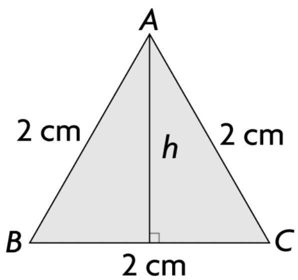
D. To solve this puzzle using trigonometry, I decided to arbitrarily set the length of the sides of $\triangle ABC$ to 2 cm.
To determine the area of $\triangle ABC$, I drew a height on the triangle and determined the height using a primary trigonometric ratio.

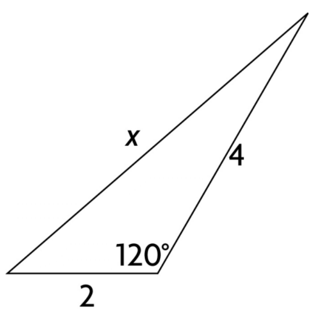
$$\begin{aligned} \sin 60° &= \frac{h}{2} \\ 2\sin 60° &= h \\ 1.732 &= h \end{aligned}$$

Area of $\triangle ABC$ = $\frac{1}{2}(2)(1.732...)$ or 1.732…



To determine the area of $\triangle DEF$, I must first determine a side length. I used the cosine law.

$x^2 = 2^2 + 4^2 - 2(2)(4)\cos 120°$
$x = 5.291…$cm

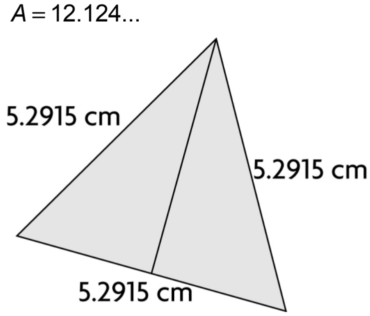


To determine the area of $\triangle DEF$, I drew a height on the triangle and determined the height using the sine ratio.

$$\begin{aligned} \sin 60° &= \frac{h}{5.291...} \\ (5.291...)\sin 60° &= h \\ 4.582... &= h \end{aligned}$$

Area of $\triangle DEF$, A:

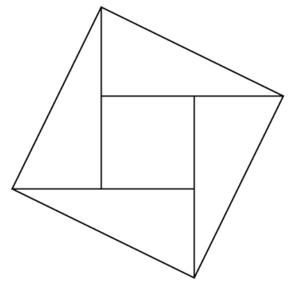
$$A = \frac{1}{2}(2)(5.291...)(4.582...)$$
$$A = 12.124...$$



To determine how many $\triangle ABC$s fit into $\triangle DEF$, I divided.

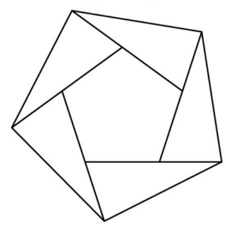
$$\frac{12.124...}{1.732...} = 7$$

E. Similar puzzle: Start with a square. Extend the side lengths to make the following shape:

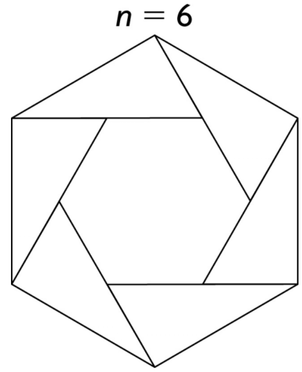


This puzzle can be solved by inspection. Five small squares fit into the larger square. However, this puzzle becomes more challenging as the degree of the polygon increases.

$n = 5$



$n = 6$



Note: There is a truly marvellous demonstration of these solutions, but the Solution Manual is not large enough to include it.

Lesson 4.4: Solving Problems Using Obtuse Triangles, page 193

1. a) i) sine law
ii) sine law
iii) cosine law
b) i)

$$\frac{x}{\sin(180° - 14° - 30°} = \frac{18}{\sin 14°}$$
$$\sin 136°\left(\frac{c}{\sin 136°}\right) = \left(\frac{18}{\sin 14°}\right)\sin 136°$$
$$c = 51.7...\text{ m}$$

ii) $\dfrac{\sin\theta}{15} = \dfrac{\sin 20°}{8}$

$15\left(\dfrac{\sin\theta}{15}\right) = \left(\dfrac{\sin 20°}{8}\right)15$

$\sin\theta = 0.6412...$

$\theta = \sin^{-1}(0.6412...)$

$\theta = 39.8879...°$

However, the angle is obtuse.
$180° - 39.8879…° = 140.1120…°$
$\theta = 140.1°$

iii) $\cos\alpha = \dfrac{1.0^2 + 0.9^2 - 1.3^2}{2(1.0)(0.9)}$

$\cos\alpha = 0.0666…$

$\alpha = \cos^{-1}(0.666…)$

$\alpha = 86.2°$

c) e.g., Our strategies were the same.

2. Since the angle from due south to AB at tower A is 80°, in the triangle $\angle A = 50°$. By alternate interior angles, the angle from due north to AB at tower B is 80°.
In the triangle,
$\angle B = 80° - 60°$
$\angle B = 20°$
If the fire is at F, then:
$\angle F = 180° - 50° - 20°$
$\angle F = 110°$

$\dfrac{b}{\sin 20°} = \dfrac{20.3}{\sin 110°}$

$\sin 20°\left(\dfrac{b}{\sin 20°}\right) = \left(\dfrac{20.3}{\sin 110°}\right)\sin 20°$

$b = 7.388...\text{ km}$

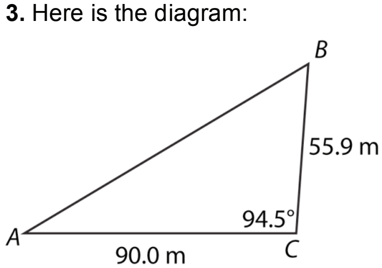
$\dfrac{a}{\sin 50°} = \dfrac{20.3}{\sin 110°}$

$\sin 50°\left(\dfrac{a}{\sin 50°}\right) = \left(\dfrac{20.3}{\sin 110°}\right)\sin 50°$

$a = 16.548...\text{ km}$

The fire is 7.4 km from tower A and 16.5 km from tower B.

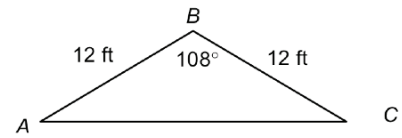
3. Here is the diagram:



$c^2 = a^2 + b^2 - 2ab\cos C$
$c^2 = 55.9^2 + 90.0^2 - 2(55.9)(90.0)\cos 94.5°$
$c = \sqrt{12014.265...}$
$c = 109.609…\text{ m}$
The distance from the top of the tower to the tip of the shadow is 110 m.

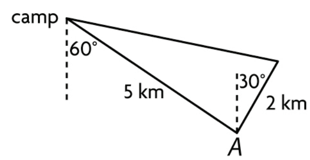
4. The angle in a regular polygon is:

$\dfrac{(5-2)180°}{5} = 108°$



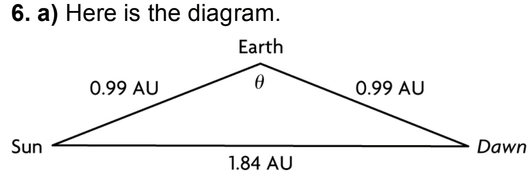
$b^2 = 12^2 + 12^2 - 2(12)(12)\cos(108°)$
$b = \sqrt{376.996...}$
$b = 19.416…\text{ ft}$
The longest plank will be about 19.4 ft long.

5. a) e.g. Bijan is 5.5 km from camp and should still be in range.



b) Let A be the point at which Bijan changed trails. The right part of $\angle A$ measures 30°. The left part of $\angle A$ measures 60° because of alternate interior angles. $\angle A$ is a right angle.
$a^2 = 5^2 + 2^2$
$a = \sqrt{29}$
$a = 5.385…\text{ km}$
Bijan is 5.4 km from camp. He is in range because he is less than 6 km away.

6. a) Here is the diagram.



b) $\cos\theta = \dfrac{0.99^2 + 0.99^2 - 1.84^2}{2(0.99)(0.99)}$

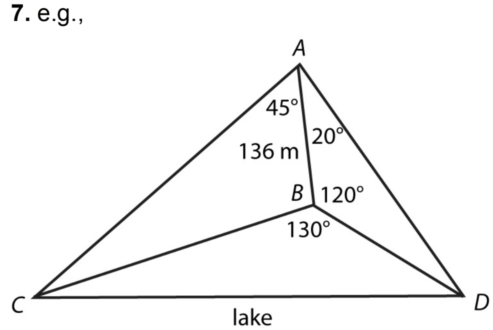
$\cos\theta = -0.7271…$

$\theta = \cos^{-1}(-0.7271…)$

$\theta = 136.6497…°$

The angle between the sight lines from Earth to Dawn and the Sun was 137°.

7. e.g.,

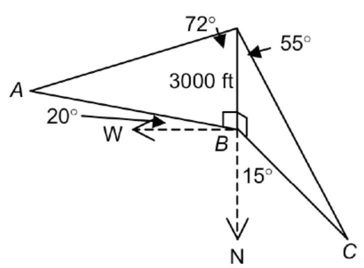


Consider $\triangle ABC$.
$\angle B = 110°$ because the angles around a point must total 360°.

$\angle C = 180° - 45° - 110°$
$\angle C = 25°$
Consider $\triangle ABD$.
$\angle D = 180° - 20° - 120°$
$\angle D = 40°$

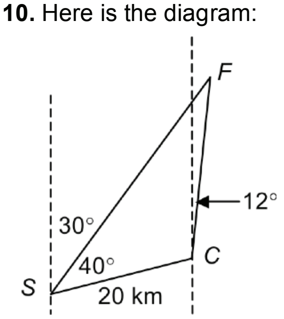
$$\frac{AD}{\sin 120°} = \frac{136}{\sin 40°}$$
$$\sin 120°\left(\frac{AD}{\sin 120°}\right) = \left(\frac{136}{\sin 40°}\right)\sin 120°$$
$$AD = 183.232...\text{ m}$$

Consider $\triangle ACD$.
$CD^2 = AC^2 + AD^2 - 2(AC)(AD)\cos 65°$
$CD = \sqrt{78\ 184.080...}$
$CD = 279.614…$ m
The lake is 280 m long.
8. e.g.,



$$\tan 72° = \frac{AB}{3000}$$
$$3000 \tan 72° = AB$$
$$9233.050…\text{ ft} = AB$$
$$\tan 55° = \frac{BC}{3000}$$
$$3000 \tan 55° = BC$$
$$4284.444…\text{ ft} = BC$$

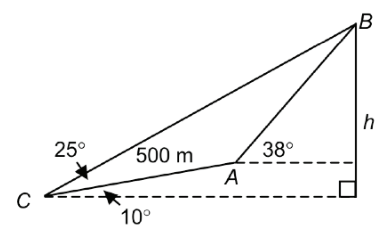
Consider $\triangle ABC$.
$\angle B = 20° + 90° + 15°$
$\angle B = 125°$
$AC^2 = AB^2 + BC^2 - 2(AB)(BC)\cos 125°$
$AC = \sqrt{148\ 985\ 317.859...}$
$AC = 12\ 205.954…$ ft
The fires are about 12 206 ft apart.
9. e.g. Yes. Bert can use the sine law and the triangle on the ground to determine the distance from A to the base of the tree. Then he can use the tangent ratio and the 28° angle to determine the height of the tree.
10. Here is the diagram:



By complementary interior angles the angle at C between SC and vertical is 110°.
In $\triangle SCF$,
$\angle C = 122°$.
$\angle F = 180° - 40° - 122°$
$\angle F = 18°$

$$\frac{FS}{\sin 122°} = \frac{20}{\sin 18°}$$
$$\sin 122°\left(\frac{FS}{\sin 122°}\right) = \left(\frac{20}{\sin 18°}\right)\sin 122°$$
$$FS = 54.886...\text{ km}$$

54.8868… – 41.6020… = 13.2843… km
Smith's Falls fire department is closer to the fire by 13 km.
11. Here is the diagram.

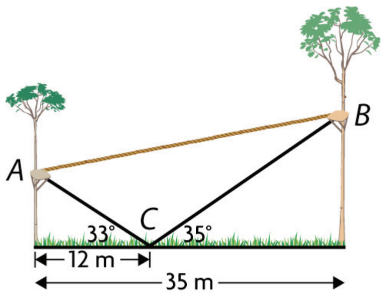


By complementary interior angles the angle at A below the horizontal measures 170°. In $\triangle ABC$, $\angle A = 152°$ because the angles around a point must total 360°.
Consider $\triangle ABC$.
$\angle B = 180° - 25° - 152°$
$\angle B = 3°$

$$\frac{BC}{\sin 152°} = \frac{500}{\sin 3°}$$
$$\sin 152°\left(\frac{BC}{\sin 152°}\right) = \left(\frac{500}{\sin 3°}\right)\sin 152°$$
$$BC = 4485.172...\text{ m}$$
$$\sin 35° = \frac{h}{4485.172...}$$

$(4485.172…) \sin 35° = h$
$2572.589…\text{ m} = h$
1834 + 2572.589… = 4406.589… m
Mount Logan is 4410 m tall.
12. e.g. I used a 3-D diagram that was made up of two right triangles. The tangent ratio can be used determine the distance from Brit to the sailboat. This is the same distance that Tara is from the boat. I would then use the cosine law and the triangle with vertices Brit, Tara, and the sailboat, to determine the angle between the boat as see from Brit's position.

13. a) e.g., I assumed the point 12 m from the base of the smaller tree is between the two trees, and the angle of elevation to the smaller tree is 33°.



b) Using the smaller right triangle:

$$\cos 33° = \frac{12}{AC}$$

$$AC = \frac{12}{\cos 33°}$$

$$AC = 14.308\ldots \text{ m}$$

Using the larger right triangle:

$$\cos 35° = \frac{35 - 12}{BC}$$

$$BC = \frac{23}{\cos 35°}$$

$$BC = 28.077\ldots \text{ m}$$

Consider $\triangle ABC$,

$\angle C = 180° - 33° - 35°$

$\angle C = 112°$

$AB^2 = AC^2 + BC^2 - 2(AC)(BC)\cos 112°$

$AB = \sqrt{1294.087\ldots}$

$AB = 35.973\ldots$ m

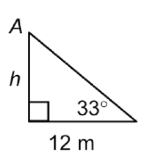
The zip line is about 36 m long.

14. Consider the smaller right triangle.

$$\tan 33° = \frac{h}{12}$$

$$12\tan 33° = h$$

$$7.792\ldots \text{ m} = h$$

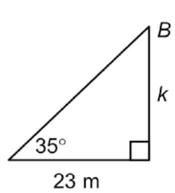


Now consider the larger right triangle.

$$\tan 35° = \frac{k}{23}$$

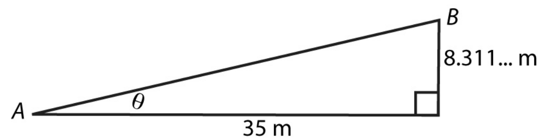
$$23\tan 35° = k$$

$$16.104\ldots \text{ m} = k$$



$k - h = 16.104\ldots - 7.792\ldots$

$k - h = 8.311\ldots$m



$$\tan\theta = \frac{8.311\ldots}{35}$$

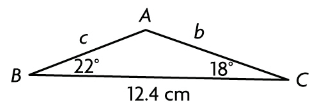
$$\tan\theta = 0.2374\ldots$$

$$\theta = \tan^{-1}(0.2374\ldots)$$

$$\theta = 13.3592\ldots°$$

The angle of depression from *B* to *A* is the same, θ, the angle of elevation from *A* to *B*. The angle of depression is about 13°.

15. e.g.,



$\angle A = 180° - 22° - 18°$

$\angle A = 140°$

$$\frac{c}{\sin 18°} = \frac{12.4}{\sin 140°}$$

$$\sin 18°\left(\frac{c}{\sin 18°}\right) = \left(\frac{12.4}{\sin 140°}\right)\sin 18°$$

$$c = 5.961\ldots \text{ m}$$

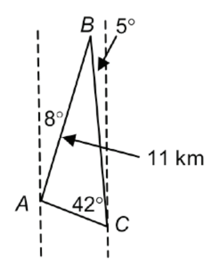
$$\frac{b}{\sin 22°} = \frac{12.4}{\sin 140°}$$

$$\sin 22°\left(\frac{b}{\sin 22°}\right) = \left(\frac{12.4}{\sin 140°}\right)\sin 22°$$

$$c = 7.226\ldots \text{ m}$$

b is about 7.2 cm. *c* is about 6.0 cm.

16. a) Here is the diagram:



The angle at *A* between *AC* and south is 47° by alternate interior angles.

Consider $\triangle ABC$.

$\angle A = 180° - 47° - 8°$

$\angle A = 125°$

$\angle B = 180° - 42° - 125°$

$\angle B = 13°$

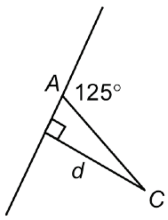
$$\frac{BC}{\sin 125°} = \frac{11}{\sin 42°}$$

$$\sin 125°\left(\frac{BC}{\sin 125°}\right) = \left(\frac{11}{\sin 42°}\right)\sin 125°$$

$$c = 13.466\ldots \text{ km}$$

The sailor is 4 km from lighthouse A and 13 km from lighthouse B.

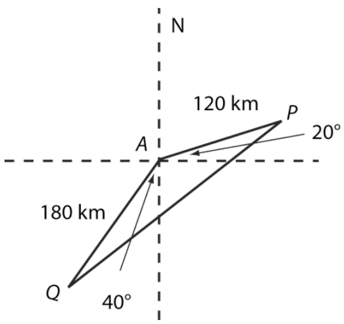
b) In the triangle, $\angle A$ = 55° because it is complementary to 125°.



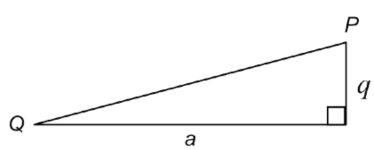
$$\sin 55° = \frac{d}{3.6980...}$$
$$(3.698\ldots) \sin 55° = d$$
$$3.029\ldots \text{ km} = d$$

The sailor is 3 km from the nearest point on shore.

17. Here is the diagram:



$\angle A = 90° + 40° + 20°$
$\angle A = 150°$
$a^2 = 180^2 + 120^2 - 2(180)(120) \cos 150°$
$a^2 = 84\ 212.297\ldots$ km
From the air:



$q = 2.7 - 1.8$
$q = 0.9$ km

$$QP^2 = a^2 + q^2$$
$$QP^2 = 84\ 212.297\ldots + 0.81$$
$$QP = \sqrt{84\ 213.107...}$$
$$QP = 290.194\ldots \text{ km}$$

The planes are 290.2 km apart.

Math in Action, page 197

Sample Solution
Plan for Measuring the Viewing Angle of a Screen

• We can walk around a television and mark on the floor the points at which the picture appears to degrade. There should be two points: one to the right of the screen and one to the left.
• The two points will make a triangle with the centre of the screen, so we will record the lengths of the sides of the triangle.
• We will be able to use the cosine law to solve for the angle at the screen.

Evaluation
The plan seemed to work, because we had a similar viewing angle to another group with the same television.

Chapter Self-Test, page 198

1. a)
$$\cos \alpha = 0.235$$
$$\alpha = \cos^{-1}(0.235)$$
$$\alpha = 76.4°$$

b)
$$\tan \alpha = 2.314$$
$$\alpha = \tan^{-1}(2.314)$$
$$\alpha = 66.6°$$

c)
$$\sin \alpha = 0.015$$
$$\alpha = \sin^{-1}(0.015)$$
$$\alpha = 0.9°$$

180.0° – 0.9° = 179.1°
$$\alpha = 0.9° \text{ or } 179.1°$$

d)
$$\cos \alpha = -\frac{3}{4}$$
$$\alpha = \cos^{-1}\left(-\frac{3}{4}\right)$$
$$\alpha = 138.6°$$

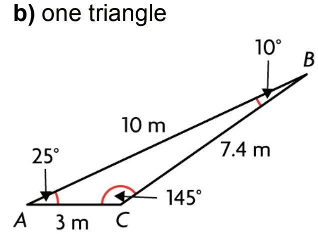
e)
$$\sin \alpha = \frac{1}{2}$$
$$\alpha = \sin^{-1}\left(\frac{1}{2}\right)$$
$$\alpha = 30.0°$$

180.0° – 30.0° = 150.0°
α = 30.0° or 150.0°

f)
$$\sin \alpha = 0.600$$
$$\alpha = \sin^{-1}(0.600)$$
$$\alpha = 36.9°$$

180.0° – 36.9° = 143.1°
$$\alpha = 36.9° \text{ or } 143.1°$$

2. a) No triangles will be formed because $e + d < f$.
b) one triangle



$a^2 = 3^2 + 10^2 - 2(3)(10) \cos(25°)$

$$\frac{AC}{\sin 13°} = \frac{11}{\sin 42°}$$
$$\sin 13°\left(\frac{AC}{\sin 13°}\right) = \left(\frac{11}{\sin 42°}\right)\sin 13°$$
$$AC = 3.698\ldots \text{ km}$$

$a = \sqrt{54.621...}$
$a = 7.390\ldots$ km
Since $\angle A$ is acute and $a > b$, there is one triangle possible.