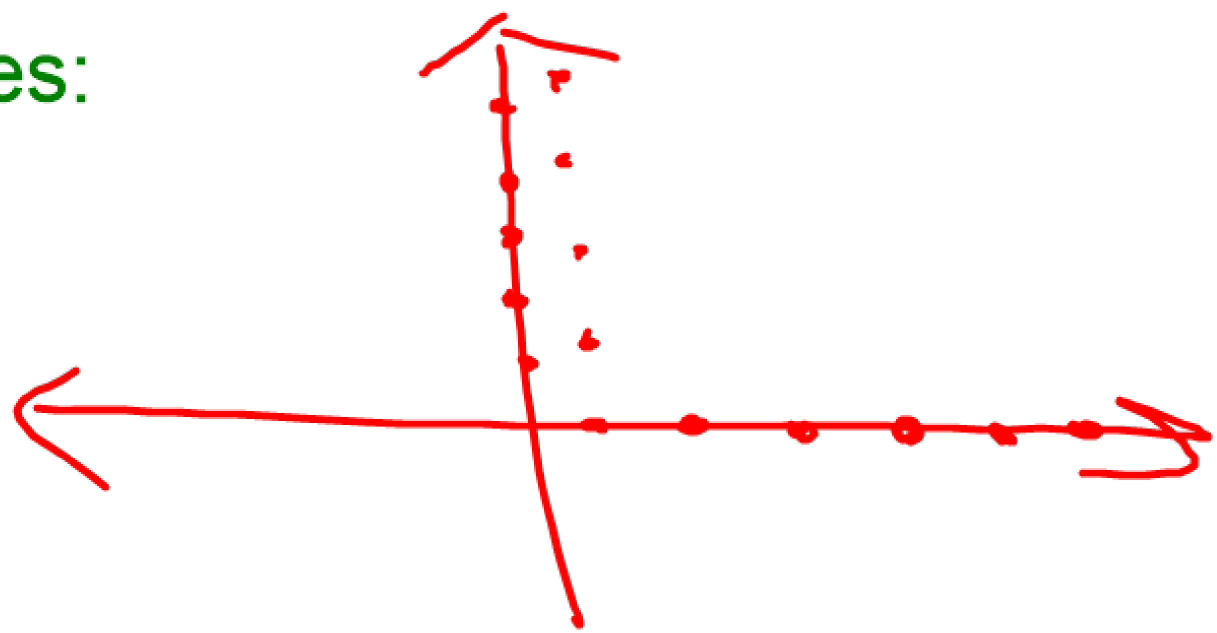
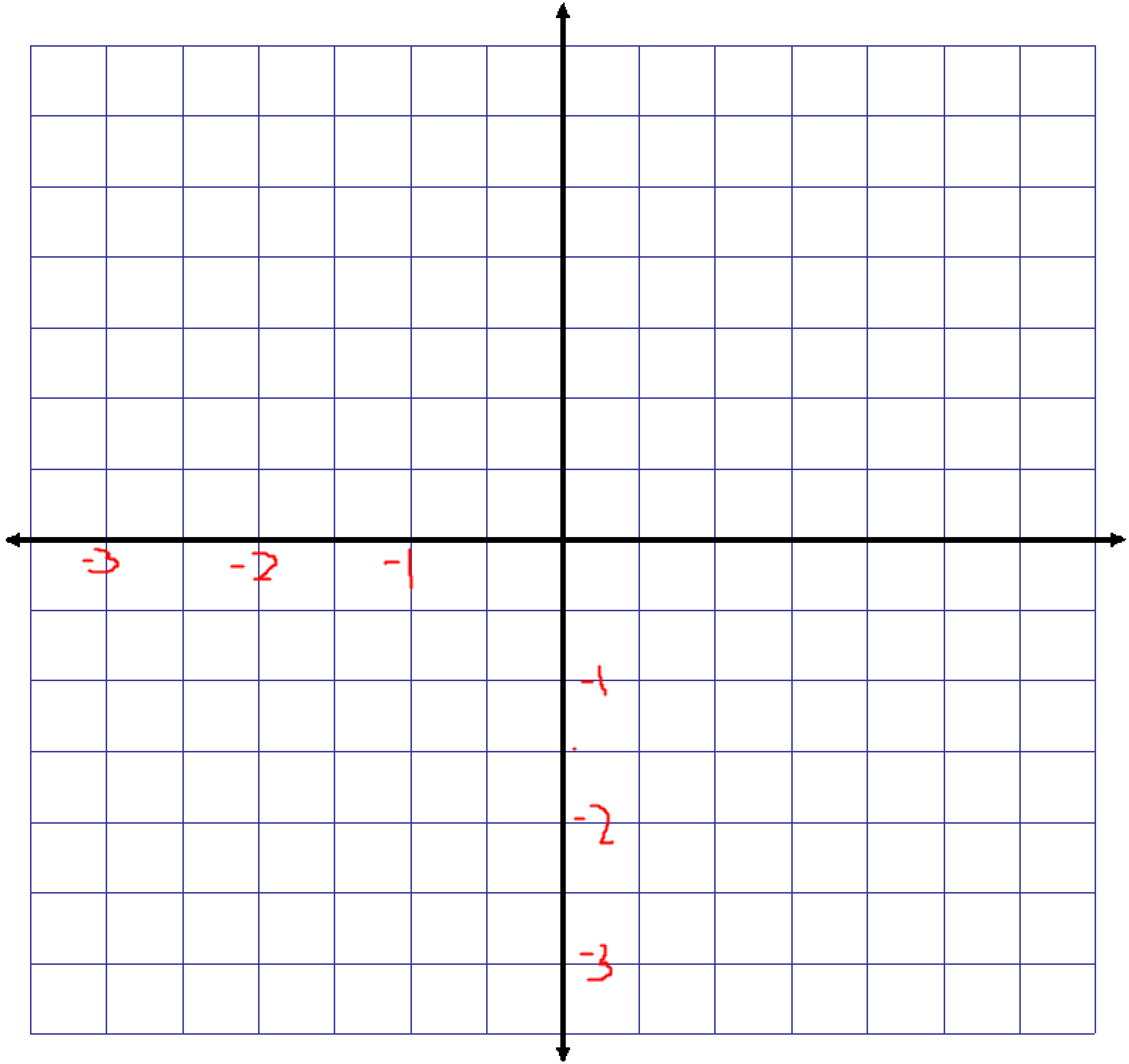
Chapter 6:
Systems of Linear Inequalities

Recall: whole numbers = { 0, 1, 2, 3, ... }

If I asked you to shade all of the points in the coordinate plane that had **WHOLE NUMBER** coordinates, what would that look like?

Examples:

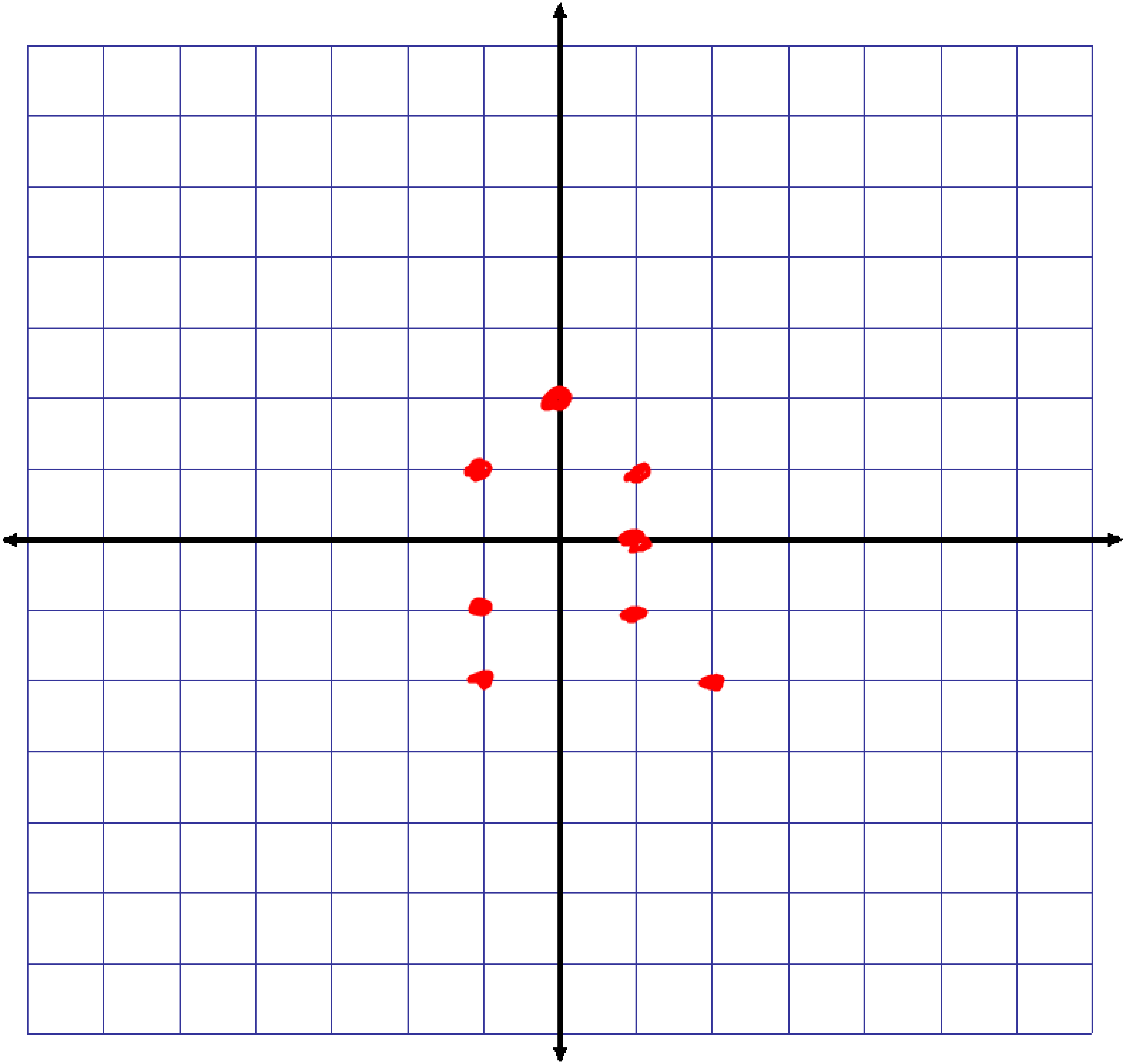



-3
-2
-1
-1
-2
-3

Recall: integers = { ...,-2, -1, 0, 1, 2, 3, ... }

If I asked you to shade all of the points in the coordinate plane that had **INTEGER** coordinates, what would that look like?

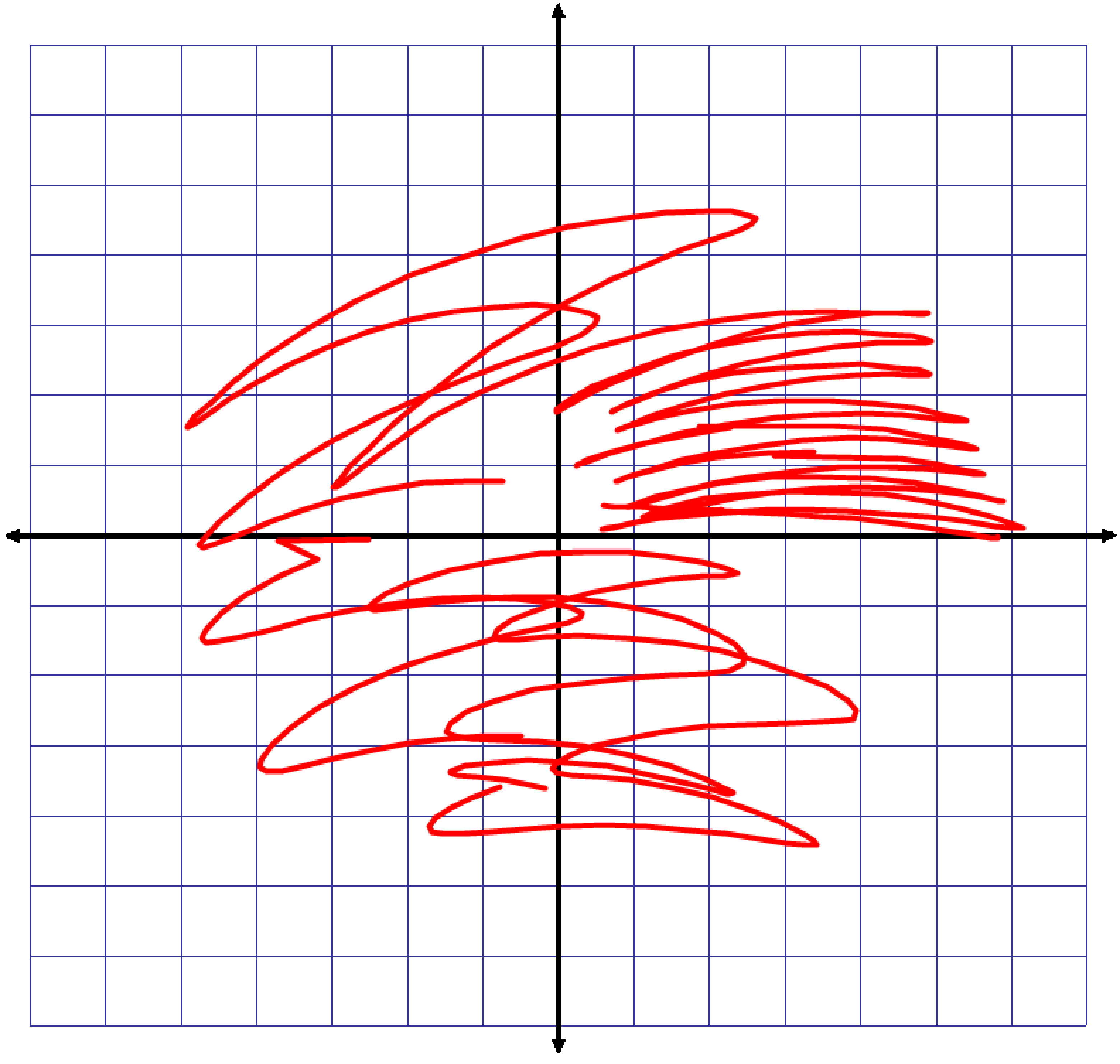
Examples:



Recall: REAL numbers = any number at all, including fractions, decimals, irrational numbers like pi and square roots, etc.

If I asked you to shade all of the points in the coordinate plane that had **REAL NUMBER** coordinates, what would that look like?

Examples:



Section 6.1
Graphing Linear Inequalities in Two Variables

Learning targets:

1. Determine whether a point satisfies a linear inequality in two variables.
2. Determine whether a boundary should be solid or dotted.
3. Determine the solution set for a linear inequality in two variables.

A **linear inequality in two variables** looks like the equation of a linear function, but instead of an "equals" sign (=), there is one of four inequality symbols $(<, >, \leq, or \; \geq)$:

$4 > 2$

$3 < 5$

- $>$ "is greater than"
- $<$ "is less than"
- $\geq$ "is greater than or equal to"
- $\leq$ "is less than or equal to"

Examples:

$y > 2x + 4$

$\{(x, y) | 5x - 3y \leq 12, x \in \mathbb{R}, y \in \mathbb{R}\}$

$\{(x, y) | x + y > 1, x \in \mathrm{I}, y \in \mathrm{I}\}$

The **solution set** for a linear inequality in two variables is the **set of all points** (x, y) in the coordinate plane that **satisfy the inequality**.

Example:

$(0, 8)$ satisfies the inequality $y > 2x + 4$ since substituting **8 for** y and **0 for** x results in a **true** statement:

$$\mathbf{8 > 2(0) + 4}$$

$$\mathbf{8 > 4}$$

(this is a true statement)

Therefore, $(0, 8)$ belongs in the solution set for the inequality $y > 2x + 4$.

When a point is **not part of the solution set**, then its coordinates **do not satisfy the inequality**.

Example:

$(1, 3)$ does not satisfy the inequality $y > 2x + 4$ since substituting **3 for y** and **1 for x** results in a **false** statement:

$$\mathbf{3 > 2(1) + 4}$$
$$\mathbf{3 > 6}$$

(this is a false statement)

Therefore, $(1, 3)$ does not belong in the solution set for the inequality $y > 2x + 4$.

Example:

(1) Would (7, -2) be in the solution set for $y \geq -2x + 8$?

$-2 \geq -2(7) + 8$

$-2 \geq -14 + 8$

$-2 \geq -6$

(2) Is the point (3, 2) a solution to the inequality

$$\{(x, y) | 2x - 5y < -1, x \in I, y \in I\}$$

$2x - 5y < -1$

$2(3) - 5(2) < -1$

$-4 < -1$

The **boundary** of a linear inequality in two variables is a **straight line** that creates two half-planes.

- One of these half-planes is the solution set of the inequality.
- The boundary itself may or may not be a part of the solution set.
- The **solution set** is represented by a **shaded half-plane**.

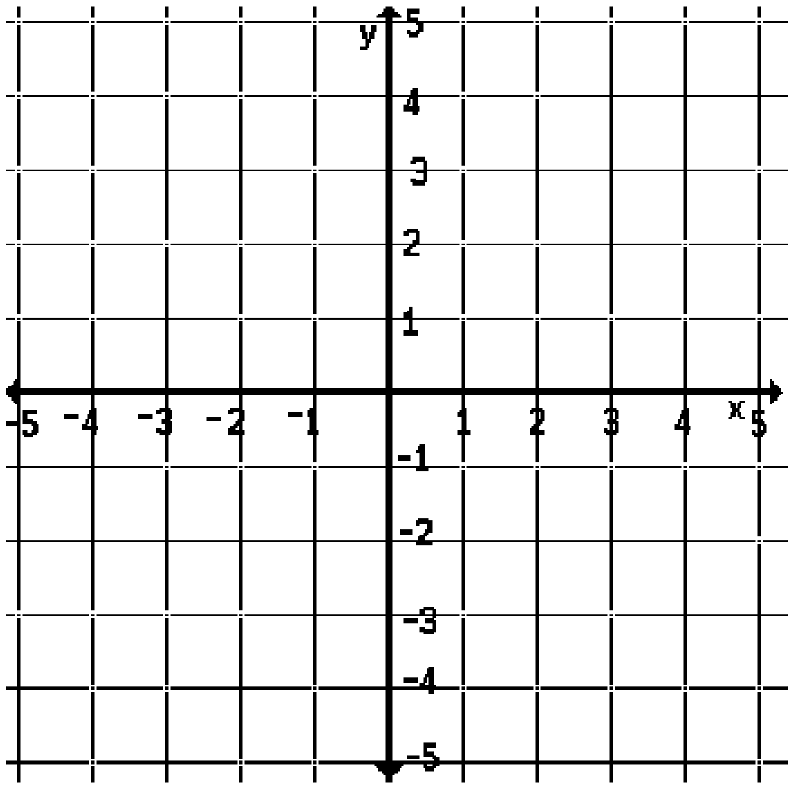
- The boundary line **IS** part of the solution set when $\geq$ **or** $\leq$ are used. The boundary line in these cases is drawn as a **solid line**.

- The boundary line **IS NOT** part of the solution set when $>$ **or** $<$ are used. The boundary line in these cases is drawn as a **dashed line**.

A solution region that is **shaded completely** represents a solution set that comes from the **REAL NUMBERS** $(x \in \mathbb{R}, y \in \mathbb{R})$.

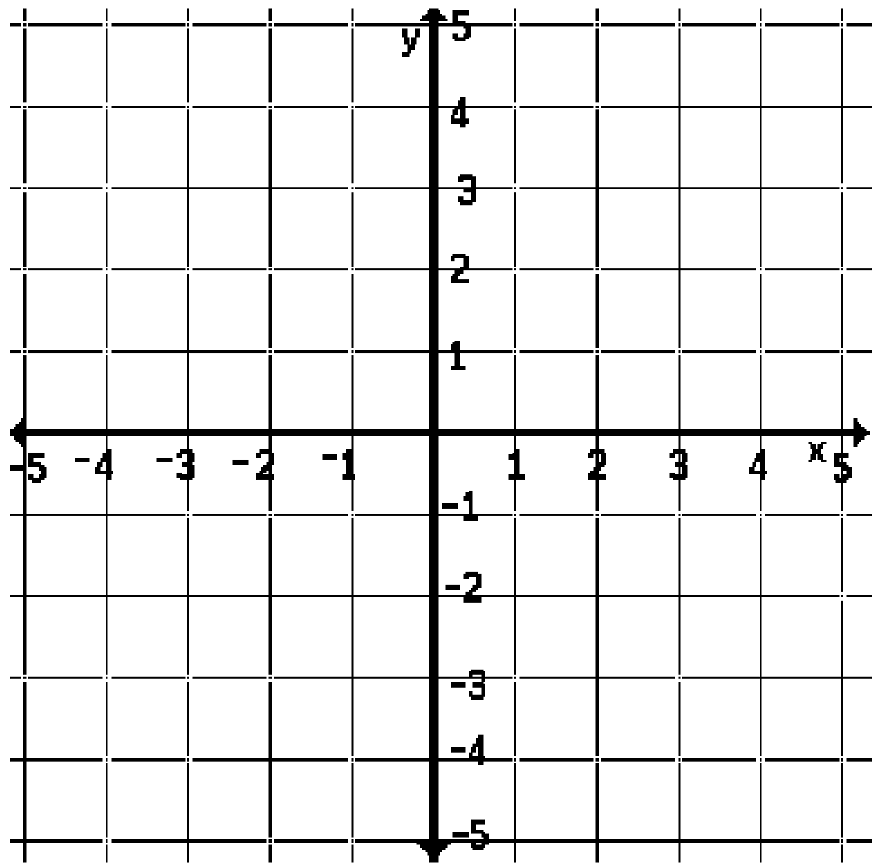
- If the inequality specifies a solution set that is made up of **INTEGERS** $(x \in \mathrm{I}, y \in \mathrm{I})$, then instead of shading completely, we simply draw dots on the **lattice points**:

Ex.



- If the inequality specifies a solution set that is made up of **WHOLE NUMBERS** $(x \in \mathbf{W}, y \in W)$, then we draw dots on the **lattice points**, BUT the solution region must be limited to **quadrant 1.**

Ex:



Steps for Graphing a Linear Inequality in Two Variables:

1. Replace the inequality symbol with an equals sign to determine the equation of the **boundary**.

2. **Graph** the boundary line (*solid* or ***dashed*** according to the inequality symbol):
   – slope and y-intercept method may be used
   – x-intercept and y-intercept values may be used

3. Choose a "**test point**" from either one of the half-planes
   (hint: always choose (0, 0) as your test point unless the boundary line goes through the origin).

4. Substitute the test point into the inequality to see if you get a true statement.
   – If the result is a **true** statement, **shade the region that does contain the test point**
   – If the result is a **false** statement, **shade the region that does NOT contain the test point**.

$W = \{0, 1, 2, 3, \ldots\}$

Example #1:

Graph the solution set for $\{(x, y) \mid -3x + 4y \leq 12, x \in \mathbf{W}, y \in \mathbf{W}\}$

$-3x + 4y = 12$

let $x = 0$

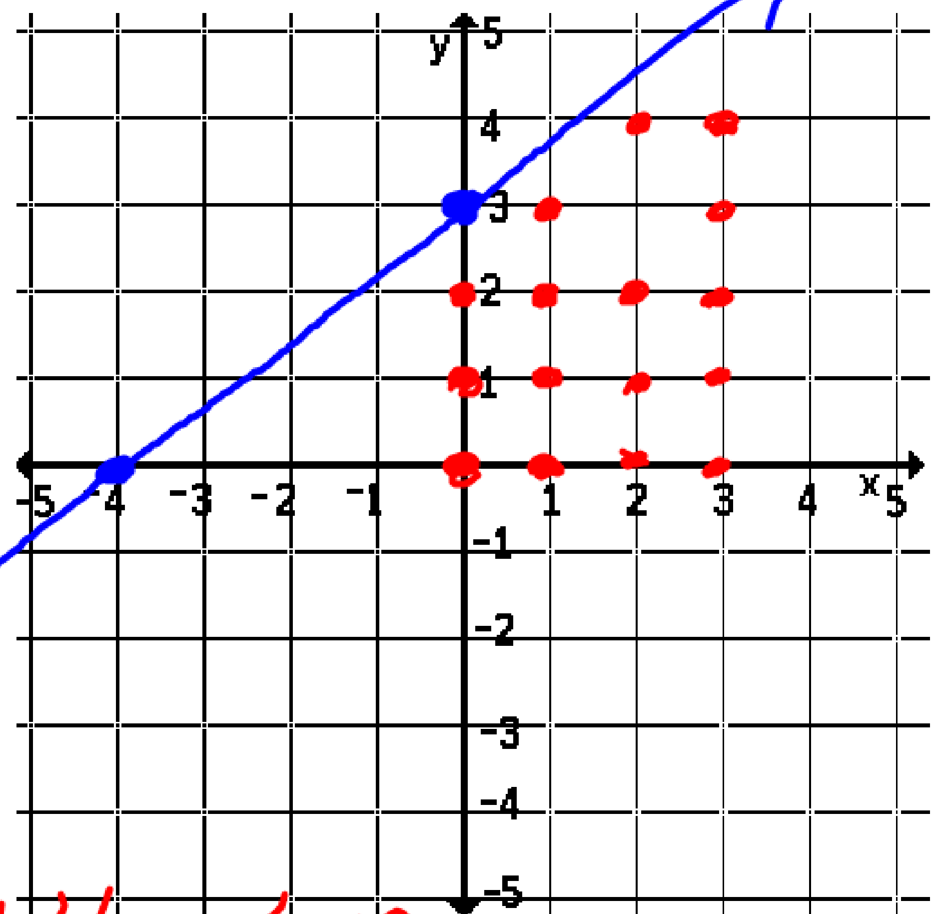
$4y = 12$

$y = 3$

let $y = 0$

$-3x = 12$

$x = -4$



$-3x + 4y \leq 12$

$-3(0) + 4(0) \leq 12$

$0 \leq 12$

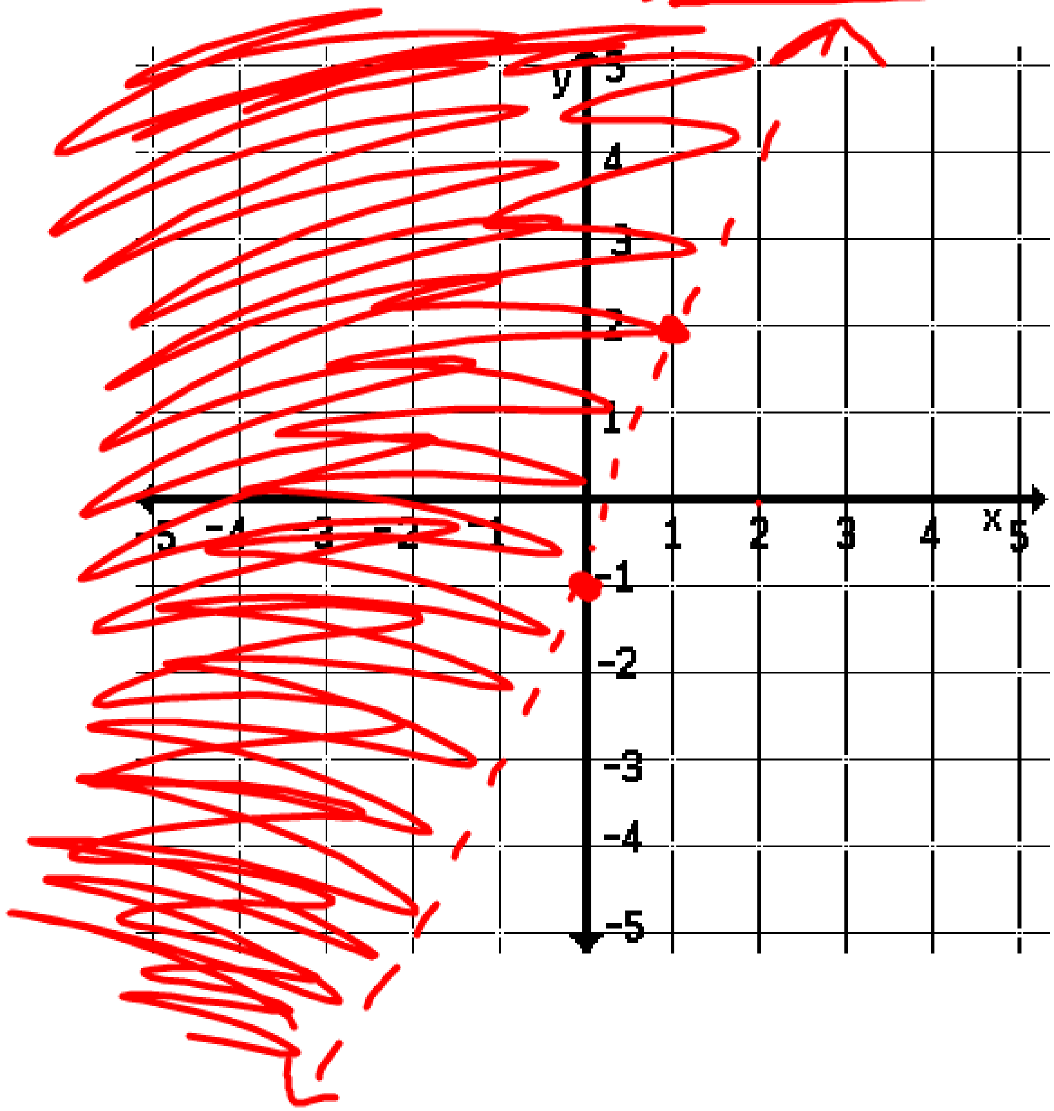
Example #2:

Graph the linear inequality: $\{(x, y) \mid y > 3x - 1, x \in \mathbb{R}, y \in \mathbb{R}\}$

$y > 3x - 1$

$0 > 3(2) - 1$

~~$0 > 5$~~



You Try: Graph the solution set for

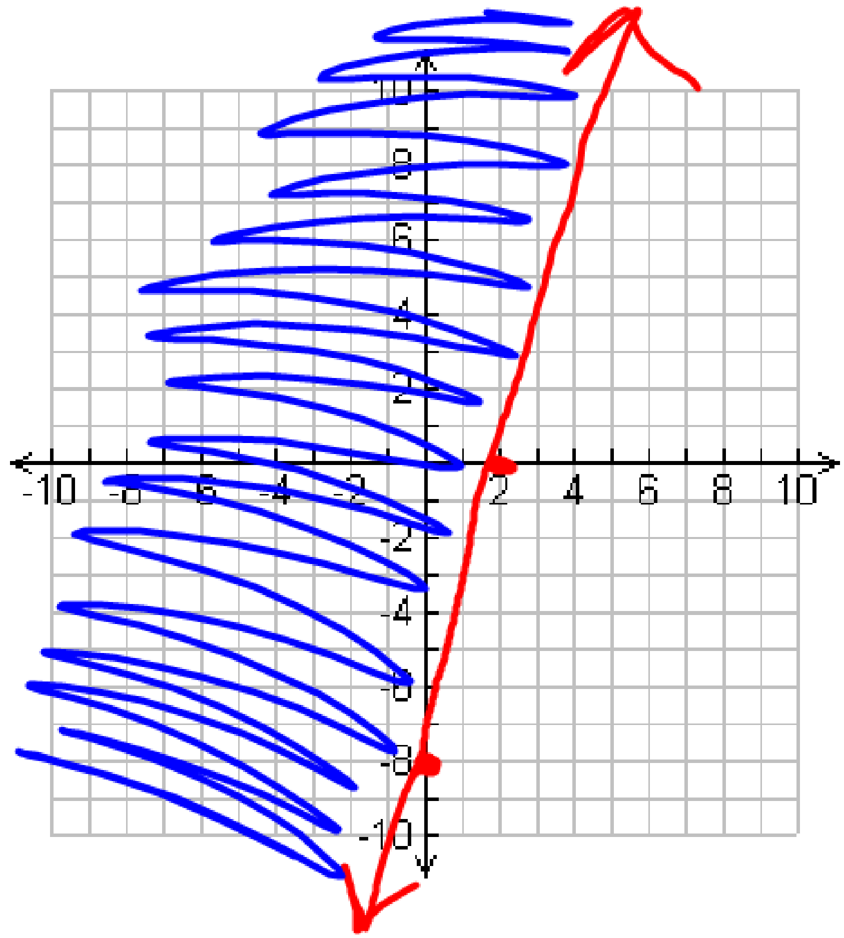
$$\{(x, y) | 4x - y \le 8, x \in \mathbb{R}, y \in \mathbb{R}\}$$

$4x - y \le 8$

Test $(4, 2)$

$4(4) - 2 \le 8$

$14 \not\le 8$



Assignment:

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