6.6 Chapter Review

3. $s(t) = 2t^3 - 21t^2 + 72t$

a) $v(t) = s'(t) = 6t^2 - 42t + 72$

$a(t) = v'(t) = 12t - 42$

b) $s(8) = 2(8)^3 - 21(8)^2 + 72(8) = 256$ units

$v(8) = 6(8)^2 - 42(8) + 72 = 120$ units/s

$a(8) = 12(8) - 42 = 54$ units/s^2

c) $v(t) = 0$ when $6t^2 - 42t + 72 = 0$

$6(t^2 - 7t + 12) = 0$

$6(t-3)(t-4) = 0$

$t = 3s$ or $t = 4s$

	+	0	−	0	+	$v(t)$
	2	3	3.5	4	5	

Moving Right $(0,3) \cup (4,\infty)$

d) Moving Left $(3,4)$

e) Distance from 0 to 5

$|s(3) - s(0)| = |81 - 0| = 81$

$|s(4) - s(3)| = |80 - 81| = 1$

$|s(5) - s(4)| = |85 - 80| = 5$

$\therefore$ Total Distance $81 + 1 + 5 = 87$ units

) ave vel $= \dfrac{s(10) - s(6)}{10 - 6} = \dfrac{620 - 108}{4} = 128$ units/s

④ $s(t) = -5t^{-} + 60t + 8$

a) $S(0) = 8m$

b) when $v(t) = 0$

$\therefore v(t) = s'(t) = -10t + 60$

$-10t + 60 = 0$

$60 = 10t$

$6s = t$

c) Max height

$S(6) = 188m$

d) when $s(t) = 0$

$-5t^2 + 60t + 8 = 0$

quad formula

$t = \cancel{-0.13}2s$ or $t = 12.132s$

e) $V(12.132) = -10(12.132) + 60$

$= -61.32 m/s$

⑤ $s(t) = 12t - t^3$

Find when $v(t) = 0$ that gives time for moving snowbile to stop.

$v(t) = s'(t) = 12 - 3t^2$

$12 - 3t^2 = 0$

$12 = 3t^2$

$4 = t^2$

$\pm 2 = t$

$2 = t$

$S(2) = 12(2) - (2)^3$

$= 16m$

$\therefore$ No collision as moving snowmobile has 18 m to stop.

⑧ Let $x = 1^{st}$ non-negative #
$y = 2^{nd}$ non-negative #

$xy = 72$

$y = \frac{72}{x}$

$x + 2y = S$

$x + 2\left(\frac{72}{x}\right) = S$

$x + 144x^{-1} = S$

$S' = 1 - 144x^{-2}$

$S' = 1 - \frac{144}{x^2}$

$S'\ \infty$ when $x = 0$

$S' = 0$

$1 - \frac{144}{x^2} = 0$

$1 = \frac{144}{x^2}$

$x^2 = 144$

$x = \pm 12$

$\boxed{x = 12}$

Proof

$S'' = 288x^{-3}$

$S'' = \frac{288}{x^3}$

$S''(12) = \frac{288}{(12)^3} > 0$

$\therefore$ ∪ concave up

$\therefore$ min

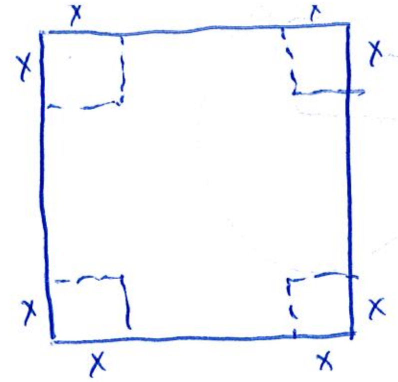
Find y

$y = \frac{72}{12}$

$y = 6.$

The numbers are 12 and 6.

(10).

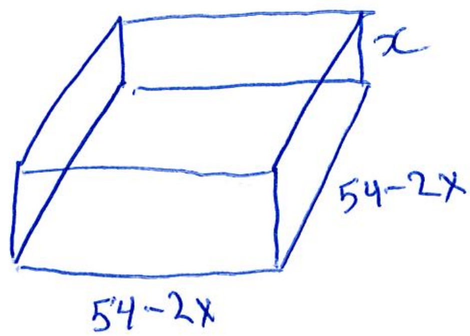


let x = length of cut.

$V = x(54-2x)^2$

$V = x(2916 - 216x + 4x^2)$

$V = 4x^3 - 216x^2 + 2916x$



$V' = 12x^2 - 432x + 2916$

Set $V'(x) = 0$

$12x^2 - 432x + 2916 = 0$

quad formula

$x = \cancel{27}$ or $x = 9$

too big

Proof

$V'' = 24x - 432$

$V''(9) = 24(9) - 432$

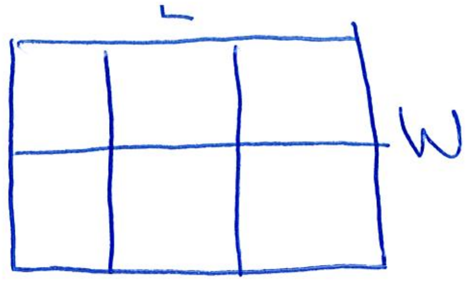
$V''(9) < 0$

$\therefore$ Concave down

$\therefore$ max.

There size of square being cut are 9cm x 9cm.

(11)



Let L = entire length
W = entire width

$A = LW$

$4W + 3L = 72$

$3L = 72 - 4W$

$L = 24 - \frac{4}{3}W$

$A = (24 - \frac{4}{3}w)w$

$A = 24w - \frac{4}{3}w^2$

$A' = 24 - \frac{8}{3}w$

$24 - \frac{8}{3}w = 0$

$24 = \frac{8}{3}w$

$72 = 8w$

$9 = w$

Proof

$A'' = -\frac{8}{3} < 0$

$\therefore$ concave down

$\therefore$ $w = 9$ max.

Find length

$L = 24 - \frac{4}{3}(9)$

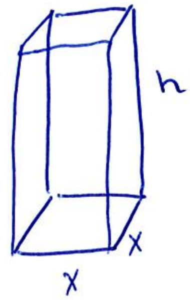
$L = 12$

So now we need to calculate dimensions of one pen.

$4\,m \times 4.5\,m$

$\therefore$ Each pen has the dimensions of 4m by 4.5m.

14.)



let $x =$ width and length
$h =$ height

$$x^2 h = 50 \rightarrow h = \frac{50}{x^2}$$

$$C(x) = 80(2x^2) + 200(4xh)$$
$$C(x) = 160x^2 + 800xh$$

$$C(x) = 160x^2 + 800x\left(\frac{50}{x^2}\right)$$
$$C(x) = 160x^2 + 4000x^{-1}$$
$$c'(x) = 320x - 4000x^{-2}$$

$\underline{c'(x)\ \infty}$
when $x = 0$

$\underline{c'(x) = 0}$
$$320x - \frac{40000}{x^2} = 0$$
$$320x = \frac{40000}{x^2}$$
$$320x^3 = 40000$$
$$x^3 = 125$$
$$x = 5$$

Proof

$$c''(x) = 320 + \frac{8000}{x^3}$$
$$c''(5) = 320 + \frac{8000}{(5)^3} > 0$$
$\therefore$ CD

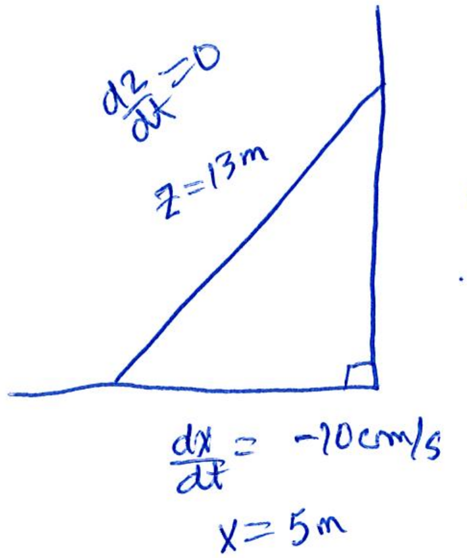
$\therefore x = 5$ min

Dimensions

The box has dimension of 5m x 5m x 2m.

Find h
$$h = \frac{50}{x^2} = \frac{50}{(5)^2} = 2$$

17.



$\frac{dy}{dt} = ?$

$y = 12m$

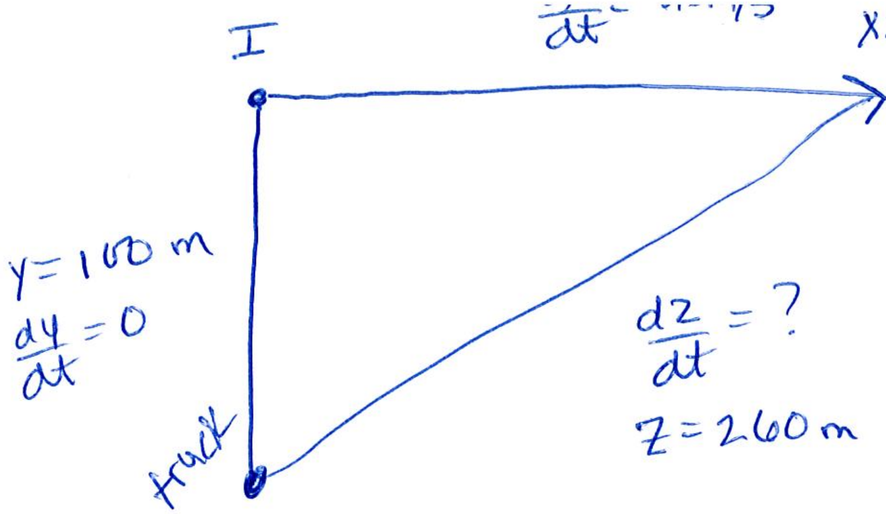
$$x^2 + y^2 = z^2$$
$$(5)^2 + y^2 = (13)^2$$
$$y^2 = 144$$
$$y = \pm 12$$
$$y = 12$$

$$x^2 + y^2 = z^2$$
$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2z\overbrace{\frac{dz}{dt}}^{=0}$$
$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$
$$\cancel{2}y\frac{dy}{dt} = -\cancel{2}x\frac{dx}{dt}$$
$$\frac{dy}{dt} = -\frac{x}{y}\frac{dx}{dt}$$
$$\frac{dy}{dt} = \left(-\frac{5\cancel{m}}{12\cancel{m}}\right)\left(-10\ cm/s\right)$$
$$\frac{dy}{dt} = \frac{25}{6}\ cm/s$$

The top of ladder slides up at a rate of $\frac{25}{6}$ cm/s.

(14)

$x = (20\,m/s)(12\,s) = 240\,m$



$x^2 + y^2 = z^2$

$(240)^2 + (100)^2 = z^2$

$z = 260\,m$

$$x^2 + y^2 = z^2$$

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2z\frac{dz}{dt}.$$

$$\cancel{2}x\frac{dx}{dt} = \cancel{2}z\frac{dz}{dt}$$

$$\frac{x}{z}\frac{dx}{dt} = \frac{dz}{dt}$$

$$\left(\frac{240\,m}{260\,m}\right)20\,m/s = \frac{dz}{dt}.$$

$$18.46\,m/s = \frac{dz}{dt}.$$

The rate at which the can is moving away from the truck is 18.46 m/s.

(23) $\frac{dr}{dt} = 2 \text{ mm/min}$

$\frac{dV}{dt} = ?$

$r = 1.5 \text{ cm} = 15 \text{ mm}$

$V = \frac{4}{3}\pi r^3$

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

$\frac{dV}{dt} = 4\pi (15 \text{ mm})^2 (2 \text{ mm/min})$

$= 1800\pi \text{ mm}^3/\text{min}$

$= 5654.87 \text{ mm}^3/\text{min}$

(24) $\frac{dV}{dt} = 20\pi \text{ m}^3/\text{h}$

$\frac{dr}{dt} = ?$

$r = 5 \text{m}$

$V = \frac{4}{3}\pi r^3$

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

$\frac{\frac{dV}{dt}}{4\pi r^2} = \frac{dr}{dt}$

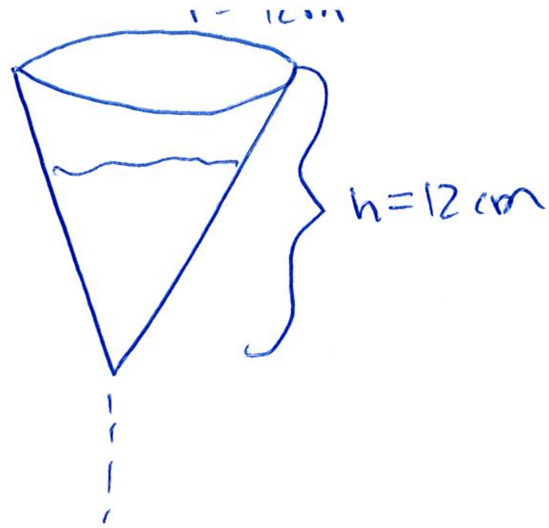
$\frac{20\cancel{\pi} \frac{m^3}{h}}{4\cancel{\pi}(5m)^2} = \frac{dr}{dt}$

$\frac{1}{5} \text{ m/h} = \frac{dr}{dt}$

$0.2 \frac{m}{h} = \frac{dr}{dt}$

The rate at which the radius is increasing is 0.2 m/h.

(25)



$$\frac{r}{h} = \frac{4}{12}$$

$$r = \frac{h}{3}$$

$$\frac{dV}{dt} = -6\,cm^3/s$$

$$h = 8\,cm$$

$$\frac{dh}{dt} = ?$$

$$V = \frac{1}{3}\pi r^2 h$$

$$V = \frac{1}{3}\pi \left(\frac{h}{3}\right)^2 h$$

$$V = \frac{\pi}{3}\left(\frac{h^2}{9}\right) h$$

$$V = \frac{\pi h^3}{27}$$

$$\frac{dV}{dt} = \frac{3\pi}{27} h^2 \frac{dh}{dt}$$

$$-6\,cm^3/s = \frac{3\pi}{27}(8\,cm)^2 \frac{dh}{dt}$$

$$-6\,cm^3/s = \frac{64\pi}{9} cm^2 \frac{dh}{dt}$$

$$-.269\,cm/s = \frac{dh}{dt}$$

The water level is falling at a rate of .269 cm/s.

(26) $\frac{dV}{dt} = 0.72\ m^3/min$

$r = h$

$\frac{dh}{dt} = ?$

$h = 6m$

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi h^3$

$\frac{dV}{dt} = \pi h^2 \frac{dh}{dt}$

$$\frac{\frac{dV}{dt}}{\pi h^2} = \frac{dh}{dt}$$

$$\frac{0.72\ m^3/min}{\pi (6m)^2} = \frac{dh}{dt}$$

$$0.0064\ m/min = \frac{dh}{dt}$$

21. $d = 12m$

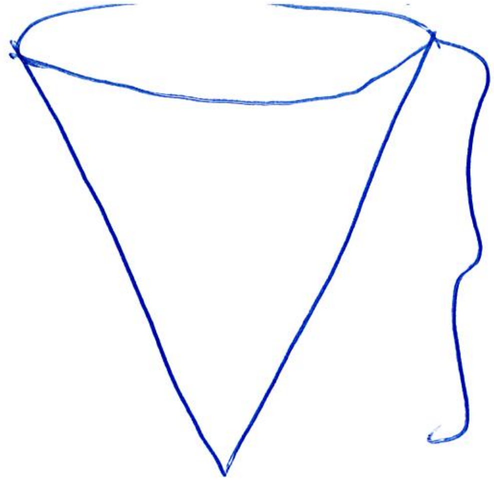
$r = 6m$

$h = 1m$

$\frac{dV}{dt} = -2m^3/min$

$\frac{dh}{dt} = ?$

$h = 25cm$



$h = 1m = 100cm$

$\frac{r}{h} = \frac{600}{100}$

$r = 6h$

$$V = \frac{1}{3}\pi r^2 h$$
$$V = \frac{1}{3}\pi (6h)^2 h$$
$$V = \frac{1}{3}\pi (36h^2) h$$
$$V = 12\pi h^3$$
$$\frac{dV}{dt} = 36\pi h^2 \frac{dh}{dt}$$

$$\frac{\frac{dV}{dt}}{36\pi h^2} = \frac{dh}{dt}$$

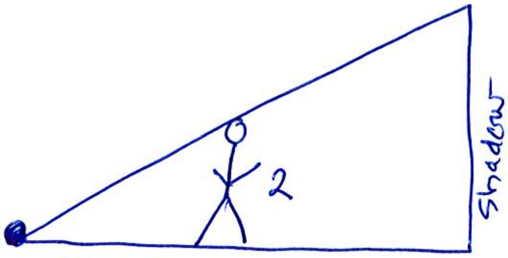
$$\frac{-2m^3/min}{36\pi(.25m)^2} = \frac{dh}{dt}$$

$$-0.283\ m/min = dh/dt$$

$$-28.3\ cm/min = \frac{dh}{dt}$$

The water level is decreasing at a rate of 28.3 cm/min.

29.



$\frac{ds}{dt} = ?$

$\frac{dx}{dt} = 1.6 m/s$

$x = 8$

$$\frac{s}{12} = \frac{2}{x}$$

$$sx = 24$$

$$s = 24x^{-1}$$

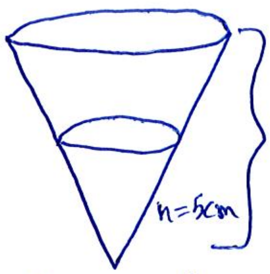
$$\frac{ds}{dt} = -24x^{-2}\frac{dx}{dt}$$

$$\frac{ds}{dt} = \frac{-24}{x^2}\frac{dx}{dt}$$

$$= \frac{-24}{(8)^2} 1.6 m/s$$

$$= -0.6 m/s$$

30.

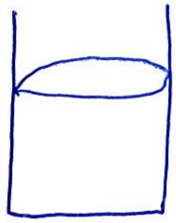


$h=12\,cm$

$h=5\,cm$

$$\frac{r}{h}=\frac{6}{12}$$

$$\boxed{r=\frac{h}{2}}$$



$$\frac{dh}{dt}=3\,cm/min$$

$r=5\,cm$

$$V=\pi r^2 h$$
$$V=\pi(25)h$$
$$\frac{dV}{dt}=25\pi\frac{dh}{dt}$$
$$\frac{dV}{dt}=25\pi(3\,cm/min)$$
$$\frac{dV}{dt}=75\pi\,cm^3/min$$

$$V=\frac{1}{3}\pi r^2 h$$
$$V=\frac{\pi}{3}\left(\frac{h}{2}\right)^2 h$$
$$V=\frac{\pi h^3}{12}$$
$$\frac{dV}{dt}=\frac{3\pi}{12}h^2\frac{dh}{dt}$$
$$-75\cancel{\pi}\,cm^3/min=\frac{\cancel{\pi}}{4}(5\,cm)^2\frac{dh}{dt}$$
$$\frac{-75\,cm^3/min}{\left(\frac{25}{4}\,cm^2\right)}=\frac{dh}{dt}=-12\,cm/min$$

Water height in cone is decreasing at a rate of 12cm/min.