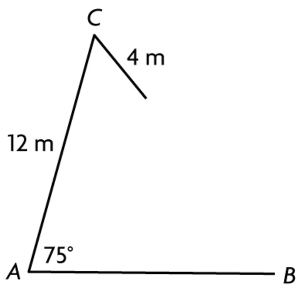
Lesson 4.3: The Ambiguous Case of the Sine Law, page 183

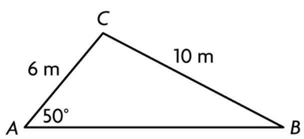
1. a)

$$\begin{aligned} \sin 75^\circ &= \frac{h}{12} \\ 12 \sin 75^\circ &= h \\ 11.591\ldots \text{ m} &= h \end{aligned}$$

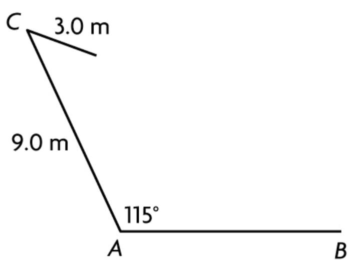
Since $\angle A$ is acute and $a < h$, there are zero possibilities.



b) Since $\angle A$ is acute and $a > b$, there is one possibility.



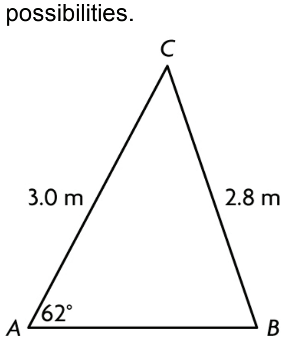
c) Since $\angle A$ is obtuse and $a < b$, there are zero possibilities.

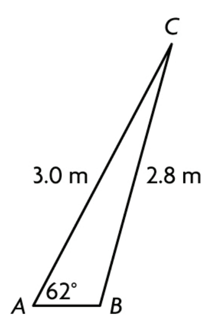


d)

$$\begin{aligned} \sin 62^\circ &= \frac{h}{3.0} \\ 3.0 \sin 62^\circ &= h \\ 2.648\ldots \text{ m} &= h \end{aligned}$$

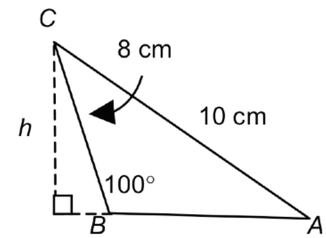
Since $\angle A$ is acute and $h < a < b$, there are two possibilities.





2. a) *SSA*
b) not *SSA*
c) *SSA*
d) not *SSA*
e) *SSA*
f) *SSA*

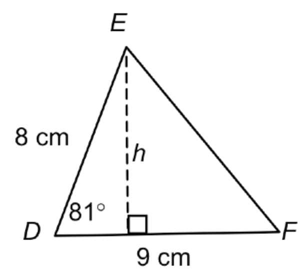
3. a) Here is the diagram:



$$\begin{aligned} 180^\circ - 100^\circ &= 80^\circ \\ \sin 80^\circ &= \frac{h}{8} \\ 8 \sin 80^\circ &= h \\ 7.878\ldots \text{ cm} &= h \end{aligned}$$

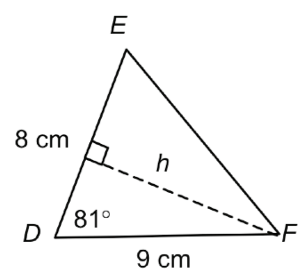
The height is 7.9 cm to the nearest tenth.
Since $\angle B$ is obtuse and $b > a$, there is one triangle.

b) Here is the diagram:



$$\begin{aligned} \sin 81^\circ &= \frac{h}{8} \\ 8 \sin 81^\circ &= h \\ 7.901\ldots \text{ cm} &= h \end{aligned}$$

The height is 7.9 cm to the nearest tenth.

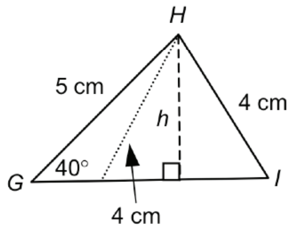


Or,

$$\begin{aligned} \sin 81^\circ &= \frac{h}{9} \\ 9 \sin 81^\circ &= h \\ 8.889\ldots \text{ cm} &= h \end{aligned}$$

The height is 8.9 cm to the nearest tenth.
There is no ambiguity with the sine law in this case, so there is only one triangle.

c) Here is the diagram:



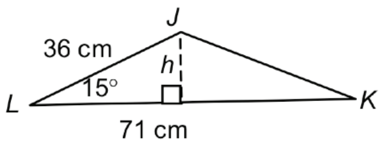
$$\sin 40° = \frac{h}{5}$$

$$5 \sin 40° = h$$

$$3.213\ldots \text{ cm} = h$$

The height is 3.2 cm to the nearest tenth.
Since $\angle G$ is acute and $h < g < i$, there are two possible triangles.

d) Here is the diagram:

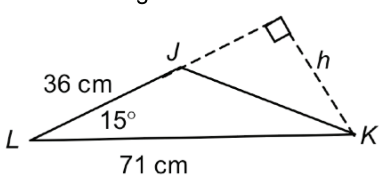


$$\sin 15° = \frac{h}{36}$$

$$36 \sin 15° = h$$

$$9.317\ldots \text{ cm} = h$$

The height is 9.3 cm to the nearest tenth.
Here is the diagram:



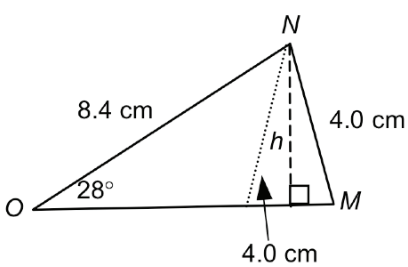
Or, $$\sin 15° = \frac{h}{71}$$

$$71 \sin 15° = h$$

$$18.376\ldots \text{ cm} = h$$

The height is 18.4 cm to the nearest tenth.
There is no ambiguity with the sine law in this case, so there is only one triangle.

e) Here is the diagram:



$$\sin 28° = \frac{h}{8.4}$$

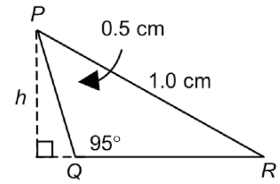
$$8.4 \sin 28° = h$$

$$3.943\ldots \text{ cm} = h$$

The height is 3.9 cm to the nearest tenth.

Since $\angle O$ is acute and $h < o < m$, there are two possible triangles.

f) Here is the diagram.



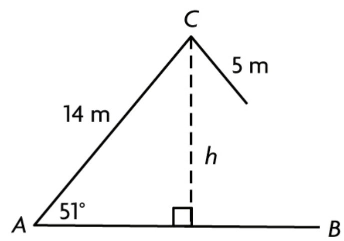
$$\sin 95° = \frac{h}{0.5}$$

$$0.5 \sin 95° = h$$

$$0.498\ldots \text{ cm} = h$$

The height is about 0.5 cm.
Since $\angle Q$ is obtuse and $q > r$, there is one triangle.

4. a) *SSA*, zero



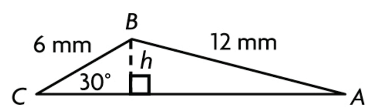
$$\sin 51° = \frac{h}{14}$$

$$14 \sin 51° = h$$

$$10.880\ldots \text{ m} = h$$

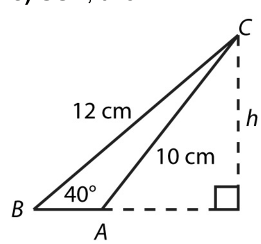
The height is about 11 m.
Since $\angle A$ is acute and $a < h$, no triangles are possible.

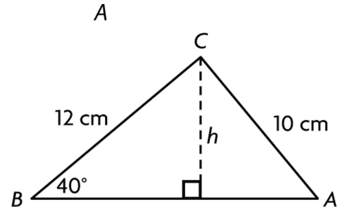
b) *SSA*, one



Since $\angle C$ is acute and $c > a$, one triangle is possible.

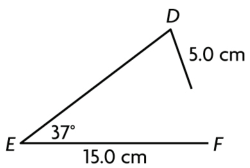
c) *SSA*, two



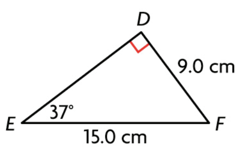


$$\sin 40° = \frac{h}{12}$$
$$12 \sin 40° = h$$
$$7.713\ldots \text{ m} = h$$
The height is about 8 cm.
Since $\angle B$ is acute and $h < b < a$, two triangles are possible.
d) not *SSA*

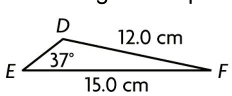
5. a) $\sin 37° = \dfrac{h}{15.0}$
$15.0 \sin 37° = h$
$9.027\ldots \text{ cm} = h$
The height is about 9.0 cm.
b) e.g., If *FD* = 5.0 cm, then $\angle E$ is acute and $FD < h$, so no triangles are possible.

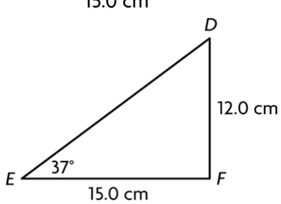


If *FD* = 9.0 cm, then $\angle E$ is acute and $FD = h$, so one right triangle is possible.

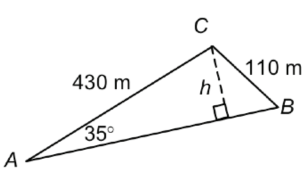


If *FD* = 12.0 cm, then $\angle E$ is acute and $h < FD < d$, so two triangles are possible.



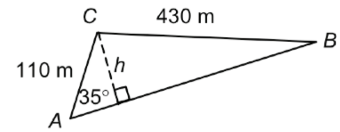


6. a) Suppose the side opposite the 35° angle is 110 m long.

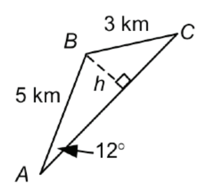


$$\sin 35° = \frac{h}{430}$$
$$430 \sin 35° = h$$
$$246.637\ldots \text{ cm} = h$$
Since the angle is acute and $a < h < b$, no triangles are possible.

The side opposite the 35° angle must be 430 m.



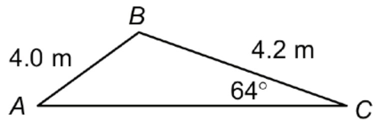
Since 430 > 110, one triangle is possible.
$$\frac{\sin B}{110} = \frac{\sin 35°}{430}$$
$$110\left(\frac{\sin B}{110}\right) = \left(\frac{\sin 35°}{430}\right)110$$
$$\sin B = 0.1467...$$
$$\angle B = \sin^{-1}(0.1467...)$$
$$\angle B = 8.437...°$$
$\angle C = 180° - 35° - 8.437\ldots°$
$\angle C = 136.562\ldots°$
$b^2 = 110^2 + 430^2 - 2(110)(430)\cos(136.5625\ldots°)$
$b = \sqrt{265\ 691.407...}$
$b = 515.452\ldots$ m
The third side of the property is 515 m long.
b) e.g. The property is triangular, with one side that is 110 m and another that is 430 m. The angle opposite the side that is 430 m is 35°.
7. a) Here is the diagram:



$$\frac{\sin C}{5} = \frac{\sin 12°}{3}$$
$$5\left(\frac{\sin C}{5}\right) = \left(\frac{\sin 12°}{3}\right)5$$
$$\sin C = 0.3465...$$
$$\angle C = \sin^{-1}(0.3465...)$$
$$\angle C = 20.2745...°$$
$\angle B = 180° - 12° - 20.2745\ldots°$
$\angle B = 147.7254\ldots°$
$b^2 = 3^2 + 5^2 - 2(3)(5)\cos(147.7254\ldots°)$
$b = \sqrt{59.364...}$
$b = 7.704\ldots$ km
The *Raven's Song* is about 7.7 km from the dock.

b) $\sin 12° = \dfrac{h}{5}$
$5 \sin 12° = h$
$1.0395\ldots \text{ km} = h$
Since $\angle A$ is acute, and $h < a < b$, there are two possible triangles. From the buoy, the canoe could have travelled 3 km toward the shore (acute angle) or away from the shore (obtuse angle).

8. a) Here is a sample diagram.



$$\frac{\sin A}{4.2} = \frac{\sin 64.0°}{4.0}$$

$$4.2\left(\frac{\sin A}{4.2}\right) = \left(\frac{\sin 64.0°}{4.0}\right)4.2$$

$$\sin A = 0.9437...$$

$$\angle A = \sin^{-1}(0.9437...)$$

$$\angle A = 70.6883...°$$

180° – 70.6883…° = 109.3116…°

$\angle A$ = 70.6883…° or $\angle A$ = 109.3116…°

If $\angle A$ = 70.6883…°, then:

$\angle B$ = 180° – 70.6883…° – 64.0°

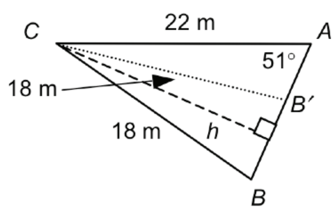
$\angle B$ = 45.3116…°

This means the triangle is acute, not obtuse.

$\angle A$ = 109.3116…° must be true; the obtuse angle measures 109.3°.

b) e.g., Yes, there is only one possible answer. The 4.2 cm side must be opposite the obtuse angle. If it is not, then the triangle is an acute triangle.

9. Here is a diagram.



$$\sin 51° = \frac{h}{22}$$

$$22 \sin 51° = h$$

$$17.097...\text{ m} = h$$

Since $\angle A$ is acute and $h < a < b$, there are two possible triangles, one acute and one obtuse.

$$\frac{\sin B}{22} = \frac{\sin 51°}{18}$$

$$22\left(\frac{\sin B}{22}\right) = \left(\frac{\sin 51°}{18}\right)22$$

$$\sin B = 0.9498...$$

$$\angle B = \sin^{-1}(0.9498...)$$

$$\angle B = 71.7767...°$$

$\angle B'$ = 180° – 71.7767…°

$\angle B'$ = 108.2232…°

In $\triangle ABC$:

$\angle C$ = 180° – 71.7767…° – 51°

$\angle C$ = 57.2232…°

$c^2 = 18^2 + 22^2 - 2(18)(22)\cos(57.2232…°)$

$c = \sqrt{379.237...}$

c = 19.474… m

In $\triangle AB'C$:

$\angle C$ = 180° – 108.2232…° – 51°

$\angle C$ = 20.7767…°

$c^2 = 18^2 + 22^2 - 2(18)(22)\cos(20.7767…°)$

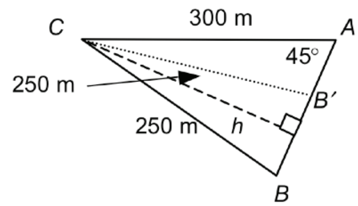
$c = \sqrt{67.503...}$

c = 8.216… m

The support post could be fastened 8.2 m down the hill or 19.5 m down the hill.

e.g., I would recommend 19.5 m so the support post would be more vertical.

10. Here is the diagram:



$$\sin 45° = \frac{h}{300}$$

300 sin 45° = h

212.132… m = h

Since $\angle A$ is acute and $h < a < b$, there are two possible triangles.

$$\frac{\sin B}{300} = \frac{\sin 45°}{250}$$

$$300\left(\frac{\sin B}{300}\right) = \left(\frac{\sin 45°}{250}\right)300$$

$$\sin B = 0.8485...$$

$$\angle B = \sin^{-1}(0.8485...)$$

$$\angle B = 58.0519...°$$

$\angle B'$ = 180° – 58.0519…°

$\angle B'$ = 121.9480…°

In $\triangle ABC$:

$\angle C$ = 180° – 58.0519…° – 45°

$\angle C$ = 76.9480…°

$c^2 = 300^2 + 250^2 - 2(300)(250)\cos(76.9480…°)$

$c = \sqrt{118.624...}$

c = 344.419… m

In $\triangle AB'C$:

$\angle C$ = 180° – 121.9480…° – 45°

$\angle C$ = 13.0519…°

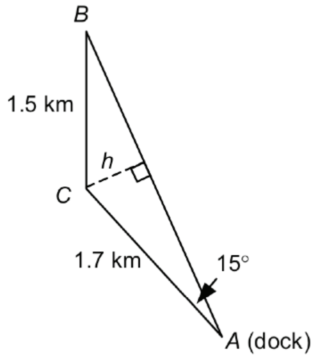
$c^2 = 300^2 + 250^2 - 2(300)(250)\cos(13.0519…°)$

$c = \sqrt{6375.139...}$

c = 79.844… m

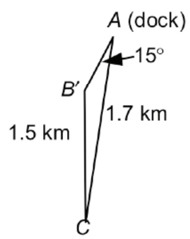
The farmer must walk either 80 m or 344 m.

11. Here is the diagram:



$$\sin 15° = \frac{h}{1.7}$$
$$1.7 \sin 15° = h$$
$$0.439\ldots \text{ km} = h$$

Since $\angle A$ is acute and $h < a < b$, there are two possible triangles. The 1.5 km side runs north-south, so the other possible triangle looks like this:



$$\frac{\sin B}{1.7} = \frac{\sin 15°}{1.5}$$
$$1.7\left(\frac{\sin B}{1.7}\right) = \left(\frac{\sin 15°}{1.5}\right)1.7$$
$$\sin B = 0.2933...$$
$$\angle B = \sin^{-1}(0.2933...)$$
$$\angle B = 17.0573...°$$

$\angle B' = 180° - 17.0573\ldots°$

$\angle B' = 162.9426\ldots°$

In $\triangle ABC$:

$\angle C = 180° - 17.0573\ldots° - 15°$

$\angle C = 147.9426\ldots°$

$c^2 = 1.5^2 + 1.7^2 - 2(1.5)(1.7)\cos(147.9426\ldots°)$

$c = \sqrt{9.462...}$

$c = 3.076\ldots$ km

In $\triangle AB'C$:

$\angle C = 180° - 162.9426\ldots° - 15°$

$\angle C = 2.0573\ldots°$

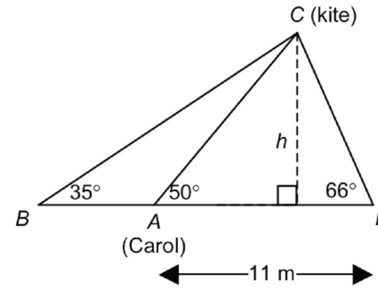
$c^2 = 1.5^2 + 1.7^2 - 2(1.5)(1.7)\cos(2.0573\ldots°)$

$c = \sqrt{0.432...}$

$c = 0.208\ldots$ km

The kayak leg of the race is either 3.1 km or 0.2 km.

12. Case 1:



$d = 11.180\ldots$ m and $h = 8.564\ldots$ m in $\triangle ACD$.

$\angle CAB = 130°$ and $\angle BCA = 15°$ in $\triangle ABC$.

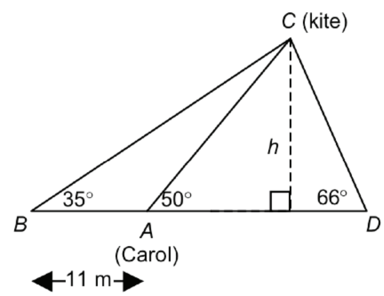
$$\frac{c}{\sin 15°} = \frac{11.180...}{\sin 35°}$$
$$\sin 15°\left(\frac{c}{\sin 15°}\right) = \left(\frac{11.180...}{\sin 35°}\right)\sin 15°$$
$$c = 5.045... \text{ m}$$

a) The kite is 9 m above the ground.

b) The string is 11 m long.

c) The second girl is 5 m from Carol.

Case 2:



$b = 24.377\ldots$ m and $h = 18.674\ldots$ m.

In $\triangle ACD$:

$\angle ACD = 180° - 50° - 66°$

$\angle ACD = 64°$

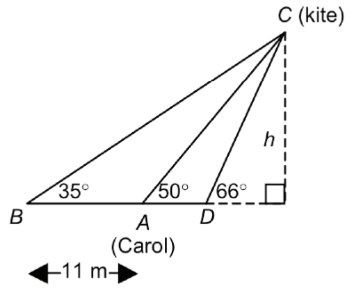
$$\frac{c}{\sin 64°} = \frac{24.377...}{\sin 66°}$$
$$\sin 64°\left(\frac{c}{\sin 64°}\right) = \left(\frac{24.377...}{\sin 66°}\right)\sin 64°$$
$$b = 23.983... \text{ m}$$

a) The kite is 19 m above the ground.

b) The string is 24 m long.

c) The second girl is 24 m from Carol.

Case 3:



$\angle BAC$ = 130° and $\angle BCA$ = 15°.

$$\frac{b}{\sin 35°} = \frac{11}{\sin 15°}$$

$$\sin 35°\left(\frac{b}{\sin 35°}\right) = \left(\frac{11}{\sin 15°}\right)\sin 35°$$

$$b = 24.377... \text{ m}$$

$$\sin 50° = \frac{h}{24.377...}$$

$$(24.377\ldots) \sin 50° = h$$

$$18.674\ldots \text{ m} = h$$

From $\triangle ACD$ in Case 3, $\angle D$ = 114° and $\angle ACD$ = 16°.

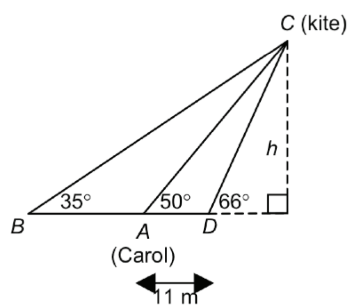
$$\frac{c}{\sin 16°} = \frac{24.377...}{\sin 114°}$$

$$\sin 16°\left(\frac{c}{\sin 16°}\right) = \left(\frac{24.377...}{\sin 114°}\right)\sin 16°$$

$$b = 7.355... \text{ m}$$

a) The kite is 19 m above the ground.
b) The string is 24 m long.
c) The second girl is 7 m from Carol.

Case 4:



From $\triangle ACD$ in Case 4, d = 36.457… m and h = 27.927… m.

In $\triangle ABC$:

$\angle BAC$ = 180° – 50°

$\angle BAC$ = 130°

$\angle BCA$ = 180° – 130° – 35°

$\angle BCA$ = 15°

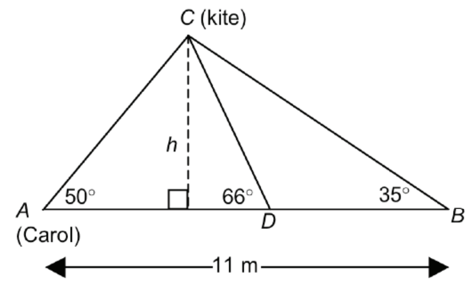
$$\frac{c}{\sin 15°} = \frac{36.457...}{\sin 35°}$$

$$\sin 15°\left(\frac{c}{\sin 15°}\right) = \left(\frac{36.457...}{\sin 35°}\right)\sin 15°$$

$$c = 16.450.. \text{ m}$$

a) The kite is 28 m above the ground.
b) The string is 36 m long.
c) The second girl is 16 m from Carol.

Case 5:



From Case 1, $\angle ACB$ = 95° in $\triangle ABC$.

$$\frac{b}{\sin 35°} = \frac{11}{\sin 95°}$$

$$\sin 35°\left(\frac{b}{\sin 35°}\right) = \left(\frac{11}{\sin 95°}\right)\sin 35°$$

$$q = 6.333... \text{ m}$$

$$\sin 50° = \frac{h}{6.333...}$$

$$(6.333\ldots) \sin 50° = h$$

$$4.851\ldots \text{ m} = h$$

From Case 1, $\angle ACD$ = 64° in $\triangle ACD$.

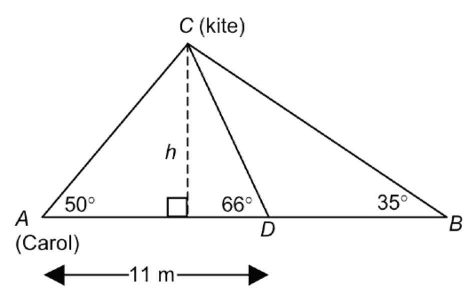
$$\frac{c}{\sin 64°} = \frac{6.333...}{\sin 66°}$$

$$\sin 64°\left(\frac{c}{\sin 64°}\right) = \left(\frac{6.333...}{\sin 66°}\right)\sin 64°$$

$$c = 6.231... \text{ m}$$

a) The kite is 5 m above the ground.
b) The string is 6 m long.
c) The second girl is 6 m from Carol.

Case 6:



In $\triangle ACD$:

$\angle ACD$ = 180° – 50° – 66°

$\angle ACD$ = 64°

$$\frac{d}{\sin 66°} = \frac{11}{\sin 64°}$$

$$\sin 66°\left(\frac{d}{\sin 66°}\right) = \left(\frac{11}{\sin 64°}\right)\sin 66°$$

$$d = 11.180... \text{ m}$$

$$\sin 50° = \frac{h}{11.180}$$

$(11.180\ldots) \sin 50° = h$

$8.564\ldots \text{ m} = h$

In $\triangle ABC$:

$\angle C = 180° - 50° - 35°$

$\angle C = 95°$

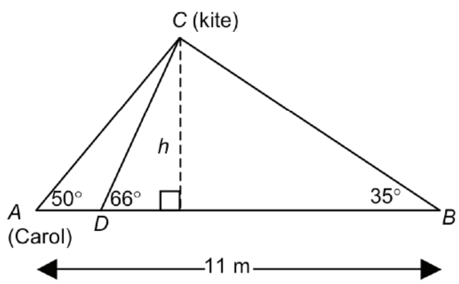
$$\frac{c}{\sin 95°} = \frac{11.180}{\sin 35°}$$

$$\sin 95°\left(\frac{c}{\sin 95°}\right) = \left(\frac{11.180}{\sin 35°}\right)\sin 95°$$

$$c = 19.418\ldots \text{ m}$$

a) The kite is 9 m above the ground.
b) The string is 11 m long.
c) The second girl is 19 m from Carol.

Case 7:



From $\triangle ABC$ in Case 2, $b = 6.333\ldots$ m, and $h = 4.851\ldots$ m

In $\triangle ACD$:

$\angle ADC = 180° - 66°$

$\angle ADC = 114°$

$\angle ACD = 180° - 50° - 114°$

$\angle ACD = 16°$

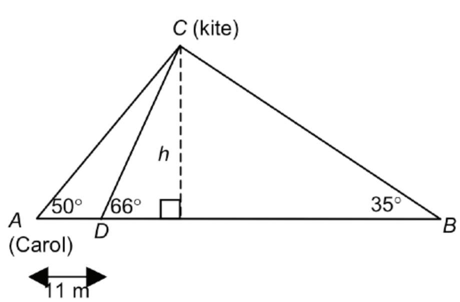
$$\frac{c}{\sin 16°} = \frac{6.333...}{\sin 114°}$$

$$\sin 16°\left(\frac{c}{\sin 16°}\right) = \left(\frac{6.333...}{\sin 114°}\right)\sin 16°$$

$$c = 1.910\ldots \text{ m}$$

a) The kite is 5 m above the ground.
b) The string is 6 m long.
c) The second girl is 2 m from Carol.

Case 8:



From $\triangle ACD$ in Case 3, $\angle ADC = 114°$ and $\angle ACD = 16°$.

$$\frac{d}{\sin 114°} = \frac{11}{\sin 16°}$$

$$\sin 114°\left(\frac{d}{\sin 114°}\right) = \left(\frac{11}{\sin 16°}\right)\sin 114°$$

$$d = 36.457\ldots \text{ m}$$

$$\sin 50° = \frac{h}{36.457...}$$

$(36.457\ldots) \sin 50° = h$

$27.927\ldots \text{ m} = h$

From the previous cases, $\angle ACB = 95°$ in $\triangle ABC$.

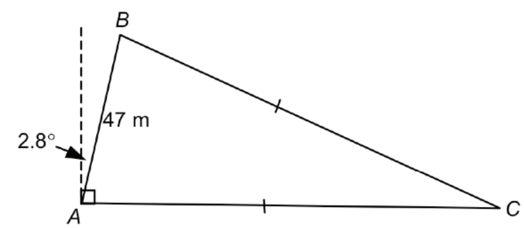
$$\frac{c}{\sin 95°} = \frac{36.457...}{\sin 35°}$$

$$\sin 95°\left(\frac{c}{\sin 95°}\right) = \left(\frac{36.457...}{\sin 35°}\right)\sin 95°$$

$$c = 63.319\ldots \text{ m}$$

a) The kite is 28 m above the ground.
b) The string is 36 m long.
c) The second girl is 63 m from Carol.

13. Here is the diagram:



In $\triangle ABC$:

$\angle A = 90.0° - 2.8°$

$\angle A = 87.2°$

Since $\triangle ABC$ is isosceles, $\angle B = 87.2°$.

$\angle C = 180.0° - 87.2° - 87.2°$

$\angle C = 5.6°$

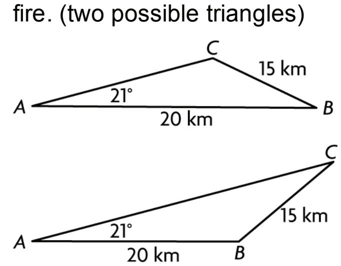
$$\frac{b}{\sin 87.2°} = \frac{47}{\sin 5.6°}$$

$$\sin 87.2°\left(\frac{b}{\sin 87.2°}\right) = \left(\frac{47}{\sin 5.6°}\right)\sin 87.2°$$

$$b = 481.066\ldots \text{ m}$$

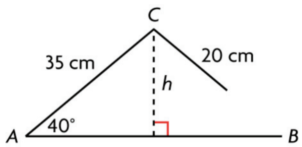
The point is 481 m from the base.

14. e.g., Two forest-fire stations, *A* and *B*, are 20 km apart. A ranger at station *B* sees a fire 15 km away. If the angle between the line *AB* and the line from *A* to the fire is 21°, determine how far station *A* is from the fire. (two possible triangles)



$\angle B$ can be acute or obtuse, two triangles are possible.

15. a) Here is the diagram:



$$\sin 40° = \frac{h}{35}$$
$$35 \sin 40° = h$$
$$22.497\ldots \text{ cm} = h$$

Since $\angle A$ is acute and $a < h$, there is no possible triangle.

b) Since $a < b$, the only way to get one possible triangle is if $\angle A$ is acute and $h = a$.

$$\sin A = \frac{20}{35}$$
$$\angle A = \sin^{-1}\left(\frac{20}{35}\right)$$
$$\angle A = 34.8499\ldots°$$

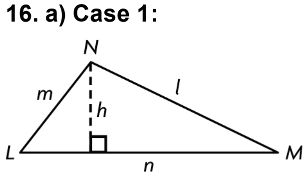
$\angle A$ must be less than 35° to the nearest degree.

c) Any acute angle where $h < 20$ cm, so that $h < a < b$.

e.g., If $h = 10$ cm, then

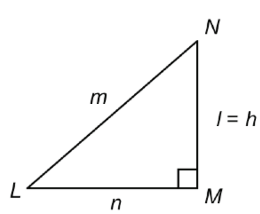
$$\sin A = \frac{10}{35}$$
$$\angle A = \sin^{-1}\left(\frac{10}{35}\right)$$
$$\angle A = 17°$$

16. a) Case 1:



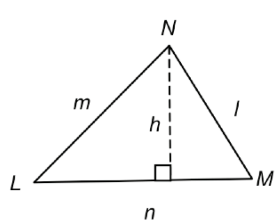
$l > m$, $\frac{\sin L}{l} = \frac{\sin M}{m}$, $h = m \sin L$

Case 2:

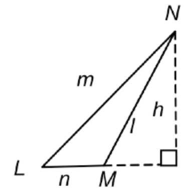


$l < m$, $l = h$, $l = m \sin L$

b) Here is the diagram:

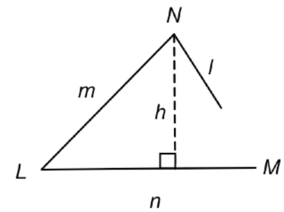


$h < m < l$, $h = m \sin L$



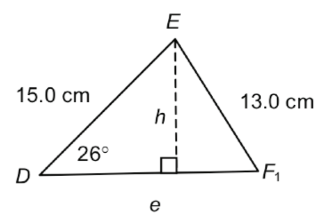
$h < l < m$, $h = m \sin L$

c) Here is the diagram:



$l < h$, $h = m \sin L$

17. a) e.g.,



$$\frac{\sin F_1}{15.0} = \frac{\sin 26°}{13.0}$$
$$15.0\left(\frac{\sin F_1}{15.0}\right) = \left(\frac{\sin 26°}{13.0}\right)15.0$$
$$\sin F_1 = 0.5058\ldots°$$
$$\angle F_1 = \sin^{-1}(0.5058\ldots)$$
$$\angle F_1 = 30.3853\ldots$$

$\angle E = 180° - 26° - 30.3853\ldots°$

$\angle E = 123.6146\ldots°$

$$\frac{e}{\sin 123.6146\ldots°} = \frac{13.0}{\sin 26°}$$
$$\sin 123.6146\ldots°\left(\frac{e}{\sin 123.6146\ldots°}\right) = \left(\frac{13.0}{\sin 26°}\right)\sin 123.6146\ldots°$$
$$e = 24.6962\ldots \text{ cm}$$

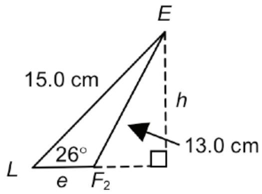
$$\sin 26° = \frac{h}{15.0}$$
$$15.0 \sin 26° = h$$
$$6.575\ldots \text{ cm} = h$$

Area($\triangle DEF_1$) = $0.5eh$

Area($\triangle DEF_1$) = $(0.5)(24.696\ldots)(6.755\ldots)$

Area($\triangle DEF_1$) = 81.2 cm^2

b) Here is the diagram.



From part a, h = 6.575… cm.

$$\frac{\sin F_2}{15.0} = \frac{\sin 26°}{13.0}$$

$$15.0\left(\frac{\sin F_2}{15.0}\right) = \left(\frac{\sin 26°}{13.0}\right)15.0$$

$$\sin F_2 = 0.5058...°$$

$$\angle F_2 = \sin^{-1}(0.5058...)$$

$$\angle F_2 = 30.3853...°$$

but $\angle F_2$ is obtuse.

$\angle F_2$ = 180° – 30.385…°

$\angle F_2$ = 149.614…°

$\angle E$ = 180° – 26° – 149.6146…°

$\angle E$ = 4.3853…°

$$\frac{e}{\sin 4.3853...°} = \frac{13.0}{\sin 26°}$$

$$\sin 4.3853...°\left(\frac{e}{\sin 4.3853...°}\right) = \left(\frac{13.0}{\sin 26°}\right)\sin 4.3853...°$$

$$e = 2.267...\text{ cm}$$

Area($\triangle DEF_2$) = 0.5eh

Area($\triangle DEF_2$) = (0.5)(2.267…)(6.755…)

Area($\triangle DEF_2$) = 7.5 cm^2

c) Area($\triangle F_1EF_2$) = Area($\triangle DEF_1$) – Area($\triangle DEF_2$)

Area($\triangle F_1EF_2$) = 81.2 – 7.5

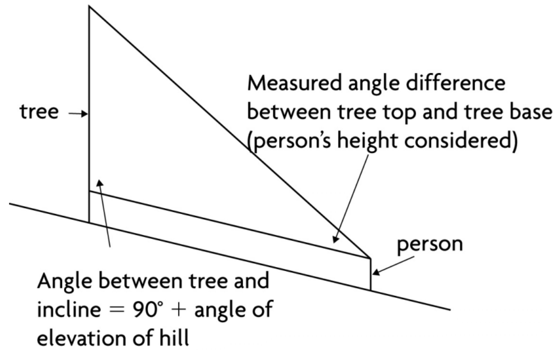
Area($\triangle F_1EF_2$) = 73.7 cm^2

d) e.g., Subtract the area of $\triangle DEF_2$ from $\triangle DEF_1$.

History Connection: Dioptras and Theodolites, page 183

A. A dioptra is easily constructed according to the diagram. However, you may wish to work in pairs, so that one student can sight the object and his or her partner can read the angle from the protractor. Most protractors have 90° at the bottom when students are looking horizontally, so you will need to account for this and adjust your measurements accordingly to achieve the traditional angles of elevation or depression.

B. If you choose to measure the height of a building or tree at a distance up a hill, the following diagram will lead to an accurate solution.

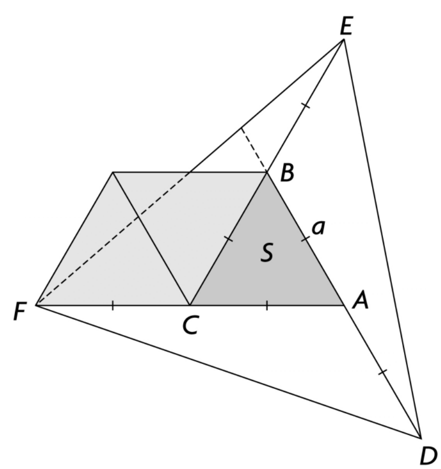


After taking the required measurements, students have an *ASA* situation that can be solved using the sine law.

Applying Problem-Solving Puzzles: Analyzing an Area Puzzle, page 187

A. I started with a cutout of $\triangle ABC$, and I used this to estimate that at least six of these triangles fit into $\triangle DEF$.

B. I used two cutouts of $\triangle ABC$ and laid them to the left of the original $\triangle ABC$. I noticed that the overlap looked approximately the same size as the remaining part of the obtuse triangle, $\triangle CEF$, that was not covered by my cutouts. I decided to cut my two triangles where they overlapped the figure (dotted lines below). These two smaller triangles fit perfectly into the part of $\triangle CEF$ that was not covered. Therefore, $\triangle ECF$ is the same size as two $\triangle ABC$s. There are three congruent obtuse triangles that surround the original $\triangle ABC$. Therefore, seven $\triangle ABC$s fit into $\triangle DEF$.

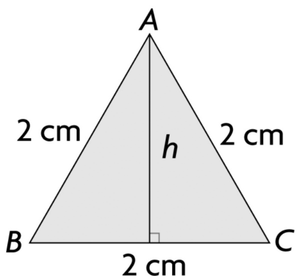


C. To solve this puzzle, I looked at the smaller problem: How many $\triangle ABC$s fit into the obtuse triangle, $\triangle CEF$? I modelled this problem with cutouts of $\triangle ABC$, and then I extended the solution to three obtuse triangles.

D. To solve this puzzle using trigonometry, I decided to arbitrarily set the length of the sides of $\triangle ABC$ to 2 cm.
To determine the area of $\triangle ABC$, I drew a height on the triangle and determined the height using a primary trigonometric ratio.

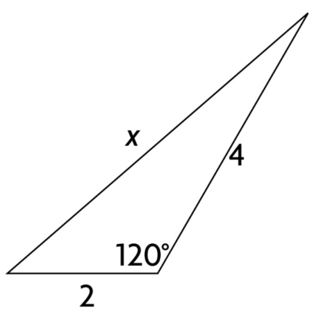
$$\begin{aligned} \sin 60° &= \frac{h}{2} \\ 2\sin 60° &= h \\ 1.732 &= h \end{aligned}$$

Area of $\triangle ABC$ = $\frac{1}{2}(2)(1.732...)$ or 1.732…



To determine the area of $\triangle DEF$, I must first determine a side length. I used the cosine law.

$x^2 = 2^2 + 4^2 - 2(2)(4)\cos 120°$
$x = 5.291…$cm

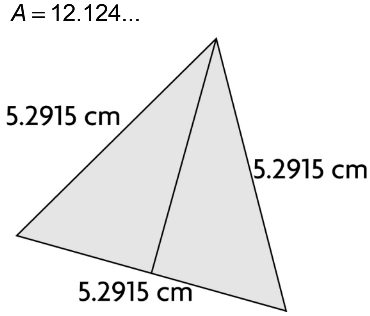


To determine the area of $\triangle DEF$, I drew a height on the triangle and determined the height using the sine ratio.

$$\begin{aligned} \sin 60° &= \frac{h}{5.291...} \\ (5.291...)\sin 60° &= h \\ 4.582... &= h \end{aligned}$$

Area of $\triangle DEF$, A:

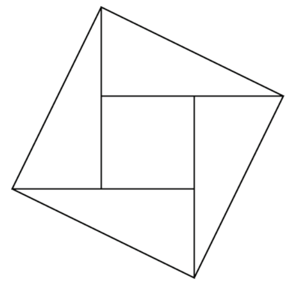
$$A = \frac{1}{2}(2)(5.291...)(4.582...)$$
$$A = 12.124...$$



To determine how many $\triangle ABC$s fit into $\triangle DEF$, I divided.

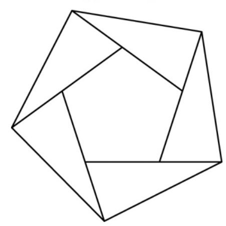
$$\frac{12.124...}{1.732...} = 7$$

E. Similar puzzle: Start with a square. Extend the side lengths to make the following shape:

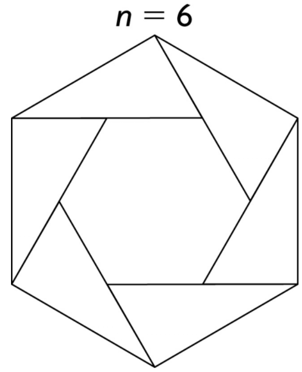


This puzzle can be solved by inspection. Five small squares fit into the larger square. However, this puzzle becomes more challenging as the degree of the polygon increases.

$n = 5$



$n = 6$



Note: There is a truly marvellous demonstration of these solutions, but the Solution Manual is not large enough to include it.

Lesson 4.4: Solving Problems Using Obtuse Triangles, page 193

1. a) i) sine law
ii) sine law
iii) cosine law
b) i)

$$\frac{x}{\sin(180° - 14° - 30°} = \frac{18}{\sin 14°}$$
$$\sin 136°\left(\frac{c}{\sin 136°}\right) = \left(\frac{18}{\sin 14°}\right)\sin 136°$$
$$c = 51.7...\text{ m}$$