**c)** e.g., One factor is that the longer the distance, the less likely to maintain a high constant speed throughout due to fatigue. By the end of the race the speed will usually be lower than at the start. Other factors include weather, competition, and training before the race.

## Lesson 8.3: Scale Diagrams, page 479

**1.** Let $k$ be the scale factor for the diagrams.

**a)** $k = \dfrac{\text{length of Y}}{\text{length of X}}$

$k = \dfrac{6 \cancel{\text{units}}}{10 \cancel{\text{units}}}$

$k = \dfrac{3}{5}$ or 60%

The scale factor is $\dfrac{3}{5}$ or 60%.

**b)** $k = \dfrac{\text{diameter of Y}}{\text{diameter of X}}$

$k = \dfrac{6 \cancel{\text{units}}}{4 \cancel{\text{units}}}$

$k = \dfrac{3}{2}$ or 150%

The scale factor is 150%.

**2. a)** Since a scale factor of 112% or 1.12 is greater than 1, the original will be smaller than the scale diagram.

**b)** Since a scale factor of 0.75 is less than 1, the original will be larger than the scale diagram.

**c)** Since a scale factor of $\dfrac{4}{9}$ or 0.444… is less than 1, the original will be larger than the scale diagram.

**3. a)** Scale as given: 5 in.:6 ft

Scale in inches:

6 ft = $(6 \cancel{\text{ft}})\left(\dfrac{12 \text{ in.}}{1 \cancel{\text{ft}}}\right)$

6 ft = 72 in.

5 in.:72 in.

**b)** Let $k$ represent the scale factor using the measurements in inches.

$k = \dfrac{\text{diagram measurement}}{\text{actual measurement}}$

$k = \dfrac{5}{72}$

The scale factor is $\dfrac{5}{72}$.

**4.** If two figures are similar, the ratios of the lengths of the corresponding sides are equal. So determine the scale factor and solve for the measures of the unknown sides.

For an enlargement:

$k = \dfrac{9.0 \cancel{\text{cm}}}{6.0 \cancel{\text{cm}}}$

$k = 1.5$

$8.0 \text{ cm} = h(1.5)$

$\dfrac{8.0 \text{ cm}}{1.5} = h$

$5.333... \text{ cm} = h$

$h$ is 5.3 cm to one tenth.

$6.0 \text{ cm} = g(1.5)$

$\dfrac{6.0 \text{ cm}}{1.5} = g$

$4.0 \text{ cm} = g$

For a reduction:

$k = \dfrac{6.0 \cancel{\text{cm}}}{9.0 \cancel{\text{cm}}}$

$k = 0.666…$

$4.0 \text{ m} = x(0.666...)$

$\dfrac{4.0 \text{ m}}{0.666...} = x$

$6.0 \text{ m} = x$

$5.0 \text{ m} = y(0.666...)$

$\dfrac{5.0 \text{ m}}{0.666...} = y$

$7.5 \text{ m} = y$

**5.** e.g.,

$k = \dfrac{\text{diagram measurement}}{\text{actual measurement}}$

**a)** length of acorn = 2.3 cm

$k = \dfrac{2.3 \cancel{\text{cm}}}{1.9 \cancel{\text{cm}}}$

$k = 1.210…$

The scale factor is 1.2.

**b)** length of acorn = 3.5 cm

$k = \dfrac{3.5 \cancel{\text{cm}}}{1.9 \cancel{\text{cm}}}$

$k = 1.842…$

The scale factor is 1.8.

**c)** length of acorn = 1.7 cm

$k = \dfrac{1.7 \cancel{\text{cm}}}{1.9 \cancel{\text{cm}}}$

$k = 0.894…$

The scale factor is 0.9.

**6.** actual measurement = $\dfrac{\text{diagram measurement}}{k}$

Let a represent the actual measurement.

**a)** For bedroom #1:

length = 2.5 cm

$a = \dfrac{2.5 \text{ cm}}{0.005}$

$a$ = 500 cm

$a$ = 5.0 m

width = 2.0 cm

$a = \dfrac{2.0 \text{ cm}}{0.005}$

$a$ = 400 cm

$a$ = 4.0 m

Bedroom 1 is 4.0 m by 5.0 m.

For bedroom #2 or #3:
length = 2.0 cm

$a = \dfrac{2.0 \text{ cm}}{0.005}$

$a$ = 400 cm
$a$ = 4.0 m
width = 4.0 cm

$a = \dfrac{2.0 \text{ cm}}{0.005}$

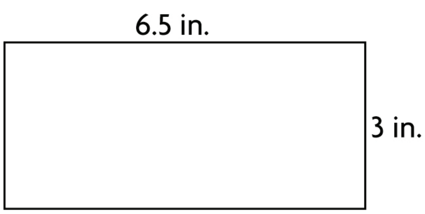
$a$ = 400 cm
$a$ = 4.0 m
Bedrooms 2 and 3 are 4.0 m by 4.0 m.
**b)** For living room:
length = 2.4 cm

$a = \dfrac{2.4 \text{ cm}}{0.005}$

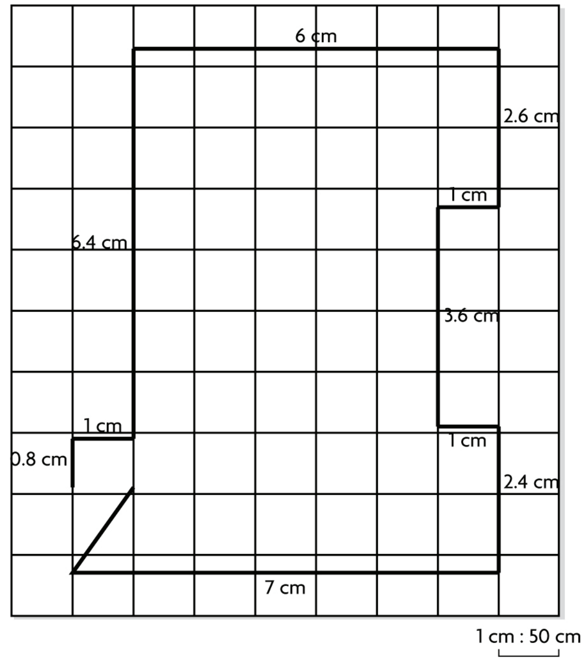
$a$ = 480 cm
$a$ = 4.8 m
width = 4.0 cm

$a = \dfrac{2.0 \text{ cm}}{0.005}$

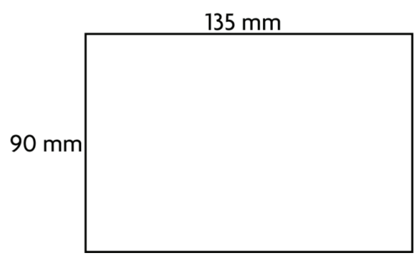
$a$ = 400 cm
$a$ = 4.0 m
The living room is 4.8 m by 4.0 m.
**c)** Area = (length)(width)
For bedroom #1:
Area = (5.0 m)(4.0 m)
Area = 20.0 $m^2$
For bedroom #2 or 3:
Area = (4.0 m)(4.0 m)
Area = 16.0 $m^2$
For living room:
Area = (4.8 m)(4.0 m)
Area = 19.2 $m^2$
Bedroom 1 has the greatest area or 20.0 $m^2$.
**7. a)** e.g., A reasonable scale would be 1 in.:100 ft.
**b)** e.g.,




**8.**

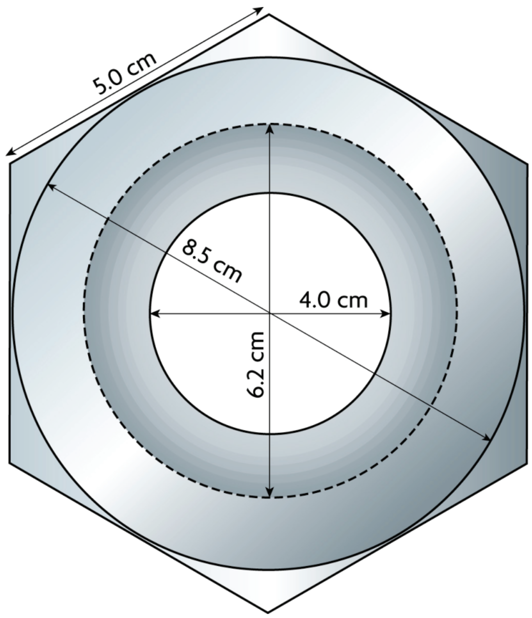



**9.** Width = 15(6 mm) or 90 mm
The width is 90 mm, or 9.0 cm.
Length = 15(9 mm) or 135 mm
The length is 135 mm, or 13.5 cm.




**10.** e.g.,
**a)** I made the following measurements:
innermost diameter = 1.6 cm
middle inner diameter = 2.5 cm
outer diameter = 3.4 cm
hexagon side = 2.0 cm
**b)** The scale factor is 2.5.
new innermost diameter = 2.5(1.6 cm)
new innermost diameter = 4.0 cm
new middle inner diameter = 2.5(2.5 cm)
new middle inner diameter = 6.25 cm
new outer diameter = 2.5(3.4 cm)
new outer diameter = 8.5 cm
new hexagon side = 2.5(2.0 cm)
new hexagon side = 5.0 cm

**c)**




**11.** $k = 40$, diagram measurement = 1 cm

$$k = \frac{\text{diagram measurement}}{\text{actual measurement}}$$

Let $a$ represent the actual measurement.

$$40 = \frac{1\text{ cm}}{a}$$

$$40a = a\left(\frac{1\text{ cm}}{a}\right)$$

$$40a = 1\text{ cm}$$

$$a = \frac{1\text{ cm}}{40}$$

$a = 0.025$ cm or 0.25 mm

The length of the onion cell is 0.25 mm.

**12.** e.g.,

**a)** scale 1.24 cm: 200 km

Set up a proportion using equivalent ratios and solve.

Let $x$ represent the actual distance.

**i)** On the map, Yellowknife to Fort Norman are about 3.9 cm apart.

$$\frac{\text{map measurement}}{\text{actual measurement}} = \frac{1.24\text{ cm}}{200\text{ km}}$$

$$\frac{3.9\text{ cm}}{x} = \frac{1.24\text{ cm}}{200\text{ km}}$$

$$\frac{x}{3.9\text{ cm}} = \frac{200\text{ km}}{1.24\text{ cm}}$$

$$\cancel{3.9\text{ cm}}\left(\frac{x}{\cancel{3.9\text{ cm}}}\right) = 3.9\ \cancel{\text{cm}}\left(\frac{200\text{ km}}{1.24\ \cancel{\text{cm}}}\right)$$

$$x = 629.032...\text{ km}$$

It is about 629 km from Yellowknife to Fort Norman.

**ii)** On the map, Fort Providence and Fort Norman are about 3.45 cm apart.

$$\frac{\text{map measurement}}{\text{actual measurement}} = \frac{1.24\text{ cm}}{200\text{ km}}$$

$$\frac{3.456\text{ cm}}{x} = \frac{1.24\text{ cm}}{200\text{ km}}$$

$$\frac{x}{3.456\text{ cm}} = \frac{200\text{ km}}{1.24\text{ cm}}$$

$$\cancel{3.456\text{ cm}}\left(\frac{x}{\cancel{3.456\text{ cm}}}\right) = 3.456\ \cancel{\text{cm}}\left(\frac{200\text{ km}}{1.24\ \cancel{\text{cm}}}\right)$$

$$x = 557.419...\text{ km}$$

It is about 557 km from Fort Providence to Fort Norman.

**b)** By looking at the map, Fort Providence and Yellowknife are the closest.

**13. a)** Side lengths from top going counter clockwise are:

12(0.5 cm) = 6.0 cm (with door closed)
6(0.5 cm) = 3.0 cm
6(0.5 cm) = 3.0 cm
2(0.5 cm) = 1.0 cm
6(0.5 cm) = 3.0 cm
8(0.5 cm) = 4.0 cm
Perimeter = sum of side lengths
Perimeter = 6.0 cm + 3(3.0 cm) + 1.0 cm + 4.0 cm
Perimeter = 20.0 cm
Scale is 1 cm to 75 cm.
Actual perimeter = 75(perimeter)
Actual perimeter = 75(20.0 cm)
Actual perimeter = 1500 cm or 15 m
The actual perimeter of the greenhouse is 15 m.

**b)** The greenhouse floor is a composite figure of a large horizontal rectangle measuring 12 units by 6 units and a smaller rectangle measuring 6 units by 2 units.

For the large rectangle,
In actual grid units:
Area = [12 units(0.5 cm)][6 units(0.5 cm)]
Area = (6.0 cm)(3.0 cm)
In actual distance:
Area = [6.0 cm(75)][3.0 cm(75)]
Area = (450 cm)(225 cm)
Area = (4.50 m)(2.25 m)
Area = 10.125 $m^2$
For the small rectangle,
In actual grid units:
Area = [6 units(0.5 cm)][2 units(0.5 cm)]
Area = (3.0 cm)(1.0 cm)
In actual distance:
Area = [3.0 cm(75)][1.0 cm(75)]
Area = (225 cm)(75 cm)
Area = (2.25 m)(0.75 m)
Area = 1.6875 $m^2$
Total area = area of large rectangle
+ area of small rectangle
Total area = 10.125 $m^2$ + 1.6875 $m^2$
Total area = 11.8125 $m^2$
The total area of the greenhouse floor is 11.8 $m^2$.

**14.** $k = \dfrac{\text{diagram measurement}}{\text{actual measurement}}$

**a)** actual measurement = 19 mm
diagram measurement = 5.7 cm or 57 mm

$k = \dfrac{57\ \cancel{\text{mm}}}{19\ \cancel{\text{mm}}}$

$k = 3$

**b)** actual measurement = 30 in.
diagram measurement = 1.5 in.

$k = \dfrac{1.5\ \cancel{\text{in.}}}{30\ \cancel{\text{in.}}}$

$k = \dfrac{1}{20}$

**c)** actual measurement = 2.5 cm
diagram measurement = 1.0 m or 100 cm

$k = \dfrac{100\ \cancel{\text{cm}}}{2.5\ \cancel{\text{cm}}}$

$k = 40$

**d)** actual measurement = 55 ft or 660 in.
diagram measurement = 6 in.

$k = \dfrac{6\ \cancel{\text{in.}}}{660\ \cancel{\text{in.}}}$

$k = \dfrac{1}{110}$

**15.** e.g., The scale diagram of the billboard could be a rectangle measuring 18 cm by 14 cm.

**16. a)** Length = 6 units, Width = 3 units
Area in units = (length)(width)
Area in units = 6 units by 3 units
Area = 18 units$^2$
Actual area = 72 m$^2$

Area per square = $\dfrac{\text{actual area}}{\text{area in units}}$

Area per square = $\dfrac{72\ \text{m}^2}{18\ \text{units}^2}$

Area per square = 4 m$^2$/unit
The area of one square is 4 m$^2$.

**b)** Each unit square measures 5 mm by 5 mm.
The area of each square is 2 m by 2 m or 4 m$^2$.
Therefore, 5 mm represents 2 m on the diagram.

**c)** The scale of the plan is 5 mm:2 m.

**d)** Scale is 5 mm:2 m or 5 mm:2000 mm.

The scale factor is $\dfrac{5\ \cancel{\text{mm}}}{2000\ \cancel{\text{mm}}}$ or $\dfrac{1}{400}$.

**17.** Width of shelf = 4 ft or 48 in.
Height of shelf = 26 in.
Scale of television is 16:9 for length : width.
The diagonal, vertical, and horizontal sides of a LCD television form a right triangle. So you can use the Pythagorean theorem to determine the lengths of the vertical and horizontal sides of a 42 in. television.
Let $x$ represent the scale factor for the actual sides.
So,
length = $16x$, width = $9x$

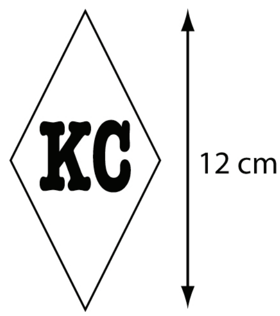
$$42^2 = (16x)^2 + (9x)^2$$
$$1764 = 256x^2 + 81x^2$$
$$1764 = 337x^2$$
$$5.234\ldots = x^2$$
$$2.287\ldots = x$$

Length = 16(2.287…) Width = 9(2.287…)
Length = 36.606… Width = 20.590…

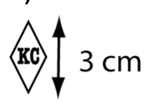
The dimensions of a 42 in. television would be 36.6 in. by 20.6 in. Therefore, the television will fit on the shelf.

**18.** e.g.,

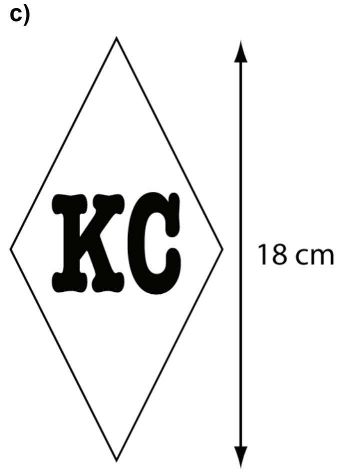
**a)**




**b)**

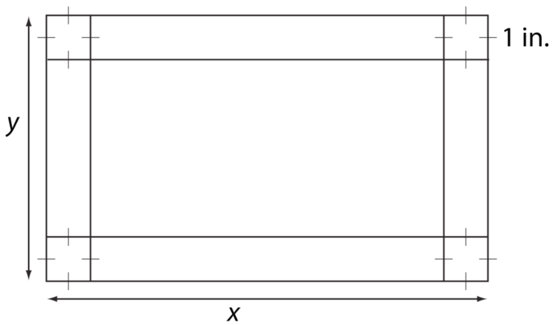



**c)**



**19.** e.g., The dimensions of the space you actually have for your scale diagram; how large you want the scale diagram to be in that space; and a comparison of the ratio of the dimensions of the available space to the ratio of the dimensions of the original.

**20. a)** Let $x$ and $y$ be the dimensions of the required frame.




Perimeter = $2x + 2y$

$34 = 2x + 2y$

Therefore, the dimensions of the photograph are $x - 2$ and $y - 2$.

The scale is 12 in.:8 in. Therefore, the scale factor is $\frac{12}{8}$ or $\frac{3}{2}$.

Set up a proportion and solve for $x$ in terms of $y$.

$$\frac{x-2}{y-2} = \frac{3}{2}$$

$$\frac{2}{3}\left(\frac{x-2}{y-2}\right) = \frac{2}{3}\left(\frac{3}{2}\right)$$

$$\frac{2x-4}{3y-6} = 1$$

$$3y-6\left(\frac{2x-4}{3y-6}\right) = 1(3y-6)$$

$$2x - 4 = 3y - 6$$

$$2x = 3y - 2$$

$$x = \frac{3}{2}y - 1$$

Substitute $x$ into the formula for perimeter.

$34 = 2x + 2y$

$34 = 2\left(\frac{3}{2}y - 1\right) + 2y$

$34 = 3y - 2 + 2y$

$36 = 5y$

$7.2 = y$

Solve for $x$.

$x = \frac{3}{2}y - 1$

$x = \frac{3}{2}(7.2) - 1$

$x = 10.8 - 1$

$x = 9.8$

New dimensions of photograph:

$x - 2 = 9.8 - 2$ $\quad$ $y - 2 = 7.2 - 2$

$x - 2 = 7.8$ $\quad$ $y - 2 = 5.2$

Scale factor = $\frac{\text{original dimension}}{\text{reduced dimension}}$

Scale factor = $\frac{7.8}{12}$

Scale factor = 0.65

**b)** New measure = (scale factor)(original measure)

New length = (0.65)(12 in.)

New length = 7.8 in.

New width = (0.65)(8 in.)

New width = 5.2 in.

The dimensions of the reduced photograph are 7.8 in. by 5.2 in.

## Lesson 8.4: Scale Factors and Areas of 2-D Shapes, page 487

**1. a)** Scale factor = $\frac{\text{enlarged dimension}}{\text{original dimension}}$

Scale factor = $\frac{8 \not{\text{cm}}}{2 \not{\text{cm}}}$

Scale factor = 4

**b)** Area = (length)(width)

Area of A = (6 cm)(2 cm)

Area of A = 12 $\text{cm}^2$

Area of B = $(\text{scale factor})^2$(Area of A)

Area of B = $4^2(12\ \text{cm}^2)$

Area of B = $16(12\ \text{cm}^2)$

Area of B = 192 $\text{cm}^2$

**c)** Number of rectangles = $\frac{\text{Area of B}}{\text{Area of A}}$

Number of rectangles = $\frac{192\ \text{cm}^2}{12\ \text{cm}^2}$

Number of rectangles = 16

**2.**

| **Length of Base (cm)** | **Height of Triangle (cm)** | **Scale Factor** | **Area ($\text{cm}^2$)** | **Area of scaled triangle / Area of original triangle** |
|---|---|---|---|---|
| 3.0 | 4.0 | 1 | 6.0 | 1 |
| **9.0** | **12.0** | 3 | **54.0** | **9** |
| 1.5 | **2.0** | **0.5** | **1.5** | **0.25** |
| **30.0** | **40.0** | **10** | 600.0 | **100** |
| **0.75** | **1.0** | 25% | **0.375** | **0.0625** |

**3.** Area of similar 2-D shape = $k^2$(Area of original shape)

Area of similar 2-D shape = $5^2(42\ \text{cm}^2)$

Area of similar 2-D shape = $25(42\ \text{cm}^2)$

Area of similar 2-D shape = 1050 $\text{cm}^2$

The area is 1050 $\text{cm}^2$.