A computer screen measures 35 cm by 55 cm. An image on the computer is projected onto a whiteboard with a screen area of $7238\ \text{cm}^2$. Determine the length and width of the whiteboard to the nearest cm.

Computer screen area: $(35\ \text{cm})(55\ \text{cm}) = 1925\ \text{cm}^2$

Projected image on whiteboard area: $7238\ \text{cm}^2$

Computer screen = original

Projected image on whiteboard = enlargement

Scale factor for the whiteboard (enlargement)

$$k^2 = \frac{7238\ \text{cm}^2}{1925\text{cm}^2}$$

$$k^2 = 3.76$$

$$k = \sqrt{3.76} = 1.939\ \ldots$$

Dimensions of the whiteboard:

Length: 55 cm x 1.939... = 106.6489... cm

Width: 35 cm x 1.939... = 67.8675... cm

$\therefore$ The whiteboard is approximately 68 cm wide by 107 cm long

Assignment

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#1, 2, 3, 6, 7, 9, 10, 13, 14