

Section 6.3

Graphing to Solve Systems of Linear Inequalities

Learning targets:

1. Determine whether a point satisfies a system of linear inequalities.
2. Determine whether a boundary intersection point should be open or solid for a system of linear inequalities.
3. Determine the solution region for a system of two linear inequalities.

$$y \geq 2x + 3$$

$$y < \frac{1}{2}x - 7$$

A **system of linear inequalities** is a set of two (or more) linear inequalities.

The **solution set** for a system of linear inequalities is the **set of all points (x, y)** in the coordinate plane that **satisfy both inequalities.**

→ it is the region in the coordinate plane where the individual solution regions for each of the inequalities **OVERLAP**.

The **boundaries** of each linear inequality in a system will **intersect** at a single point (unless the boundaries are parallel).

- Use an **open dot** to show that an intersection point of a system's boundaries is **excluded** from the solution set.



– An intersection point is excluded when a dotted line intersects either a dotted line or a solid line.



- Use a **solid dot** to show that an intersection point of a system's boundaries is **included** in the solution set.

– An intersection point is included only when both boundaries are solid lines.

Not a
solution to
the system.

Example:

without graphing

(1) Would $(7, 5)$ be in the solution set for the following system?

✓ $y \geq -2x + 8$

✗ $y > 3x - 2$

$$5 \geq -2(7) + 8$$

$$5 \geq -6$$

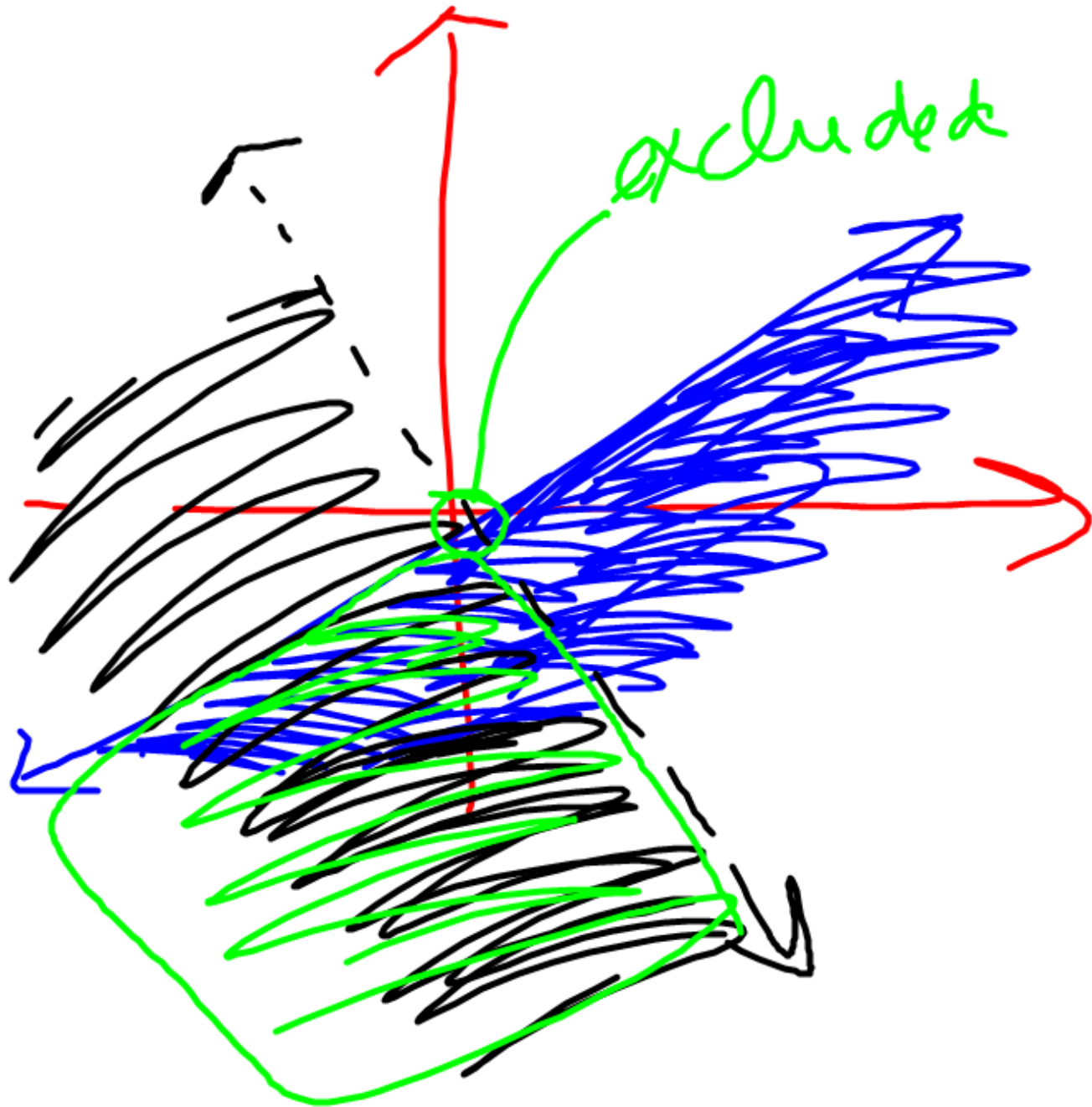
$$5 > 3(7) - 2$$

$$5 > 19$$

(2) Would the intersection point of the boundaries of this system be included or excluded?

$$\{(x, y) \mid 2x - 5y < -1, x \in \mathbb{R}, y \in \mathbb{R}\}$$

$$\{(x, y) \mid y \geq 6x + 1, x \in \mathbb{R}, y \in \mathbb{R}\}$$



Steps for Graphing a System of Linear Inequalities

1. Replace the inequality symbols with equals signs to determine the equations of each **boundary**.
2. Graph each boundary line (*solid* or *dashed* according to the inequality symbol). With two lines, the coordinate plane has been divided into **four regions**.
3. Choose a **test point**. Choose **(0,0)** if possible to make it easier.
4. Substitute the test point into the first inequality to see if you get a true statement.
 - If the result is a **true** statement, **imagine you shaded the entire half-plane on that side of the boundary line.**
 - If the result is a **false** statement, **imagine you shaded the other half-plane.**
5. Now substitute the test point into the second inequality to see if you get a true statement.
 - If the result is a **true** statement, **imagine you shaded the entire half-plane on that side of the boundary line.**
 - If the result is a **false** statement, **imagine you shaded the other half-plane.**
6. One of the four regions would contain **no shading**.

Two of the regions would contain **shading from only one** of the inequalities.

One region would contain shading from both inequalities → this is the OVERLAP we are looking for. This is the region that contains our solution set.

Example #1: Graph the solution to the system:

$$\{(x, y) \mid 2x + 6 > y, x \in \mathbb{R}, y \in \mathbb{R}\}$$

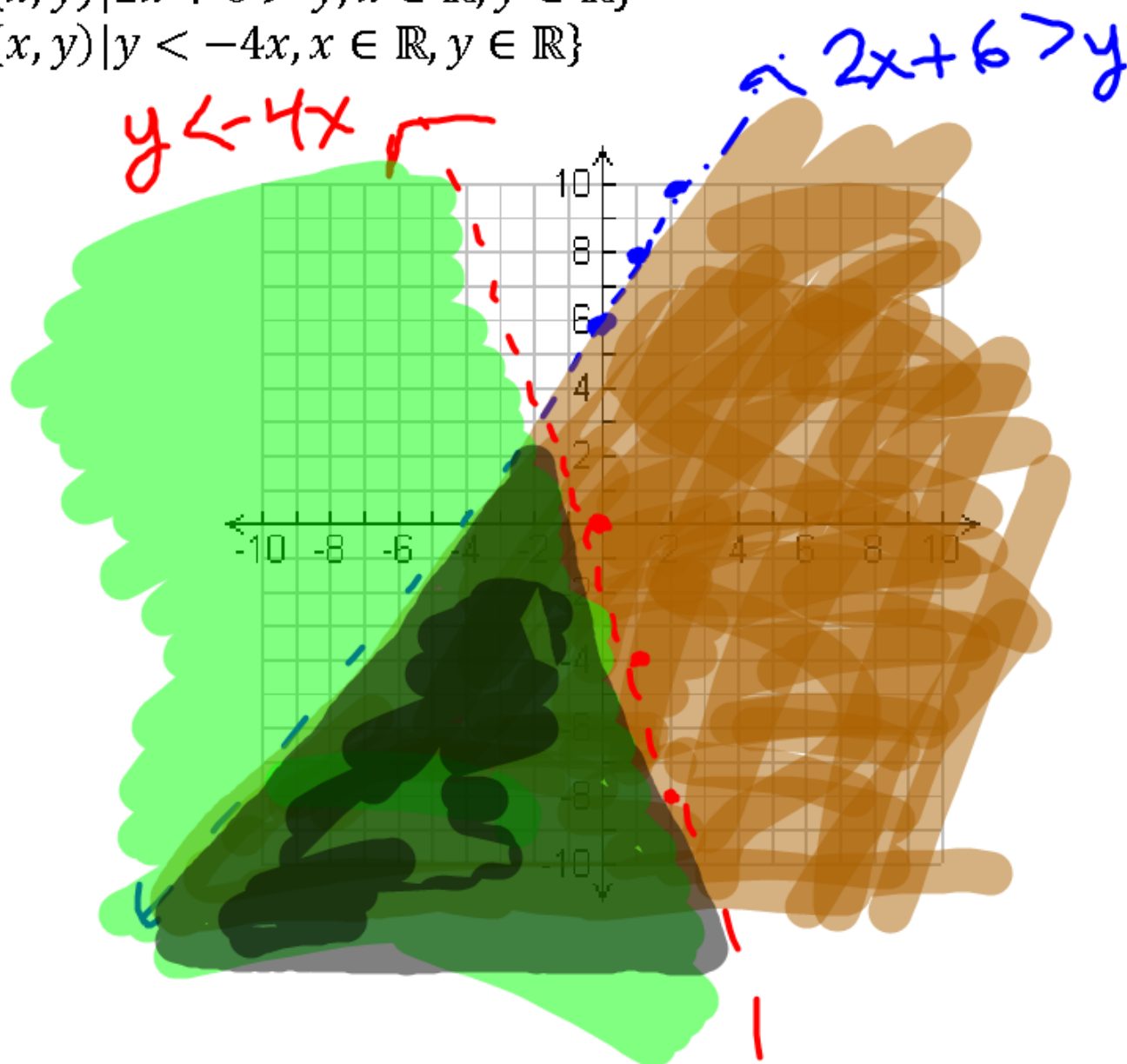
$$\{(x, y) \mid y < -4x, x \in \mathbb{R}, y \in \mathbb{R}\}$$

$$2x + 6 > y$$

Test Point
 $(0, 0)$
 $2(0) + 6 > 0$
 $6 > 0$

$$y < -4x$$

Test $(4, 0)$
 $0 < -4(4)$
 $0 < -16$



Example #2: Graph the solution to the system:

$$\{(x, y) \mid 2x + y \geq 6, x \in W, y \in W\}$$

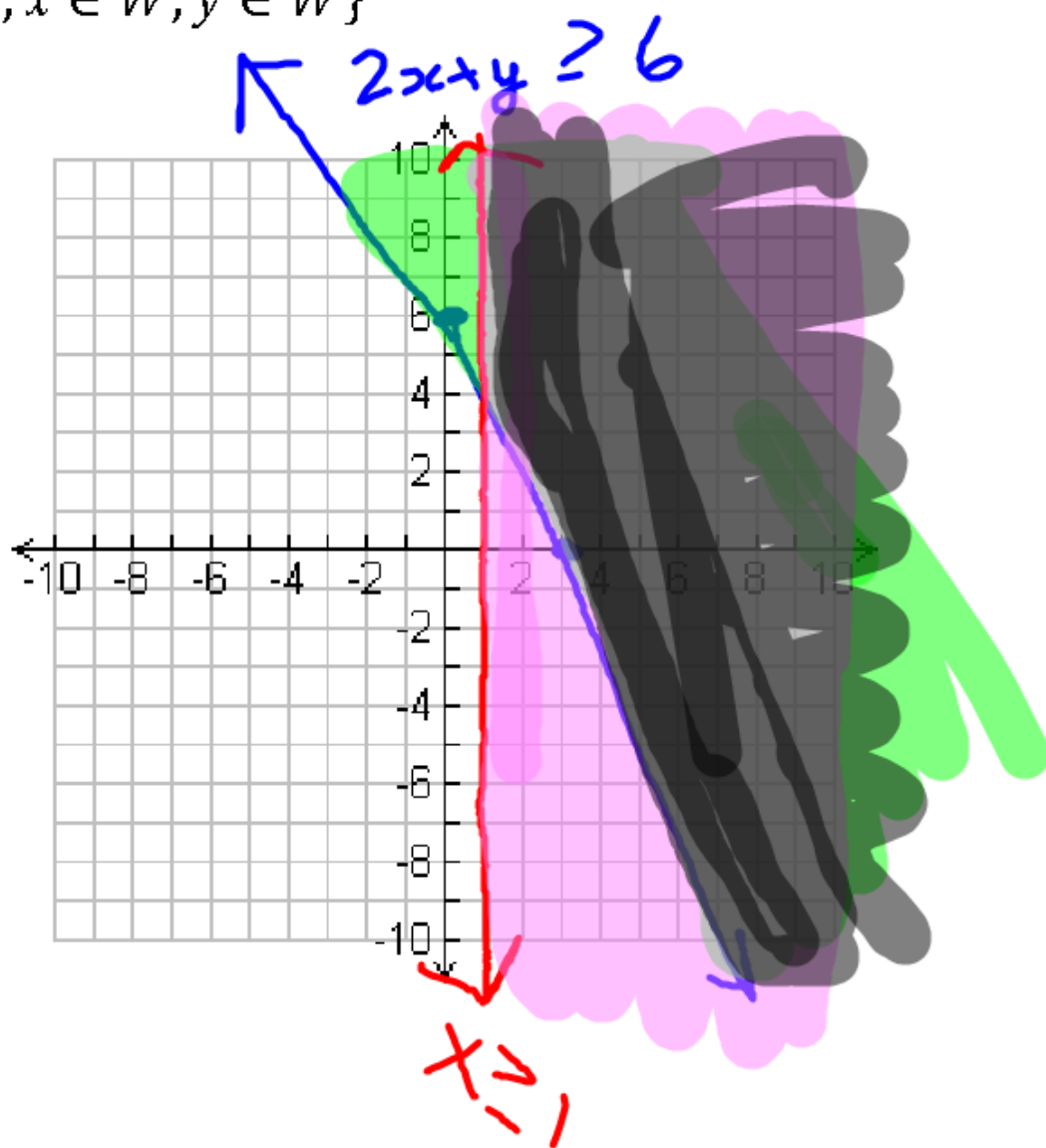
$$\{(x, y) \mid x \geq 1, x \in W, y \in W\}$$

$$2x + y \geq 6$$
$$\begin{array}{l|l} \text{let } x=0 & \text{let } y=0 \\ 2(0) + y = 6 & 2x = 6 \\ y = 6 & x = 3 \end{array}$$

Test

$$\begin{array}{l} (0, 0) \\ 2(0) + 0 \geq 6 \\ 0 \geq 6 \end{array}$$

$$x \geq 1$$



You Try: Graph the solution to the system:

$$\{(x, y) \mid x + y \leq 3, x \in \mathbb{R}, y \in \mathbb{R}\}$$

$$\{(x, y) \mid y \geq x - 2, x \in \mathbb{R}, y \in \mathbb{R}\}$$

$$x + y \leq 3$$

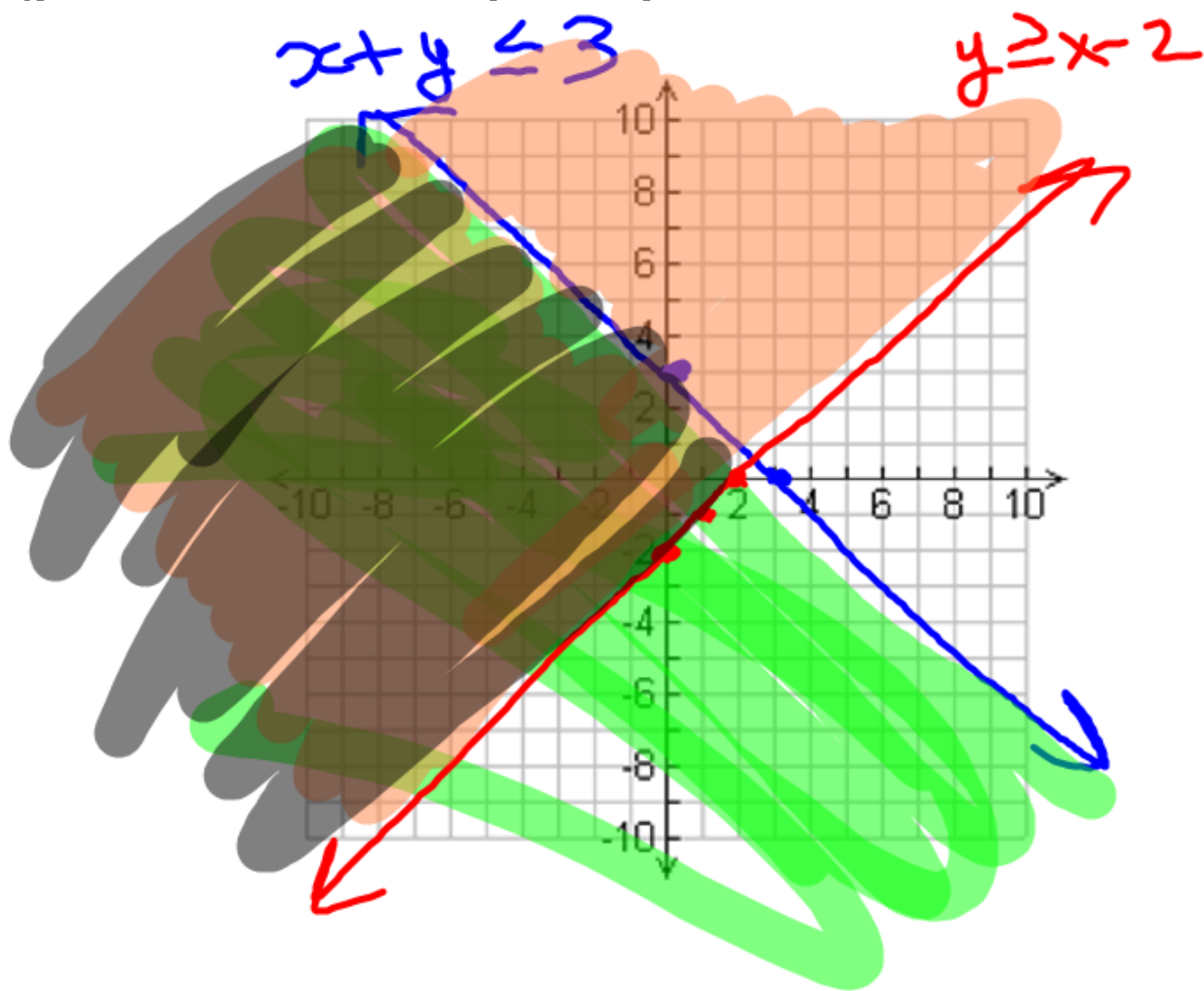
Test $(0, 0)$

$$0 + 0 \leq 3$$

$$y \geq x - 2$$

Test $(0, 0)$

$$0 \geq 0 - 2$$



Assignment:

Page 317 – 319

#1, 2, 3(a)(b), 4(a)(b)(d), 5(i), 7

Handout Extra Practice