Chapter 4: Oblique Triangle Trigonometry

Lesson 4.1: Exploring the Primary Trigonometric Ratios of Obtuse Angles, page 163

1. a) not valid; $180° - 25° \neq 65°$
b) valid; $180° - 70° = 110°$
c) not valid; $\tan 46° = -\tan 134°$
d) valid; $180° - 122° = 58°$
e) not valid; $\cos 135° = -\cos 45°$
f) valid; $180° - 175° = 5°$
2. a) $\sin 15° = 0.2588$; $\sin 165° = 0.2588$
b) $\cos 62° = 0.4695$; $\cos 118° = -0.4695$
c) $\tan 35° = 0.7002$; $\tan 145° = -0.7002$
d) $\sin 170° = 0.1736$; $\sin 10° = 0.1736$
3. a) $\sin\theta = 0.64$
$\sin^{-1}(0.64) = 40°$
$180° - 40° = 140°$
$\theta = 40°$ or $140°$

b) $\sin\theta = \frac{1}{3}$

$\sin^{-1}\left(\frac{1}{3}\right) = 19°$

$180° - 19° = 161°$
$\theta = 19°$ or $161°$
c) $\sin\theta = 0.95$
$\sin^{-1}(0.95) = 72°$
$180° - 72° = 108°$
$\theta = 72°$ or $108°$

d) $\sin\theta = \frac{7}{23}$

$\sin^{-1}\left(\frac{7}{23}\right) = 18°$

$180° - 18° = 162°$
$\theta = 18°$ or $162°$
4. a) $\sin D = \sin K$, $\sin H = \sin M$, $\sin H = \sin N$, $\sin H = \sin O$, $\sin M = \sin N$, $\sin M = \sin O$, $\sin N = \sin O$
b) The cosine and tangent ratios for $\angle D$ and $\angle K$, *and for* $\angle M$ and $\angle H$ are opposites. The other angles have equal cosine and tangent ratios.

Lesson 4.2: Proving and Applying the Sine and Cosine Laws for Obtuse Triangles, page 170

1. a) Reverse $\sin 100°$ and $\sin 32°$.
b) On the left side of the equation change 12 to x; on the right side of the equation, change x to 12.
2. a) sine law; two side lengths and the measure of one opposite angle are known.
b) cosine law; all three side lengths are known.
c) cosine law; two side lengths and the measure of the contained angle are known.
d) sine law; the measures of two angles and one side length are known.
e) neither; none of the side lengths are known

3. a) $\frac{x}{\sin 101°} = \frac{4.0}{\sin 28°}$

$\sin 101°\left(\frac{x}{\sin 101°}\right) = \left(\frac{4.0}{\sin 28°}\right)\sin 101°$

$x = 8.363...$ cm

To the nearest tenth of a centimetre, x is 8.4 cm.
b) $x^2 = 30.0^2 + 24.0^2 - 2(30.0)(24.0)\cos(32.0°)$
$x = \sqrt{254.810...}$
$x = 15.962\ldots$
To the nearest tenth of a centimetre, x is 16.0 cm.
c) $x^2 = 1.4^2 + 2.0^2 - 2(1.4)(2.0)\cos(130.0°)$
$x = \sqrt{9.559...}$
$x = 3.091\ldots$
To the nearest tenth of a centimetre, x is 3.1 cm.

4. a) $\frac{\sin x}{44} = \frac{\sin 118°}{68}$

$44\left(\frac{\sin x}{44}\right) = \left(\frac{\sin 118°}{68}\right)44$

$\sin x = 0.5713...$

$x = \sin^{-1}(0.5713...)$

$x = 34.8409...°$

To the nearest degree, x is 35°.

b) $\cos x = \frac{2^2 + 4^2 - 5^2}{(2)(2)(4)}$

$\cos x = -0.3125$

$x = \cos^{-1}(-0.3125)$

$x = 108.2099...°$

To the nearest degree, x is 108°.

c) $\frac{\sin x}{106} = \frac{\sin 150°}{180}$

$106\left(\frac{\sin x}{106}\right) = \left(\frac{\sin 150°}{180}\right)106$

$\sin x = 0.2944...$

$x = \sin^{-1}(0.2944...)$

$x = 17.1215...°$

To the nearest degree, x is 17°.
5. a) $m^2 = 7.5^2 + 11.2^2 - 2(7.5)(11.2)\cos(105°)$
$m = \sqrt{225.171...}$
$m = 15.005\ldots$
To the nearest tenth of a centimetre, m is 15.0 cm.

$\frac{\sin L}{11.2} = \frac{\sin 105°}{15.0}$

$11.2\left(\frac{\sin L}{11.2}\right) = \left(\frac{\sin 105°}{15.0}\right)11.2$

$\sin L = 0.7212...$

$\angle L = \sin^{-1}(0.7212...)$

$\angle L = 46.1536...°$

To the nearest degree, $\angle L$ is 46°.

$\angle N = 180° - 46.1536…° - 105°$
$\angle N = 28.8464…°$
To the nearest degree, $\angle N$ is 29°.

b) $\angle R = 180° - 120° - 28°$
$\angle R = 32°$

$$\frac{r}{\sin 32°} = \frac{25.6}{\sin 120°}$$
$$\sin 32°\left(\frac{r}{\sin 32°}\right) = \left(\frac{25.6}{\sin 120°}\right)\sin 32°$$
$$r = 15.664... \text{ cm}$$

To the nearest tenth of a centimetre, r is 15.7 cm.

$$\frac{t}{\sin 28°} = \frac{25.6}{\sin 120°}$$
$$\sin 28°\left(\frac{t}{\sin 28°}\right) = \left(\frac{25.6}{\sin 120°}\right)\sin 28°$$
$$t = 13.877... \text{ cm}$$

To the nearest tenth of a centimetre, t is 13.9 cm.

c) $\cos A = \dfrac{8^2 + 5^2 - 10^2}{2(8)(5)}$
$\cos A = -0.1375$
$\angle A = \cos^{-1}(-0.1375)$
$\angle A = 97.9032...°$

To the nearest degree, $\angle A$ is 98°.

$$\cos B = \frac{8^2 + 10^2 - 5^2}{2(8)(10)}$$
$\cos B = 0.868\ 75$
$\angle B = \cos^{-1}(0.868\ 75)$
$\angle B = 29.6862...°$

To the nearest degree, $\angle B$ is 30°.
$\angle C = 180° - 29.6862…° - 97.9032…°$
$\angle C = 52.4105…°$
To the nearest degree, $\angle C$ is 52°.

d) $\angle X = 180° - 21° - 35°$
$\angle X = 124°$

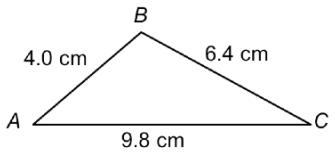
$$\frac{y}{\sin 21°} = \frac{18.7}{\sin 124°}$$
$$\sin 21°\left(\frac{y}{\sin 21°}\right) = \left(\frac{18.7}{\sin 124°}\right)\sin 21°$$
$$y = 8.083... \text{ cm}$$

To the nearest tenth of a centimetre, y is 8.1 cm.

$$\frac{z}{\sin 35°} = \frac{18.7}{\sin 124°}$$
$$\sin 35°\left(\frac{z}{\sin 35°}\right) = \left(\frac{18.7}{\sin 124°}\right)\sin 35°$$
$$z = 12.937... \text{ cm}$$

To the nearest tenth of a centimetre, z is 12.9 cm.

6. a) e.g., about 135°



b) $\cos B = \dfrac{4.0^2 + 6.4^2 - 9.8^2}{2(4.0)(6.4)}$
$\cos B = -0.7632...$
$\angle B = \cos^{-1}(-0.7632...)$
$\angle B = 139.7471...°$

To the nearest tenth of a degree, $\angle B$ is 139.7°.

c) e.g., The estimate was reasonable. It could be improved by rounding the side lengths and using the cosine law.

7. Wei-Ting made a mistake from line 3 to line 4. The domain of inverse cosine is –1 to 1, and 100 is outside the domain.

$$400 = 244 - 240\cos\theta$$
$$156 = -240\cos\theta$$
$$\frac{156}{-240} = \cos\theta$$
$$\cos^{-1}\left(\frac{156}{-240}\right) = \theta$$
$$130.5° = \theta$$

8. $\cos R = \dfrac{12.8^2 + 10.2^2 - 20.5^2}{2(12.8)(10.2)}$
$\cos R = -0.5835...$
$\angle R = \cos^{-1}(-0.5835...)$
$\angle R = 125.8277...°$

To the nearest tenth of a degree, $\angle R$ is 125.7°.

$$\cos S = \frac{20.5^2 + 10.2^2 - 12.8^2}{2(20.5)(10.2)}$$
$\cos S = 0.8619...$
$\angle S = \cos^{-1}(0.8619...)$
$\angle S = 30.4684...°$

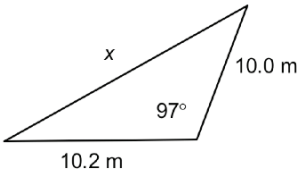
To the nearest tenth of a degree, $\angle S$ is 30.5°.
$\angle Q = 180° - 30.4684…° - 125.8277…°$
$\angle Q = 23.7039…°$
To the nearest tenth of a degree, $\angle Q$ is 23.7°.

9. $t^2 = 175^2 + 295^2 - 2(175)(295)\cos(23°)$
$t = \sqrt{22\ 607.873...}$
$t = 150.359…$ yd
The ball is 150 yards from the hole.

10. $\cos\theta = \dfrac{21^2 + 26^2 - 6^2}{2(21)(26)}$
$\cos\theta = 0.9899…$
$\theta = \cos^{-1}(0.9899…)$
$\theta = 8.1393…°$
The shot should be made from any angle between 0° and 8.1°.

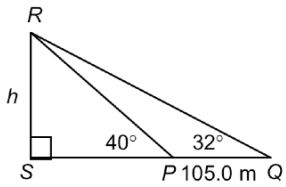
11.
$$\frac{\sin D}{68.4} = \frac{\sin 136°}{124.0}$$
$$68.4\left(\frac{\sin D}{68.4}\right) = \left(\frac{\sin 136°}{124.0}\right)68.4$$
$$\sin D = 0.3831...$$
$$\angle D = \sin^{-1}(0.3831...)$$
$$\angle D = 22.5309...°$$
To the nearest tenth of a degree, $\angle D$ is 22.5°.
$\angle F = 180° - 22.5309…° - 136°$
$\angle F = 21.4690…°$
To the nearest tenth of a degree, $\angle F$ = 21.5°.
$$\frac{f}{\sin 21.4690...°} = \frac{124.0}{\sin 136°}$$
$$\sin 21.4690...°\left(\frac{f}{\sin 21.4690...°}\right) = \left(\frac{124.0}{\sin 136°}\right)\sin 21.4690...°$$
$$f = 65.332...\text{ m}$$
To the nearest tenth of a metre, f is 65.3 m.

12. Here is the diagram:



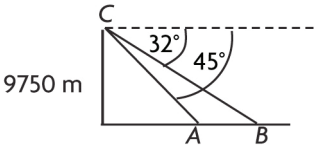
$x^2 = 10.2^2 + 10.0^2 - 2(10.2)(10.0)\cos(97°)$
$x = \sqrt{228.901...}$
$x = 15.129…$ m
The length of cable needed is 15.1 m.

13. Here is the diagram:



$\angle PRS = 90° - 40° = 50°$
$\angle QRS = 90° - 32° = 58°$
$\angle QRP = 58° - 50° = 8°$
In $\triangle PQR$,
$$\frac{q}{\sin 32°} = \frac{105.0}{\sin 8°}$$
$$\sin 32°\left(\frac{q}{\sin 32°}\right) = \left(\frac{105.0}{\sin 8°}\right)\sin 32°$$
$$q = 399.800...$$
In $\triangle PRS$,
$$\sin 40° = \frac{h}{399.800...}$$
$$(399.800...)(\sin 40°) = \left(\frac{h}{399.800...}\right)(399.800...)$$
$$256.9864...\text{ m} = h$$
The building is 257.0 m tall.

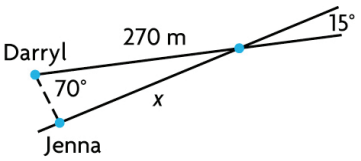
14. Here is the diagram:



Consider $\triangle ABC$. Since the plane flies parallel to the ground, $\angle B$ = 32°.
$\angle C$ = 45° – 32° or 13°
$$\frac{AB}{\sin 13°} = \frac{9750}{\sin 32°}$$
$$\sin 13°\left(\frac{AB}{\sin 13°}\right) = \left(\frac{9750}{\sin 32°}\right)\sin 13°$$
$$AB = 4138.881...$$
The lake is 4139 m wide.

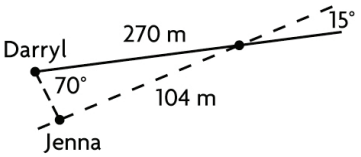
15. e.g., Use the sine law to write an equation with q over sin Q on the left side and r over sin R on the right side. Solve for sin Q. Determine Q using inverse sine.

16. a) e.g., Darryl sees his friend Jenna standing on the other road. He estimates the angle between the road he is on and his line of sight to Jenna to be 70°. How far is Jenna from the intersection?



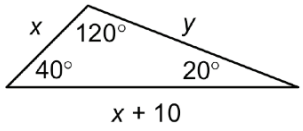
Solution:
180° – 70° – 15° = 95°
The angle between the road Jenna is on and her line of sight to Darryl is 95°.
$$\frac{x}{\sin 70°} = \frac{270}{\sin 95°}$$
$$\sin 70°\left(\frac{x}{\sin 70°}\right) = \left(\frac{270}{\sin 95°}\right)\sin 70°$$
$$q = 254.686...$$
Jenna is 255 m from the intersection.

b) e.g., Darryl sees his friend Jenna standing on the other road. If Jenna is 104 m from the intersection, how far apart are Jenna and Darryl?



Solution:
$x^2 = 270^2 + 104^2 - 2(270)(104)\cos(15°)$
$x = \sqrt{29\ 469.605...}$
$x = 171.667…$ m
Jenna and Darryl are 172 m apart.

17. Here is the diagram:



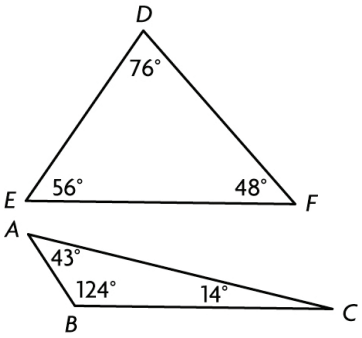
$$\frac{x}{\sin 20^\circ} = \frac{x+10}{\sin 120^\circ}$$
$$\sin 20^\circ\left(\frac{x}{\sin 20^\circ}\right) = \left(\frac{x+10}{\sin 120^\circ}\right)\sin 20^\circ$$
$$x = \left(\frac{x+10}{\sin 120^\circ}\right)\sin 20^\circ$$
$$(\sin 120^\circ)x = \cancel{(\sin 120^\circ)}\left(\frac{x+10}{\cancel{\sin 120^\circ}}\right)(\sin 20^\circ)$$
$$(\sin 120^\circ)x = (x+10)(\sin 20^\circ)$$
$$(\sin 120^\circ)x = x(\sin 20^\circ) + 10\sin 20^\circ$$
$$(\sin 120^\circ - \sin 20^\circ)x = 10\sin 20^\circ$$
$$x = \frac{10\sin 20^\circ}{\sin 120^\circ - \sin 20^\circ}$$
$$x = 6.527...\text{ cm}$$
$$\frac{y}{\sin 40^\circ} = \frac{6.527...}{\sin 20^\circ}$$
$$\sin 40^\circ\left(\frac{y}{\sin 40^\circ}\right) = \left(\frac{6.527...}{\sin 20^\circ}\right)\sin 40^\circ$$
$$y = 12.266...$$

$P = x + (x + 10) + y$
$P = 6.527\ldots + 16.527\ldots + 12.266\ldots$
$P = 35.320\ldots$
The perimeter is 35 cm.

Mid-Chapter Review, page 175

1. a) sin 75° = 0.9659
180° – 75° = 105°
sin 105° = 0.9659
b) cos 100° = –0.1736
180° – 100° = 80°
cos 80° = 0.1736
c) tan 32° = 0.6249
180° – 32° = 148°
tan 148° = –0.6249
d) sin 172° = 0.1392
180° – 172° = 8°
sin 8° = 0.1392
e) cos 38.5° = 0.7826
180° – 38.5° = 141.5°
cos 141.5° = –0.7826
f) tan 122.3° = –1.5818
180° – 122.3° = 57.7°
tan 57.7° = 1.5818

2. e.g., sin 124° = sin 56°



3. a) $\theta = \sin^{-1}(0.362)$
$\theta = 21°$
180° – 21° = 159°
b) $\theta = \cos^{-1}(-0.75)$
$\theta = 139°$
c) $\theta = \tan^{-1}\left(\frac{5}{2}\right)$
$\theta = 68°$
d) $\theta = \sin^{-1}\left(\frac{1}{2}\right)$
$\theta = 30°$
180° – 30° = 150°
e) $\theta = \cos^{-1}(-0.214)$
$\theta = 78°$
f) $\theta = \tan^{-1}(1)$
$\theta = 45°$

4. a) $$\cos\theta = \frac{11.0^2 + 15.0^2 - 10.7^2}{2(11.0)(15.0)}$$
$$\cos\theta = 0.7015\ldots$$
$$\theta = \cos^{-1}(0.7015\ldots)$$
$$\theta = 45.4488\ldots°$$
To the nearest tenth of a degree, angle θ is 45.4°
b) 180° – 155° = 25°
25° ÷ 2 = 12.5°
$$\frac{y}{\sin 155^\circ} = \frac{2.5}{\sin 12.5^\circ}$$
$$\sin 155^\circ\left(\frac{y}{\sin 155^\circ}\right) = \left(\frac{2.5}{\sin 12.5^\circ}\right)\sin 155^\circ$$
$$y = 4.881...\text{ km}$$
To the nearest tenth of a kilometre, y is 4.9 km.
c)
$$\frac{x}{\sin 55.0^\circ} = \frac{2.0}{\sin 121.0^\circ}$$
$$\sin 55.0^\circ\left(\frac{x}{\sin 55.0^\circ}\right) = \left(\frac{2.0}{\sin 121^\circ}\right)\sin 55.0^\circ$$
$$x = 1.911...\text{ cm}$$
To the nearest tenth of a centimetre, x is 1.9 cm.