Applications: Quadratic Functions and Equations in Vertex Form

Learning targets:

1. Use quadratic graphing skills to solve quadratic applications problems dealing with maximums and minimums.
2. Use real-world information to develop the equation of a quadratic function in vertex form.

Quadratic Application: Height versus Time

When an object is thrown, kicked, hit or dropped, its height, $f(t)$, above the ground as a function of time, t, can be modelled by a quadratic function where $f(t)$ is the **dependent** variable (replaces y) and t is the **independent** variable (replaces x).

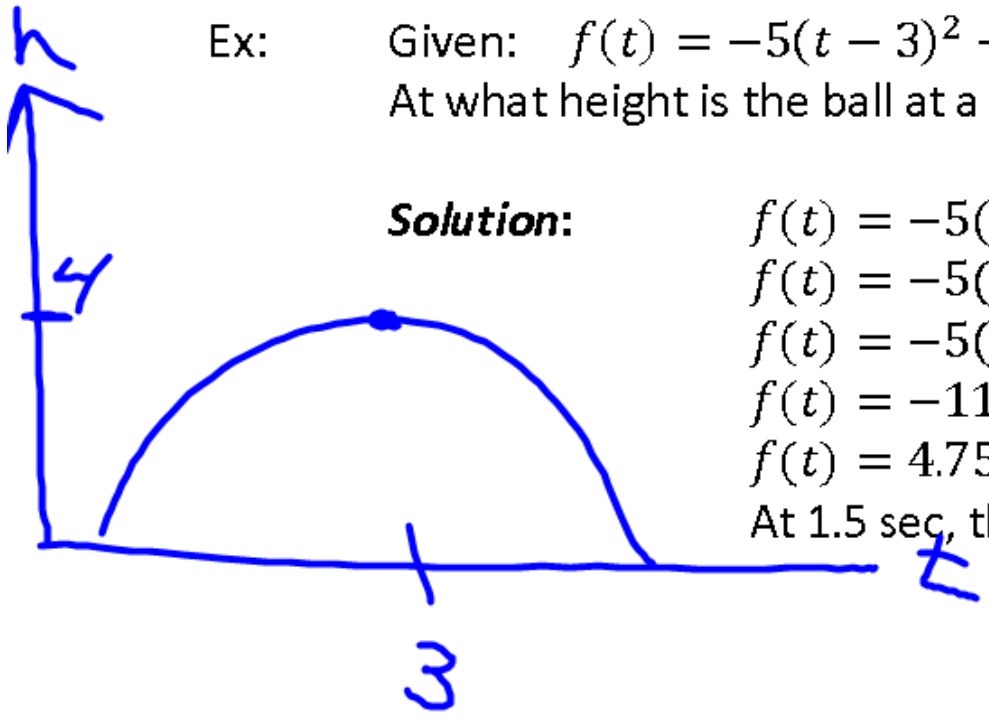
$f(x) = a(x-h)^2 + k$ becomes $f(t) = a(t-h)^2 + k$

$$\boldsymbol{height = f(t) = a(t-h)^2 + k}$$

Every point on this graph has coordinates of $(\boldsymbol{time, height}) = (\boldsymbol{t, f(t)})$

The height of the object **at any given time** can be calculated by substituting in the t-value and evaluating $f(t)$.

Ex: Given: $f(t) = -5(t-3)^2 + 16$ ($f(t)$ = metres, t = sec)
At what height is the ball at a time of 1.5 seconds?



***Solution*:**

$$f(t) = -5(1.5-3)^2 + 16$$
$$f(t) = -5(-1.5)^2 + 16$$
$$f(t) = -5(2.25) + 16$$
$$f(t) = -11.25 + 16$$
$$f(t) = 4.75$$

At 1.5 sec, the ball is at a height of 4.75 m.

$$height = f(t) = a(t - h)^2 + k$$

There are <u>three key features</u> that we often need to solve for in these types of applications problems:

Initial height:

- this is the height when time = 0
- Calculated by setting t to zero and evaluating $f(t)$

Maximum height:

- this is the **height** value of the vertex of the graph (in other words, k)

When the object reaches its maximum height:

- this is the **time** value of the vertex of the graph (in other words, h)

Example #1:

The height reached by a ball thrown upwards determined by the equation $f(t) = -5(t-3)^2 + 16$, where $f(t)$ represents the height of the ball in metres at the time t in seconds.

$t - 3 = 0$

What is the **maximum height** reached by the ball and **at what time** does it occur?

$t = 3$

Solution:

The maximum height is the ***maximum*** of the function. Therefore it occurs at the vertex. The corresponding time for this maximum is the other coordinate of the vertex.

The vertex is (3, 16).

Therefore, the ball reaches a maximum height of 16 metres, 3 seconds after being thrown.

Example #2:

In a game of football, a team can score a field goal by kicking the ball over a bar and between two uprights. For a kick in a particular game, the height of the ball above the ground, $f(t)$, in metres can be modelled by the function $f(t) = -5(t - 2.5)^2 + 31.25$

a) What is the initial height of the football?

$t = 0$

$f(0) = -5(0 - 2.5)^2 + 31.25$

$= -5(6.25) + 31.25$

$= -31.25 + 31.25$

$f(0) = 0$ m

Example #2 (cont'd):

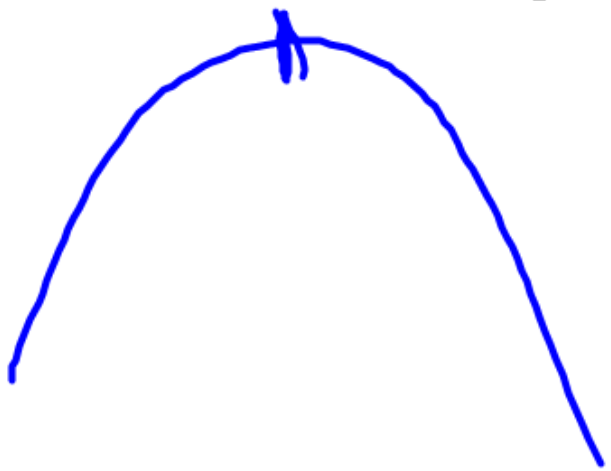
In a game of football, a team can score a field goal by kicking the ball over a bar and between two uprights. For a kick in a particular game, the height of the ball above the ground, $f(t)$, in metres can be modelled by the function $f(t) = -5(t - 2.5)^2 + 31.25$

b) What is the maximum height reached by the football?

31.25 m

c) At what point in time does the football reach its maximum height?

$t = 2.5$ s



Example #2 (cont'd):

In a game of football, a team can score a field goal by kicking the ball over a bar and between two uprights. For a kick in a particular game, the height of the ball above the ground, $f(t)$, in metres can be modelled by the function $f(t) = -5(t - 2.5)^2 + 31.25$

d) How high above the ground is the ball 4 seconds after being kicked?

$$f(4) = -5(4 - 2.5)^2 + 31.25$$

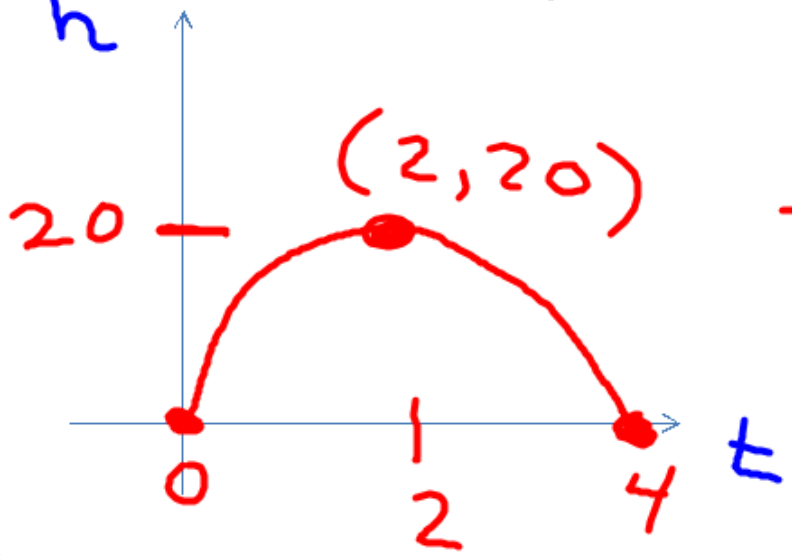
$$= -5(1.5)^2 + 31.25$$

$$= 20\text{ m}$$

Writing Equations:

A soccer ball is kicked from the ground. After 2 seconds, the ball reaches a maximum height of 20 metres. It lands on the ground at 4 seconds.

a) Draw a sketch of the parabola that represents this situation:



$f(t) = -5(t-h)^2 + k$

$f(t) = -5(t-2)^2 + 20$

$0 = a(0-2)^2 + 20$

$-20 = a(-2)^2$

$-20 = 4a$

$-5 = a$

Continued…

A soccer ball is kicked from the ground. After 2 seconds, the ball reaches a maximum height of 20 metres. It lands on the ground at 4 seconds.

b) Determine the quadratic function in vertex form that models the height of the kick.

Continued...

A soccer ball is kicked from the ground. After 2 seconds, the ball reaches a maximum height of 20 metres. It lands on the ground at 4 seconds.

c) What was the height of the ball at 1 second?

$$f(t) = -5(t-2)^2 + 20$$

$$f(1) = -5(1-2)^2 + 20$$

$$= -5(-1)^2 + 20$$

$$= -5 + 20$$

$$= 15 \text{ m}$$

Continued…

A soccer ball is kicked from the ground. After 2 seconds, the ball reaches a maximum height of 20 metres. It lands on the ground at 4 seconds.

d) At 1 second the ball was still on its way up. When was the ball at the same height on the way down?

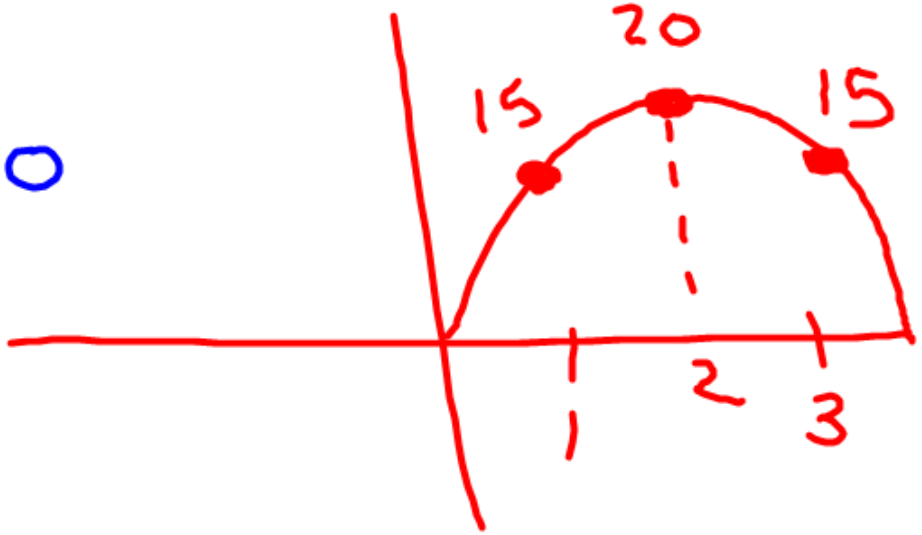
$h = 15$ m

$15 = -5(t-2)^2 + 20$

$\frac{-5}{-5} = \frac{-5(t-2)^2}{-5}$

$1 = (t-2)^2$

$\pm 1 = t - 2$



$2 \pm 1 = t$

Assignment

Handout #1 – 4