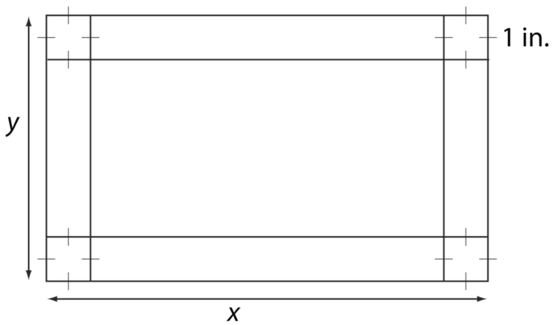
19. e.g., The dimensions of the space you actually have for your scale diagram; how large you want the scale diagram to be in that space; and a comparison of the ratio of the dimensions of the available space to the ratio of the dimensions of the original.

20. a) Let x and y be the dimensions of the required frame.



Perimeter = $2x + 2y$

$34 = 2x + 2y$

Therefore, the dimensions of the photograph are $x - 2$ and $y - 2$.

The scale is 12 in.:8 in. Therefore, the scale factor is $\frac{12}{8}$ or $\frac{3}{2}$.

Set up a proportion and solve for x in terms of y.

$$\frac{x-2}{y-2} = \frac{3}{2}$$

$$\frac{2}{3}\left(\frac{x-2}{y-2}\right) = \frac{2}{3}\left(\frac{3}{2}\right)$$

$$\frac{2x-4}{3y-6} = 1$$

$$3y-6\left(\frac{2x-4}{3y-6}\right) = 1(3y-6)$$

$$2x-4 = 3y-6$$

$$2x = 3y-2$$

$$x = \frac{3}{2}y - 1$$

Substitute x into the formula for perimeter.

$$34 = 2x + 2y$$

$$34 = 2\left(\frac{3}{2}y - 1\right) + 2y$$

$$34 = 3y - 2 + 2y$$

$$36 = 5y$$

$$7.2 = y$$

Solve for x.

$$x = \frac{3}{2}y - 1$$

$$x = \frac{3}{2}(7.2) - 1$$

$$x = 10.8 - 1$$

$$x = 9.8$$

New dimensions of photograph:

$x - 2 = 9.8 - 2$ $\quad$ $y - 2 = 7.2 - 2$

$x - 2 = 7.8$ $\quad$ $y - 2 = 5.2$

$$\text{Scale factor} = \frac{\text{original dimension}}{\text{reduced dimension}}$$

$$\text{Scale factor} = \frac{7.8}{12}$$

Scale factor = 0.65

b) New measure = (scale factor)(original measure)

New length = (0.65)(12 in.)

New length = 7.8 in.

New width = (0.65)(8 in.)

New width = 5.2 in.

The dimensions of the reduced photograph are 7.8 in. by 5.2 in.

Lesson 8.4: Scale Factors and Areas of 2-D Shapes, page 487

1. a) $\text{Scale factor} = \frac{\text{enlarged dimension}}{\text{original dimension}}$

$$\text{Scale factor} = \frac{8\ \cancel{\text{cm}}}{2\ \cancel{\text{cm}}}$$

Scale factor = 4

b) Area = (length)(width)

Area of A = (6 cm)(2 cm)

Area of A = 12 cm^2

Area of B = $(\text{scale factor})^2$(Area of A)

Area of B = 4^2(12 cm^2)

Area of B = 16(12 cm^2)

Area of B = 192 cm^2

c) $\text{Number of rectangles} = \frac{\text{Area of B}}{\text{Area of A}}$

$$\text{Number of rectangles} = \frac{192\ \text{cm}^2}{12\ \text{cm}^2}$$

Number of rectangles = 16

2.

Length of Base (cm)	Height of Triangle (cm)	Scale Factor	Area (cm^2)	$\frac{\text{Area of scaled triangle}}{\text{Area of original triangle}}$
3.0	4.0	1	6.0	1
9.0	**12.0**	3	**54.0**	**9**
1.5	**2.0**	**0.5**	**1.5**	**0.25**
30.0	**40.0**	**10**	600.0	**100**
0.75	**1.0**	25%	**0.375**	**0.0625**

3. Area of similar 2-D shape = k^2(Area of original shape)

Area of similar 2-D shape = 5^2(42 cm^2)

Area of similar 2-D shape = 25(42 cm^2)

Area of similar 2-D shape = 1050 cm^2

The area is 1050 cm^2.

4. Area of similar 2-D shape = k^2(Area of original shape)
a) Area of original shape = 11 unit^2
Area of similar 2-D shape = 2^2(11 unit^2)
Area of similar 2-D shape = 4(11 unit^2)
Area of similar 2-D shape = 44 unit^2
The area is 44 unit^2.
b) Area of original shape = 13 unit^2
Area of similar 2-D shape = 2^2(13 unit^2)
Area of similar 2-D shape = 4(13 unit^2)
Area of similar 2-D shape = 52 unit^2
The area is 52 unit^2.
c) Area of original shape = 12.5 unit^2
Area of similar 2-D shape = 2^2(12.5 unit^2)
Area of similar 2-D shape = 4(12.5 unit^2)
Area of similar 2-D shape = 50 unit^2
The area is 50 unit^2.
5. a) Area of original shape = 22.5 unit^2

Area of similar 2-D shape = $\left(\frac{1}{3}\right)^2$(22.5 unit^2)

Area of similar 2-D shape = $\frac{1}{9}$(22.5 unit^2)

Area of similar 2-D shape = 2.5 unit^2
The area is 2.5 unit^2.

b) Area of original shape = $1.5^2\pi + \frac{1}{2}(3)(3)$

Area of original shape = 11.568… unit^2

Area of similar 2-D shape = $\left(\frac{1}{3}\right)^2$(11.568… unit^2)

Area of similar 2-D shape = $\frac{1}{9}$(11.568… unit^2)

Area of similar 2-D shape = 1.285… unit^2
The area is 1.3 unit^2.
6. a) The dimensions are enlarged by 150%. Therefore, the scale ratio is 1:1.5.
Dimensions of enlarged photograph:
(4 in.)(1.5) = 6.0 in.
(6 in.)(1.5) = 9.0 in.
Since the enlarged photograph fits Tammy's frame, the frame size is 6 in. by 9 in.
b) The original dimensions are 4 in. by 6 in., so the area is 24 in.^{2}.
$k^2 = 1.5^2$
k^2 = 2.25 or 225%
c) One different strategy: Determine the area of the original photograph, 24 in.^{2}. Multiply the original area by the square of the scale factor, k^2(24) or $(1.5)^2$(24), which gives the area of 54 in.^{2}.
Another different strategy: Multiply each dimension of the original photograph by 150%, or 1.5, to get 6 in. by 9 in. Then multiply the enlarged dimensions to get the area of 54 in.^{2}.
7. I would enlarge each side using a scale factor of 2 because if the scale factor is 2, then the area will increase by the square of the scale factor, 4.

8. a) $AB + DE$ = 35 cm
14 cm + DE = 35 cm
DE = 21 cm

Scale factor = $\frac{AB}{DE}$

Scale factor = $\frac{14\ \cancel{\text{cm}}}{21\ \cancel{\text{cm}}}$

Scale factor = $\frac{2}{3}$

The scale factor that relates $\triangle ABC$ to $\triangle DEF$ is $\frac{2}{3}$, which indicates $\triangle ABC$ is a reduction of $\triangle DEF$.
b) $\triangle DEF$ is an enlargement of $\triangle ABC$, by a scale factor of $\frac{3}{2}$.

Area of $\triangle DEF = k^2$(area of $\triangle ABC$)

144 cm^2 = $\left(\frac{3}{2}\right)^2$(area of $\triangle ABC$)

144 cm^2 = $\frac{9}{4}$(area of $\triangle ABC$)

$\frac{4}{9}$(144 cm^2) = $\frac{4}{9}\left(\frac{9}{4}\right)$(area of $\triangle ABC$)

64 cm^2 = area of $\triangle ABC$
The area of $\triangle ABC$ is 64 cm^2.
9. Scale is 1 cm: 500 cm.
For an enlargement, the scale factor is 500.
Let G represent the area of the actual garage.
$G = k^2$(area of original garage)
$G = 500^2$(24 cm^2)
G = 250 000(24 cm^2)
G = 6 000 000 cm^2

$G = 6\,000\,000\,(1\ \cancel{\text{cm}})(1\ \cancel{\text{cm}})\left(\frac{1\text{ m}}{100\ \cancel{\text{cm}}}\right)\left(\frac{1\text{ m}}{100\ \cancel{\text{cm}}}\right)$

G = 600 m^2
The area of the actual garage is 600 m^2.
Let A represent the area of the actual office.
$A = k^2$(area of original office)
$A = 500^2$(4 cm^2)
A = 250 000(4 cm^2)
A = 1 000 000 cm^2

$A = 1\,000\,000\,(1\ \cancel{\text{cm}})(1\ \cancel{\text{cm}})\left(\frac{1\text{ m}}{100\ \cancel{\text{cm}}}\right)\left(\frac{1\text{ m}}{100\ \cancel{\text{cm}}}\right)$

A = 100 m^2
The area of the actual office is 100 m^2.

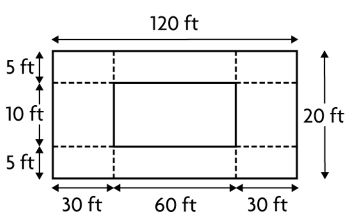
10. a) Area of original shape = (2 m)(3 m)
Area of original shape = 6 m^2
Scale factor is 1:120.
For an enlargement, the scale factor is 120.
Area of actual park = k^2(area of original shape)
Area of actual park = 120^2(6 m^2)
Area of actual park = 14 400(6 m^2)
Area of actual park = 86 400 m^2
Total cost = (area of actual park)(unit cost)
Total cost = (86 400 m^2)(\$0.75/m^2)

$$\text{Total cost} = (86\,400\ \cancel{m^2})\left(\frac{\$0.75}{1\ \cancel{m^2}}\right)$$

Total cost = \$64 800
The cost to maintain People's Park is about \$65 000.
b) Area of original shape = 6 m^2
Scale factor is 1:250.
For an enlargement, the scale factor is 250.
Area of actual park = k^2(area of original shape)
Area of actual park = 250^2(6 m^2)
Area of actual park = 62 500(6 m^2)
Area of actual park = 375 000 m^2
Total cost = (area of actual park)(unit cost)
Total cost = (375 000 m^2)(\$0.75/m^2)

$$\text{Total cost} = (375\,000\ \cancel{m^2})\left(\frac{\$0.75}{1\ \cancel{m^2}}\right)$$

Total cost = \$281 250
The cost to maintain Meadow Park is about \$280 000.
11. The dimensions of the mural must be half the dimensions of the wall because that would make the area of the mural one quarter the area of the wall. To centre the mural, have it take up the middle two quarters of the height and the middle two quarters of the length of the wall.



12. If the scale ratio is 1:2, then the larger rectangle will have an area 4 times the area of the smaller rectangle. Let x represent the area of the smaller rectangle, then $4x$ represents the area of the larger rectangle.

$x + 4x = 40\ cm^2$
$5x = 40\ cm^2$
$x = 8\ cm^2$
$4x = 4(8\ cm^2)$
$4x = 32\ cm^2$

The areas of the rectangles are 8 cm^2 and 32 cm^2.

13. Area of similar shape = k^2(area of original shape)

$$\frac{\text{Area of similar shape}}{\text{area of original shape}} = \frac{k^2(\text{area of original shape})}{\text{area of original shape}}$$

$$\frac{\text{Area of similar shape}}{\text{area of original shape}} = k^2$$

$$\sqrt{\frac{\text{Area of similar shape}}{\text{area of original shape}}} = k$$

a) $k = \sqrt{\dfrac{11.25\ \cancel{cm^2}}{5.00\ \cancel{cm^2}}}$
$k = \sqrt{2.25}$
$k = 1.5$

b) $k = \sqrt{\dfrac{3.00\ \cancel{cm^2}}{12.00\ \cancel{cm^2}}}$
$k = \sqrt{0.25}$
$k = 0.5$

14. a) Since there are 4 triangles congruent to the small triangle along the base of the large triangle, $k = 4$.
b) The perimeter of the large triangle is 4 times the perimeter of the small triangle; the area of the large triangle is 4^2 = 16 times the area of the small triangle.
15. a) Scale is 0.152 m:7600 m, or 1 m:50 000 m.
b) The dimensions of the parcel in the photograph of land are 2.6 cm by 0.75 cm.
(2.6 cm)(0.75 cm) = 1.95 cm^2

$$(2.6\text{ cm})(0.75\text{ cm}) = (1.95\ \cancel{cm^2})\left(\frac{1\ m^2}{10\,000\ \cancel{cm^2}}\right)$$

(2.6 cm)(0.75 cm) = 0.000 195 m^2
Since the actual land is a enlargement, the scale factor is 50 000.
Area of actual land = k^2(area of original shape)
Area of actual land = $(50\ 000)^2$(0.000 195 m^2)
Area of actual land = 2 500 000 000(0.000 195 m^2)
Area of actual land = 487 500 m^2

$$\text{Area in hectares} = (\text{area of land})\left(\frac{1\text{ ha}}{10\,000\ m^2}\right)$$

$$\text{Area in hectares} = (487\,500\ \cancel{m^2})\left(\frac{1\text{ ha}}{10\,000\ \cancel{m^2}}\right)$$

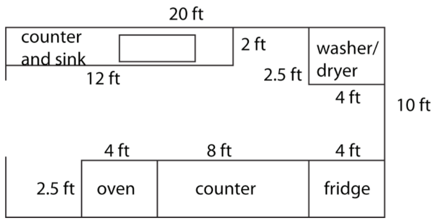
Area in hectares = 48.75 ha
The area is about 49 ha.
c) Land value = (area in hectares)(unit price)

$$\text{Land value} = (48.75\ \cancel{ha})\left(\frac{\$375}{1\ \cancel{ha}}\right)$$

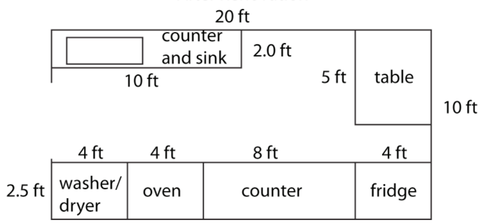
Land value = \$18 281.25
The land value is about \$18 300.
16. a), e.g., If kitchen is about 10 ft by 20 ft and scale diagrams are drawn on 8.5 in. by 11 in. paper, scale factor could be $\dfrac{1}{48}$.

b) e.g.,

Before Renovation



After Renovation



c) e.g., Estimate or measure the open floor space areas in each diagram and compare. In this case, The kitchen had 126 square feet of free area before it was renovated and had 110 square feet of free area after it was renovated, so the kitchen was more spacious before it was renovated. However, it was possible to use the space more fully by rearranging things.

17. e.g., A rectangle is 3 m by 4 m.

A. The area of the original rectangle is 12 m^2.

The shape is reduced by a scale factor of $\frac{1}{2}$ means the dimensions are reduced by $\frac{1}{2}$ or are now 1.5 m by 2 m.

The area of the reduced shape is 3 m.

The area is divided by 4 in process A.

B. If you divide the area of the original rectangle by 2 the new area is 6 m^2 and is just $\frac{1}{2}$ of the original area. Side lengths are not reduced by $\frac{1}{2}$.

18. Use an example of a rectangle to help you.

Rectangle 1 has the dimensions 3 m by 4 m.

Area of rectangle 1 = 12 m^2

Rectangle 2 has dimensions 180% or 1.8 times rectangle 1 dimensions.

Rectangle 2 has dimensions 5.4 m by 7.2 m.

Area of rectangle 2 = 38.88 m^2

Rectangle 3 has dimensions 50% or 0.5 times rectangle 2 dimensions.

Rectangle 3 has dimensions 2.7 m by 3.6 m.

Area of rectangle 3 = 9.72 m^2

$$\frac{\text{area of rectangle 3}}{\text{area of rectangle 1}} = \frac{9.72\ \cancel{m^2}}{12\ \cancel{m^2}}$$

$$\frac{\text{area of rectangle 3}}{\text{area of rectangle 1}} = 0.81$$

The area of the third polygon is 81% the area of the original polygon.

Alternate solution:

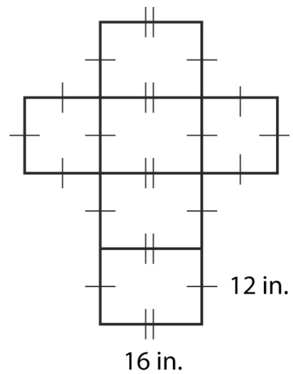
The first scale factor is 1.8. The second scale factor is 0.5. The total scale factor is (1.8)(0.5) = 0.9.

The change in area is the square of the total scale factor:

$0.9^2 = 0.81$.

The area of the third polygon is 81% the area of the original polygon.

19. The original box will have the following panels with these dimensions:



16 in. = 1.333… ft, 12 in. = 1 ft

Total surface area = 4(1.333… ft)(1 ft) + 2(1 ft)(1 ft)

Total surface area = 4(1.333… ft^2) + 2(1 ft^2)

Total surface area = 5.333… ft^2 + 2 ft^2

Total surface area = 7.333… ft^2

$$\text{Cost} = (7.333...\ \cancel{ft^2})\left(\frac{\$0.05}{1\ \cancel{ft^2}}\right)$$

Cost = \$0.366…

New surface area = k^2(old surface area)

New surface area = $(1.5)^2$(7.333… ft^2)

New surface area = (2.25)(7.333… ft^2)

New surface area = 16.5 ft^2

$$\text{New cost} = (16.5\ \cancel{ft^2})\left(\frac{\$0.05}{1\ \cancel{ft^2}}\right)$$

New cost = \$0.825

Difference = new cost – cost

Difference = \$0.825 – \$0.366…

Difference = \$0.458…

The difference in cost is about \$0.46.

Lesson 8.5: Similar Objects: Scale Models and Scale Diagrams, page 497

1. Determine the scale factor for pairs of corresponding sides.

a) The scale factor for pairs of corresponding sides is $\frac{2}{3}$. Therefore, the two objects are similar.