Section 6.6
Optimization Problems III: Linear Programming

Learning targets:

1. Determine the **optimal solution** for an **optimization** problem.
2. Summarize the steps in **linear programming** to solve an optimization problem.

- The **optimal solution** is a point in the **feasible region** that represents the **maximum** or **minimum** value of the objective function.
  - **These will always occur at a vertex of the feasible region**

- To solve an optimization problem using **linear programming**, begin by creating algebraic models of the problem (as in Section 6.4). Then use the objective function to determine which vertex of the feasible region results in the optimal solution.

Example #1:

The graph represents the feasible region for the example we looked at yesterday regarding the toy company.

Objective function: $\boxed{C = 12s + 8r}$

1) **Determine the coordinates of the vertices of the feasible region.**
2) **Determine the optimal solution for the optimization of the objective function.**

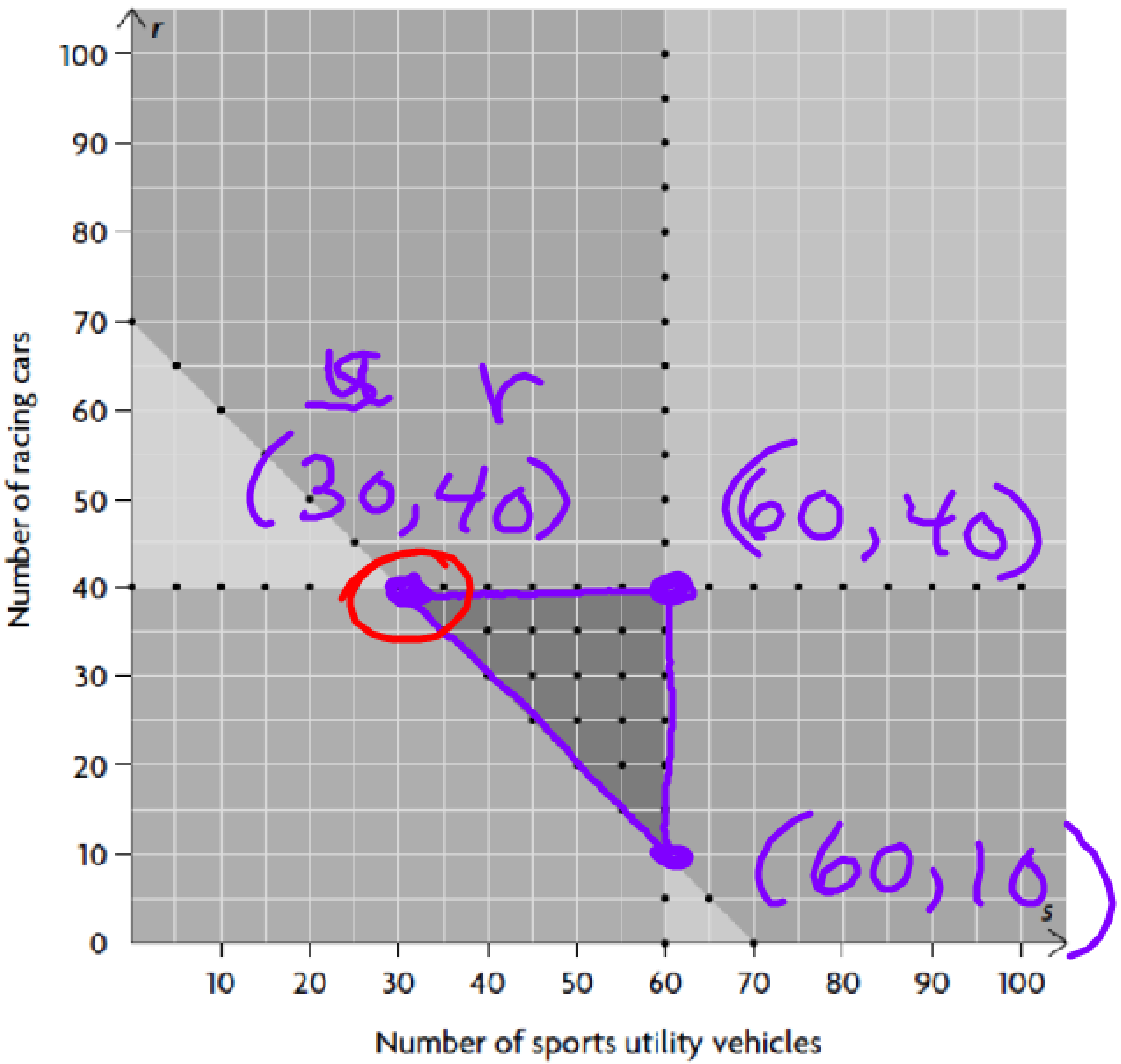
lowest cost

$C = 12(30) + 8(40) = 360 + 320 = \boxed{\$680}$

$C = 12(60) + 8(40) = 720 + 320 = \$1040$

$C = 12(60) + 8(10) = 720 + 80 = \$800$

Racing Cars and Sports Utility Vehicles



Example #2:

Three teams are travelling to a basketball tournament in cars and minivans.

- Each team has no more than 2 coaches and 14 athletes.
- Each car can take 4 team members, and each minivan can take 6 team members.
- No more than 4 minivans and 12 cars are available.

There are many possible combinations of cars and minivans that could be used to transport the teams.

The school wants to know what combinations of cars and minivans will require the minimum and maximum number of vehicles.

Let's recall our work from Section 6.4 on this example:

let m = # minivans

c = # cars

constraints

$m \in W \qquad c \in W$

inequalities

$m \leq 4 \qquad 4c + 6m \leq 48$

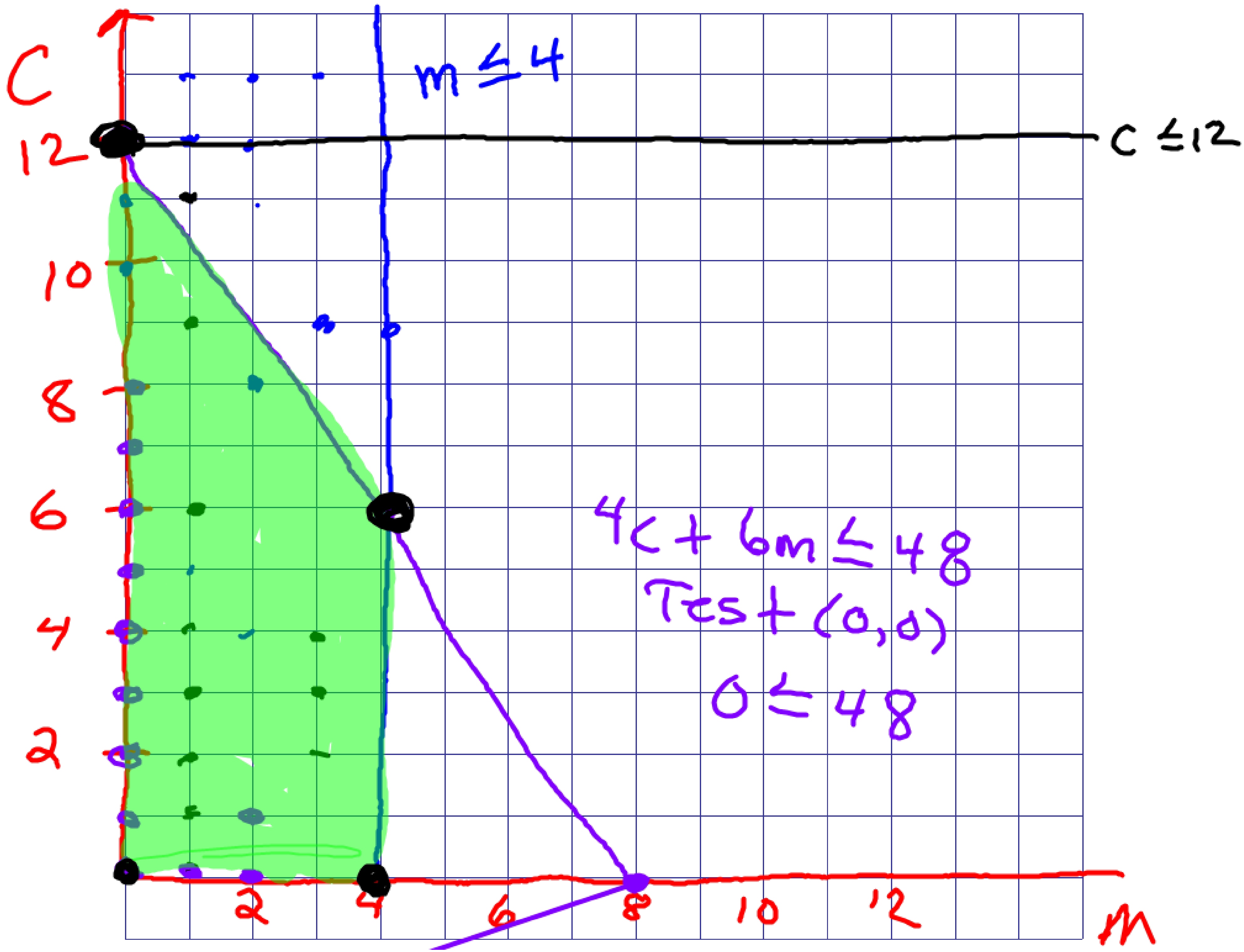
$c \leq 12$

Objective Function

$$V = m + c$$

(Don't graph this equation)

- Graph the system of inequalities.
- Determine the coordinates of the vertices of the feasible region.
- Determine the optimal solution by evaluating the objective function using the coordinates of each vertex.


C
12
10
8
6
4
2
2
4
6
8
10
12
M
m ≤ 4
c ≤ 12
4c + 6m ≤ 48
Test (0,0)
0 ≤ 48

$V = c + m$

(4, 0) $V = 0 + 4 = 4$ only 24

(4, 6) $V = 4 + 6 = 10$ 48 travel

m c

(0, 12) $V = 0 + 12 = 12$ 48 travel

~~(0, 0)~~ $\cancel{V = 0 + 0 = 0}$

- Verify that your choice(s) for the optimal solution satisfy the constraints of the problem situation.

Example #3:

L&G Construction is competing for a contract to build a fence.

- The fence will be no longer than 50 yd. and will consist of narrow boards that are 6 in. wide and wide boards that are 8 in. wide.
- There must be no fewer than 100 wide boards and no more than 80 narrow boards.
- The narrow boards cost $3.56 each and the wide boards cost $4.36 each.

There are many possible combinations of narrow boards and wide boards that could be used to build the fence.

Determine the maximum and minimum costs for the lumber to build the fence.

Define your variables:

Let n represent the number of narrow boards

Let w represent the number of wide boards

Restrictions:

We are working with WHOLE NUMBER solutions.

We need to use at least 100 wide boards.

We can't use more than 80 narrow boards.

The fence is at most 50 yards and made up of only 6 inch and 8 inch boards (and nothing else).

Write the constraints based on the restrictions as linear inequalities using our variables.

Let $n = \#$ narrow boards

$w = \#$ wide boards

$n \in W$, $w \in W$

$w \geq 100$

$n \leq 80$

$6n + 8w \leq 1800$

80

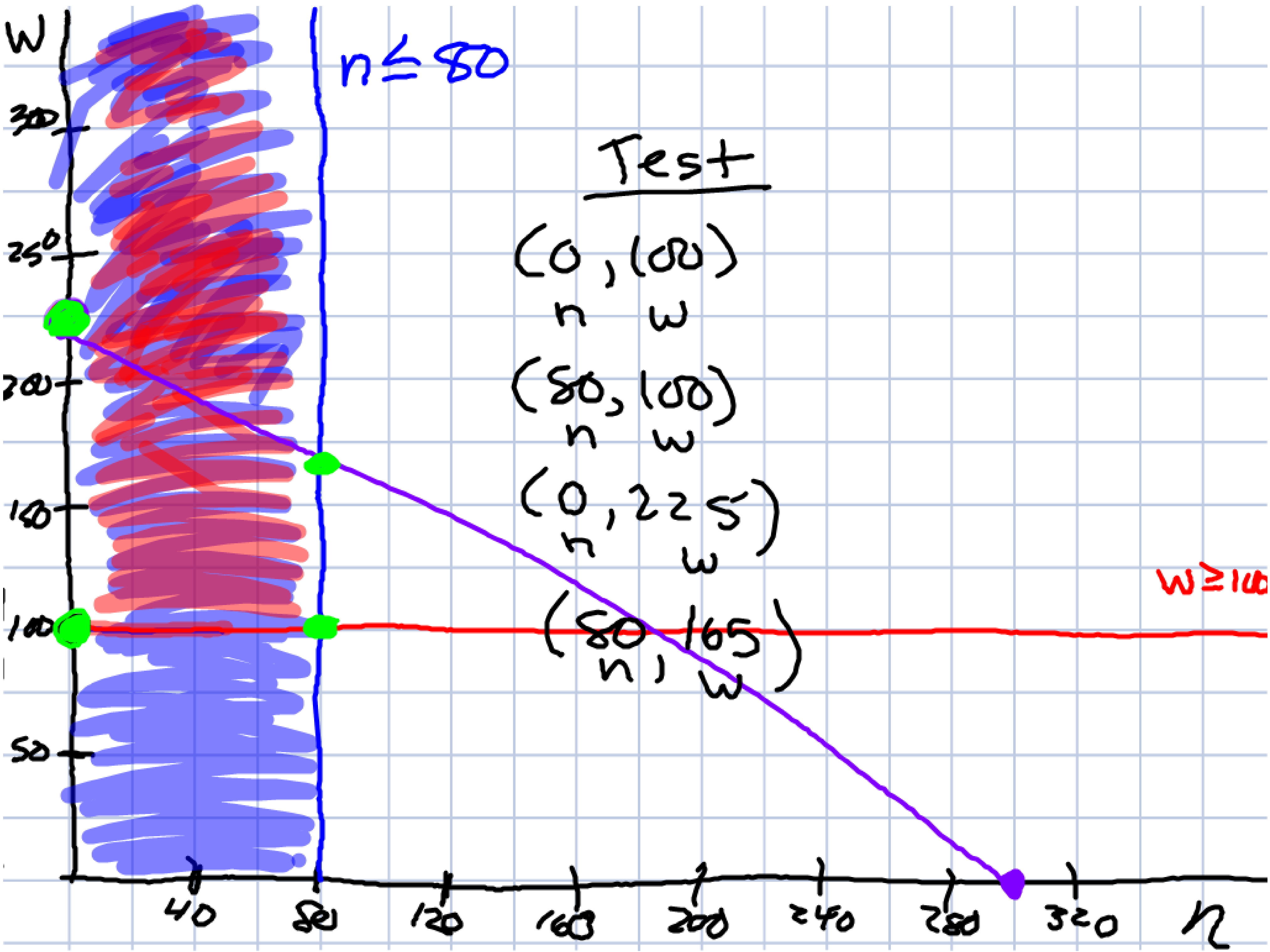
Write the equation for the quantity being optimized (the objective function) - COST.

$C = 3.56n + 4.36w$

- Graph the system of inequalities.

- Determine the coordinates of the vertices of the feasible region.

- Determine the optimal solution by evaluating the objective function using the coordinates of each vertex.


W
300
250
200
150
100
50
n≤80
Test
(0, 100)
n w
(80, 100)
n w
(0, 225)
n w
(80, 165)
n w
w≥10
40
80
120
160
200
240
280
320
n

$$C = 3.56n + 4.36w$$

$$C = 3.56(0) + 4.36(100) = \$436$$ min

$$C = 3.56(0) + 4.36(225) = \$981$$

$$C = 3.56(80) + 4.36(100) = \$720.80$$

$$C = 3.56(80) + 4.36(165) = \$1004.20$$

max

- Verify that your choice(s) for the optimal solution satisfy the constraints of the problem situation.

Assignment Pasge 341-345
#'s 1,2,4-7, 11-13