11. Determine the measurements of the passenger jet:
Length = 200(16.8 cm) or 3360 cm or 33.6 m
Width = 200(17.2 cm) or 3440 cm or 34.4 m
The is 33.6 m long and 34.4 m wide.
Determine the number of jets that could fit along the width of the hangar:

$$\frac{71.9\ \cancel{m}}{34.4\ \cancel{m}} = 2.066\ldots$$

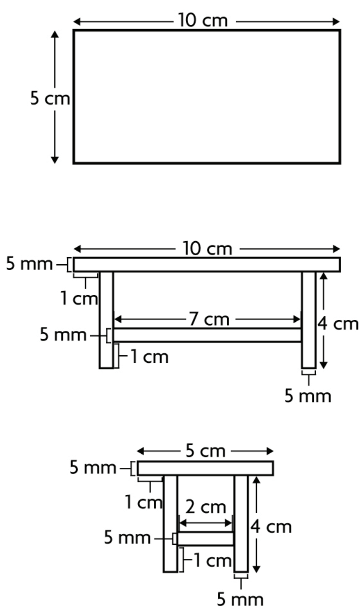
Two jets can fit along the width.
Only one jet can fit along the length of the hangar:

$$\frac{46.6\ \cancel{m}}{33.6\ \cancel{m}} = 1.386\ldots$$

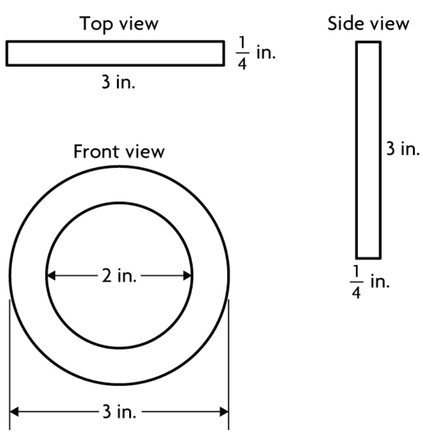
Two passenger jets can fit inside Hangar 77.

12. a) e.g., I used a photograph of a two storey house. The estimated dimensions are 6 m tall and 5 m wide.
b) Measure the metre stick in the photograph to determine the scale factor. Then multiply the building measurements by the scale factor to determine the building dimensions in metres.

13.

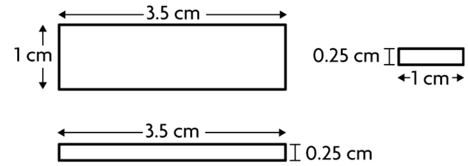


14.



15. e.g., Using a scale factor of $\frac{1}{20}$, the views would have the following dimensions:
Top view rectangle: 6.7 cm by 3.3 cm
Side view rectangle: 3.3 cm by 4.4 cm
Front view rectangle: 6.7 cm by 4.4 cm

16. e.g., I chose an eraser measuring 7.0 cm by 2.0 cm by 0.5 cm and I used a scale factor of $\frac{1}{2}$.



17. a) No; The area of the base increases by the square of the scale factor.
b) No; The volume increases by the cube of the scale factor.

18. e.g., Both involve multiplying each dimension by a scale factor; shapes have two dimensions while objects have three dimensions.

19. a) Since you have to determine the area of metal used for the can, you must determine k^2.
$k^2 = 1.5^2$
$k^2 = 2.25$
The scale factor is 2.25.
b) Since you need to determine the area of the smaller can, you must make a reduction. So the scale factor is

$$\frac{1}{2.25} \text{ or } \frac{100}{225} \text{ or } \frac{4}{9}.$$

Cost of small can = (cost of large can)(scale factor)

Cost of small can = $\$0.045\left(\frac{4}{9}\right)$

Cost of small can = \$0.02
It will cost \$0.02 to make the small can.

20. Reduction is 50%, so the scale factor is 0.5.
Smaller area = k^2(larger area)
Smaller area = $(0.5)^2(100\text{ cm}^2)$
Smaller area = $0.25(100\text{ cm}^2)$
Smaller area = 25 cm^2
The smaller cone has an area of 25 cm^2.

Lesson 8.6: Scale Factors and 3-D Objects, page 508

1. For an enlargement,

$$k = \frac{\text{measure of one dimension in enlargement}}{\text{measure of dimension in original}}$$

For surface area, use k^2. For volume, use k^3.

a) $k = \frac{3.0\ \cancel{cm}}{1.5\ \cancel{cm}}$
$k = 2$

i) $k^2 = 2^2$
$k^2 = 4$
The surface area of the larger object is 4 times the surface area of the smaller object.

ii) $k^3 = 2^3$
$k^3 = 8$
The volume of the larger object is 8 times the volume of the smaller object.

b) $k = \dfrac{9.0 \cancel{cm}}{6.0 \cancel{cm}}$

$k = \dfrac{3}{2}$ or 1.5

i) $k^2 = \left(\dfrac{3}{2}\right)^2$ or 1.5^2

$k^2 = \dfrac{9}{4}$ or 2.25

The surface area of the larger object is $\dfrac{9}{4}$ or 2.25 times the surface area of the smaller object.

ii) $k^3 = \left(\dfrac{3}{2}\right)^3$ or 1.5^3

$k^3 = \dfrac{27}{8}$ or 3.375

The volume of the larger object is $\dfrac{27}{8}$ or 3.375 times the volume of the smaller object.

c) $k = \dfrac{12.0 \cancel{cm}}{3.0 \cancel{cm}}$

$k = 4$

i) $k^2 = 4^2$
$k^2 = 16$
The surface area of the larger object is 16 times the surface area of the smaller object.

ii) $k^3 = 4^3$
$k^3 = 64$
The volume of the larger object is 64 times the volume of the smaller object.

d) $k = \dfrac{5.0 \cancel{cm}}{3.0 \cancel{cm}}$

$k = \dfrac{5}{3}$ or 1.666…

i) $k^2 = \left(\dfrac{5}{3}\right)^2$ or $1.666\ldots^2$

$k^2 = \dfrac{25}{9}$ or 2.777…

The surface area of the larger object is $\dfrac{25}{9}$ or about 2.8 times the surface area of the smaller object.

ii) $k^3 = \left(\dfrac{5}{3}\right)^3$ or $1.666\ldots^3$

$k^3 = \dfrac{125}{27}$ or 4.629…

The volume of the larger object is $\dfrac{125}{27}$ or about 4.6 times the volume of the smaller object.

2. a) For an enlargement,

$k = \dfrac{\text{measure of one dimension in enlargement}}{\text{measure of dimension in original}}$

$k = \dfrac{600 \cancel{mm}}{12 \cancel{mm}}$

$k = 50$

b) For surface area, use k^2.
$k^2 = 50^2$
$k^2 = 2500$
The surface area of the enlarged dice is 2500 times the surface area of the actual dice.

c) For volume, use k^3.
$k^3 = 50^3$
$k^3 = 125\ 000$
The volume of the enlarged dice is 125 000 times the volume of the actual dice.

3. Scale is 1:30. For an enlargement, the scale factor, k, is 30.
For the height of the sail:
Actual height = (model height)(k)
Actual height = (16 cm)(30)
Actual height = 480 cm
The actual height of the sail is 480 cm.
For the area of the sail:
New area = k^2(area of model sail)
New area = $(30)^2$(8.5 cm^2)
New area = 900(8.5 cm^2)
New area = 7650 cm^2
The area of the actual sail is 7650 cm^2.

4. Capacity of similar tank = k^3(capacity of original)
Capacity of similar tank = 3^3(32 m^3)
Capacity of similar tank = 27(32 m^3)
Capacity of similar tank = 864 m^3
The capacity of the larger tank is 864 m^3.

5. a) For an enlargement, the scale factor, k is 3.
New area = k^2(area of original)
New area = 3^2(500 cm^2)
New area = 9(500 cm^2)
New area = 4500 cm^2
The area of the larger page will be 4500 cm^2.

b) If the paper is the same, the third dimension, thickness does not change. You can assign a value of one to this dimension.
For volume, use k^2.
$k^2 = 3^2$
$k^2 = 9$
The volume will change by a factor of 9.

6. The scale factor for the reduction is $\dfrac{3}{4}$.

Lesser volume = k^3(greater volume)

Lesser volume = $\left(\dfrac{3}{4}\right)^3$(9420 cm^3)

Lesser volume = $\dfrac{27}{64}$(9420 cm^3)

Lesser volume = 3974.0625 cm^3
The volume of the smaller vase is 3974 cm^3.

7. a) For volume, use k^3.
$k^3 = 1.5^3$
$k^3 = 3.375$
The ratio of the volumes is 3.375.
b) For surface area, use k^2.
$k^2 = 1.5^2$
$k^2 = 2.25$
The ratio of the surface areas is 2.25.
c) For base perimeter, use k.
$k = 1.5$
The ratio of the base perimeters is 1.5.
8. For surface area, use k^2.
$k^2 = 3^2$
$k^2 = 9$
The surface area of the lid would change by a factor of 9.
For volume, use k^3.
$k^3 = 3^3$
$k^3 = 27$
The volume of the box would change by a factor of 27.
9. To obtain the next tallest doll, you are must enlarge the previous shorter one, so the scale factor, k will be

$$k = \frac{\text{height of taller doll}}{\text{height of shorter doll}}$$

$$k = \frac{3.5 \cancel{\text{cm}}}{2.0 \cancel{\text{cm}}}$$

$$k = \frac{3.5 \cancel{\text{cm}}}{2.0 \cancel{\text{cm}}}$$

$k = 1.75$
Volume of next doll
= (volume of previous doll)(scale factor)^3
Volume of doll 2 = (volume of doll 1)k^3
Volume of doll 2 = (8 cm^3)1.75^3
Volume of doll 2 = 42.875 cm^3
Volume of doll 3 = (volume of doll 2)k^3
Volume of doll 3 = (42.875 cm^3)1.75^3
Volume of doll 3 = 229.783… cm^3
Volume of doll 4 = (volume of doll 3)k^3
Volume of doll 4 = (229.783… cm^3)1.75^3
Volume of doll 4 = 1231.494… cm^3
Volume of doll 5 = (volume of doll 4)k^3
Volume of doll 5 = (1231.494… cm^3)1.75^3
Volume of doll 5 = 6600.040… cm^3
The volume of the fifth doll is about 6600 cm^3.
10. a) By counting, the model is made up of 14 linking cubes.
Each cube represents one unit of volume because it is an object that has three dimensions. So the volume of the model is 14 cubes.
For an enlargement, the scale factor is 5.
New volume = k^3(old volume)
New volume = 5^3(14 cubes)
New volume = 125(14 cubes)
New volume = 1750 cubes
The number of cubes needed for the larger model is 1750.
b) For surface area, use k^2.
$k^2 = 5^2$
$k^2 = 25$
The surface area of the new model is greater by a factor of 25.

11. a) For the Earth:

$$10.0 \text{ cm} = (10 \cancel{\text{cm}})\left(\frac{1 \text{ km}}{100\,000 \cancel{\text{cm}}}\right)$$

$$10.0 \text{ cm} = \frac{1}{10\,000} \text{ km}$$

Since the model is a reduction of the actual Earth, then the scale factor, k is

$$k = \frac{\text{model radius}}{\text{Earth's radius}}$$

$$k = \frac{\frac{1}{10\,000} \text{ km}}{6378.1 \text{ km}}$$

$$k = \left(\frac{1}{10\,000} \cancel{\text{km}}\right)\left(\frac{1}{6378.1 \cancel{\text{km}}}\right)$$

$$k = \frac{1}{63\,781\,000}$$

For the Moon:
Radius of model = k(radius of moon)

$$\text{Radius of model} = \frac{1}{63\,781\,000}(1737.4 \text{ km})$$

Radius of model = 0.000 027 2… km or 2.724… cm
The radius of the model of the Moon should be 2.7 cm.
b) Let C represent the circumference and r represent radius.
$C = 2\pi r$
$C_{\text{model of Earth}}$ = 2(10.0 cm)π or 62.831… cm
$C_{\text{model of Moon}}$ = 2(2.724… cm)π or 17.115… cm

$$\frac{C_{\text{model of Earth}}}{C_{\text{model of Moon}}} = \frac{62.831...}{17.115...} \text{ or } 3.671\ldots$$

The ratio of the circumferences of the Earth and the Moon is about 3.7:1.
c) The ratio for the surface area uses the ratio from part a) before rounding, squared:
$3.671\ldots^2 = 13.476\ldots$
The ratio is about 13.5:1.
d) The ratio for the volume uses the ratio from part a) before rounding, cubed:
$3.671\ldots^3 = 49.473\ldots$
The ratio is about 49.5:1.

12. $k = \dfrac{\text{grapefruit radius}}{\text{orange radius}}$

$$k = \frac{7 \cancel{\text{cm}}}{5 \cancel{\text{cm}}}$$

$$k = \frac{7}{5}$$

If you consider the grapefruit to be a larger but similar object to the orange, then use k^3 to compare their volumes.

$k^3 = \left(\frac{7}{5}\right)^3$

$k^3 = \frac{343}{125}$ or 2.744

The volume of the grapefruit is about 2.7 times the volume of the orange.

13. $k = \frac{\text{softball diameter}}{\text{baseball diameter}}$

$k = \frac{3.8\ \cancel{\text{in.}}}{2.9\ \cancel{\text{in.}}}$

$k = 1.310\ldots$

The amount of leather is a measure of surface area, so use k^2.

$k^2 = (1.310\ldots)^2$

$k^2 = 1.717\ldots$

The amount of leather needed to cover a softball is 1.717… or about 172% of the amount of leather needed to cover the baseball.

14. a) $k = \frac{\text{greater radius}}{\text{lesser radius}}$

$k = \frac{10.2\ \cancel{\text{cm}}}{6.8\ \cancel{\text{cm}}}$

$k = 1.5$

The difference in the height of the labels is by a factor of 1.5.

b) For surface area, use k^2.

$k^2 = 1.5^2$

$k^2 = 2.25$

The surface areas differ by a factor of 2.25.

c) For volume, use k^3.

$k^3 = 1.5^3$

$k^3 = 3.375$

The capacities differ by a factor of 3.375.

d) Volume of cylinder = π(radius)^2(height)

Volume of small cylinder = $\pi(6.8\text{ cm})^2(6.8\text{ cm})$

Volume of small cylinder = 987.817… cm^3

The small container holds about 988 cm^3 of ice cream.

Volume of large cylinder = $\pi(10.2\text{ cm})^2(10.2\text{ cm})$

Volume of large cylinder = 3333.883… cm^3

The large container holds about 3334 cm^3 of ice cream.

15. a) $k = \frac{\text{greater length}}{\text{lesser length}}$

$k = \frac{36.0\ \cancel{\text{cm}}}{18.0\ \cancel{\text{cm}}}$

$k = 2$

For surface area, use k^2.

$k^2 = 2^2$

$k^2 = 4$

Disagree. It will take 4 times as much wrapping paper to wrap the large box relative to the small box.

b) For volume, use k^3.

$k^3 = 2^3$

$k^3 = 8$

No. The volume of the large box is 8 times the volume of the small boxes, so the volume of the small box is $\frac{1}{8}$ the volume of the large box.

16. a) Let *SA* represent the surface area.

SA of cylinder = $2\pi r^2 + 2\pi rh$

SA of scaled cylinder = $2\pi(kr)(kr) + 2\pi(kr)(kh)$

SA of scaled cylinder = $k^2(2\pi r^2) + k^2(2\pi rh)$

SA of scaled cylinder = $k^2(2\pi r^2 + 2\pi rh)$

Let *V* represent the volume.

V of cylinder = $\pi r^2 h$

V of scaled cylinder = $\pi(kr)(kr)(kh)$

V of scaled cylinder = $k^3(\pi r^2 h)$

b) Let *SA* represent the surface area.

SA of cone = $\pi r^2 + \pi rs$

SA of scaled cone = $\pi(kr)(kr) + \pi(kr)(ks)$

SA of scaled cone = $k^2(\pi r^2) + k^2(\pi rs)$

SA of scaled cone = $k^2(\pi r^2 + \pi rs)$

Let *V* represent the volume.

V of cone = $\frac{1}{3}\pi r^2 h$

V of scaled cone = $\frac{1}{3}\pi(kr)(kr)(kh)$

V of scaled cone = $k^3\left(\frac{1}{3}\pi r^2 h\right)$

17. e.g., Consider the relationship between the volumes. The scale factor is 2, so the larger prism has a volume that is 8 times the volume of the smaller prism. Eight of the smaller prisms will fit inside the larger prism.

18. $k^3 = \frac{\text{volume of large tent}}{\text{volume of small tent}}$

$k^3 = \frac{15\,660\ \cancel{\text{m}^3}}{580\ \cancel{\text{m}^3}}$

$k^3 = 27$

$k = 3$

For surface area, use k^2.

$k^2 = 3^2$

$k^2 = 9$

The floor area of the larger tent is 9 times as great as the floor area of the smaller tent.

19. a) Volume of cube = (length)(width)(height)

27 000 cm^3 = (length)^3

30 cm = length

Therefore, the diameter of the model is 30 cm.

Therefore, the radius of the model is 15 cm.

Scale factor is 1:11 580 000.

Since the model must be enlarged to the size of the actual Moon, the scale factor is 11 580 000.

Radius of Moon = (model radius)(scale factor)

Radius of Moon = (15 cm)(11 580 000)

Radius of Moon = 173 700 000 cm or 1737 km

SA of sphere = $4\pi r^2$
SA of sphere = $4\pi(1737\text{ km})^2$
SA of sphere = 37 914 863.860… km^2
The surface area of the Moon is 37 914 864 km^2.

b) V of sphere = $\frac{4}{3}\pi r^3$

V of sphere = $\frac{4}{3}\pi(1737\text{ km})^3$

V of sphere = 21 952 706 175.029… km^3
The volume of the Moon is about
21 952 700 000 km^3.

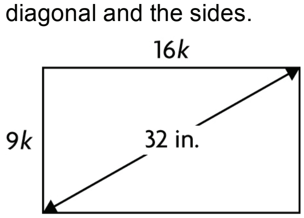
20. $k = \frac{\text{greater diameter}}{\text{lesser diameter}}$

$k = \frac{12\ \cancel{\text{in.}}}{10\ \cancel{\text{in.}}}$

$k = 1.2$
I assumed that the height of the cake would be the same and the cost of the frosting is the same as for the interior of the cake, so the scale factor for height is 1.
Price of large cake = (price of small cake)(k^2)
Price of large cake = (\$14.00)(1.2^2)
Price of large cake = \$20.16
The price of the large cake should be \$20.16.

Math in Action, page 511

e.g., I drew a diagram to represent the 32″ television screen. The width-to-height ratio of the screen is 16:9. Let k represent the scale factor. Using the Pythagorean theorem, I represented the relationship between the diagonal and the sides.



$$32^2 = (16k)^2 + (9k)^2$$
$$1024 = 256k^2 + 81k^2$$
$$1024 = 337k^2$$
$$\frac{1024}{337} = k^2$$
$$3.038\ldots = k^2$$
$$\sqrt{3038...} = k$$
$$1.743\ldots = k$$

I determined the dimensions using k.

Width = $16k$	Height = $9k$
Width = 16(1.743…)	Height = 9(1.743…)
Width = 27.879…	Height = 15.688…

The 32″ television screen is about 28 in. wide and 16 in. high.

I wrote similar equations for the 40″ and 52″ screens and determined their dimensions. I researched the prices for televisions from the same manufacturer and put my data in a table.

Diagonal (in.)	Width (in.)	Height (in.)	Scale Factor	Price (\$)
32	28	16	1.743…	480
40	35	20	2.178…	750
52	45	25	2.832…	1600

I compared the prices using the scale factors. I did not have a television with a scale factor of 1, so I set up a proportion using the scale factor and price of the smallest television.

$\frac{480(2.178...)}{1.743...} \doteq 600$ $\frac{480(2.832...)}{1.743...} \doteq 780$

The prices of the televisions are not related by the same scale factor as the length. The price per inch increases as the television size increases.
I chose to compare the prices and sizes of paint cans, since I noticed that the smaller cans were similar to the larger cans. I made a table to compare two sizes of paint cans.

Volume (L)	Scale Factor	Price (\$)
0.946	0.250…	17.46
3.78	1	44.96

I determined the price of the smaller can based on the price of the larger can, using the scale factor:
$44.96(0.250) \doteq 11.251\ldots$
If the products were priced proportionally, the smaller can would cost \$11.25. I noticed two different relationships in my research. For televisions, the price per inch increases as the television size increases. This might happen because the screens for larger televisions are more expensive to produce or because consumers are willing to pay more for a bigger television. Companies may also make and sell more of the smaller sizes, which would bring the cost of the smaller sizes down. For paint, the price per litre decreases as the size of the container increases. It may be cheaper for a paint company to sell larger containers, since the cost of the packaging goes down per litre.

Chapter Self-Test, page 512

1. a) Intervals of increase are the periods where the slope of the line is going upward from left to right:
1993–1995, 1996–2000, 2001–2004, 2005–2008
Intervals of decrease are the periods where the slope of the line is going downward from left to right:
1995–1996, 2000-2001, 2008–mid-2009
b) No change is indicated by a flat or horizontal slope:
2004–2005