

**2017 AP<sup>®</sup> CALCULUS AB FREE-RESPONSE QUESTIONS**

**CALCULUS AB**  
**SECTION II, Part A**  
**Time—30 minutes**  
**Number of questions—2**

**A GRAPHING CALCULATOR IS REQUIRED FOR THESE QUESTIONS.**

$h$ (feet)	0	2	5	10
$A(h)$ (square feet)	50.3	14.4	6.5	2.9

1. A tank has a height of 10 feet. The area of the horizontal cross section of the tank at height  $h$  feet is given by the function  $A$ , where  $A(h)$  is measured in square feet. The function  $A$  is continuous and decreases as  $h$  increases. Selected values for  $A(h)$  are given in the table above.
- (a) Use a left Riemann sum with the three subintervals indicated by the data in the table to approximate the volume of the tank. Indicate units of measure.
- (b) Does the approximation in part (a) overestimate or underestimate the volume of the tank? Explain your reasoning.
- (c) The area, in square feet, of the horizontal cross section at height  $h$  feet is modeled by the function  $f$  given by  $f(h) = \frac{50.3}{e^{0.2h} + h}$ . Based on this model, find the volume of the tank. Indicate units of measure.
- (d) Water is pumped into the tank. When the height of the water is 5 feet, the height is increasing at the rate of 0.26 foot per minute. Using the model from part (c), find the rate at which the volume of water is changing with respect to time when the height of the water is 5 feet. Indicate units of measure.
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2. When a certain grocery store opens, it has 50 pounds of bananas on a display table. Customers remove bananas from the display table at a rate modeled by

$$f(t) = 10 + (0.8t)\sin\left(\frac{t^3}{100}\right) \text{ for } 0 < t \leq 12,$$

where  $f(t)$  is measured in pounds per hour and  $t$  is the number of hours after the store opened. After the store has been open for three hours, store employees add bananas to the display table at a rate modeled by

$$g(t) = 3 + 2.4 \ln(t^2 + 2t) \text{ for } 3 < t \leq 12,$$

where  $g(t)$  is measured in pounds per hour and  $t$  is the number of hours after the store opened.

- (a) How many pounds of bananas are removed from the display table during the first 2 hours the store is open?
- (b) Find  $f'(7)$ . Using correct units, explain the meaning of  $f'(7)$  in the context of the problem.
- (c) Is the number of pounds of bananas on the display table increasing or decreasing at time  $t = 5$ ? Give a reason for your answer.
- (d) How many pounds of bananas are on the display table at time  $t = 8$ ?
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**END OF PART A OF SECTION II**

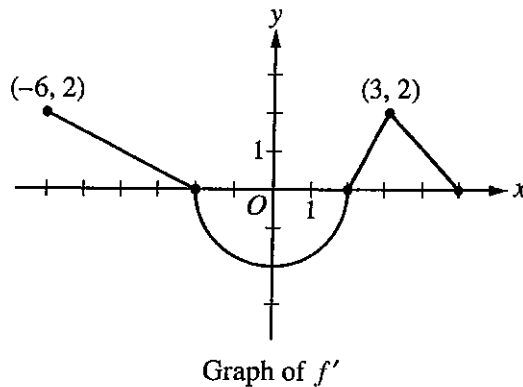
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**CALCULUS AB  
SECTION II, Part B**

**Time—1 hour**

**Number of questions—4**

**NO CALCULATOR IS ALLOWED FOR THESE QUESTIONS.**



3. The function  $f$  is differentiable on the closed interval  $[-6, 5]$  and satisfies  $f(-2) = 7$ . The graph of  $f'$ , the derivative of  $f$ , consists of a semicircle and three line segments, as shown in the figure above.
- (a) Find the values of  $f(-6)$  and  $f(5)$ .
- (b) On what intervals is  $f$  increasing? Justify your answer.
- (c) Find the absolute minimum value of  $f$  on the closed interval  $[-6, 5]$ . Justify your answer.
- (d) For each of  $f''(-5)$  and  $f''(3)$ , find the value or explain why it does not exist.
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4. At time  $t = 0$ , a boiled potato is taken from a pot on a stove and left to cool in a kitchen. The internal temperature of the potato is 91 degrees Celsius ( $^{\circ}\text{C}$ ) at time  $t = 0$ , and the internal temperature of the potato is greater than  $27^{\circ}\text{C}$  for all times  $t > 0$ . The internal temperature of the potato at time  $t$  minutes can be modeled by the function  $H$  that satisfies the differential equation  $\frac{dH}{dt} = -\frac{1}{4}(H - 27)$ , where  $H(t)$  is measured in degrees Celsius and  $H(0) = 91$ .

(a) Write an equation for the line tangent to the graph of  $H$  at  $t = 0$ . Use this equation to approximate the internal temperature of the potato at time  $t = 3$ .

(b) Use  $\frac{d^2H}{dt^2}$  to determine whether your answer in part (a) is an underestimate or an overestimate of the internal temperature of the potato at time  $t = 3$ .

(c) For  $t < 10$ , an alternate model for the internal temperature of the potato at time  $t$  minutes is the function  $G$  that satisfies the differential equation  $\frac{dG}{dt} = -(G - 27)^{2/3}$ , where  $G(t)$  is measured in degrees Celsius and  $G(0) = 91$ . Find an expression for  $G(t)$ . Based on this model, what is the internal temperature of the potato at time  $t = 3$  ?

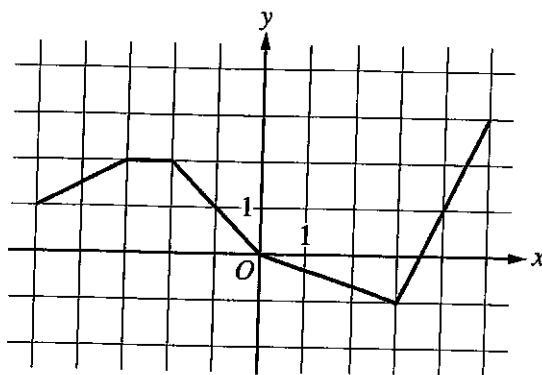
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5. Two particles move along the  $x$ -axis. For  $0 \leq t \leq 8$ , the position of particle  $P$  at time  $t$  is given by  $x_P(t) = \ln(t^2 - 2t + 10)$ , while the velocity of particle  $Q$  at time  $t$  is given by  $v_Q(t) = t^2 - 8t + 15$ . Particle  $Q$  is at position  $x = 5$  at time  $t = 0$ .
- (a) For  $0 \leq t \leq 8$ , when is particle  $P$  moving to the left?
- (b) For  $0 \leq t \leq 8$ , find all times  $t$  during which the two particles travel in the same direction.
- (c) Find the acceleration of particle  $Q$  at time  $t = 2$ . Is the speed of particle  $Q$  increasing, decreasing, or neither at time  $t = 2$ ? Explain your reasoning.
- (d) Find the position of particle  $Q$  the first time it changes direction.
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$x$	$g(x)$	$g'(x)$
-5	10	-3
-4	5	-1
-3	2	4
-2	3	1
-1	1	-2
0	0	-3



Graph of  $h$

6. Let  $f$  be the function defined by  $f(x) = \cos(2x) + e^{\sin x}$ .

Let  $g$  be a differentiable function. The table above gives values of  $g$  and its derivative  $g'$  at selected values of  $x$ .

Let  $h$  be the function whose graph, consisting of five line segments, is shown in the figure above.

- Find the slope of the line tangent to the graph of  $f$  at  $x = \pi$ .
- Let  $k$  be the function defined by  $k(x) = h(f(x))$ . Find  $k'(\pi)$ .
- Let  $m$  be the function defined by  $m(x) = g(-2x) \cdot h(x)$ . Find  $m'(2)$ .
- Is there a number  $c$  in the closed interval  $[-5, -3]$  such that  $g'(c) = -4$ ? Justify your answer.

**STOP**  
**END OF EXAM**

**2016 AP<sup>®</sup> CALCULUS AB FREE-RESPONSE QUESTIONS****CALCULUS AB  
SECTION II, Part A  
Time—30 minutes  
Number of problems—2**

A graphing calculator is required for these problems.

$t$ (hours)	0	1	3	6	8
$R(t)$ (liters / hour)	1340	1190	950	740	700

1. Water is pumped into a tank at a rate modeled by  $W(t) = 2000e^{-t^2/20}$  liters per hour for  $0 \leq t \leq 8$ , where  $t$  is measured in hours. Water is removed from the tank at a rate modeled by  $R(t)$  liters per hour, where  $R$  is differentiable and decreasing on  $0 \leq t \leq 8$ . Selected values of  $R(t)$  are shown in the table above. At time  $t = 0$ , there are 50,000 liters of water in the tank.
- (a) Estimate  $R'(2)$ . Show the work that leads to your answer. Indicate units of measure.
- (b) Use a left Riemann sum with the four subintervals indicated by the table to estimate the total amount of water removed from the tank during the 8 hours. Is this an overestimate or an underestimate of the total amount of water removed? Give a reason for your answer.
- (c) Use your answer from part (b) to find an estimate of the total amount of water in the tank, to the nearest liter, at the end of 8 hours.
- (d) For  $0 \leq t \leq 8$ , is there a time  $t$  when the rate at which water is pumped into the tank is the same as the rate at which water is removed from the tank? Explain why or why not.
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2. For  $t \geq 0$ , a particle moves along the  $x$ -axis. The velocity of the particle at time  $t$  is given by

$v(t) = 1 + 2\sin\left(\frac{t^2}{2}\right)$ . The particle is at position  $x = 2$  at time  $t = 4$ .

- (a) At time  $t = 4$ , is the particle speeding up or slowing down?
  - (b) Find all times  $t$  in the interval  $0 < t < 3$  when the particle changes direction. Justify your answer.
  - (c) Find the position of the particle at time  $t = 0$ .
  - (d) Find the total distance the particle travels from time  $t = 0$  to time  $t = 3$ .
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**END OF PART A OF SECTION II**



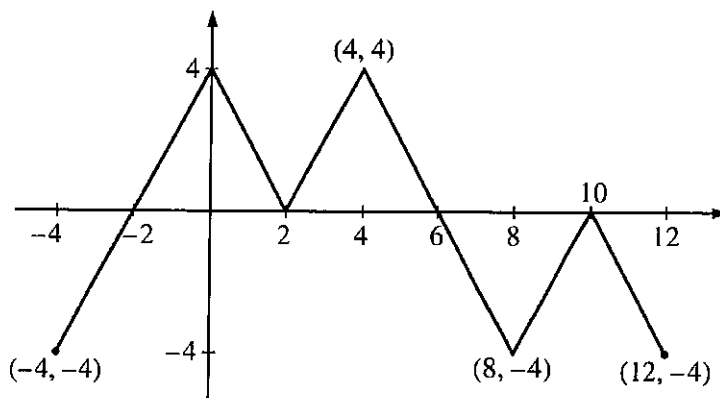
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**CALCULUS AB  
SECTION II, Part B**

**Time—60 minutes**

**Number of problems—4**

**No calculator is allowed for these problems.**



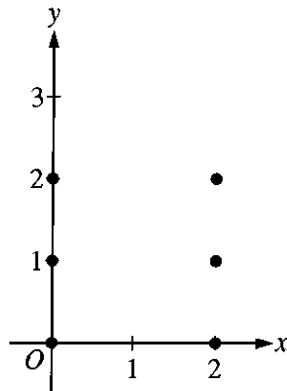
Graph of  $f$

3. The figure above shows the graph of the piecewise-linear function  $f$ . For  $-4 \leq x \leq 12$ , the function  $g$  is defined by  $g(x) = \int_2^x f(t) dt$ .
- (a) Does  $g$  have a relative minimum, a relative maximum, or neither at  $x = 10$ ? Justify your answer.
  - (b) Does the graph of  $g$  have a point of inflection at  $x = 4$ ? Justify your answer.
  - (c) Find the absolute minimum value and the absolute maximum value of  $g$  on the interval  $-4 \leq x \leq 12$ . Justify your answers.
  - (d) For  $-4 \leq x \leq 12$ , find all intervals for which  $g(x) \leq 0$ .
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**2016 AP<sup>®</sup> CALCULUS AB FREE-RESPONSE QUESTIONS**

4. Consider the differential equation  $\frac{dy}{dx} = \frac{y^2}{x-1}$ .

(a) On the axes provided, sketch a slope field for the given differential equation at the six points indicated.

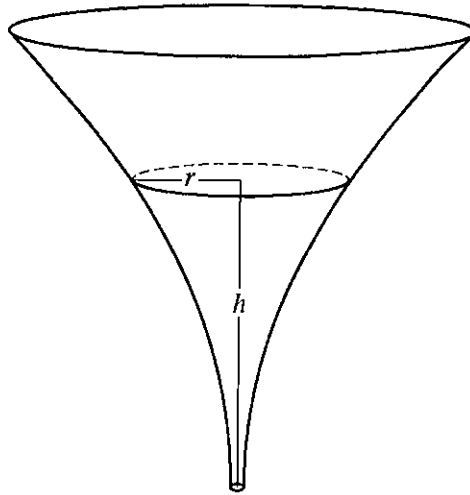


(b) Let  $y = f(x)$  be the particular solution to the given differential equation with the initial condition  $f(2) = 3$ . Write an equation for the line tangent to the graph of  $y = f(x)$  at  $x = 2$ .  
Use your equation to approximate  $f(2.1)$ .

(c) Find the particular solution  $y = f(x)$  to the given differential equation with the initial condition  $f(2) = 3$ .

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5. The inside of a funnel of height 10 inches has circular cross sections, as shown in the figure above. At height  $h$ , the radius of the funnel is given by  $r = \frac{1}{20}(3 + h^2)$ , where  $0 \leq h \leq 10$ . The units of  $r$  and  $h$  are inches.
- (a) Find the average value of the radius of the funnel.
- (b) Find the volume of the funnel.
- (c) The funnel contains liquid that is draining from the bottom. At the instant when the height of the liquid is  $h = 3$  inches, the radius of the surface of the liquid is decreasing at a rate of  $\frac{1}{5}$  inch per second. At this instant, what is the rate of change of the height of the liquid with respect to time?
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**2016 AP<sup>®</sup> CALCULUS AB FREE-RESPONSE QUESTIONS**

$x$	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	-6	3	2	8
2	2	-2	-3	0
3	8	7	6	2
6	4	5	3	-1

6. The functions  $f$  and  $g$  have continuous second derivatives. The table above gives values of the functions and their derivatives at selected values of  $x$ .

(a) Let  $k(x) = f(g(x))$ . Write an equation for the line tangent to the graph of  $k$  at  $x = 3$ .

(b) Let  $h(x) = \frac{g(x)}{f(x)}$ . Find  $h'(1)$ .

(c) Evaluate  $\int_1^3 f''(2x) dx$ .

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**STOP**

**END OF EXAM**

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**CALCULUS AB  
SECTION II, Part A**

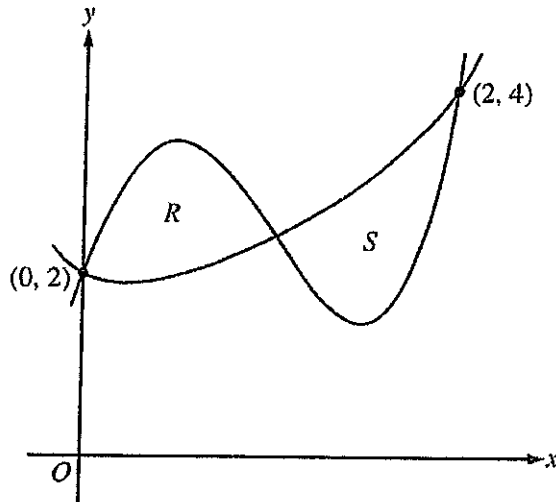
**Time—30 minutes**

**Number of problems—2**

**A graphing calculator is required for these problems.**

1. The rate at which rainwater flows into a drainpipe is modeled by the function  $R$ , where  $R(t) = 20\sin\left(\frac{t^2}{35}\right)$  cubic feet per hour,  $t$  is measured in hours, and  $0 \leq t \leq 8$ . The pipe is partially blocked, allowing water to drain out the other end of the pipe at a rate modeled by  $D(t) = -0.04t^3 + 0.4t^2 + 0.96t$  cubic feet per hour, for  $0 \leq t \leq 8$ . There are 30 cubic feet of water in the pipe at time  $t = 0$ .
- (a) How many cubic feet of rainwater flow into the pipe during the 8-hour time interval  $0 \leq t \leq 8$ ?
- (b) Is the amount of water in the pipe increasing or decreasing at time  $t = 3$  hours? Give a reason for your answer.
- (c) At what time  $t$ ,  $0 \leq t \leq 8$ , is the amount of water in the pipe at a minimum? Justify your answer.
- (d) The pipe can hold 50 cubic feet of water before overflowing. For  $t > 8$ , water continues to flow into and out of the pipe at the given rates until the pipe begins to overflow. Write, but do not solve, an equation involving one or more integrals that gives the time  $w$  when the pipe will begin to overflow.
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2. Let  $f$  and  $g$  be the functions defined by  $f(x) = 1 + x + e^{x^2 - 2x}$  and  $g(x) = x^4 - 6.5x^2 + 6x + 2$ . Let  $R$  and  $S$  be the two regions enclosed by the graphs of  $f$  and  $g$  shown in the figure above.
- Find the sum of the areas of regions  $R$  and  $S$ .
  - Region  $S$  is the base of a solid whose cross sections perpendicular to the  $x$ -axis are squares. Find the volume of the solid.
  - Let  $h$  be the vertical distance between the graphs of  $f$  and  $g$  in region  $S$ . Find the rate at which  $h$  changes with respect to  $x$  when  $x = 1.8$ .
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END OF PART A OF SECTION II

**2015 AP<sup>®</sup> CALCULUS AB FREE-RESPONSE QUESTIONS****CALCULUS AB  
SECTION II, Part B  
Time—60 minutes  
Number of problems—4**

No calculator is allowed for these problems.

$t$ (minutes)	0	12	20	24	40
$v(t)$ (meters per minute)	0	200	240	-220	150

3. Johanna jogs along a straight path. For  $0 \leq t \leq 40$ , Johanna's velocity is given by a differentiable function  $v$ . Selected values of  $v(t)$ , where  $t$  is measured in minutes and  $v(t)$  is measured in meters per minute, are given in the table above.

(a) Use the data in the table to estimate the value of  $v'(16)$ .

(b) Using correct units, explain the meaning of the definite integral  $\int_0^{40} |v(t)| dt$  in the context of the problem.

Approximate the value of  $\int_0^{40} |v(t)| dt$  using a right Riemann sum with the four subintervals indicated in the table.

(c) Bob is riding his bicycle along the same path. For  $0 \leq t \leq 10$ , Bob's velocity is modeled by  $B(t) = t^3 - 6t^2 + 300$ , where  $t$  is measured in minutes and  $B(t)$  is measured in meters per minute.

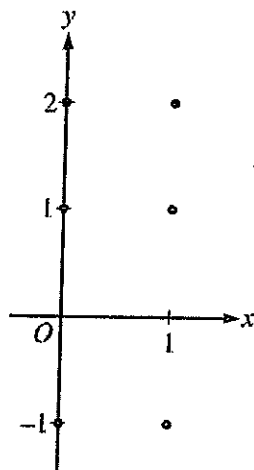
Find Bob's acceleration at time  $t = 5$ .

(d) Based on the model  $B$  from part (c), find Bob's average velocity during the interval  $0 \leq t \leq 10$ .

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4. Consider the differential equation  $\frac{dy}{dx} = 2x - y$ .

(a) On the axes provided, sketch a slope field for the given differential equation at the six points indicated.



(b) Find  $\frac{d^2y}{dx^2}$  in terms of  $x$  and  $y$ . Determine the concavity of all solution curves for the given differential equation in Quadrant II. Give a reason for your answer.

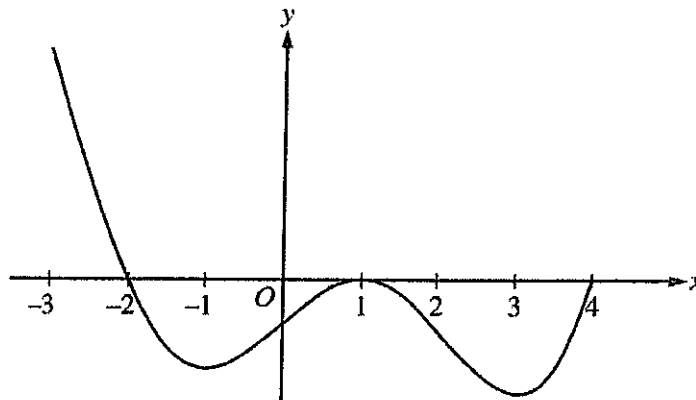
(c) Let  $y = f(x)$  be the particular solution to the differential equation with the initial condition  $f(2) = 3$ . Does  $f$  have a relative minimum, a relative maximum, or neither at  $x = 2$ ? Justify your answer.

(d) Find the values of the constants  $m$  and  $b$  for which  $y = mx + b$  is a solution to the differential equation.

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Graph of  $f'$

5. The figure above shows the graph of  $f'$ , the derivative of a twice-differentiable function  $f$ , on the interval  $[-3, 4]$ . The graph of  $f'$  has horizontal tangents at  $x = -1$ ,  $x = 1$ , and  $x = 3$ . The areas of the regions bounded by the  $x$ -axis and the graph of  $f'$  on the intervals  $[-2, 1]$  and  $[1, 4]$  are 9 and 12, respectively.
- Find all  $x$ -coordinates at which  $f$  has a relative maximum. Give a reason for your answer.
  - On what open intervals contained in  $-3 < x < 4$  is the graph of  $f$  both concave down and decreasing? Give a reason for your answer.
  - Find the  $x$ -coordinates of all points of inflection for the graph of  $f$ . Give a reason for your answer.
  - Given that  $f(1) = 3$ , write an expression for  $f(x)$  that involves an integral. Find  $f(4)$  and  $f(-2)$ .

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**CALCULUS AB  
SECTION II, Part A**

**Time—30 minutes**

**Number of problems—2**

**A graphing calculator is required for these problems.**

1. Grass clippings are placed in a bin, where they decompose. For  $0 \leq t \leq 30$ , the amount of grass clippings remaining in the bin is modeled by  $A(t) = 6.687(0.931)^t$ , where  $A(t)$  is measured in pounds and  $t$  is measured in days.
- (a) Find the average rate of change of  $A(t)$  over the interval  $0 \leq t \leq 30$ . Indicate units of measure.
  - (b) Find the value of  $A'(15)$ . Using correct units, interpret the meaning of the value in the context of the problem.
  - (c) Find the time  $t$  for which the amount of grass clippings in the bin is equal to the average amount of grass clippings in the bin over the interval  $0 \leq t \leq 30$ .
  - (d) For  $t > 30$ ,  $L(t)$ , the linear approximation to  $A$  at  $t = 30$ , is a better model for the amount of grass clippings remaining in the bin. Use  $L(t)$  to predict the time at which there will be 0.5 pound of grass clippings remaining in the bin. Show the work that leads to your answer.
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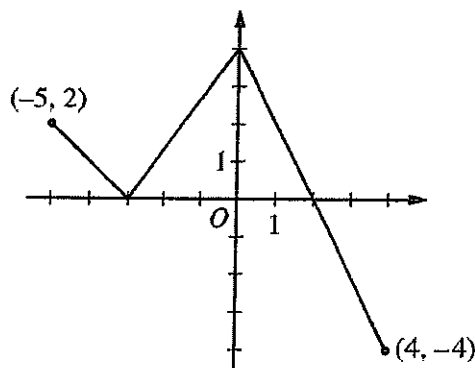
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CALCULUS AB  
SECTION II, Part B

Time—60 minutes

Number of problems—4

No calculator is allowed for these problems.



Graph of  $f$

3. The function  $f$  is defined on the closed interval  $[-5, 4]$ . The graph of  $f$  consists of three line segments and is shown in the figure above. Let  $g$  be the function defined by  $g(x) = \int_{-3}^x f(t) dt$ .
- (a) Find  $g(3)$ .
- (b) On what open intervals contained in  $-5 < x < 4$  is the graph of  $g$  both increasing and concave down? Give a reason for your answer.
- (c) The function  $h$  is defined by  $h(x) = \frac{g(x)}{5x}$ . Find  $h'(3)$ .
- (d) The function  $p$  is defined by  $p(x) = f(x^2 - x)$ . Find the slope of the line tangent to the graph of  $p$  at the point where  $x = -1$ .
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2014 AP<sup>®</sup> CALCULUS AB FREE-RESPONSE QUESTIONS

$t$ (minutes)	0	2	5	8	12
$v_A(t)$ (meters/minute)	0	100	40	-120	-150

4. Train  $A$  runs back and forth on an east-west section of railroad track. Train  $A$ 's velocity, measured in meters per minute, is given by a differentiable function  $v_A(t)$ , where time  $t$  is measured in minutes. Selected values for  $v_A(t)$  are given in the table above.
- Find the average acceleration of train  $A$  over the interval  $2 \leq t \leq 8$ .
  - Do the data in the table support the conclusion that train  $A$ 's velocity is  $-100$  meters per minute at some time  $t$  with  $5 < t < 8$ ? Give a reason for your answer.
  - At time  $t = 2$ , train  $A$ 's position is 300 meters east of the Origin Station, and the train is moving to the east. Write an expression involving an integral that gives the position of train  $A$ , in meters from the Origin Station, at time  $t = 12$ . Use a trapezoidal sum with three subintervals indicated by the table to approximate the position of the train at time  $t = 12$ .
  - A second train, train  $B$ , travels north from the Origin Station. At time  $t$  the velocity of train  $B$  is given by  $v_B(t) = -5t^2 + 60t + 25$ , and at time  $t = 2$  the train is 400 meters north of the station. Find the rate, in meters per minute, at which the distance between train  $A$  and train  $B$  is changing at time  $t = 2$ .

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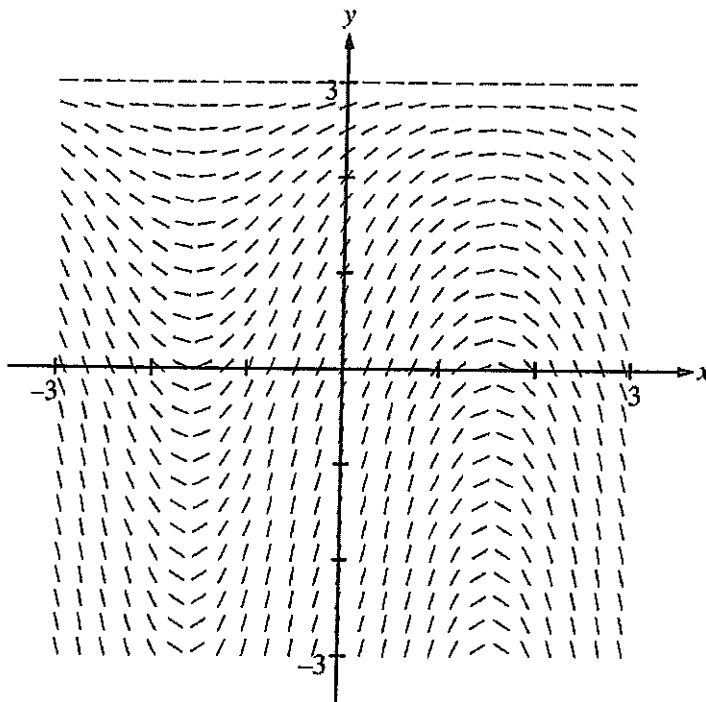
$x$	-2	$-2 < x < -1$	-1	$-1 < x < 1$	1	$1 < x < 3$	3
$f(x)$	12	Positive	8	Positive	2	Positive	7
$f'(x)$	-5	Negative	0	Negative	0	Positive	$\frac{1}{2}$
$g(x)$	-1	Negative	0	Positive	3	Positive	1
$g'(x)$	2	Positive	$\frac{3}{2}$	Positive	0	Negative	-2

5. The twice-differentiable functions  $f$  and  $g$  are defined for all real numbers  $x$ . Values of  $f$ ,  $f'$ ,  $g$ , and  $g'$  for various values of  $x$  are given in the table above.
- (a) Find the  $x$ -coordinate of each relative minimum of  $f$  on the interval  $[-2, 3]$ . Justify your answers.
- (b) Explain why there must be a value  $c$ , for  $-1 < c < 1$ , such that  $f''(c) = 0$ .
- (c) The function  $h$  is defined by  $h(x) = \ln(f(x))$ . Find  $h'(3)$ . Show the computations that lead to your answer.
- (d) Evaluate  $\int_{-2}^3 f'(g(x))g'(x) dx$ .
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6. Consider the differential equation  $\frac{dy}{dx} = (3 - y)\cos x$ . Let  $y = f(x)$  be the particular solution to the differential equation with the initial condition  $f(0) = 1$ . The function  $f$  is defined for all real numbers.

(a) A portion of the slope field of the differential equation is given below. Sketch the solution curve through the point  $(0, 1)$ .



(b) Write an equation for the line tangent to the solution curve in part (a) at the point  $(0, 1)$ . Use the equation to approximate  $f(0.2)$ .

(c) Find  $y = f(x)$ , the particular solution to the differential equation with the initial condition  $f(0) = 1$ .

**STOP**

**END OF EXAM**

**2013 AP<sup>®</sup> CALCULUS AB FREE-RESPONSE QUESTIONS**

**CALCULUS AB  
SECTION II, Part A**

**Time—30 minutes**

**Number of problems—2**

**A graphing calculator is required for these problems.**

1. On a certain workday, the rate, in tons per hour, at which unprocessed gravel arrives at a gravel processing plant is modeled by  $G(t) = 90 + 45\cos\left(\frac{t^2}{18}\right)$ , where  $t$  is measured in hours and  $0 \leq t \leq 8$ . At the beginning of the workday ( $t = 0$ ), the plant has 500 tons of unprocessed gravel. During the hours of operation,  $0 \leq t \leq 8$ , the plant processes gravel at a constant rate of 100 tons per hour.
- (a) Find  $G'(5)$ . Using correct units, interpret your answer in the context of the problem.
  - (b) Find the total amount of unprocessed gravel that arrives at the plant during the hours of operation on this workday.
  - (c) Is the amount of unprocessed gravel at the plant increasing or decreasing at time  $t = 5$  hours? Show the work that leads to your answer.
  - (d) What is the maximum amount of unprocessed gravel at the plant during the hours of operation on this workday? Justify your answer.
-

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2. A particle moves along a straight line. For  $0 \leq t \leq 5$ , the velocity of the particle is given by  $v(t) = -2 + (t^2 + 3t)^{6/5} - t^3$ , and the position of the particle is given by  $s(t)$ . It is known that  $s(0) = 10$ .
- (a) Find all values of  $t$  in the interval  $2 \leq t \leq 4$  for which the speed of the particle is 2.
  - (b) Write an expression involving an integral that gives the position  $s(t)$ . Use this expression to find the position of the particle at time  $t = 5$ .
  - (c) Find all times  $t$  in the interval  $0 \leq t \leq 5$  at which the particle changes direction. Justify your answer.
  - (d) Is the speed of the particle increasing or decreasing at time  $t = 4$ ? Give a reason for your answer.
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**END OF PART A OF SECTION II**

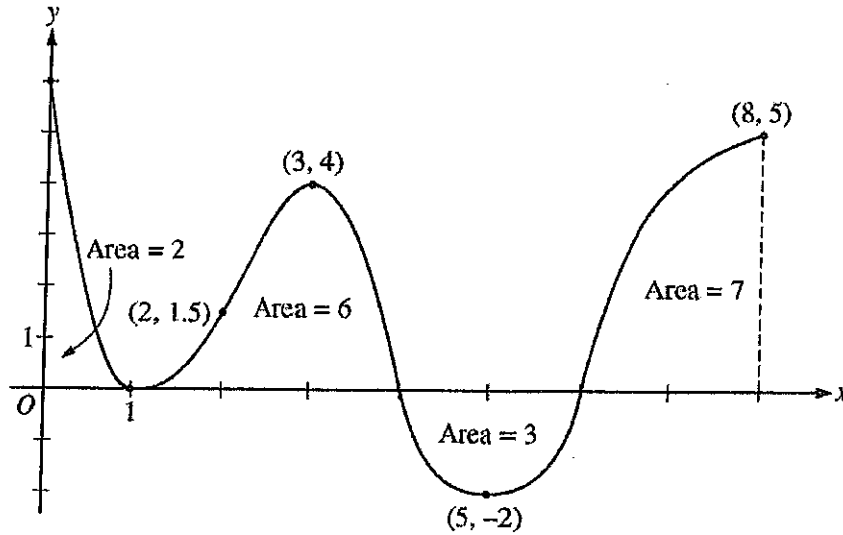


**2013 AP<sup>®</sup> CALCULUS AB FREE-RESPONSE QUESTIONS****CALCULUS AB  
SECTION II, Part B****Time—60 minutes****Number of problems—4****No calculator is allowed for these problems.**

$t$ (minutes)	0	1	2	3	4	5	6
$C(t)$ (ounces)	0	5.3	8.8	11.2	12.8	13.8	14.5

3. Hot water is dripping through a coffeemaker, filling a large cup with coffee. The amount of coffee in the cup at time  $t$ ,  $0 \leq t \leq 6$ , is given by a differentiable function  $C$ , where  $t$  is measured in minutes. Selected values of  $C(t)$ , measured in ounces, are given in the table above.
- (a) Use the data in the table to approximate  $C'(3.5)$ . Show the computations that lead to your answer, and indicate units of measure.
- (b) Is there a time  $t$ ,  $2 \leq t \leq 4$ , at which  $C'(t) = 2$ ? Justify your answer.
- (c) Use a midpoint sum with three subintervals of equal length indicated by the data in the table to approximate the value of  $\frac{1}{6} \int_0^6 C(t) dt$ . Using correct units, explain the meaning of  $\frac{1}{6} \int_0^6 C(t) dt$  in the context of the problem.
- (d) The amount of coffee in the cup, in ounces, is modeled by  $B(t) = 16 - 16e^{-0.4t}$ . Using this model, find the rate at which the amount of coffee in the cup is changing when  $t = 5$ .
-

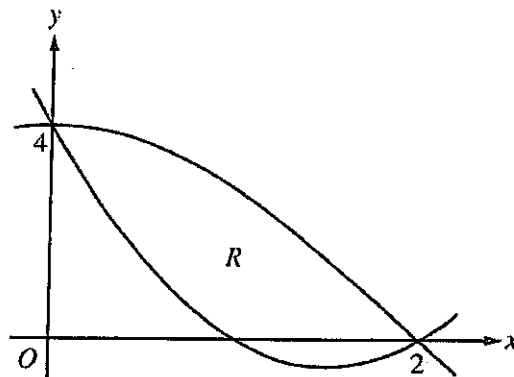
2013 AP<sup>®</sup> CALCULUS AB FREE-RESPONSE QUESTIONS



Graph of  $f'$

4. The figure above shows the graph of  $f'$ , the derivative of a twice-differentiable function  $f$ , on the closed interval  $0 \leq x \leq 8$ . The graph of  $f'$  has horizontal tangent lines at  $x = 1$ ,  $x = 3$ , and  $x = 5$ . The areas of the regions between the graph of  $f'$  and the  $x$ -axis are labeled in the figure. The function  $f$  is defined for all real numbers and satisfies  $f(8) = 4$ .
- Find all values of  $x$  on the open interval  $0 < x < 8$  for which the function  $f$  has a local minimum. Justify your answer.
  - Determine the absolute minimum value of  $f$  on the closed interval  $0 \leq x \leq 8$ . Justify your answer.
  - On what open intervals contained in  $0 < x < 8$  is the graph of  $f$  both concave down and increasing? Explain your reasoning.
  - The function  $g$  is defined by  $g(x) = (f(x))^3$ . If  $f(3) = -\frac{5}{2}$ , find the slope of the line tangent to the graph of  $g$  at  $x = 3$ .

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5. Let  $f(x) = 2x^2 - 6x + 4$  and  $g(x) = 4\cos\left(\frac{1}{4}\pi x\right)$ . Let  $R$  be the region bounded by the graphs of  $f$  and  $g$ , as shown in the figure above.
- Find the area of  $R$ .
  - Write, but do not evaluate, an integral expression that gives the volume of the solid generated when  $R$  is rotated about the horizontal line  $y = 4$ .
  - The region  $R$  is the base of a solid. For this solid, each cross section perpendicular to the  $x$ -axis is a square. Write, but do not evaluate, an integral expression that gives the volume of the solid.
-

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6. Consider the differential equation  $\frac{dy}{dx} = e^y(3x^2 - 6x)$ . Let  $y = f(x)$  be the particular solution to the differential equation that passes through  $(1, 0)$ .
- (a) Write an equation for the line tangent to the graph of  $f$  at the point  $(1, 0)$ . Use the tangent line to approximate  $f(1.2)$ .
- (b) Find  $y = f(x)$ , the particular solution to the differential equation that passes through  $(1, 0)$ .
- 

**STOP**

**END OF EXAM**

2012 AP<sup>®</sup> CALCULUS AB FREE-RESPONSE QUESTIONS

4. The function  $f$  is defined by  $f(x) = \sqrt{25 - x^2}$  for  $-5 \leq x \leq 5$ .

(a) Find  $f'(x)$ .

(b) Write an equation for the line tangent to the graph of  $f$  at  $x = -3$ .

(c) Let  $g$  be the function defined by  $g(x) = \begin{cases} f(x) & \text{for } -5 \leq x \leq -3 \\ x + 7 & \text{for } -3 < x \leq 5. \end{cases}$

Is  $g$  continuous at  $x = -3$ ? Use the definition of continuity to explain your answer.

(d) Find the value of  $\int_0^5 x\sqrt{25 - x^2} \, dx$ .

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**2012 AP<sup>®</sup> CALCULUS AB FREE-RESPONSE QUESTIONS**

6. For  $0 \leq t \leq 12$ , a particle moves along the  $x$ -axis. The velocity of the particle at time  $t$  is given by  $v(t) = \cos\left(\frac{\pi}{6}t\right)$ . The particle is at position  $x = -2$  at time  $t = 0$ .
- (a) For  $0 \leq t \leq 12$ , when is the particle moving to the left?
  - (b) Write, but do not evaluate, an integral expression that gives the total distance traveled by the particle from time  $t = 0$  to time  $t = 6$ .
  - (c) Find the acceleration of the particle at time  $t$ . Is the speed of the particle increasing, decreasing, or neither at time  $t = 4$ ? Explain your reasoning.
  - (d) Find the position of the particle at time  $t = 4$ .
- 

**STOP**

**END OF EXAM**

2011 AP<sup>®</sup> CALCULUS AB FREE-RESPONSE QUESTIONS

CALCULUS AB  
SECTION II, Part A

Time—30 minutes

Number of problems—2

A graphing calculator is required for these problems.

1. For  $0 \leq t \leq 6$ , a particle is moving along the  $x$ -axis. The particle's position,  $x(t)$ , is not explicitly given. The velocity of the particle is given by  $v(t) = 2\sin(e^{t/4}) + 1$ . The acceleration of the particle is given by  $a(t) = \frac{1}{2}e^{t/4}\cos(e^{t/4})$  and  $x(0) = 2$ .
- (a) Is the speed of the particle increasing or decreasing at time  $t = 5.5$ ? Give a reason for your answer.
- (b) Find the average velocity of the particle for the time period  $0 \leq t \leq 6$ .
- (c) Find the total distance traveled by the particle from time  $t = 0$  to  $t = 6$ .
- (d) For  $0 \leq t \leq 6$ , the particle changes direction exactly once. Find the position of the particle at that time.
- 

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2011 AP<sup>®</sup> CALCULUS AB FREE-RESPONSE QUESTIONS

$t$ (minutes)	0	2	5	9	10
$H(t)$ (degrees Celsius)	66	60	52	44	43

2. As a pot of tea cools, the temperature of the tea is modeled by a differentiable function  $H$  for  $0 \leq t \leq 10$ , where time  $t$  is measured in minutes and temperature  $H(t)$  is measured in degrees Celsius. Values of  $H(t)$  at selected values of time  $t$  are shown in the table above.
- (a) Use the data in the table to approximate the rate at which the temperature of the tea is changing at time  $t = 3.5$ . Show the computations that lead to your answer.
- (b) Using correct units, explain the meaning of  $\frac{1}{10} \int_0^{10} H(t) dt$  in the context of this problem. Use a trapezoidal sum with the four subintervals indicated by the table to estimate  $\frac{1}{10} \int_0^{10} H(t) dt$ .
- (c) Evaluate  $\int_0^{10} H'(t) dt$ . Using correct units, explain the meaning of the expression in the context of this problem.
- (d) At time  $t = 0$ , biscuits with temperature  $100^\circ\text{C}$  were removed from an oven. The temperature of the biscuits at time  $t$  is modeled by a differentiable function  $B$  for which it is known that  $B'(t) = -13.84e^{-0.173t}$ . Using the given models, at time  $t = 10$ , how much cooler are the biscuits than the tea?

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END OF PART A OF SECTION II



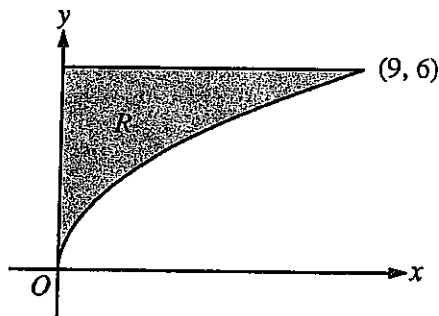
2010 AP<sup>®</sup> CALCULUS AB FREE-RESPONSE QUESTIONS

CALCULUS AB  
SECTION II, Part B

Time—45 minutes

Number of problems—3

No calculator is allowed for these problems.

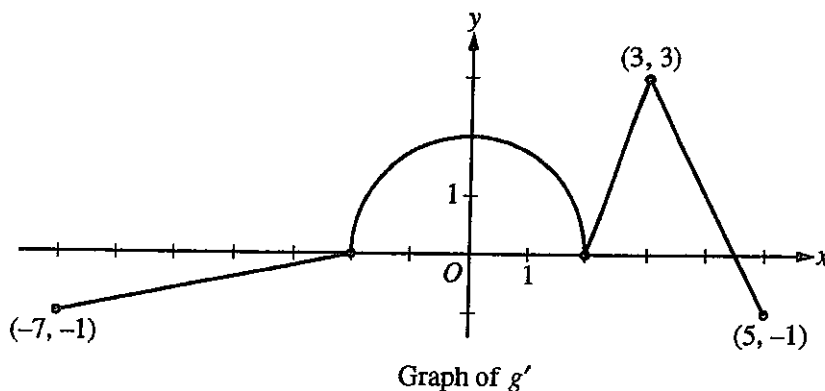


4. Let  $R$  be the region in the first quadrant bounded by the graph of  $y = 2\sqrt{x}$ , the horizontal line  $y = 6$ , and the  $y$ -axis, as shown in the figure above.
- Find the area of  $R$ .
  - Write, but do not evaluate, an integral expression that gives the volume of the solid generated when  $R$  is rotated about the horizontal line  $y = 7$ .
  - Region  $R$  is the base of a solid. For each  $y$ , where  $0 \leq y \leq 6$ , the cross section of the solid taken perpendicular to the  $y$ -axis is a rectangle whose height is 3 times the length of its base in region  $R$ . Write, but do not evaluate, an integral expression that gives the volume of the solid.

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2010 AP<sup>®</sup> CALCULUS AB FREE-RESPONSE QUESTIONS



5. The function  $g$  is defined and differentiable on the closed interval  $[-7, 5]$  and satisfies  $g(0) = 5$ . The graph of  $y = g'(x)$ , the derivative of  $g$ , consists of a semicircle and three line segments, as shown in the figure above.
- Find  $g(3)$  and  $g(-2)$ .
  - Find the  $x$ -coordinate of each point of inflection of the graph of  $y = g(x)$  on the interval  $-7 < x < 5$ . Explain your reasoning.
  - The function  $h$  is defined by  $h(x) = g(x) - \frac{1}{2}x^2$ . Find the  $x$ -coordinate of each critical point of  $h$ , where  $-7 < x < 5$ , and classify each critical point as the location of a relative minimum, relative maximum, or neither a minimum nor a maximum. Explain your reasoning.

6. Solutions to the differential equation  $\frac{dy}{dx} = xy^3$  also satisfy  $\frac{d^2y}{dx^2} = y^3(1 + 3x^2y^2)$ . Let  $y = f(x)$  be a particular solution to the differential equation  $\frac{dy}{dx} = xy^3$  with  $f(1) = 2$ .
- Write an equation for the line tangent to the graph of  $y = f(x)$  at  $x = 1$ .
  - Use the tangent line equation from part (a) to approximate  $f(1.1)$ . Given that  $f(x) > 0$  for  $1 < x < 1.1$ , is the approximation for  $f(1.1)$  greater than or less than  $f(1.1)$ ? Explain your reasoning.
  - Find the particular solution  $y = f(x)$  with initial condition  $f(1) = 2$ .

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END OF EXAM

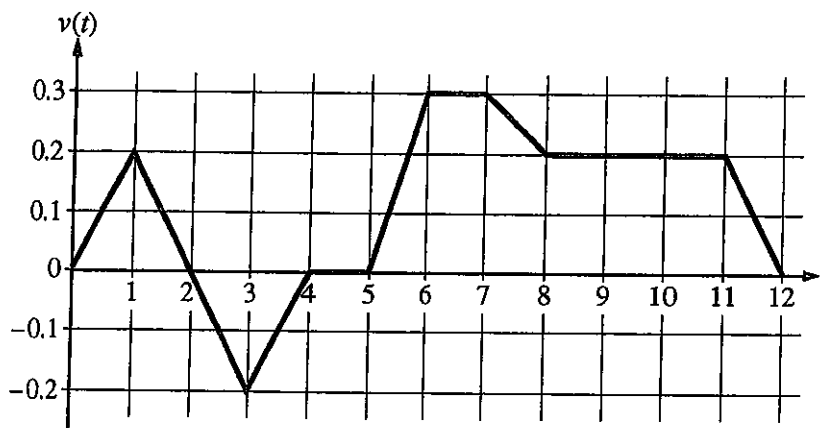
2009 AP<sup>®</sup> CALCULUS AB FREE-RESPONSE QUESTIONS

CALCULUS AB  
SECTION II, Part A

Time—45 minutes

Number of problems—3

A graphing calculator is required for some problems or parts of problems.



1. Caren rides her bicycle along a straight road from home to school, starting at home at time  $t = 0$  minutes and arriving at school at time  $t = 12$  minutes. During the time interval  $0 \leq t \leq 12$  minutes, her velocity  $v(t)$ , in miles per minute, is modeled by the piecewise-linear function whose graph is shown above.
- (a) Find the acceleration of Caren's bicycle at time  $t = 7.5$  minutes. Indicate units of measure.
- (b) Using correct units, explain the meaning of  $\int_0^{12} |v(t)| dt$  in terms of Caren's trip. Find the value of  $\int_0^{12} |v(t)| dt$ .
- (c) Shortly after leaving home, Caren realizes she left her calculus homework at home, and she returns to get it. At what time does she turn around to go back home? Give a reason for your answer.
- (d) Larry also rides his bicycle along a straight road from home to school in 12 minutes. His velocity is modeled by the function  $w$  given by  $w(t) = \frac{\pi}{15} \sin\left(\frac{\pi}{12}t\right)$ , where  $w(t)$  is in miles per minute for  $0 \leq t \leq 12$  minutes. Who lives closer to school: Caren or Larry? Show the work that leads to your answer.

WRITE ALL WORK IN THE PINK EXAM BOOKLET.

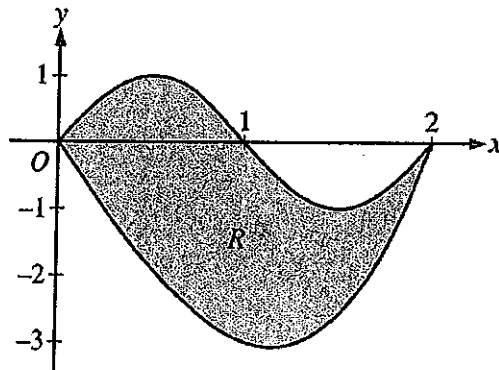
**2008 AP<sup>®</sup> CALCULUS AB FREE-RESPONSE QUESTIONS**

**CALCULUS AB  
SECTION II, Part A**

**Time—45 minutes**

**Number of problems—3**

A graphing calculator is required for some problems or parts of problems.



1. Let  $R$  be the region bounded by the graphs of  $y = \sin(\pi x)$  and  $y = x^3 - 4x$ , as shown in the figure above.
    - (a) Find the area of  $R$ .
    - (b) The horizontal line  $y = -2$  splits the region  $R$  into two parts. Write, but do not evaluate, an integral expression for the area of the part of  $R$  that is below this horizontal line.
    - (c) The region  $R$  is the base of a solid. For this solid, each cross section perpendicular to the  $x$ -axis is a square. Find the volume of this solid.
    - (d) The region  $R$  models the surface of a small pond. At all points in  $R$  at a distance  $x$  from the  $y$ -axis, the depth of the water is given by  $h(x) = 3 - x$ . Find the volume of water in the pond.
- 

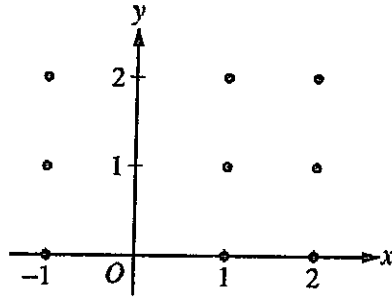
**WRITE ALL WORK IN THE PINK EXAM BOOKLET.**

**2008 AP<sup>®</sup> CALCULUS AB FREE-RESPONSE QUESTIONS**

5. Consider the differential equation  $\frac{dy}{dx} = \frac{y-1}{x^2}$ , where  $x \neq 0$ .

(a) On the axes provided, sketch a slope field for the given differential equation at the nine points indicated.

(Note: Use the axes provided in the exam booklet.)



(b) Find the particular solution  $y = f(x)$  to the differential equation with the initial condition  $f(2) = 0$ .

(c) For the particular solution  $y = f(x)$  described in part (b), find  $\lim_{x \rightarrow \infty} f(x)$ .

6. Let  $f$  be the function given by  $f(x) = \frac{\ln x}{x}$  for all  $x > 0$ . The derivative of  $f$  is given by  $f'(x) = \frac{1 - \ln x}{x^2}$ .

(a) Write an equation for the line tangent to the graph of  $f$  at  $x = e^2$ .

(b) Find the  $x$ -coordinate of the critical point of  $f$ . Determine whether this point is a relative minimum, a relative maximum, or neither for the function  $f$ . Justify your answer.

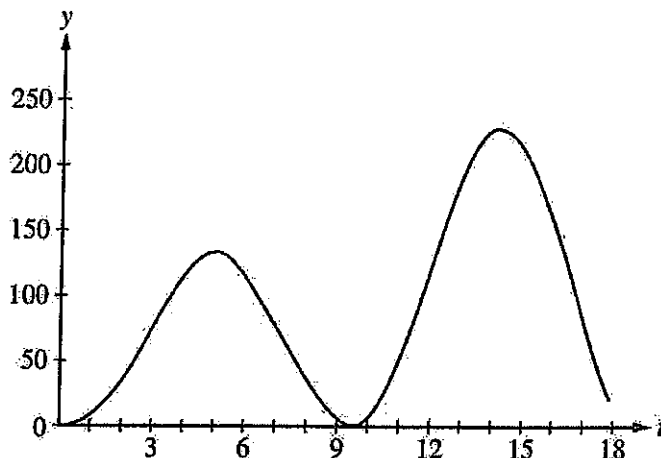
(c) The graph of the function  $f$  has exactly one point of inflection. Find the  $x$ -coordinate of this point.

(d) Find  $\lim_{x \rightarrow 0^+} f(x)$ .

**WRITE ALL WORK IN THE PINK EXAM BOOKLET.**

**END OF EXAM**

**2006 AP<sup>®</sup> CALCULUS AB FREE-RESPONSE QUESTIONS**



2. At an intersection in Thomasville, Oregon, cars turn left at the rate  $L(t) = 60\sqrt{t} \sin^2\left(\frac{t}{3}\right)$  cars per hour over the time interval  $0 \leq t \leq 18$  hours. The graph of  $y = L(t)$  is shown above.
- To the nearest whole number, find the total number of cars turning left at the intersection over the time interval  $0 \leq t \leq 18$  hours.
  - Traffic engineers will consider turn restrictions when  $L(t) \geq 150$  cars per hour. Find all values of  $t$  for which  $L(t) \geq 150$  and compute the average value of  $L$  over this time interval. Indicate units of measure.
  - Traffic engineers will install a signal if there is any two-hour time interval during which the product of the total number of cars turning left and the total number of oncoming cars traveling straight through the intersection is greater than 200,000. In every two-hour time interval, 500 oncoming cars travel straight through the intersection. Does this intersection require a traffic signal? Explain the reasoning that leads to your conclusion.

**WRITE ALL WORK IN THE PINK EXAM BOOKLET.**