b) $\mu = 5.02$ kg

$$z = \frac{5 - 5.02}{0.065}$$

$z = -0.307\ldots$

e.g., Using a z-score table, a z-score of –0.307… means that 37.83% or 37.8% of the bags will be below 5 kg or that 62.17%, or 62.2% will be over 5 kg. This percentage is too high, so the set mean should be rejected. (Using a calculator, the value is 62.1%.)

23. a) $z = \frac{x - \mu}{\sigma}$

$x1_{40\%} = 145$
$x2_{40\%} = 155$
$x1_{70\%} = 140$
$x2_{70\%} = 160$

70% of the data is almost 68% of the data.

$$140 = \mu - 1\sigma$$
$$140 - \mu = -1\sigma$$
$$-140 + \mu = \sigma$$
$$160 = \mu + 1\sigma$$
$$160 - \mu = \sigma$$
$$-140 + \mu = 160 - \mu$$
$$2\mu = 300$$
$$\mu = 150$$

At 140, 15% of the scores are to the left of the 140. Using the z-score table, 0.15 corresponds to a z-score of –1.035.

$$-1.035 = \frac{140 - 150}{\sigma}$$
$$-1.035\sigma = -10$$
$$\sigma = 9.661\ldots$$

At 145, 30% of the scores are to the left of the 145. Using the z-score table, 0.30 corresponds to a z-score of –0.525.

$$-0.525 = \frac{145 - 150}{\sigma}$$
$$-0.525\sigma = -5$$
$$\sigma = 9.523\ldots$$

The standard deviation is about 9.6.

b) $x = 172$

$\mu = 150$

$$z = \frac{172 - 150}{9.6}$$

$z = 2.291\ldots$

Using a z-score table, 0.9890 of the scores is to the left of 172, or 98.9%. So, 1.1% of the scores is to the right of 172.

Approximately, 1.1% of the applicants would be considered.

24. e.g., *Problem:* If the ABC Company from Example 4 wants its process to meet 6-Sigma standards, that is, to reject fewer than 1 bungee cord per 300 produced, what standard deviation does the company need to have in its manufacturing process?

Solution:

$\mu = 45.2$ cm
Maximum length = 48.0 cm
Minimum length = 42.0 cm

If the maximum and minimum lengths were within 3 standard deviations of the mean, ABC Company would reject fewer than 1 in every 300 bungee cords, or 0.333…%.

Since the mean is closer to the maximum acceptable length, determine one third of the difference:

$$\sigma = \frac{1}{3}(48.0 - 45.2)$$
$$\sigma = \frac{1}{3}(2.8)$$
$$\sigma = 0.933\ldots$$

Reducing the standard deviation to 0.9 cm will definitely mean that fewer than 1 bungee cord per 300 produced is rejected.

Test reducing the standard deviation to 1.0 cm:

$$z = \frac{48.0 - 45.2}{1.0}$$

$z = 2.8$

Using a z-score table, a z-score of –2.8 means that 00.26% of the bungee cords will be longer than 48.0 cm.

$$z = \frac{42.0 - 45.2}{1.0}$$

$z = -3.2$

Using a z-score table, a z-score of –3.2 means that 0.07% of the bungee cords will be shorter than 42.0 cm.

0.26% + 0.07% = 0.33%

This value is just less than 1 in every 300 bungee cords, so the ABC Company needs to reduce its standard deviation to 1.0 cm (or even less) if it wants to reject only 0.33% of bungee cords.

Lesson 5.6: Confidence Intervals, page 274

1. a) The confidence level is 19 times out of 20, or 95%.

b) The confidence interval is 81% ± 3.1%.

81% – 3.1% = 77.9%
81% + 3.1% = 84.1%

The confidence interval is 77.9% to 84.3%.

c) Population, $N = 33.5 \times 10^6$ people
Number of people, $n = N \times$ percent
At 77.9%,
$n = (33.5 \times 10^6 \text{ people})(0.779)$
$n = 26.0965 \times 10^6$ people
At 84.3%,
$n = (33.5 \times 10^6 \text{ people})(0.843)$
$n = 28.2405 \times 10^6$ people
The range of the number of people who knew about climate change was 26.1 million to 28.2 million.
2. a) The confidence interval is 542 g ± 1.9 g.
542 g – 1.9 g = 540.1 g
542 g + 1.9 g = 543.9 g
The confidence interval is 540.1 g to 543.9 g.
b) As the sample size increases, the margin of error decreases.

Sample Size	Margin of Error (g)
50	± 3.9
100	± 2.7
500	± 1.2

3. a) The confidence level is 9 times out of 10, or 0.90, or 90%.
b) The confidence interval is 64% ± 3.4%.
64% – 3.4% = 60.6%
64% + 3.4% = 67.4%
The confidence interval is 60.6% to 67.4%.
c) Population, N = 32 people
Number of people, $n = N \times$ percent

At 60.6%,
n = (32 people)(0.606)
n = 19.392 people

At 67.4%,
n = (32 people)(0.674)
n = 21.568 people

The range of the number of students who would expect better dental checkups is 19 students to 22 students.
4. a) The confidence level is 19 times out of 20 or 95%.
The confidence interval is 81% ± 2.2%.
81% – 2.2% = 78.8%
81% + 2.2% = 83.2%
The confidence interval is 78.8% to 83.2%.

With 95% confidence, you could expect that 78.8% to 83.2% of those polled would be in favour of supporting bilingualism in Canada and that Canadians would want Canada to remain a bilingual country.
b) e.g., No, I disagree with Mark. There is no information about how the poll was conducted. No additional information about whether or not the participants in the poll were bilingual was given.
5. a) The confidence interval is 58% ± 3.1%.
58% – 3.1% = 54.9%
58% + 3.1% = 61.1%
The confidence interval is 54.9% to 61.1%.

b) e.g.,
Population of Swift Current, SK, N = 16 000
Number of people, $n = N \times$ percent

At 54.9%,
n = (16 000 people)(0.549)
n = 8784 people

At 61.1%,
n = (16 000 people)(0.611)
n = 9776 people

You could expect 8784 people to 9776 people in Canada to know about the plight of the average Inuit so they would have answered the question correctly.
6. a) The confidence level is 99 times out of 100, or 99%.
The confidence interval is 89% ± 4.3%.
89% – 4.3% = 84.7%
89% + 4.3% = 93.3%
The confidence interval is 84.7% to 93.3%.
b) Population, N = 23 500 000 people
Number of people, $n = N \times$ percent
At 84.7%,
n = (23 500 000 people)(0.847)
n = 19 904 500 people

At 93.3%,
n = (23 500 000 people)(0.933)
n = 21 925 500 people

You could expect 19 904 500 people to 21 925 500 people to recycle their cellphones.
7. a) e.g., A recent Nanos Research poll reported that a majority (65.3%) of Canadians believe that immigration is advantageous to Canada. A sample of 1008 Canadians was polled, and the margin of error is 3.1 percentage points, 19 times out of 20.
b) e.g., The confidence interval is 62.2% to 68.4%.
c) e.g., I agree with the concluding statement in the survey. The confidence interval lies entirely above 50%. Therefore, a majority of Canadians believe that immigration is advantageous to Canada.
8. a) The confidence interval is 174.8 g to 175.2 g.
Margin of error = mean – confidence interval limit
Margin of error = 175 g – 174.8 g
Margin of error = 0.2 g
The margin of error is ± 0.2 g.
b) At a 99% confidence level, the sample size needed is 135 discs.
c) At a 90% confidence level, the sample size needed is 55 discs.
d) At a confidence level of 19 times out or 20, or 95%, the sample size needed is 78 discs.

9. a) The confidence level is 9 times out of 10, or 90%.
The confidence interval is 54% ± 4.5%.
54% – 4.5% = 49.5%
54% + 4.5% = 58.5%
The confidence interval is 49.5% to 58.5%.

With 90% confidence, you could say that 49.5% to 58.5% of post-secondary graduates can expect to earn at least $100 000/year by the time they retire.

b) The confidence level is 99%.
The confidence interval is 63% ± 2.1%.
63% – 2.1% = 60.9%
63% + 2.1% = 65.1%
The confidence interval is 60.9% to 65.1%.
With 99% confidence, you could say that 60.9% to 65.1% of online shoppers search for online coupons or deals.

c) The confidence level is 19 times out of 20, or 95%.
The margin of error is ± 3.38% of 18.1 h, or 0.6 h.
The confidence interval for weekly online time is 18.1 h ± 0.6 h.
18.1 h – 0.6 h = 17.5 h
18.1 h + 0.6 h = 18.7 h
The confidence interval for weekly online-time is 17.5 h to 18.7 h.

The confidence interval for weekly television-watching time is 16.9 h ± 0.6 h.
16.9 h – 0.6 h = 16.3 h
16.9 h + 0.6 h = 17.5 h
The confidence interval for weekly television-watching time is 16.3 h to 17.5 h.

With 95% confidence, you could say that Canadians spend 17.5 h to 18.7 h online, compared with 16.3 h to 17.5 h watching television per week.

d) The confidence level is 95%.
The confidence interval is 39% ± 3%.
39% – 3% = 36%
39% + 3% = 42%
The confidence interval is 36% to 42%.

With 95% confidence, you could say that 36% to 42% of decided voters will not vote for the incumbent political party in the next election.

10. a) The margin of error increases as the confidence level increases (with a constant sample size). The sample size that is needed also increases as the confidence level increases (with a constant margin of error).
The sample size affects the margin of error. A larger sample results in a smaller margin of error, assuming that the same confidence level is required.

b) As the confidence level increases, the *z*-score in the formula for the margin of error increases, so the margin of error increases.

11. a) e.g.,

Sample Size, *n*	Pattern $\sqrt{\frac{n_{x-1}}{n_x}}$	Margin of Error $\left(\sqrt{\frac{n_{x-1}}{n_x}}\right)(\text{error}_{x-1})$
100		9.80%
400	$\sqrt{\frac{100}{400}} = \frac{1}{2}$	$\frac{1}{2}(9.80\%) = 4.90\%$
900	$\sqrt{\frac{400}{900}} = \frac{2}{3}$	$\frac{2}{3}(4.90\%) = 3.27\%$
1600	$\sqrt{\frac{900}{1600}} = \frac{3}{4}$	$\frac{3}{4}(3.27\%) = 2.45\%$
2500	$\sqrt{\frac{1600}{2500}} = \frac{4}{5}$	$\frac{4}{5}(2.45\%) = 1.96\%$
3600	$\sqrt{\frac{2500}{3600}} = \frac{5}{6}$	$\frac{5}{6}(1.96\%) = 1.63\%$

Note: The sample sizes are perfect square numbers.

b) i) For 4900,

$$\left(\sqrt{\frac{n_{x-1}}{n_x}}\right)(\text{error}_{x-1}) = \left(\sqrt{\frac{3600}{4900}}\right)(1.63\%)$$

$$\left(\sqrt{\frac{n_{x-1}}{n_x}}\right)(\text{error}_{x-1}) = \frac{6}{7}(1.63\%)$$

$$\left(\sqrt{\frac{n_{x-1}}{n_x}}\right)(\text{error}_{x-1}) = 0.01397\ldots$$

The margin of error for a sample size of 4900 is 1.40%.

ii) For 2000,

$$\left(\sqrt{\frac{n_{x-1}}{n_x}}\right)(\text{error}_{x-1}) = \left(\sqrt{\frac{1600}{2000}}\right)(2.45\%)$$

$$\left(\sqrt{\frac{n_{x-1}}{n_x}}\right)(\text{error}_{x-1}) = \sqrt{\frac{4}{5}}(2.45\%)$$

$$\left(\sqrt{\frac{n_{x-1}}{n_x}}\right)(\text{error}_{x-1}) = 0.02191\ldots$$

The margin of error for a sample size of 2000 is 2.19%.

c) e.g., The margin of error gets smaller as sample size increases. Therefore, a relatively small sample size is needed to get a small margin of error.

Math in Action, page 276

e.g., Canadians see the environment as the most important issue facing the world today (29%), followed closely by war and conflict (28%).
When presented with a slate of issues and asked to indicate their level of concern about each, 61% of Canadians said they were very concerned about global warming and other environmental problems.
The research was conducted by Environics Research and consisted of a comprehensive national public-opinion telephone survey, with a representative sample of 2001 Canadians. The results were accurate to within ±2.2 percentage points, in 95 out of 100 samples.
The confidence interval for each issue is given below.

Environment:	29% ± 2.2%	26.8% to 31.2%
War and conflict:	28% ± 2.2%	25.8% to 30.2%
Global warming and other environmental problems:	61% ± 2.2%	58.8% to 63.2%

- I agree with the media statements. The sum of the maximum values for the first two issues is less than 100%.
- If Canadians focused on ways to reduce harm to the environment, then the environment might not be the issue of greatest concern in future surveys.
- For the results to be noticeably different if the same survey were taken again, using the same sample size, the population mean would have to lie outside the confidence interval of the first survey. This would mean that concern about environmental problems would be an issue for 55% of people surveyed.

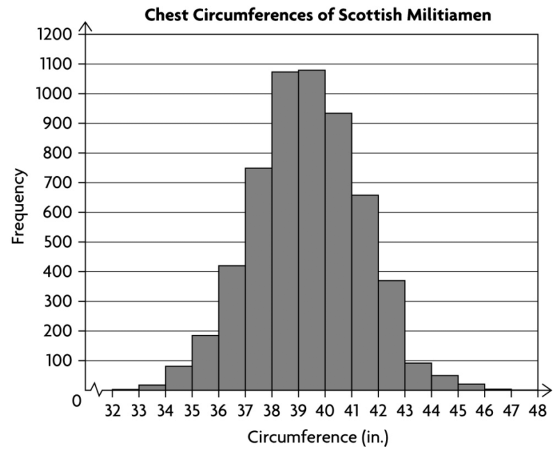
Chapter Self-Test, page 277

1. a) For 1999:
Mean, $\bar{x}$ = 43.4 in.
Standard deviation, σ = 3.0 in.

For 2011:
Mean, $\bar{x}$ = 67.8 in.
Standard deviation, σ = 5.6 in.

b) The standard deviation for the heights in 2011 is greater than the standard deviation for the heights in 1999.
e.g., The difference could be due to varying growth spurts in children from ages 13 years and older. The growth spurts could vary less in children under the age of 13.

2. a) e.g., The graph of the data shows a normal distribution.



b) $z = \frac{x - \mu}{\sigma}$

Measurement, x = 42 in.
Mean, μ = 39.83 in.
Standard deviation, σ = 2.05 in.

$z = \frac{42 - 39.83}{2.05}$

z = 1.06

3. e.g., Edmonton's average daily temperature is lower than Calgary's average daily temperature. However, standard deviation for Edmonton is higher. This means the average daily temperatures were less consistent with the mean average daily temperature relative to Calgary.

4. a) The confidence level is "19 times out of 20", or 95%.
The confidence intervals are:
For flag: 88% ± 3.1%
88% – 3.1% = 84.9%
88% + 3.1% = 91.1%
The confidence interval for respondents being proud of the flag is 84.9% to 91.1%.

For hockey: 80% ± 3.1%
80% – 3.1% = 76.9%
80% + 3.1% = 83.1%
The confidence interval for respondents being proud of the hockey is 76.9% to 83.1%.

For justice system: 44% ± 3.1%
44% – 3.1% = 40.9%
44% + 3.1% = 47.1%
The confidence interval for respondents being proud of the Canadian justice system is 40.9% to 47.1%.