6. For females:

Midpoint Salary ($)	Frequency
22 500	92
27 500	52
32 500	19
37 500	10
42 500	4
47 500	1
52 500	3
57 500	3

$\bar{x}$ = $27 391.30
σ = $7241.12

For males:

Midpoint Salary ($)	Frequency
25 000	86
35 000	78
45 000	28
55 000	20
65 000	22
75 000	10
85 000	4
95 000	5
105 000	2
115 000	1
125 000	0
135 000	1

$\bar{x}$ = $41 614.79
σ = $19 542.92
The average salary for males was greater than the average salary for females. The standard deviation for male salaries was greater than for females. The salaries for males varied more around the mean, possibly indicating a greater range in salaries.

Lesson 5.4: The Normal Distribution, page 251

1. a) Mean, μ = 63 years
Standard deviation, σ = 4 years
2σ = 8 years

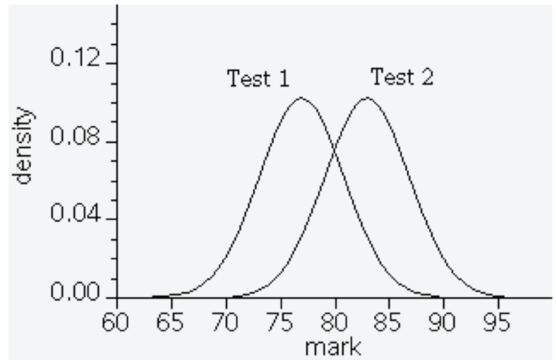
$55 = \mu - 2\sigma$
55 years to 63 years = 47.5% of the curlers
Of the entire group of curlers, 47.5% are between the ages of 55 years to 63 years.
b) μ = 63 years
σ = 4 years
2σ = 8 years

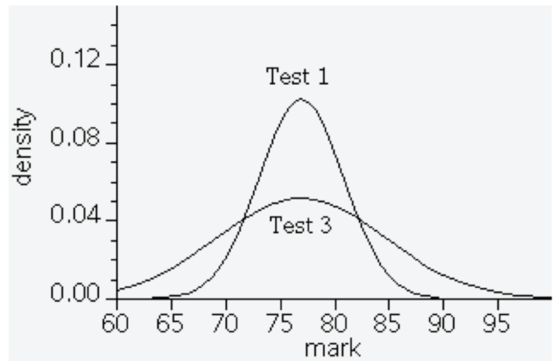
$67 = \mu + 1\sigma$
63 years to 67 years = 34% of the curlers

$75 = \mu + 3\sigma$
75 years to end = 50% of the curlers

67 years to 75 years = 49.85% – 34%
67 years to 75 years = 15.85%
Of the entire group of curlers, 15.85% are between the ages of 67 years to 75 years.

c) Those older than 75 years are in the last standard deviation unit of the normal distribution. Those older than 75 years represent 0.15% of the curler population.
2. a) Use a graphing calculator.

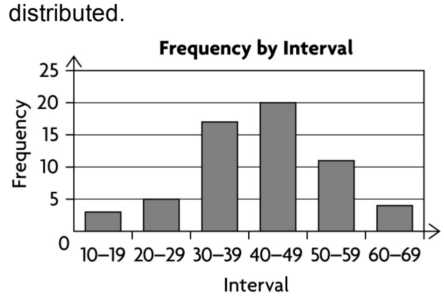




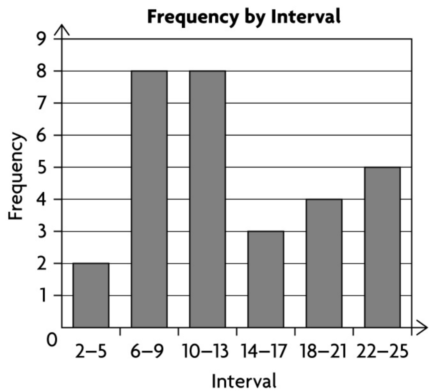
b) e.g., Test 1 and test 2 have different means but the same standard deviation. So the widths of the graphs are the same.
Test 1 and test 3 have the same mean but test 3's standard deviation is twice that of test 1. So the graph for test 3 is flatter than test 1.
c) Create a table.

Test	Oliver's Mark	Oliver's Score (%)
1	$\mu + 2\sigma$	77 + 2(3.9) = 84.8
2	$\mu - 1\sigma$	83 – 3.9 = 79.1
3	$\mu + 3\sigma$	77 + 3(7.4) = 99.2

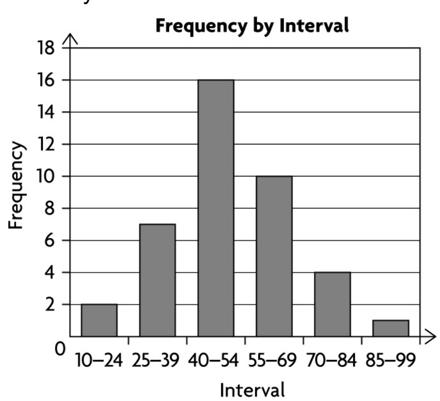
3. a) e.g., The two middle intervals, 30–39 and 40–49, represent 61% of the data. The four middle intervals from 20–29 to 50–59 represent 88% of the data. Therefore, the data is normally distributed since the intervals are close to 68% and 95% of the entire data, respectively. A graph of the data approximates a bell shape, so the data is normally distributed.



b) e.g., The two middle intervals, 10–13 and 14–17, represent 37% of the data. The four middle intervals from 6–9 to 18–21 represent 77% of the data. Therefore, the data is not normally distributed since the intervals should represent 68% and 95% of the entire data, respectively. A graph of the data confirms the data is not normally distributed, as the graph does not have a bell shape.



c) e.g., The two middle intervals, 40–54 and 55–69, represent 65% of the data. The four middle intervals from 25–39 to 70–84 represent 92.5% of the data. Therefore, the data is normally distributed since the intervals are relatively close to 68% and 95% representations of the entire data, respectively. A graph of the data approximates a bell shape, so the data is normally distributed.



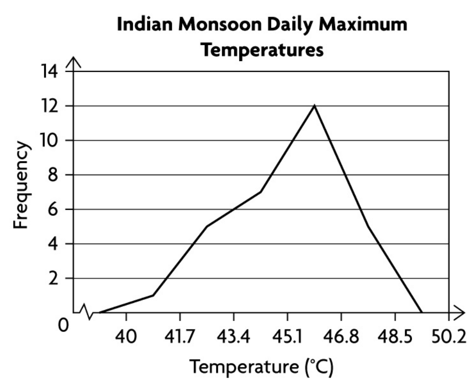
4. a) Mean, μ = 104.5 min
Standard deviation, σ = 22.3 min
b) Create a table.

Movie Length (min)	Frequency
$\mu - 2\sigma$ = 59.5 Interval: 59.5–82.0	3
$\mu - 1\sigma$ = 82.0 Interval: 82.0–104.5	33
$\mu + 1\sigma$ = 127.0 Interval: 104.5–127.0	7
$\mu + 2\sigma$ = 149.5 Interval: 127.0–149.5	3
$\mu + 3\sigma$ = 172.0 Interval: 149.5–172.0	3
$\mu + 4\sigma$ = 194.5 Interval: 172.0–194.5	1

c) Between one standard deviation from the mean, 40 of 50 movie times, or 80% of the data, is contained. So the data is not normally distributed.

5. a) i) Mean, μ = 45.2 °C
Median = 45.5 °C
Standard deviation, σ = 1.7 °C
ii) Create a table.

Temperature (°C)	Frequency
$\mu - 4\sigma$ = 38.4 °C Interval: 38.4–40.1 Midpoint: 39.25	0
$\mu - 3\sigma$ = 40.1 °C Interval: 40.1–41.8 Midpoint: 40.95	1
$\mu - 2\sigma$ = 41.8 °C Interval: 41.8–43.5 Midpoint: 42.65	5
$\mu - 1\sigma$ = 43.5 °C Interval: 43.5–45.2 Midpoint: 44.35	7
$\mu + 1\sigma$ = 46.9 °C Interval: 45.2–46.9 Midpoint: 46.05	12
$\mu + 2\sigma$ = 48.6 °C Interval: 46.9–48.6 Midpoint: 47.75	5
$\mu + 3\sigma$ = 50.3 °C Interval: 48.6–50.3 Midpoint: 49.95	0
$\mu + 4\sigma$ = 52.0 °C Interval: 50.3–52.0 Midpoint: 51.15	0

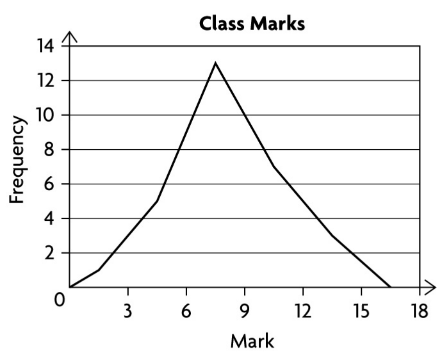


iii) The two middle intervals, 43.5–45.2 and 45.2–46.9, represent 63% of the data. The four middle intervals from 41.8–43.5 to 46.9–48.6 represent 97% of the data. But the data is not symmetric about the mean, so the data is not a normal distribution. This conclusion is confirmed by the graph in part ii).

b) i) Mean, μ = 8.6
Median = 8
Standard deviation, σ = 2.8

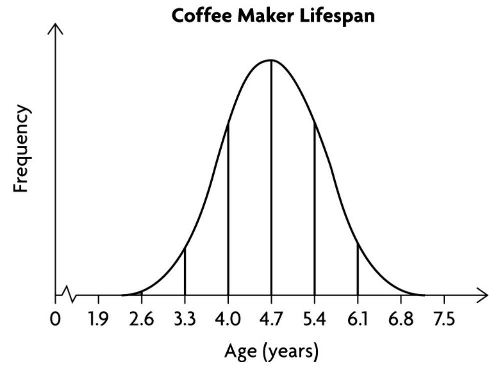
ii) Create a table.

Class Marks	Frequency
$\mu - 4\sigma$ = NA Interval: NA Midpoint: NA	NA
$\mu - 3\sigma = 0.2$ Interval: 0.2–3.0 Midpoint: 1.6	1
$\mu - 2\sigma = 3.0$ Interval: 3.0–5.8 Midpoint: 4.4	2
$\mu - 1\sigma = 5.8$ Interval: 5.8–8.6 Midpoint: 7.2	13
$\mu + 1\sigma = 11.4$ Interval: 8.6–11.4 Midpoint: 10.0	8
$\mu + 2\sigma = 14.2$ Interval: 11.4–14.2 Midpoint: 12.1	4
$\mu + 3\sigma = 17.0$ Interval: 14.2–17.0 Midpoint: 15.6	1
$\mu + 4\sigma = 19.8$ Interval: 17.0–19.8 Midpoint: 18.4	0



iii) e.g., The two middle intervals, 5.8–8.6 and 8.6–11.4, represent 72% of the data. The four middle intervals from 3.0–5.8 to 11.4–14.2 represent 90% of the data. The majority of the data clusters around the mean. Therefore, the data is somewhat normally distributed.

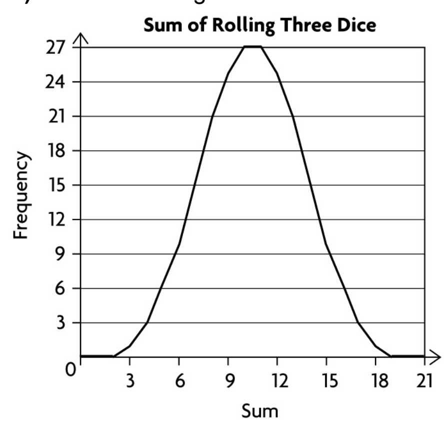
6. A rate of 2.5% repairs means that 97.5% of the coffee makers do not need repairs. This represents three standard deviations from the mean. The number of years three standard deviations from the mean is 2.6 years. The warranty should be for 2.6 years, or more reasonably 3 years.



7. a) Mean, $\mu = 10.5$

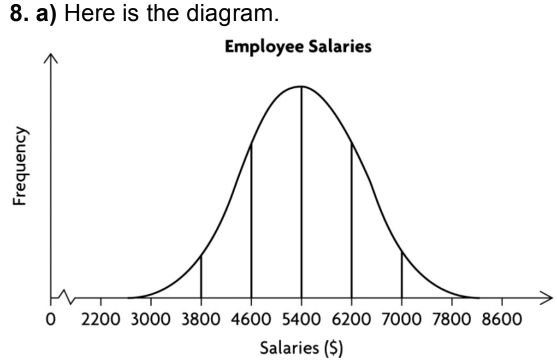
Standard deviation, $\sigma = 2.96$

b) Here is the diagram.

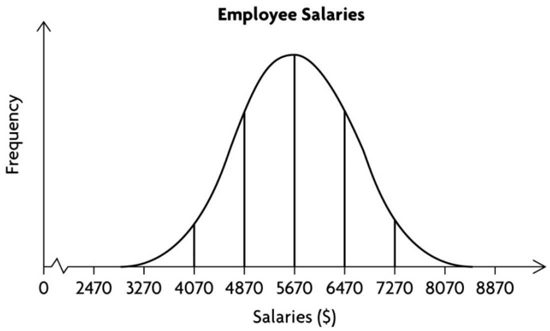


c) The frequency polygon is a bell-shaped curve that is symmetrical about the mean. The data has a normal distribution.

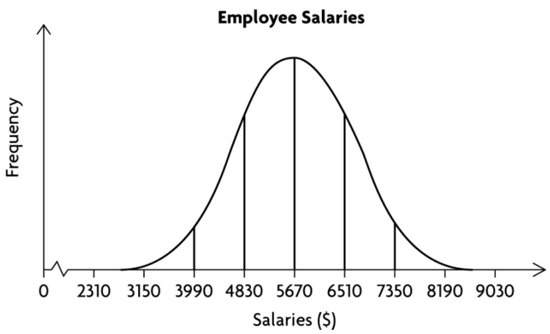
8. a) Here is the diagram.



b) Here is the diagram.



c) Here is the diagram.



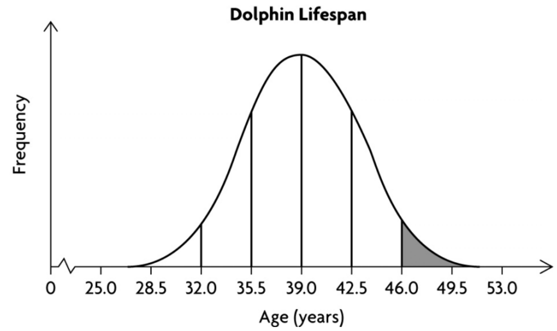
9. a) Mean, $\mu = 72.3$
Standard deviation, $\sigma = 3.2$

Interval	Score	Frequency
$\mu - 3\sigma$ 62.7–65.9	62.7	1
$\mu - 2\sigma$ 65.9–69.1	65.9	18
$\mu - 1\sigma$ 69.1–72.3	69.1	31
$\mu + 1\sigma$ 72.3–75.5	75.5	32
$\mu + 2\sigma$ 75.5–78.7	78.7	9
$\mu + 3\sigma$ 78.7–81.9	81.9	4

Within one standard deviation of the mean, 66% of the data is represented. Within two standard deviations of the mean, 94% of the data is represented. So the data is a normal distribution. The shape of the frequency polygon supports this conclusion.

b) $\mu = 72.25$, median = 72, mode = 73
The three measures of central tendency are approximately the same and the standard deviation is the same for each measure, so the golf scores seem to be a normal distribution.

10. $46 = \mu + 2\sigma$
So for a dolphin to live 46 years or more, their age data would be in the interval $\mu + 2\sigma$ and greater. The interval greater than and equal to $\mu + 2\sigma$ represents 2.5% of the population.



Number of dolphins = percent $\times$ population
Number of dolphins = 0.025×130 dolphins
Number of dolphins = 3.25 dolphins
You can expect 3 dolphins to live to 46 years and older.

11. a) Mean, $\mu = 71.8$ kg
Standard deviation, $\sigma = 13.6$ kg
It is given that 95% of the masses are represented within two standard deviations of the mean mass.

Interval	Mass (kg)
$\mu - 2\sigma$	44.6
$\mu + 2\sigma$	99.0

Julie should consider a range of 44.6 kg to 99.0 kg in her design.

b) It is given that 99.7% of the masses are represented within three standard deviations of the mean mass.

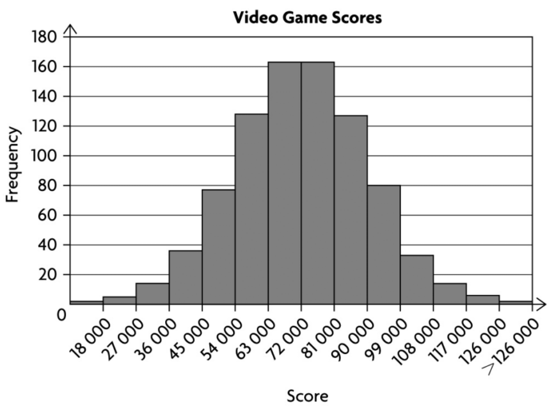
Interval	Mass (kg)
$\mu - 3\sigma$	31.0
$\mu + 3\sigma$	112.6

Julie should consider a range of 31.0 kg to 112.6 kg in her design.

c) The answers to parts a) and b) are valid if the mass data is a normal distribution for separate gender groups. But owing to differences between the mean masses of men and women, the answers to part a) and b) are not valid for the combined gender groups. The group data should be separated by gender and a separate analysis on each group should be performed.

12. a) e.g., Since I do not know the midpoint scores for the first and last intervals, I will use the same interval width as for the other intervals to determine the mean and standard deviation.

Midpoint Score	Frequency
13 500	2
22 500	5
31 500	14
40 500	36
49 500	77
58 500	128
67 500	163
76 500	163
85 500	127
94 500	80
103 500	33
112 500	14
121 500	6
130 500	2



Yes. The graph is bell shaped. The data appears to have a normal distribution.

b) μ = 72 010 points

σ = 18 394 points

Interval	Range of Scores	Frequency
$\mu - 3\sigma$	16 828–35 222	21
$\mu - 2\sigma$	35 222–53 616	113
$\mu - 1\sigma$	53 616–72 010	291
$\mu + 1\sigma$	72 010–90 404	290
$\mu + 2\sigma$	90 404–108 798	113
$\mu + 3\sigma$	108 798–127 192	22

Within one standard deviation of the mean, about 68% of the data is represented. Within two standard deviations of the mean, about 95% of the data is represented. Within three standard deviations of the mean, about 99.5% of the data is represented. So the data is a normal distribution and my answer to part a) is valid.

13. Mean, μ = 11.2 kg

Standard deviation, σ = 2.8 kg

a) 8.4 kg = $\mu - 1\sigma$

14 kg = $\mu + 1\sigma$

The interval between $\mu - 1\sigma$ and $\mu + 1\sigma$ represents 68% of the population.

Number of dogs = percent $\times$ population

Number of dogs = 0.68 $\times$ 60 dogs

Number of dogs = 40.8 dogs

The number of dogs is 41.

b) 5.6 kg = $\mu - 2\sigma$

16.8 kg = $\mu + 2\sigma$

The interval between $\mu - 2\sigma$ and $\mu + 2\sigma$ represents 95% of the population.

Number of dogs = percent $\times$ population

Number of dogs = 0.95 $\times$ 60 dogs

Number of dogs = 57 dogs

The number of dogs is 57.

c) 2.8 kg = $\mu - 3\sigma$

19.6 kg = $\mu + 3\sigma$

The interval between $\mu - 3\sigma$ and $\mu + 3\sigma$ represents 99.7% of the population.

Number of dogs = percent $\times$ population

Number of dogs = 0.997 $\times$ 60 dogs

Number of dogs = 59.8 dogs

The number of dogs is 60.

d) Less than 11.2 kg represents the intervals below the mean or 50% of the population.

So 30 dogs will have a mass less than 11.2 kg.

14. If the data is normally distributed, you can use the midpoint of the range of values as the mean.

$$\mu = \frac{\text{greatest mass} - \text{least mass}}{2}$$

$$\mu = \frac{533 \text{ kg} + 431 \text{ kg}}{2}$$

μ = 482 kg

Range = greatest mass – least mass

Range = 533 kg – 431 kg

Range = 51 kg

Then 51 kg must be distributed over 4 equal intervals or standard deviations for the entire data distribution.

$$\sigma = \frac{51 \text{ kg}}{4}$$

σ = 17 kg

15. e.g., For the smaller sample to be normally distributed, the number of selected marks from various intervals within the population must correspond proportionally to the frequency of those numbers in the same intervals for the entire population. The top ten or bottom ten students could have been selected for the sample.

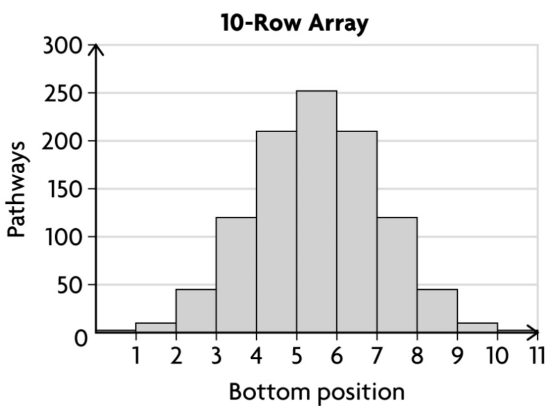
16. e.g., No. If the data is normally distributed, then a mass of 78.9 kg for a male dog means this mass is 10 standard deviations above the mean. A mass of 29.9 kg is 13 standard deviations below the mean. It is highly unlikely the information is truthful.

Applying Problem-Solving Strategies, page 254

A. 15
B., C. Create a table.

Row	Number of Pegs	Number of Ways
1	1	2
2	3	4
3	6	8
4	10	16
5	15	32
6	21	64

D., E. e.g., The number in the third column doubles as you move down the rows. Algebraically, the number of ways is 2^{10} or 1024.
F. e.g., Balls dropped onto an array (given sufficient pegs) will form the following symmetrical distribution: 1, 10, 45, 120, 210, 252, 210, 120, 45, 10, 1. This is approximately a normal distribution.



Lesson 5.5: *Z*-Scores, page 264

1. $z = \frac{x - \mu}{\sigma}$

a) $\mu = 112$, $\sigma = 15.5$, $x = 174$

$z = \frac{174 - 112}{15.5}$

$z = 4$
The z-score is 4.

b) $\mu = 53.46$, $\sigma = 8.24$, $x = 47.28$

$z = \frac{47.28 - 53.46}{8.24}$

$z = -0.75$
The z-score is –0.75.

c) $\mu = 82$, $\sigma = 12.5$, $x = 58$

$z = \frac{58 - 82}{12.5}$

$z = -1.92$
The z-score is –1.92.

d) $\mu = 245$, $\sigma = 22.4$, $x = 300$

$z = \frac{300 - 245}{22.4}$

$z = 2.455\ldots$
The z-score is 2.455….

2. a) $z = 1.24$
The z-score of 1.24 corresponds to a 0.8925.
This means 89.25% of the scores are to the left of the z-score.

b) $z = -2.35$
A z-score of –2.35 corresponds to 0.0094.
This means 0.94% of the scores are to the left of the z-score.

c) $z = 2.17$
A z-score of 2.17 corresponds to 0.9850.
This means 98.50% of the scores are to the left of the z-score.

d) $z = -0.64$
A z-score of –0.64 corresponds to 0.2611.
This means 26.11% of the scores are to the left of the z-score.

3. a) For $z = -2.88$, the percentage is 0.20%.
For $z = -1.47$, the percentage is 7.08%.
The percent of data between these scores is 6.88%.

b) For $z = -0.85$, the percentage is 19.77%.
For $z = 1.64$, the percentage is 94.95%.
The percent of data between these scores is 75.18%.

4. a) 10% of the data to the left of the z-score
= 0.10 of the data to the left of the z-score
Using a calculator, 0.10 corresponds to a z-score of –1.28.

b) 10% of the data to the right of the z-score
= 0.90 of the data to the left of the z-score
Using the z-score table, 0.90 corresponds to a z-score
of 1.28.

c) 60% of the data is below the z-score
= 0.60 of the data to the left of the z-score
Using the z-score table, 0.60 corresponds to a z-score
of 0.25.

d) 60% of the data is above the z-score
= 0.40 of the data to the left of the z-score
Using the z-score table, 0.40 corresponds to a z-score
of –0.25.

5. $z = \frac{x - \mu}{\sigma}$

a) $\mu = 24$, $\sigma = 2.8$, $x = 29.3$

$z = \frac{29.3 - 24}{2.8}$

$z = 1.892\ldots$
The z-score is 1.892….