Chapter 8: Proportional Reasoning

Lesson 8.1: Comparing and Interpreting Rates, page 458

1. a) Rate = $\frac{\text{cost}}{\text{mass}}$

Store A rate = $\frac{\$68}{8 \text{ kg}}$

Store A rate = $8.50/kg

Store B rate = $\frac{\$88.20}{12 \text{ kg}}$

Store B rate = $7.35/kg

Store B charges the lower rate.

b) Rate = $\frac{\text{cost}}{\text{volume}}$

Station A rate = $\frac{\$41.36}{44 \text{ L}}$

Station A rate = $0.94/L

Station B rate = $\frac{\$31.36}{32 \text{ L}}$

Station B rate = $0.98/L

Station A charges the lower rate.

2. Rate = $\frac{\text{volume}}{\text{time}}$

a) For Tank A:

4 h 15 min = $4 \text{ h} + (15 \cancel{\text{min}})\left(\frac{1 \text{ h}}{60 \cancel{\text{min}}}\right)$

4 h 15 min = 4 h + 0.25 h

4 h 15 min = 4.25 h

Rate = $\frac{300 \text{ L}}{4.25 \text{ h}}$

Rate = 70.588… L/h

The rate of flow for tank A is about 71 L/h.

For Tank B:

2 h 10 min = $2 \text{ h} + (10 \cancel{\text{min}})\left(\frac{1 \text{ h}}{60 \cancel{\text{min}}}\right)$

2 h 10 min = 2 h + 0.166… h

2 h 10 min = 2.166… h

Rate = $\frac{150 \text{ L}}{2.166... \text{ h}}$

Rate = 69.230… L/h

The rate of flow for tank B is about 69 L/h.

Tank A has the greater rate of flow.

b) Rate = $\frac{\text{distance}}{\text{time}}$

For Person A:

1 min 15 s = $(1 \cancel{\text{min}})\left(\frac{60 \text{ s}}{1 \cancel{\text{min}}}\right) + 15 \text{ s}$

1 min 15 s = 60 s + 15 s

1 min 15 s = 75 s

Rate = $\frac{400 \text{ m}}{75 \text{ s}}$

Rate = 5.333… m/s

Person A's running rate is about 5 m/s.

For Person B:

5 min 20 s = $(5 \cancel{\text{min}})\left(\frac{60 \text{ s}}{1 \cancel{\text{min}}}\right) + 20 \text{ s}$

5 min 20 s = 300 s + 20 s

5 min 20 s = 320 s

1 km = 1000 m

Rate = $\frac{1000 \text{ m}}{320 \text{ s}}$

Rate = 3.125 m/s

Person B's running rate is about 3 m/s.

Person A has the greater running rate.

3. a) e.g., Answers should be within this range.

The segment on the graph where the distance of travel is constant is where the ATV is travelling the slowest—between 20 s to 28 s.

The segment where the slope of the line segment is steepest indicates the ATV is travelling the fastest—between 28 s to 32 s.

b) The starting position is distance 0 m or when the line graph reaches 0 again. The start of the return is at 28 s. The ATV reaches 0 m again at 32 s.

c) A slope of zero means the distance travelled by the ATV over an interval of time does not change. That is the speed of the ATV is 0 m/s.

4. a) Rate = $\frac{\text{cost}}{\text{volume}}$

For bottle: 1 L = 1000 mL

Rate = $\frac{\$1.75}{1000 \text{ mL}}$

Rate = $0.001 75/mL

The unit rate for a bottle is $0.001 75/mL.

For boxes:

15(200 mL) = 3000 mL

Rate = $\frac{\$4.99}{3000 \text{ mL}}$

Rate = $0.001 66…/mL

The unit rate for a box is about $0.001 66/mL.

b) The boxes have the lower cost per millilitre.

5. For 925 mL container:

Rate = $\frac{\$20.09}{925 \text{ mL}}$

Rate = $0.021…/mL

The unit rate for the 925 mL container is about $0.022/mL.

For 3.54 L container:

1 L = 1000 mL

3.54 L = 3540 mL

Rate = $\frac{\$52.99}{3540 \text{ mL}}$

Rate = $0.016…/mL

The unit rate for the 3.54 L container is about $0.016/mL.

The 3.25 L container has the lower unit cost.

6. For aerobics:

$$\text{Rate} = \frac{140\text{ Cal}}{20\text{ min}}$$

Rate = 7 Cal/min
For hockey:
1.5 h = 1 h + 0.5 h
1.5 h = 60 min + 30 min
1.5 h = 90 min

$$\text{Rate} = \frac{720\text{ Cal}}{90\text{ min}}$$

Rate = 8 Cal/min
Rupi burns more calories playing hockey.

7. a) Whole chickens by the kilogram:
Rate = $3.61/kg
Whole chickens by 10 lb units:

$$10\text{ lb} = (10\text{ lb})\left(\frac{1\text{ kg}}{2.2\text{ lb}}\right)$$

10 lb = 4.545… kg

$$\text{Rate} = \frac{\$17.40}{4.545...\text{ kg}}$$

Rate = $3.828/kg
For 10 lbs of chickens it costs $3.83/kg.
The lower rate is $3.61/kg.

b) For 6 mph:

$$6\text{ mi} = (6\text{ mi})\left(\frac{1\text{ km}}{0.6\text{ mi}}\right)$$

6 mi = 10 km
Time = 1 h
Rate = 10 km/h
The rate of 6 mph is the same as 10 km/h.
For 2 km in 10 min:

$$10\text{ min} = (10\text{ min})\left(\frac{1\text{ h}}{60\text{ min}}\right)$$

10 min = 0.166… h

$$\text{Rate} = \frac{2\text{ km}}{0.166...\text{ h}}$$

Rate = 12 km/h
The rate is 2 km in 10 min is the same as 12 km/h.
The jogging speed of 6 mph is the lower rate.

c) For 10.6 L/100 km:
Rate = 10.6 L/100 km
Rate = 0.106 L/km
For 35.1 L for 450 km:

$$\text{Rate} = \frac{35.1\text{ L}}{450\text{ km}}$$

Rate = 0.078 L/km or 7.8 L/100 km
The fuel efficiency of 35.1 L in 450 km is the lower rate.

d) For 30 m/s:

$$\text{Rate} = \left(\frac{30\text{ m}}{1\text{ s}}\right)\left(\frac{1\text{ km}}{1000\text{ m}}\right)\left(\frac{3600\text{ s}}{1\text{ h}}\right)$$

$$\text{Rate} = \frac{108\,000\text{ km}}{1000\text{ h}}$$

Rate = 108 km/h
For 100 km/h: Rate = 100 km/h
The driving speed of 100 km/h is the lower rate.

8. I calculated the unit cost of the seed mixture at each store in dollars per kilogram ($/kg) and then compared the two unit costs. The conversion rate for kilograms and pounds is 2.2 lb/1 kg.
Farmers Co-op: cost of 1 kg

$$\left(\frac{\$21.30}{25\text{ lb}}\right)\left(\frac{2.2\text{ lb}}{1\text{ kg}}\right) = \$1.8744/\text{kg}$$

Pet store: 18 kg for $24.69

$$\frac{\$24.69}{18\text{ kg}} = \$1.371\ldots/\text{kg}$$

$1.874…/kg – $1.371…/kg = $0.502…/kg
The seed mixture costs about $0.50/kg less at the pet store.

9. For telephone company per year:

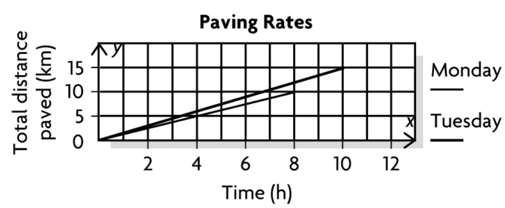
$$\text{Charge} = \left(\frac{\$0.04}{1\text{ min}}\right)\left(\frac{50\text{ min}}{1\text{ month}}\right)\left(\frac{12\text{ month}}{1\text{ year}}\right)$$

Charge = $24.00/year
For Internet device per year:

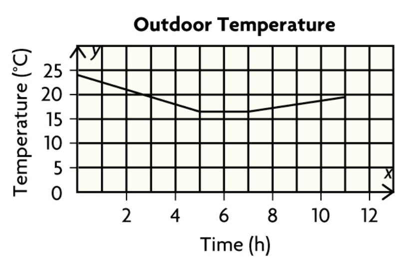
$$\text{Charge} = \$19.95 + \left(\frac{\$0.015}{1\text{ min}}\right)\left(\frac{50\text{ min}}{1\text{ month}}\right)\left(\frac{12\text{ month}}{1\text{ year}}\right)$$

Charge = $28.95
It will be cheaper to go with the telephone company by $4.95 per year.

10.



11.



12. e.g., In the first 10 min, the shuttle was driven away from the airport to drop off passengers at three different hotels; the farthest hotel was about 9 km away. Then the shuttle was driven toward the airport for one more pick-up/drop-off about 7.5 km from the airport. If continued toward the airport and stopped at one more hotel, where David disembarked. The whole trip took about 22 min.

13. e.g.,

a) This graph shows a steep but steady or constant increase in water depth. The graduated cylinder is constant in width all along its height and it is a tall, narrow vessel. As the water fills the cylinder, the change in water depth will be constant but steep. Therefore, this graph models the filling of the graduated cylinder.

b) This graph shows a slow increase in the depth of the water then a rapid increase. The flask is a wide bottom vessel with a narrow neck. As water fills the bottom, the depth will slowly increase. As the water reaches the start of the neck of the flask, the water depth will increase more rapidly. Therefore, this graph models the filling of the flask.

c) This graph shows a gradual but steady increase in water depth. The flask is constant in width all along its height and it is a wide vessel. As the water fills the vessel, the change in water depth will be constant but gradual. Therefore, this graph models the filling of the beaker.

d) This graph shows a rapid increase in depth followed by a slow increase in water depth then a very little change in water depth. The drinking glass is narrow at the bottom and widens at its mouth. As the water fills the vessel, the change in water depth will be steep near the bottom, the increases in depth are lesser and lesser as the water reaches the mouth. Therefore, this graph models the filling of the drinking glass.

14. $\text{Rate} = \dfrac{\text{year 2 emissions} - \text{year 1 emissions}}{\text{difference in years}}$

For 1900 to 1995:

$\text{Rate} = \dfrac{127 - 103}{5}$

$\text{Rate} = \dfrac{24}{5}$

Rate = 4.8

From 1900 to 1995, the amount of emissions was increasing by 4.8 megatonnes/year.

For 1995 to 2000:

$\text{Rate} = \dfrac{151 - 127}{5}$

$\text{Rate} = \dfrac{24}{5}$

Rate = 4.8

From 1995 to 2000, the amount of emissions was increasing by 4.8 megatonnes/year.

For 2000 to 2003:

$\text{Rate} = \dfrac{161 - 151}{5}$

$\text{Rate} = \dfrac{10}{5}$

Rate = 2

From 1995 to 2000, the amount of emissions was increasing by 2.0 megatonnes/year.

For 2003 to 2006:

$\text{Rate} = \dfrac{158 - 161}{5}$

$\text{Rate} = -\dfrac{3}{5}$

Rate = –0.6

From 2003 to 2006, the amount of emissions was decreasing by 0.6 megatonnes/year.

Between 1900 to 1995 and 1995 and 2000, the greatest rates of emission increase occurred.

15. Let the *R* represent the skating speed.

$R = \dfrac{\text{difference in distance}}{\text{time 2} - \text{time 1}}$

For 0 m to 300 m:	
In 2002	In 2006
$R = \dfrac{300\text{ m}}{25.65\text{ s}}$ R = 11.695… m/s	$R = \dfrac{300\text{ m}}{25.42\text{ s}}$ R = 11.801… m/s
Difference = 11.801… m/s – 11.695…m/s Difference = 0.105… m/s The difference in speeds is about 0.106 m/s.	

For 300 m to 700 m:	
In 2002	In 2006
$R = \dfrac{700\text{ m} - 300\text{ m}}{54.17\text{ s} - 25.65\text{ s}}$ $R = \dfrac{400\text{ m}}{28.52\text{ s}}$ R = 14.025… m/s	$R = \dfrac{700\text{ m} - 300\text{ m}}{53.83\text{ s} - 25.42\text{ s}}$ $R = \dfrac{400\text{ m}}{28.41\text{ s}}$ R = 14.079…m/s
Difference = 14.079… m/s – 14.025…m/s Difference = 0.054… m/s The difference in the speeds is about 0.054 m/s.	

For 700 m to 1100 m:	
In 2002	In 2006
1:24.09 s = 60 s + 24.09 s 1:24.09 s = 84.09 s $R = \dfrac{1100\text{ m} - 700\text{ m}}{84.09\text{ s} - 54.17\text{ s}}$ $R = \dfrac{400\text{ m}}{29.92\text{ s}}$ R = 13.368… m/s	1:23.50 s = 60 s + 23.50 s 1:23.50 s = 83.50 s $R = \dfrac{1100\text{ m} - 700\text{ m}}{83.50\text{ s} - 53.83\text{ s}}$ $R = \dfrac{400\text{ m}}{29.67\text{ s}}$ R = 13.481… m/s
Difference = 13.481… m/s – 13.368…m/s Difference = 0.112… m/s The difference in speeds is about 0.113 m/s.	

For 1100 m to 1500 m:	
In 2002	In 2006
1:55.59 s = 60 s + 55.59 s 1:55.59 s = 115.59 s $R = \frac{1500\text{ m} - 1100\text{ m}}{115.59\text{ s} - 84.09\text{ s}}$ $R = \frac{400\text{ m}}{31.5\text{ s}}$ R = 12.698… m/s	1:55.27 s = 60 s + 55.27 s 1:55.27 s = 115.27 s $R = \frac{1500\text{ m} - 1100\text{ m}}{115.27\text{ s} - 83.50\text{ s}}$ $R = \frac{400\text{ m}}{31.77\text{ s}}$ R = 12.590… m/s
Difference = 12.698… m/s – 12.590…m/s Difference = 0.107… m/s The difference in speed is about 0.108 m/s.	

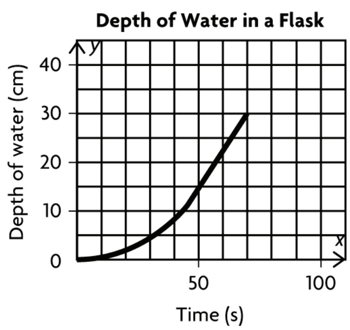
The greatest difference in speed was between 700 m to 1100 m by 0.113 m/s.

16. e.g.,

a) An estimate is sufficient when you only need to know which rate is better, such as which car uses less fuel per kilometre. A precise answer is needed if you want to know how much fuel you will save for a particular trip.

b) A graphing strategy is a good approach for comparing rates because you can visually compare the slopes. For example, a steeper slope for one lap of a car race on a graph of distance versus time means a faster speed. A numerical strategy is better if you want to know exactly how much faster one lap was compared to another.

17. Here is the diagram.



18. a) Number of computers = $\frac{\text{brain rate}}{\text{computer rate}}$

Number of computers = $\frac{100\ 000\ 000\ \cancel{\text{MIPS}}}{7000\ \cancel{\text{MIPS}}}$

Number of computers = 14 285.714…

The processing power of the human brain is equivalent to about 14 300 computers.

b) e.g., My computer has a processing power of 27 079 MIPS.

Number of computers = $\frac{100\ 000\ 000\ \cancel{\text{MIPS}}}{27\ 079\ \cancel{\text{MIPS}}}$

Number of computers = 3692.898…

About 3693 of these computers have the processing power equivalent to the human brain.

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A. e.g., Convert Ivy's time, 10:33.40, to seconds:

$(10\ \cancel{\text{min}})\left(\frac{60\text{ s}}{1\ \cancel{\text{min}}}\right) + 33.40\text{ s} = 633.40\text{ s}$

Therefore Ivy's rate, in metres per second, is

$\frac{1500\text{ m}}{633.40\text{ s}}$ or 2.368… m/s.

For the mile race, convert the distance from miles to metres: 1 mi = 1609 m

Convert the record time, 11:03.11 for 1 mi, to seconds:

$(11\ \cancel{\text{min}})\left(\frac{60\text{ s}}{1\ \cancel{\text{min}}}\right) + 3.11\text{ s} = 663.11\text{ s}$

So the record rate for the mile race is

$\frac{1609\text{ m}}{633.11\text{ s}}$ or 2.426… m/s.

2.368… m/s < 2.426… m/s

Ivy's rate for 1500 m is less than the rate for the mile. If Ivy ran at her world record rate, she would not break the record for the mile race.

Lesson 8.2: Solving Problems That Involve Rates, page 466

1. a) Price per litre = $\frac{\$163}{50\text{ L}}$

Price per litre = \$3.26/L

Number of litres = $\frac{\$30}{\$3.26/\text{L}}$

Number of litres = 9.202… L

You can buy 9 L of oil.

b) 3 min 25 s = 3 min + $\frac{25\ \cancel{\text{s}}}{60\ \cancel{\text{s}}/\text{min}}$

3 min 25 s = 3 min + 0.416… min

3 min 25 s = 3.416… min

Rate = $\frac{\text{time}}{\text{volume}}$

Rate = $\frac{3.416...\text{ min}}{75\text{ L}}$

Rate = 0.045… min/L

Time taken for 55 L = (rate)(volume)

Time taken for 55 L = $\left(\frac{0.045...\text{ min}}{1\ \cancel{\text{L}}}\right)(55\ \cancel{\text{L}})$

Time taken for 55 L = 2.505… min

It will take 3 min to fill a 55 L tank.