Introducing the Vertex Form of a Quadratic Function

$y = ax^2 + bx + c$

Learning targets:

1. Become familiar with vertex form equations – determine the values of a, h and k.
2. Understand the effect that the a value has on the shape of the parabola.
3. Determine the coordinates of the vertex of a parabola from the vertex form equation.

Vertex Form: $y = a(x - h)^2 + k$

When a quadratic function is written in vertex form, we can tell that it is a **degree 2** function because x is found in the expression **inside the brackets**, which is being **squared**.

The letters a, h and k are called the parameters:

- a is always the number in front of the brackets
- h is always the number **subtracted** from x
- k is always the number **added** on at the end

Example #1: Identify a, h and k

(1) $f(x) = (x-5)^2 + 6$

$a = 1$ $h = 5$ $K = 6$

(2) $f(x) = -3(x-1)^2 - 4$

$a = -3$ $h = 1$ $k = -4$

(3) $f(x) = 2(x+8)^2 + 3$

(4) $f(x) = -0.2(x+1)^2$

$a = -0.2$ $h = -1$ $K = 0$

(5) $f(x) = \frac{5}{2}(x-3)^2$

(6) $f(x) = (x+5)^2$

The Value of a and what it tells us:

The graph of a quadratic function is called a parabola. Some parabolas open upwards, some downwards. Some parabolas are narrower, some are wider.

The value of a tells us important information about these characteristics of the graph:

(1) If a is **positive**, the parabola opens **upwards**

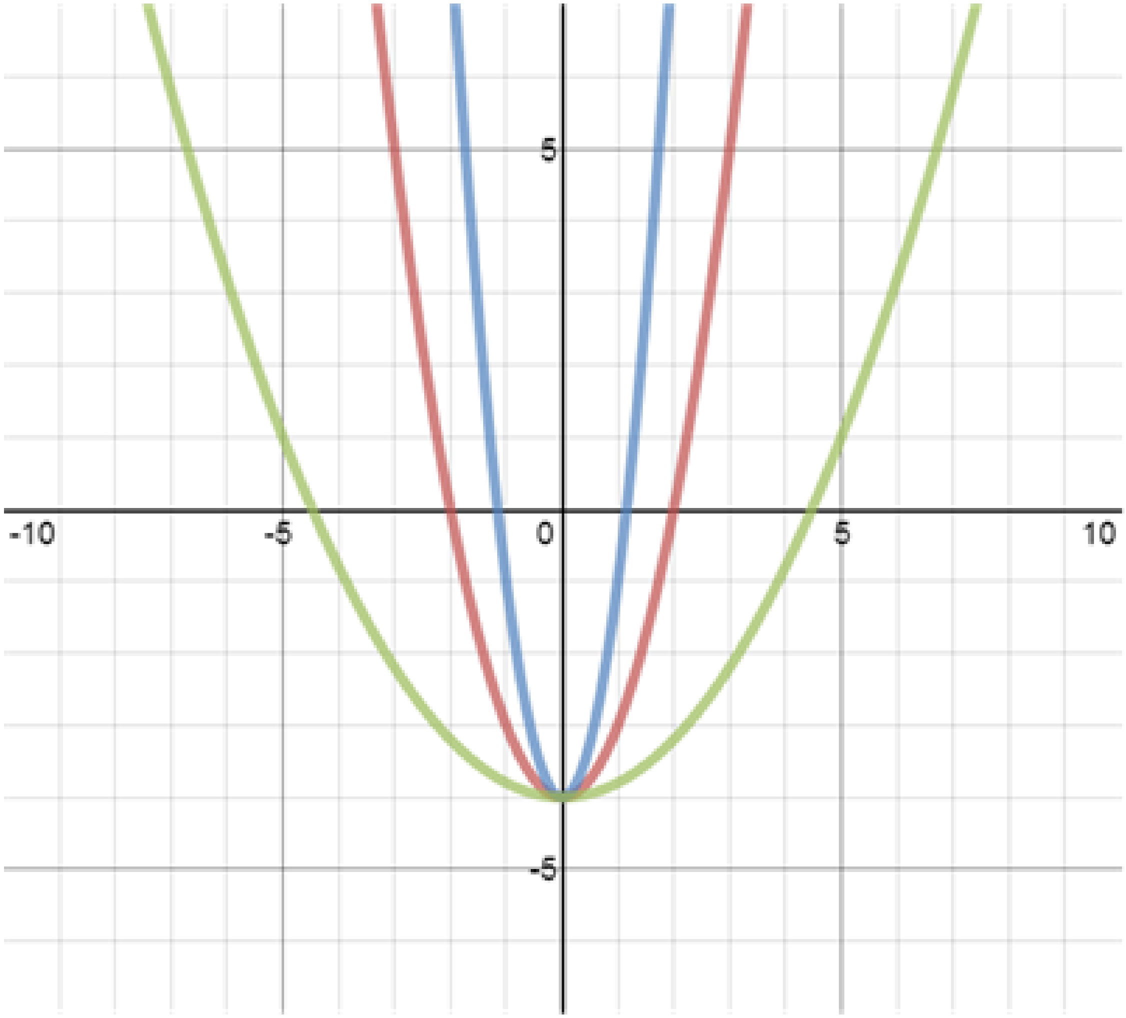
(2) If a is **negative**, the parabola opens **downwards**

(3) If $a = 1$ or if $a = -1$, the parabola has a "**normal**" width

(4) If $-1 < a < 1$, the parabola gets **wider** than normal

(5) If $a > 1$ or if $a < -1$, the parabola gets **narrower** than normal

Normal, narrow, wide



Normal: $a = 1$

Narrow: $a = 3$

Wide: $a = \frac{1}{5}$

Example: In which direction does the parabola open?Describe the width.

(1) $f(x) = \frac{1}{3}(x - 9)^2 + 2$

up wide

(2) $g(x) = -4(x + 5)^2$

down narrow

(3) $h(x) = (x - 3)^2 - 1$

up normal

Determining the coordinates of the vertex:

When a quadratic function is written in vertex form, the **parameters** $\boldsymbol{h}$ **and** $\boldsymbol{k}$ tell us the location of the **vertex**:

$$\boxed{f(x) = a(x-h)^2 + k}$$

The **vertex** is located at the point $(\boldsymbol{h}, \boldsymbol{k})$

Examples:

$k(x) = (x-5)^2 + 3$, the vertex is $(5, 3)$

$f(x) = -(x-1)^2 - 4$, the vertex is $(1, -4)$

$g(x) = \frac{2}{5}(x+2)^2 + 7$, the vertex is $(-2, 7)$

$j(x) = 3(x-8)^2$, the vertex is $(8, 0)$

$f(x) = -2x^2 + 6$, the vertex is $(0, 6)$

Determining the axis of symmetry:

The equation of the axis of symmetry can also be determined using the parameter, h:

The equation of the axis of symmetry is $\boldsymbol{x = h}$

Example:

$(-3, 7)$

Given $f(x) = \frac{1}{2}(x+3)^2 + 7$

The equation of the axis of symmetry is $x = h$ and the value of h is -3

$x = -3$

Therefore, the equation of the axis of symmetry for the given parabola is $\boldsymbol{x = -3}$

You Try:

Determine the vertex coordinates and the equation of the axis of symmetry:

$g(x) = -2(x-4)^2 - 1$

V(4,-1)   x=4

$h(x) = 3.5(x+9)^2 + 2$

V(-9,2)   x=-9

Maximum or minimum?

A quadratic function that opens **upward** will have a **minimum**.

A quadratic function that opens **downward** will have a **maximum**.

The value of the maximum or minimum is always equal to k from the vertex.

Example:

Given $f(x) = \frac{1}{2}(x+3)^2 + 7 \rightarrow a = \frac{1}{2},\ h = -3,\ k = 7$

min value 7

The function **opens upwards** because the value of **a is positive**.
Therefore the function has a **minimum**.
The **value of the minimum is 7** because $k = 7$

You Try:

Determine whether the function has a maximum or a minimum, and what its value is:

$$f(x) = 3(x-2)^2 + 15$$

min 15

$$h(x) = -(x+1)^2 - 4$$

max -4

Assignment

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Additional practice on the Handout