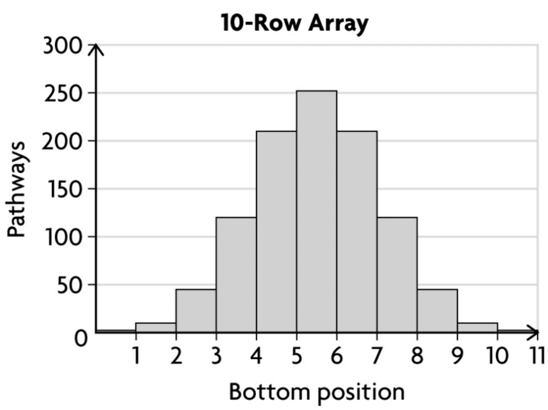
Applying Problem-Solving Strategies, page 254

A. 15
B., C. Create a table.

Row	Number of Pegs	Number of Ways
1	1	2
2	3	4
3	6	8
4	10	16
5	15	32
6	21	64

D., E. e.g., The number in the third column doubles as you move down the rows. Algebraically, the number of ways is 2^{10} or 1024.
F. e.g., Balls dropped onto an array (given sufficient pegs) will form the following symmetrical distribution: 1, 10, 45, 120, 210, 252, 210, 120, 45, 10, 1. This is approximately a normal distribution.



Lesson 5.5: *Z*-Scores, page 264

1. $z = \frac{x-\mu}{\sigma}$
a) $\mu = 112$, $\sigma = 15.5$, $x = 174$
$z = \frac{174-112}{15.5}$
$z = 4$
The z-score is 4.
b) $\mu = 53.46$, $\sigma = 8.24$, $x = 47.28$
$z = \frac{47.28-53.46}{8.24}$
$z = -0.75$
The z-score is –0.75.
c) $\mu = 82$, $\sigma = 12.5$, $x = 58$
$z = \frac{58-82}{12.5}$
$z = -1.92$
The z-score is –1.92.
d) $\mu = 245$, $\sigma = 22.4$, $x = 300$
$z = \frac{300-245}{22.4}$
$z = 2.455\ldots$
The z-score is 2.455….
2. a) $z = 1.24$
The z-score of 1.24 corresponds to a 0.8925.
This means 89.25% of the scores are to the left of the z-score.
b) $z = -2.35$
A z-score of –2.35 corresponds to 0.0094.
This means 0.94% of the scores are to the left of the z-score.
c) $z = 2.17$
A z-score of 2.17 corresponds to 0.9850.
This means 98.50% of the scores are to the left of the z-score.
d) $z = -0.64$
A z-score of –0.64 corresponds to 0.2611.
This means 26.11% of the scores are to the left of the z-score.
3. a) For $z = -2.88$, the percentage is 0.20%.
For $z = -1.47$, the percentage is 7.08%.
The percent of data between these scores is 6.88%.
b) For $z = -0.85$, the percentage is 19.77%.
For $z = 1.64$, the percentage is 94.95%.
The percent of data between these scores is 75.18%.
4. a) 10% of the data to the left of the z-score
= 0.10 of the data to the left of the z-score
Using a calculator, 0.10 corresponds to a z-score of –1.28.
b) 10% of the data to the right of the z-score
= 0.90 of the data to the left of the z-score
Using the z-score table, 0.90 corresponds to a z-score
of 1.28.
c) 60% of the data is below the z-score
= 0.60 of the data to the left of the z-score
Using the z-score table, 0.60 corresponds to a z-score
of 0.25.
d) 60% of the data is above the z-score
= 0.40 of the data to the left of the z-score
Using the z-score table, 0.40 corresponds to a z-score
of –0.25.

5. $z = \frac{x-\mu}{\sigma}$
a) $\mu = 24$, $\sigma = 2.8$, $x = 29.3$
$z = \frac{29.3-24}{2.8}$
$z = 1.892\ldots$
The z-score is 1.892….

b) $\mu = 165$, $\sigma = 48$, $x = 36$

$z = \frac{36-165}{48}$

$z = -2.6875$

The z-score is –2.6875.

c) $\mu = 784$, $\sigma = 65.3$, $x = 817$

$z = \frac{817-784}{65.3}$

$z = 0.505\ldots$

The z-score 0.505….

d) $\mu = 2.9$, $\sigma = 0.3$, $x = 3.4$

$z = \frac{3.4-2.9}{0.3}$

$z = 1.666\ldots$

The z-score is 1.666….

6. a) $z = 0.56$

Using the z-score table, the z-score indicates 71.23% of the data is to the left of the z-score.

b) $z = -1.76$

Using the z-score table, the z-score indicates 3.92% of the data is to the left of the z-score.

c) $z = -2.98$

Using the z-score table, the z-score indicates 0.14% of the data is to the left of the z-score.

d) $z = 2.39$

Using a z-score table, the z-score indicates 99.16% of the data is to the left of the z-score.

7. a) $z = -1.35$

Using the z-score table, the z-score indicates 8.85% of the data is to the left of the z-score. So, 91.15% of the data must be to the right of the z-score.

b) $z = 2.63$

Using the z-score table, the z-score indicates 99.57% of the data is to the left of the z-score. So, 0.43% of the data must be to the right of the z-score.

c) $z = 0.68$

Using the z-score table, the z-score indicates 75.17% of the data is to the left of the z-score. So, 24.83% of the data must be to the right of the z-score.

d) $z = -3.14$

Using a z-score table, the z-score indicates 0.08% of the data is to the left of the z-score. So, 99.92% of the data must be to the right of the z-score.

8. a) For $z = 0.24$, the percentage is 59.48%.

For $z = 2.53$, the percentage is 99.43%.

The percent of data between these scores is 39.95%.

b) For $z = -1.64$, the percentage is 5.05%.

For $z = 1.64$, the percentage is 94.95%.

The percent of data between these scores is 89.90%.

9. a) 33% of the data to the left of the z-score

= 0.33 of the data to the left of the z-score

Using a calculator, 0.33 corresponds to a z-score of –0.439….

b) 20% of the data to the right of the z-score

= 0.80 of the data to the left of the z-score

Using a calculator, 0.80 corresponds to a z-score of 0.841….

10. a) $z = \frac{x-\mu}{\sigma}$

Subject	Test Results (%) μ	Test Results (%) σ	Meg's Mark (%)	z-score
English	77	6.8	93	$\frac{93-77}{6.8} = 2.352...$
Math	74	5.4	91	$\frac{91-74}{5.4} = 3.148...$

b) Meg's math z-score is greater than her English z-score. A z-score of 3.148… represents approximately 99.92% of the math marks that were less than 91%. So, Meg does better in math relative to her peers.

c) e.g., Although Meg does better on math relative to her peers, her English mark is higher. If she enjoys English more than mathematics, then journalism might be a preferred career relative to science. Another consideration is the job market---in which field will Meg be more likely to get a job?

11. $\mu = 175$ mm

$\sigma = 0.4$ mm

$z = \frac{x-\mu}{\sigma}$

$z_{174} = \frac{174-175}{0.4}$

$z_{174} = -2.5$

$z_{175.6} = \frac{175.6-175}{0.4}$

$z_{175.6} = 1.5$

Using the z-score table, a z-score of –2.5 means 0.62% of the data to the left of the z-score. A z-score of 1.5 means 93.32% of the data is to the left of the z-score. The percent difference is 92.70%. So 92.70% of the flooring production can be classified as premium flooring.

12. $z = \frac{x-\mu}{\sigma}$

For 2.55 km/h on treadmill:

treadmill population,

$\mu = 76$ bpm

$\sigma = 9.15$ bpm

Violeta's results,

$x = 68$ bpm

$z = \frac{68-76}{9.15}$

$z = -0.874\ldots$

A z-score of –0.874… means 19.21% of the population had heart rates less than 68 bpm or 80.79% had heart rates greater than 68 bpm.

For 3.02 km/h in water:

water walking population,

$\mu = 160$ bpm

$\sigma = 13.50$ bpm

Violeta's results,
x = 145 bpm

$z = \dfrac{145-160}{13.50}$

$z = -1.11$

A z-score of –1.11 means 13.35% of the population had heart rates less than 145 bpm or 86.65% had heart rates greater than 145 bpm.

So, for water walking, Violeta's heart rate was lower, compared to the others who took part in the study.

13. $z = \dfrac{x-\mu}{\sigma}$

μ = 32 years

σ = 5.9 years

a) x = 40 years

$z = \dfrac{40-32}{5.9}$

$z = 1.355\ldots$

Using the z-score table, a z-score of 1.355… means 91.24% of the population were less than 40 years old.

b) x = 21 years

$z = \dfrac{21-32}{5.9}$

$z = -1.864\ldots$

Using the z-score table, a z-score of –1.864… means 3.14% of the population were less than 21 years old.

c) x = 18 years or less

$z = \dfrac{18-32}{5.9}$

$z = -2.372\ldots$

Using the z-score table, a z-score of –2.372… means 0.89% of the population were 18 years old or less. Using a calculator, 0.88% of the population were 18 years old or less.

e.g., Health educators at schools might want to know this information to determine whether or not there would be a need for planned parenting courses. Governments might want to determine how many mothers need income supplements because they are still of high school age.

14. Using the z-score table, 50% or 0.50 corresponds to a z-score of 0.0. This is the mean height of the population. So the mean height is 180 cm.
Then 10% of the population to the right of the mean is 90% or 0.90 of the population to the left of the mean. Using the z-score table, 0.90 corresponds to a z-score of 1.29.

$z = \dfrac{x-\mu}{\sigma}$

For 200 cm,
$z = 1.29$
x = 200 cm

$$1.28 = \frac{200-180}{\sigma}$$
$$1.28\sigma = 20$$
$$\sigma = 15.625$$

The standard deviation is 15.6 cm.

15. $z = \dfrac{x-\mu}{\sigma}$

μ = 4.8 million cells per cubic microlitre

σ = 0.3 million cells per cubic microlitre

a) $z = \dfrac{4-4.8}{0.3}$

$z = -2.67$

Using the z-score table, a z-score of –2.67 corresponds to 0.38% of the values less than 4.

b) $z = \dfrac{4.7-4.8}{0.3}$

$z = -0.333\ldots$

Using the z-score table, a z-score of –0.333… corresponds to 37.07% of the values less than 4.7.

$z = \dfrac{5.0-4.8}{0.3}$

$z = 0.666\ldots$

Using the z-score table, a z-score of 0.666… corresponds to 74.86% of the values less than 5.0.

Percent difference = 74.86% – 37.07%
Percent difference = 37.79%

So 37.79% of the people have a count between 4.7 million cells per cubic microlitre and 5.0 million cells per cubic microlitre. Or, using a calculator, 37.81% of people have a count between 4.7 million cells and 5.0 million cells per cubic microlitre.

c) 95% of the people = 0.95 of the people.
Using the z-score table, 0.95 corresponds to a z-score of 1.645.

$$1.645 = \frac{x-4.8}{0.3}$$
$$0.4935 = x - 4.8$$
$$5.2935 = x$$

So, 95% of the people would have a blood cell count lower than 5.29 million cells per cubic microlitre.

16. $z = \dfrac{x-\mu}{\sigma}$

μ = 2.6 years

σ = 0.48 years

Warranty period, x = 1.0 year
Number to MP3s, n = 4000

a) $z = \dfrac{1.0-2.6}{0.48}$

$z = -3.33$

Using a calculator, a z-score of –3.33 corresponds to 0.0434% of MP3 life spans being less than 1 year.

Number of defective = $n \times$ percent
Number of defective = 4000(0.000434)
Number of defective = 1.7

Out of 4000 MP3s, you could expect two to be defective.

b) New warranty period, x = 2 years

$z = \frac{2.0 - 2.6}{0.48}$

$z = -1.25$

Using the z-score table, a z-score of –1.25 corresponds to 10.56%. So Tyler has a 10.56% probability of making a claim on his warranty in less than two years from the purchase of the MP3 player.

17. $z = \frac{x - \mu}{\sigma}$

μ = 67 months

σ = 7.2 months

Repair rate, r = 1%

Using the z-score table, 1% or 0.01 corresponds to a z-score of –2.33.

$$-2.33 = \frac{x - 67}{7.2}$$
$$-16.776 = x - 67$$
$$50.22 = x$$

The warranty period should be 50 months.

18. $z = \frac{x - \mu}{\sigma}$

μ = 68

σ = 6

Success rate, r = 10%

Using the z-score table, 10% success means 90% or 0.90 failure to get an A on every test. A rate of 0.90 corresponds to a z-score of 1.28.

$$1.28 = \frac{x - 68}{6}$$
$$7.68 = x - 68$$
$$75.68 = x$$

A mark of 76% is needed to get an A on the test.

19. $z = \frac{x - \mu}{\sigma}$

μ = \$2500

σ_{high} = \$1000

σ_{mid} = \$400

x = \$3000

For the high-priced car,

$z = \frac{3000 - 2500}{1000}$

$z = 0.5$

Using the z-score table, a z-score of 0.50 corresponds to 69.15% probability that the repairs will be less than \$3000. This means the probability that the repairs will cost more than \$3000 is 30.85% or approximately 31%.

For the mid-priced car,

$z = \frac{3000 - 2500}{400}$

$z = 1.25$

Using the z-score table, a z-score of 1.25 corresponds to 89.44% probability that the repairs will be less than \$400. This means the probability that the repairs will cost more than \$400 is 10.56% or approximately 11%.

20. $z = \frac{x - \mu}{\sigma}$

μ = 100

σ = 15

a) Using the z-score table, a value of 98% or 0.98 corresponds to a z-score of 2.05.

$$2.05 = \frac{x - 100}{15}$$
$$30.75 = x - 100$$
$$130.75 = x$$

An IQ score of 131 is greater than 98% of the scores of the population.

b) Only 0.38% of the population are geniuses. This means 99.62% or 0.9962 of the population are not geniuses. Using the z-score table, 0.9962 corresponds to a z-score of 2.67.

$$2.67 = \frac{x - 100}{15}$$
$$40.05 = x - 100$$
$$140.05 = x$$

You need an IQ score of 140 to be a genius.

c) If Jarod's score is the top 30%, this means 70% or 0.70 of the population scores are below Jarod's. Using the z-score table, 0.70 corresponds to a z-score of 0.525.

$$0.525 = \frac{x - 100}{15}$$
$$7.875 = x - 100$$
$$107.875 = x$$

Jarod's IQ score is at least 108.

21. A z-score indicates the number of standard deviations that a data value lies from the mean. It is determined using this formula:

$z = \frac{x - \mu}{\sigma}$

A positive z-score indicates that the data value lies above the mean. A negative z-score indicates that the data value lies below the mean.

Z-scores can be used to compare data from different normally distributed sets by converting their distributions to the standard normal distribution.

22. $z = \frac{x - \mu}{\sigma}$

μ = 5 kg

σ = 0.065 kg

a) x = 4.9 kg

Rejection rate = 3%

Using the z-score table, the rate of 3% or 0.03 corresponds to a z-score of –1.88.

$$-1.88 = \frac{4.9 - \mu}{0.065}$$
$$-0.1222 = 4.9 - \mu$$
$$5.0222 = \mu$$

The new mean mass should be set at 5.02 kg.

b) $\mu = 5.02$ kg

$z = \frac{5 - 5.02}{0.065}$

$z = -0.307\ldots$

e.g., Using a z-score table, a z-score of –0.307… means that 37.83% or 37.8% of the bags will be below 5 kg or that 62.17%, or 62.2% will be over 5 kg. This percentage is too high, so the set mean should be rejected. (Using a calculator, the value is 62.1%.)

23. a) $z = \frac{x - \mu}{\sigma}$

$x1_{40\%} = 145$

$x2_{40\%} = 155$

$x1_{70\%} = 140$

$x2_{70\%} = 160$

70% of the data is almost 68% of the data.

$140 = \mu - 1\sigma$

$140 - \mu = -1\sigma$

$-140 + \mu = \sigma$

$160 = \mu + 1\sigma$

$160 - \mu = \sigma$

$-140 + \mu = 160 - \mu$

$2\mu = 300$

$\mu = 150$

At 140, 15% of the scores are to the left of the 140. Using the z-score table, 0.15 corresponds to a z-score of –1.035.

$-1.035 = \frac{140 - 150}{\sigma}$

$-1.035\sigma = -10$

$\sigma = 9.661\ldots$

At 145, 30% of the scores are to the left of the 145. Using the z-score table, 0.30 corresponds to a z-score of –0.525.

$-0.525 = \frac{145 - 150}{\sigma}$

$-0.525\sigma = -5$

$\sigma = 9.523\ldots$

The standard deviation is about 9.6.

b) $x = 172$

$\mu = 150$

$z = \frac{172 - 150}{9.6}$

$z = 2.291\ldots$

Using a z-score table, 0.9890 of the scores is to the left of 172, or 98.9%. So, 1.1% of the scores is to the right of 172.

Approximately, 1.1% of the applicants would be considered.

24. e.g., *Problem:* If the ABC Company from Example 4 wants its process to meet 6-Sigma standards, that is, to reject fewer than 1 bungee cord per 300 produced, what standard deviation does the company need to have in its manufacturing process?

Solution:

$\mu = 45.2$ cm

Maximum length = 48.0 cm

Minimum length = 42.0 cm

If the maximum and minimum lengths were within 3 standard deviations of the mean, ABC Company would reject fewer than 1 in every 300 bungee cords, or 0.333…%.

Since the mean is closer to the maximum acceptable length, determine one third of the difference:

$\sigma = \frac{1}{3}(48.0 - 45.2)$

$\sigma = \frac{1}{3}(2.8)$

$\sigma = 0.933\ldots$

Reducing the standard deviation to 0.9 cm will definitely mean that fewer than 1 bungee cord per 300 produced is rejected.

Test reducing the standard deviation to 1.0 cm:

$z = \frac{48.0 - 45.2}{1.0}$

$z = 2.8$

Using a z-score table, a z-score of –2.8 means that 00.26% of the bungee cords will be longer than 48.0 cm.

$z = \frac{42.0 - 45.2}{1.0}$

$z = -3.2$

Using a z-score table, a z-score of –3.2 means that 0.07% of the bungee cords will be shorter than 42.0 cm.

0.26% + 0.07% = 0.33%

This value is just less than 1 in every 300 bungee cords, so the ABC Company needs to reduce its standard deviation to 1.0 cm (or even less) if it wants to reject only 0.33% of bungee cords.

Lesson 5.6: Confidence Intervals, page 274

1. a) The confidence level is 19 times out of 20, or 95%.

b) The confidence interval is 81% ± 3.1%.

81% – 3.1% = 77.9%

81% + 3.1% = 84.1%

The confidence interval is 77.9% to 84.3%.