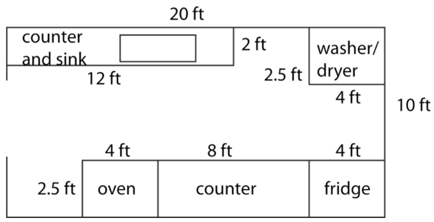
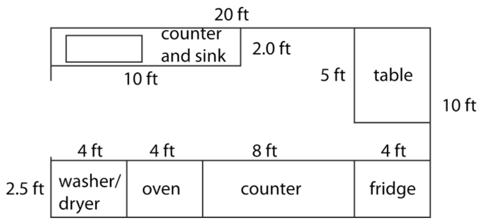
b) e.g.,

Before Renovation



After Renovation



c) e.g., Estimate or measure the open floor space areas in each diagram and compare. In this case, The kitchen had 126 square feet of free area before it was renovated and had 110 square feet of free area after it was renovated, so the kitchen was more spacious before it was renovated. However, it was possible to use the space more fully by rearranging things.

17. e.g., A rectangle is 3 m by 4 m.

A. The area of the original rectangle is 12 m^2.

The shape is reduced by a scale factor of $\frac{1}{2}$ means the dimensions are reduced by $\frac{1}{2}$ or are now 1.5 m by 2 m.

The area of the reduced shape is 3 m.

The area is divided by 4 in process A.

B. If you divide the area of the original rectangle by 2 the new area is 6 m^2 and is just $\frac{1}{2}$ of the original area. Side lengths are not reduced by $\frac{1}{2}$.

18. Use an example of a rectangle to help you.

Rectangle 1 has the dimensions 3 m by 4 m.

Area of rectangle 1 = 12 m^2

Rectangle 2 has dimensions 180% or 1.8 times rectangle 1 dimensions.

Rectangle 2 has dimensions 5.4 m by 7.2 m.

Area of rectangle 2 = 38.88 m^2

Rectangle 3 has dimensions 50% or 0.5 times rectangle 2 dimensions.

Rectangle 3 has dimensions 2.7 m by 3.6 m.

Area of rectangle 3 = 9.72 m^2

$$\frac{\text{area of rectangle 3}}{\text{area of rectangle 1}} = \frac{9.72\ \cancel{m^2}}{12\ \cancel{m^2}}$$

$$\frac{\text{area of rectangle 3}}{\text{area of rectangle 1}} = 0.81$$

The area of the third polygon is 81% the area of the original polygon.

Alternate solution:

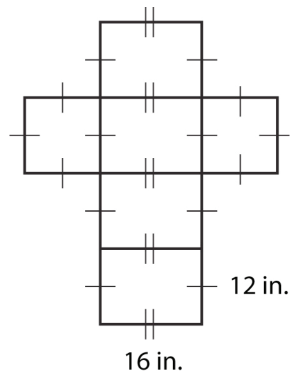
The first scale factor is 1.8. The second scale factor is 0.5. The total scale factor is (1.8)(0.5) = 0.9.

The change in area is the square of the total scale factor:

$0.9^2 = 0.81$.

The area of the third polygon is 81% the area of the original polygon.

19. The original box will have the following panels with these dimensions:



16 in. = 1.333… ft, 12 in. = 1 ft

Total surface area = 4(1.333… ft)(1 ft) + 2(1 ft)(1 ft)

Total surface area = 4(1.333… ft^2) + 2(1 ft^2)

Total surface area = 5.333… ft^2 + 2 ft^2

Total surface area = 7.333… ft^2

$$\text{Cost} = (7.333...\ \cancel{ft^2})\left(\frac{\$0.05}{1\ \cancel{ft^2}}\right)$$

Cost = $0.366…

New surface area = k^2(old surface area)

New surface area = $(1.5)^2$(7.333… ft^2)

New surface area = (2.25)(7.333… ft^2)

New surface area = 16.5 ft^2

$$\text{New cost} = (16.5\ \cancel{ft^2})\left(\frac{\$0.05}{1\ \cancel{ft^2}}\right)$$

New cost = $0.825

Difference = new cost – cost

Difference = $0.825 – $0.366…

Difference = $0.458…

The difference in cost is about $0.46.

Lesson 8.5: Similar Objects: Scale Models and Scale Diagrams, page 497

1. Determine the scale factor for pairs of corresponding sides.

a) The scale factor for pairs of corresponding sides is $\frac{2}{3}$. Therefore, the two objects are similar.

b) Height scale factor = $\dfrac{1\ \cancel{cm}}{0.5\ \cancel{cm}}$

Height scale factor = 2

Width scale factor = $\dfrac{2\ \cancel{cm}}{1\ \cancel{cm}}$

Width scale factor = 2

Length scale factor = $\dfrac{4\ \cancel{cm}}{2\ \cancel{cm}}$

Length scale factor = 2

The scale factor for pairs of corresponding sides is 2. Therefore, the two objects are similar.

c) Height scale factor = $\dfrac{9\ \cancel{cm}}{3\ \cancel{cm}}$

Height scale factor = 3

Base scale factor = $\dfrac{6\ \cancel{cm}}{2\ \cancel{cm}}$

Base scale factor = 3

The scale factor for pairs of corresponding sides is 3. Therefore, the two objects are similar.

d) Diameter scale factor = $\dfrac{5\ \cancel{cm}}{2\ \cancel{cm}}$

Diameter scale factor = 2.5

Length scale factor = $\dfrac{10\ \cancel{cm}}{5\ \cancel{cm}}$

Length scale factor = 2

The scale factor for pairs of corresponding sides is not the same for all side pairs. Therefore, the two objects are not similar.

2. a) Yes, all spheres are similar because they have only one linear dimension in common, i.e., diameter.

b) i) Scale factor for NBA to WNBA = $\dfrac{25\ \cancel{cm}}{22\ \cancel{cm}}$

Scale factor for NBA to WNBA = $\dfrac{25}{22}$

ii) Scale factor for WNBA to NBA = $\dfrac{22\ \cancel{cm}}{25\ \cancel{cm}}$

Scale factor for WNBA to NBA = $\dfrac{22}{25}$

3. Scale is 1 cm:100 cm.
Scale factor is 1:100.
Since the actual boat is an enlargement of the model, the boat is an enlargement. The scale factor is 100.
Actual dimension = (model dimension)(scale factor)
Actual length = (52 cm)(100)
Actual length = 5200 cm or 52 m
Actual width = (8.5 cm)(100)
Actual width = 850 cm or 8.5 m
Actual height = (43 cm)(100)
Actual height = 4300 cm or 43 m

4. a) Compare the corresponding dimensions.
For large box, L:
Length = 80.0 cm, width = 60.0 cm, height = 70.0 cm
For medium box, M:
Length = 60.0 cm, width = 45.0 cm, height = 52.5 cm
For small box, S:
Length = 40.0 cm, width = 30.0 cm, height = 35.0 cm

	Length	**Width**	**Height**
L to M	$\dfrac{80.0}{60.0}=\dfrac{4}{3}$	$\dfrac{60.0}{45.0}=\dfrac{4}{3}$	$\dfrac{70.0}{52.5}=\dfrac{4}{3}$
M to S	$\dfrac{60.0}{40.0}=\dfrac{3}{2}$	$\dfrac{45.0}{30.0}=\dfrac{3}{2}$	$\dfrac{52.5}{35.0}=\dfrac{3}{2}$
L to S	$\dfrac{80.0}{40.0}=\dfrac{2}{1}$	$\dfrac{60.0}{30.0}=\dfrac{2}{1}$	$\dfrac{70.0}{35.0}=\dfrac{2}{1}$

Yes, the boxes are similar, since all dimensions are proportional.

b) Use the scale factor from the height column in the table from part a).
For L to M:
Let x represent the height of the L. box.

$$\frac{x}{24}=\frac{4}{3}$$
$$24\left(\frac{x}{24}\right)=24\left(\frac{4}{3}\right)$$
$$x=32$$

The height of the L is 32 cm.
For M to S:
Let y represent the height of the S.

$$\frac{24}{y}=\frac{3}{2}$$
$$\frac{y}{24}=\frac{2}{3}$$
$$24\left(\frac{y}{24}\right)=24\left(\frac{2}{3}\right)$$
$$y=16$$

The height of the S is 16 cm.

5. Scale is 1 cm:40 cm.
Scale ratio is 1:40.
Since the actual dimensions of the dinosaur are enlargements of the model dimensions, the scale factor is 40.
Actual dimension = (model dimension)(scale factor)
Actual length = (21.5 cm)(40)
Actual length = 860 cm or 8.6 m
Actual height = (9.5 cm)(40)
Actual height = 380 cm or 3.8 m

6. Scale is 1 mm:18 mm.
Scale ratio is 1:18.
Since the actual dimensions of the car are enlargements of the model dimensions, the scale factor is 18.
Actual dimension = (model dimension)(scale factor)
Actual length = (206.3 mm)(18)
Actual length = 3713.4 mm or 3.7134 m
The actual length of the car is about 3.71 m.
Actual width = (93.5 mm)(18)
Actual width = 1683 mm or 1.683 m
The actual width of the car is 1.68 m.
Actual height = (78.2 mm)(18)
Actual height = 1407.6 mm or 1.4076 m
The actual height of the car is 1.41 m.
7. a) 150 cm is two times 75 cm.
200 cm is almost two times 90 cm.
A good scale factor to use is 2.
b) Actual dimension = (model dimension)(scale factor)
Actual length = (90 cm)(2)
Actual length = 180 cm
Actual height = (75 cm)(2)
Actual height = 150 cm
8. a) Height of carving = 6.5 ft or 78 in.
Width of carving = 2.5 ft or 30 in.

$$\text{model dimension} = (\text{actual dimension})(\text{scale factor})$$

$$\frac{\text{model dimension}}{\text{actual dimension}} = \text{scale factor}$$

$$\frac{26\text{ in.}}{78\text{ in.}} = \text{scale factor}$$

$$\frac{1}{3} = \text{scale factor}$$

The scale factor to use is $\frac{1}{3}$.
b) Since the model is a reduction of the actual carving, the scale factor is $\frac{1}{3}$.
Model width = (actual width)(scale factor)
Model width = (30 in.)$\left(\frac{1}{3}\right)$
Model width = 10 in.
The width of the model should be 10 in.
9. Assume the model scale is 1 in.: 24 in. The scale ratio is 1:24.
Actual length = 32 ft or 384 in.
Actual width = 48 in.
Since the model is a reduction of the actual umiak, the scale factor is $\frac{1}{24}$.
Model dimension = (actual dimension)(scale factor)
Model length = (384 in.)$\left(\frac{1}{24}\right)$
Model length = 16 in.
Model width = (48 in.)$\left(\frac{1}{24}\right)$
Model width = 2 in.

10. a) Compare scale ratios.
The scale factor 87 is almost 90.
So a good scale factor to use is $\frac{90}{160}$ or $\frac{9}{16}$.
Estimate dimension
= (model dimension)(estimated scale factor)
Estimated length = (6 in.)$\left(\frac{9}{16}\right)$
Estimated length = 3.375 in.
Estimated height = (8.5 in.)$\left(\frac{9}{16}\right)$
Estimated height = 4.781 25 in.
Estimated width = (4 in.)$\left(\frac{9}{16}\right)$
Estimated width = 2.25 in.
The estimated dimensions are 3 in. by 5 in. by 2 in.
b) Since the model dimensions must be enlarged for the actual dimensions, the scale factors for HO is 87, and the scale factor for N is 160.

$$\text{Conversion ratio} = \frac{\text{scale factor for HO enlargement}}{\text{scale factor for N enlargement}}$$

$$\text{Conversion ratio} = \frac{87}{160}$$

c) Converted dimension
= (model dimension)(conversion ratio)
Converted length = (6 in.)$\left(\frac{87}{160}\right)$
Converted length = 3.2625 in.
(0.2625 ÷ $\frac{1}{8}$ = 2.1 times $\frac{1}{8}$ or about 2 times $\frac{1}{8}$)
Converted length = $3\frac{2}{8}$ in. or $3\frac{1}{4}$ in.
Converted height = (8.5 in.)$\left(\frac{87}{160}\right)$
Converted height = 4.621 875 in.
(0.621 875 ÷ $\frac{1}{8}$ = 4.975 times $\frac{1}{8}$ or about 5 times $\frac{1}{8}$)
Converted height = $4\frac{5}{8}$ in.
Converted width = (4 in.)$\left(\frac{87}{160}\right)$
Converted width = 2.175 in.
(0.175 ÷ $\frac{1}{8}$ = 1.4 times $\frac{1}{8}$ or about 1 times $\frac{1}{8}$)
Converted width = $2\frac{1}{8}$ in.
The converted dimensions are about
$3\frac{1}{4}$ in. by $4\frac{5}{8}$ in. by $2\frac{1}{8}$ in.

11. Determine the measurements of the passenger jet:
Length = 200(16.8 cm) or 3360 cm or 33.6 m
Width = 200(17.2 cm) or 3440 cm or 34.4 m
The is 33.6 m long and 34.4 m wide.
Determine the number of jets that could fit along the width of the hangar:

$$\frac{71.9\ \cancel{m}}{34.4\ \cancel{m}} = 2.066\ldots$$

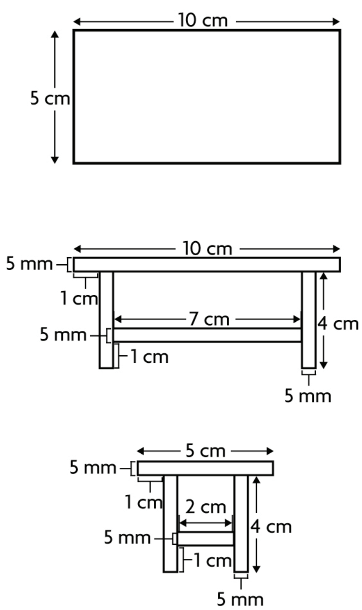
Two jets can fit along the width.
Only one jet can fit along the length of the hangar:

$$\frac{46.6\ \cancel{m}}{33.6\ \cancel{m}} = 1.386\ldots$$

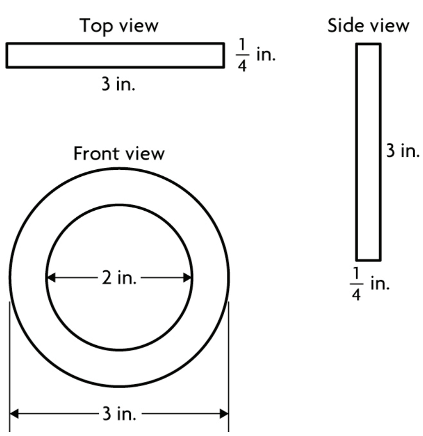
Two passenger jets can fit inside Hangar 77.

12. a) e.g., I used a photograph of a two storey house. The estimated dimensions are 6 m tall and 5 m wide.
b) Measure the metre stick in the photograph to determine the scale factor. Then multiply the building measurements by the scale factor to determine the building dimensions in metres.

13.

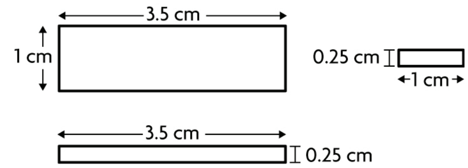


14.



15. e.g., Using a scale factor of $\frac{1}{20}$, the views would have the following dimensions:
Top view rectangle: 6.7 cm by 3.3 cm
Side view rectangle: 3.3 cm by 4.4 cm
Front view rectangle: 6.7 cm by 4.4 cm

16. e.g., I chose an eraser measuring 7.0 cm by 2.0 cm by 0.5 cm and I used a scale factor of $\frac{1}{2}$.



17. a) No; The area of the base increases by the square of the scale factor.
b) No; The volume increases by the cube of the scale factor.

18. e.g., Both involve multiplying each dimension by a scale factor; shapes have two dimensions while objects have three dimensions.

19. a) Since you have to determine the area of metal used for the can, you must determine k^2.
$k^2 = 1.5^2$
$k^2 = 2.25$
The scale factor is 2.25.
b) Since you need to determine the area of the smaller can, you must make a reduction. So the scale factor is

$$\frac{1}{2.25} \text{ or } \frac{100}{225} \text{ or } \frac{4}{9}.$$

Cost of small can = (cost of large can)(scale factor)
Cost of small can = $\$0.045\left(\frac{4}{9}\right)$
Cost of small can = \$0.02
It will cost \$0.02 to make the small can.

20. Reduction is 50%, so the scale factor is 0.5.
Smaller area = k^2(larger area)
Smaller area = $(0.5)^2(100\text{ cm}^2)$
Smaller area = $0.25(100\text{ cm}^2)$
Smaller area = 25 cm^2
The smaller cone has an area of 25 cm^2.

Lesson 8.6: Scale Factors and 3-D Objects, page 508

1. For an enlargement,

$$k = \frac{\text{measure of one dimension in enlargement}}{\text{measure of dimension in original}}$$

For surface area, use k^2. For volume, use k^3.

a) $k = \frac{3.0\ \cancel{cm}}{1.5\ \cancel{cm}}$
$k = 2$
i) $k^2 = 2^2$
$k^2 = 4$
The surface area of the larger object is 4 times the surface area of the smaller object.