Section 6.4
Optimization Problems I: Creating the Model

Learning targets:

1. Interpret given information to determine the constraints in an optimization problem.
2. Interpret given information to determine the objective function for an optimization problem.
3. Determine the feasible region for an optimization problem.

- An **optimization problem** involves a quantity that must be either maximized or minimized following a set of guidelines or conditions.
- **Constraints** are the limiting conditions in the problem. Each constraint is represented by a linear inequality.
- The **objective function** is the equation that represents the relationship between the variables in the system of linear inequalities and the quantity that needs to be optimized.
- The **feasible region** is the solution region for the system of linear inequalities modelling the optimization problem.

answer

Example #1:

A toy company manufactures two types of toy vehicles: racing cars and SUVs.

- Because the supply of materials is limited, no more than 40 racing cars and 60 SUVs can be made each day.
- However, the company can make 70 or more vehicles, in total, each day.
- It costs $8 to make a racing car and $12 to make an SUV.

There are many possible combinations of racing cars and SUVs that could be made.

The company wants to know what combinations will result in the minimum and maximum costs, and what those costs will be.

Constraints:

let r = # racing cars

s = # suv's

$x + y \geq 70$

$\boxed{s + r \geq 70}$

$\boxed{\boxed{r \leq 40} \quad r \geq 0}$

$\boxed{\boxed{s \leq 60} \quad s \geq 0}$

What quantity is being optimized?

$\boxed{C = 8r + 12s}$

Cost

Write the constraints as linear inequalities (choose two appropriate variables):

$s \in$ whole #'s $(0, 1, 2, \ldots)$

$r \in$ whole #'s

Write the equation for the quantity being optimized (the objective function).

$C = 8r + 12s$

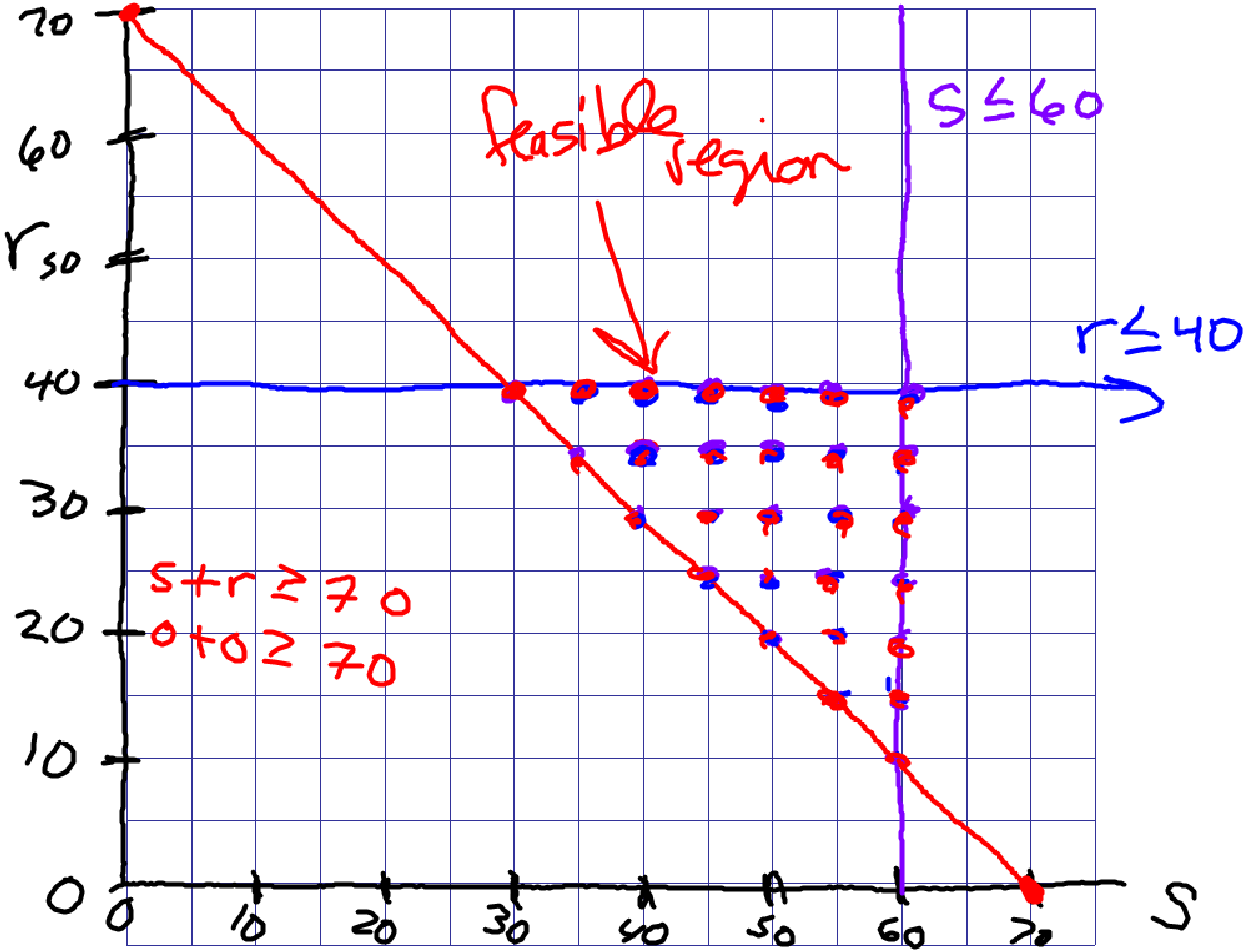
Something to think about…

If we graph the system of constraints, would our feasible region be **shaded solid**?

Or **lattice points**?

Would the feasible region be **anywhere** in the coordinate plane?

Or would be limit ourselves to **Quadrant 1**?


feasible region
s ≤ 60
r ≤ 40
s+r ≥ 70
0+0 ≥ 70
r
s
0
10
20
30
40
50
60
70

Example #2:

Three teams are travelling to a basketball tournament in cars and minivans.

- Each team has no more than 2 coaches and 14 athletes.
- Each car can take 4 team members, and each minivan can take 6 team members.
- No more than 4 minivans and 12 cars are available.

There are many possible combinations of cars and minivans that could be used to transport the teams.

The school wants to know what combinations of cars and minivans will require the minimum and maximum number of vehicles.

Constraints:

Whole # solutions

Cars limited max of 12

Minivans limited to max 4

Max # people to transport 48.

What quantity is being optimized?

vehicles used

Write the constraints as linear inequalities (choose two appropriate variables):

let c = # cars m = # minivans

$c \le 12$ $m \le 4$

$c \ge 0$ $m \ge 0$

$\boxed{6m + 4c \le 48}$

Write the equation for the quantity being optimized (the objective function).

$V = c + m$

let $m = 0$

$4c \le 48$

$c = 12$

$6(3) + 4(11) \le 48$

Something to think about…

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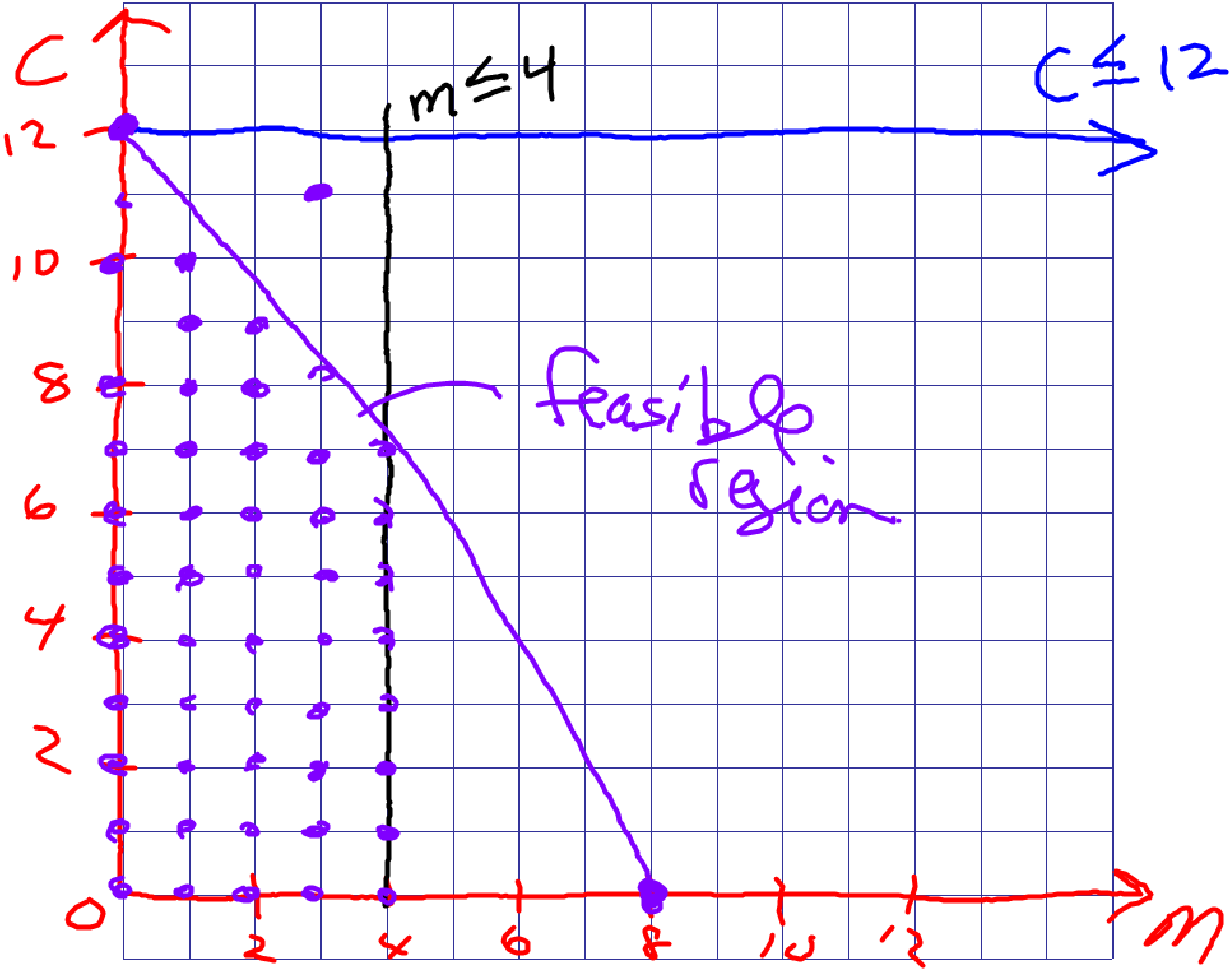
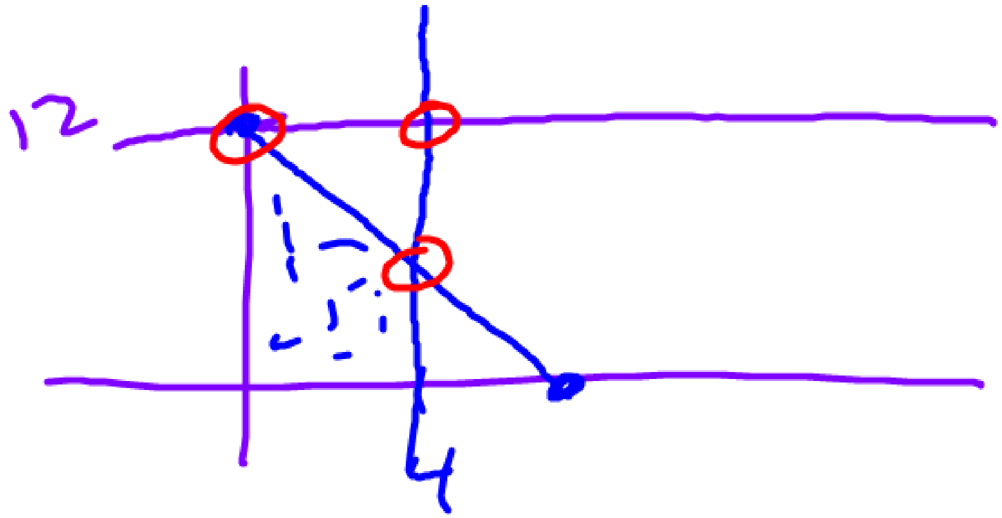


c
m ≤ 4
c ≤ 12
feasible region
m
0
2
4
6
8
10
12



Assignment:

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