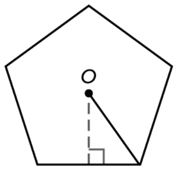
16.

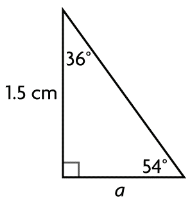


Each of the five triangles in the pentagon is isosceles.

The angle at the centre of the pentagon is $\frac{1}{5}$ of 360° or 72°.

The base angle of each triangle is 54°.

The perpendicular bisector bisects the vertex angle, so the angle in a right triangle is 36°.



Let *a* represent the base of the triangle.

$$\frac{a}{\sin 36°} = \frac{1.5}{\sin 54°}$$
$$\sin 36°\left(\frac{a}{\sin 36°}\right) = \sin 36°\left(\frac{1.5}{\sin 54°}\right)$$
$$a = \sin 36°\left(\frac{1.5}{\sin 54°}\right)$$
$$a = 1.089...$$

The length of a side of a pentagon is 2*a* or 2.179… cm.

Perimeter = 5(side length of pentagon)
Perimeter = 5(2.179…)
Perimeter = 10.898…

The perimeter of the pentagon is 10.9 cm.

Area of one triangle = $\frac{1}{2}a(1.5)$

Area of one triangle = $\frac{1}{2}(2.179\ldots)(1.5)$

Area of one triangle = 1.634…
Area of five triangles = 5(area of one triangle)
Area of five triangles = 5(1.634…)
Area of five triangles = 8.173…

The area of the pentagon is 8.2 cm^2.

17. e.g., The vertex angle at the handle of the knife is about 110°. Each of the sides of the knife is about 8.5 cm in length.

Lesson 3.4: Solving Problems Using Acute Triangles, page 147

1. a) The measures of two sides and an angle opposite one of the sides are given, so use the sine law to solve for θ.

b) The triangle is a right triangle. The length of the side adjacent to the right angle is given along with the measure of one acute angle. You can use the tangent ratio to solve for *c*. You could also use the sine ratio to solve for *c* by finding the measure of $\angle A$ first.

c) The lengths of three sides are given, so you can use the cosine law to solve for θ.

2. a) part a:

$$\frac{\sin\theta}{3.1} = \frac{\sin 60°}{2.7}$$
$$3.1\left(\frac{\sin\theta}{3.1}\right) = 3.1\left(\frac{\sin 60°}{2.7}\right)$$
$$\sin\theta = 3.1\left(\frac{\sin 60°}{2.7}\right)$$
$$\theta = \sin^{-1}(0.9943...)$$
$$\theta = 83.893...°$$

The measure of θ is 83.9°.

part b: $\tan 47° = \frac{c}{1.8}$

$1.8(\tan 47°) = c$
$1.8(1.0723\ldots) = c$
$1.930\ldots = c$

The length of *c* is 1.9 cm.

part c: $a^2 = b^2 + c^2 - 2bc\cos A$

$2.0^2 = 2.9^2 + 3.0^2 - 2(2.9)(3.0)\cos\theta$
$4.0 = 8.41 + 9.0 - 17.4\cos\theta$
$-13.41 = -17.4\cos\theta$
$0.7706\ldots = \cos\theta$
$\theta = \cos^{-1}(0.7706\ldots)$
$\theta = 39.584\ldots°$

The measure of θ is 39.6°.

b) e.g., Using a trigonometric ratio is more efficient because you have fewer calculations to do.

3. a) Since the measures of two sides and a contained angle are given, I would use the cosine law.

b) Let *d* represent the distance between the kayaks.

$d^2 = 4.0^2 + 5.0^2 - 2(4.0)(5.0)\cos 30°$
$d^2 = 16.0 + 25.0 - 40.0(0.8660\ldots)$
$d^2 = 6.358\ldots$
$d = 2.521\ldots$

The distance the kayaks are apart is 2.5 km.

4. Let x represent the measure of the remaining unknown angle.

$$x + 48° + 44° = 180°$$
$$x = 88°$$

Let a represent the length of the rafter opposite the 44° angle.

$$\frac{a}{\sin 44°} = \frac{42}{\sin 88°}$$
$$\sin 44°\left(\frac{a}{\sin 44°}\right) = \sin 44°\left(\frac{42}{\sin 88°}\right)$$
$$a = \sin 44°\left(\frac{42}{\sin 88°}\right)$$
$$a = 29.193...$$

Convert the decimal to inches:

0.193… ft · 12 in./ft = 2.321… in.

Let b represent the length of the rafter opposite the 48° angle.

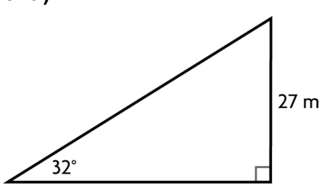
$$\frac{b}{\sin 48°} = \frac{42}{\sin 88°}$$
$$\sin 48°\left(\frac{b}{\sin 48°}\right) = \sin 48°\left(\frac{42}{\sin 88°}\right)$$
$$b = \sin 48°\left(\frac{42}{\sin 88°}\right)$$
$$b = 31.231...$$

Convert the decimal to inches:

0.231… ft · 12 in./ft = 2.773… in.

The rafters are 29′ 2″ and 31′ 3″.

5. a)

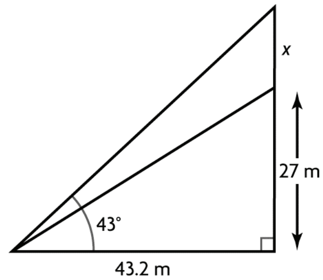


Let d represent the distance from the point to the base of the tower.

$$\tan 32° = \frac{27}{d}$$
$$d(\tan 32°) = d\left(\frac{27}{d}\right)$$
$$d(\tan 32°) = 27$$
$$d = \frac{27}{\tan 32°}$$
$$d = 43.209…$$

The distance from a point to the base of the tower is 43.2 m.

b)

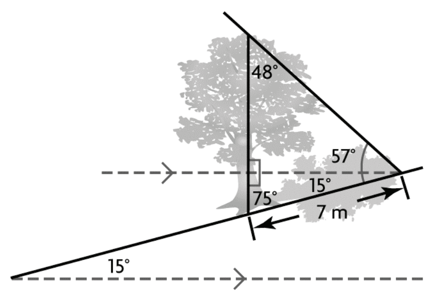


Let x represent the height of the crane.

$$\tan 43° = \frac{27 + x}{43.2}$$
$$43.2(\tan 43°) = 43.2\left(\frac{27 + x}{43.2}\right)$$
$$43.2(\tan 43°) = 27 + x$$
$$40.284… = 27 + x$$
$$13.284… = x$$

The height of the crane is about 13.3 m.

6. a) e.g., Use the properties of parallel lines to determine the angle from the shadow up to the horizontal. Subtract that angle from 57° to determine the angle from the horizontal up to the sun. Both of these are angles of right triangles with one side along the tree. Subtract each angle from 90° to determine the third angle in each right triangle. Use the sine law to determine the height of the tree.

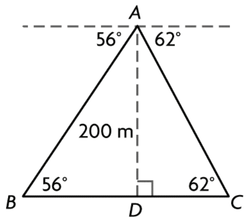


b) Let h represent the height of the tree.

$$\frac{h}{\sin 57°} = \frac{7}{\sin 48°}$$
$$\sin 57°\left(\frac{h}{\sin 57°}\right) = \sin 57°\left(\frac{7}{\sin 48°}\right)$$
$$h = \sin 57°\left(\frac{7}{\sin 48°}\right)$$
$$h = 7.899...$$

The height of the tree is 8 m.

7.



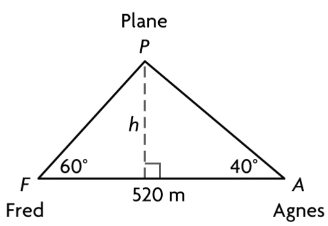
$$\angle A + \angle B + \angle C = 180°$$
$$\angle A + 56° + 62° = 180°$$
$$\angle A = 62°$$
$$\angle A = \angle C$$

Therefore, △ABC is isosceles.
So AB is equal to BC owing to the properties of an isosceles triangle.

$$\sin B = \frac{AD}{AB}$$
$$\sin 56° = \frac{200}{AB}$$
$$AB(\sin 56°) = AB\left(\frac{200}{AB}\right)$$
$$AB(\sin 56°) = 200$$
$$AB = \frac{200}{\sin 56°}$$
$$AB = 241.243\ldots$$

The length of the sunken ship is 241.2 m.

8. Let h represent the altitude of the airplane.



$$\angle P + \angle F + \angle A = 180°$$
$$\angle P + 60° + 40° = 180°$$
$$\angle P = 80°$$

Determine the length of PF:

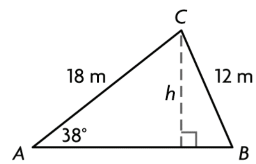
$$\frac{PF}{\sin A} = \frac{FA}{\sin P}$$
$$\frac{PF}{\sin 40°} = \frac{520}{\sin 80°}$$
$$\sin 40°\left(\frac{PF}{\sin 40°}\right) = \sin 40°\left(\frac{520}{\sin 80°}\right)$$
$$PF = \sin 40°\left(\frac{520}{\sin 80°}\right)$$
$$PF = 339.405...$$

Determine the length of h:

$$\sin F = \frac{h}{PF}$$
$$PF(\sin F) = PF\left(\frac{h}{PF}\right)$$
$$PF(\sin F) = h$$
$$339.405\ldots(\sin 60°) = h$$
$$293.934\ldots = h$$

The altitude of the airplane is 293.9 m.

9. a) Let h represent the height of the tower.



$$\sin A = \frac{h}{CA}$$
$$CA(\sin A) = CA\left(\frac{h}{CA}\right)$$
$$CA(\sin A) = h$$
$$18(\sin 38°) = h$$
$$11.081\ldots = h$$

The height of the tower is 11.1 m.

b)
$$\frac{\sin B}{CA} = \frac{\sin A}{BC}$$
$$\frac{\sin B}{18} = \frac{\sin 38°}{12}$$
$$18\left(\frac{\sin B}{18}\right) = 18\left(\frac{\sin 38°}{12}\right)$$
$$\sin B = 18\left(\frac{\sin 38°}{12}\right)$$
$$\angle B = \sin^{-1}(0.9234...)$$
$$\angle B = 67.442...°$$

Determine the measure of $\angle C$:

$$\angle C + \angle A + \angle B = 180°$$
$$\angle C + 38° + 67.442\ldots° = 180°$$
$$\angle C = 74.557\ldots°$$

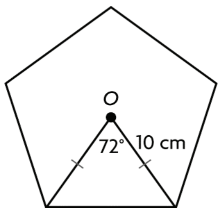
Determine the length of AB:

$$\frac{AB}{\sin C} = \frac{BC}{\sin A}$$
$$\frac{AB}{\sin 74.557...°} = \frac{12}{\sin 38°}$$
$$\sin 74.557...°\left(\frac{AB}{\sin 74.557...°}\right) = \sin 74.557...°\left(\frac{12}{\sin 38°}\right)$$
$$AB = \sin 74.557...°\left(\frac{12}{\sin 38°}\right)$$
$$AB = 18.787...$$

The two wires are 18.8 m apart.

10. a) e.g., Connect the centre to the vertices to create five congruent isosceles triangles and determine the measure of the angles at the centre. In one triangle, use the cosine law to determine the pentagon side length and multiply that answer by five.

b)



The angle at the centre of the regular pentagon is $\frac{1}{5}$ of 360° or 72°.

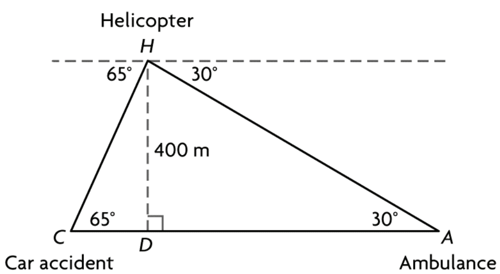
Let x represent the length of a side of the pentagon.

$x^2 = 10^2 + 10^2 - 2(10)(10)\cos 72°$
$x^2 = 100 + 100 - 200(0.3090\ldots)$
$x^2 = 138.196\ldots$
$x = 11.755\ldots$

Perimeter = $5x$
Perimeter = $5(11.755\ldots)$
Perimeter = $58.778\ldots$

The perimeter of the pentagon is 58.8 cm.

11. a)



$$\sin A = \frac{400}{AH}$$
$$AH(\sin 30°) = AH\left(\frac{400}{AH}\right)$$
$$AH(\sin 30°) = 400$$
$$AH = \frac{400}{\sin 30°}$$
$$AH = 800$$

In $\triangle AHC$,

$\angle H + \angle C + \angle A = 180°$
$\angle H + 65° + 30° = 180°$
$\angle H = 85°$

$$\frac{AC}{\sin H} = \frac{AH}{\sin C}$$
$$\frac{AC}{\sin 85°} = \frac{800}{\sin 65°}$$
$$\sin 85°\left(\frac{AC}{\sin 85°}\right) = \sin 85°\left(\frac{800}{\sin 65°}\right)$$
$$AC = \sin 85°\left(\frac{800}{\sin 65°}\right)$$
$$AC = 879.343...$$

The distance between the car accident and ambulance is 879.3 m.

b) $\text{Time} = \frac{\text{distance}}{\text{speed}}$ Time is in hours.

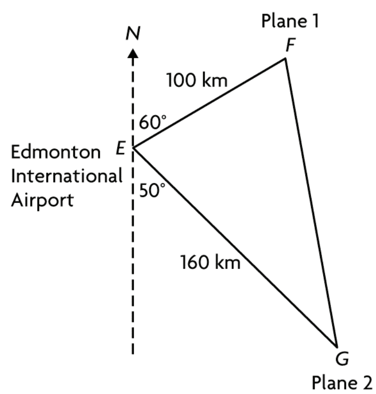
$\text{Time} = \frac{0.879\ldots}{80}$ Use distance in kilometres.

Time = 0.0109…

0.0109… h · 3600 s/h = 39.570… s

The ambulance reaches the accident scene in about 40 s.

12. a)



In $\triangle EFG$,

$\angle E + 60° + 50° = 180°$
$\angle E = 70°$

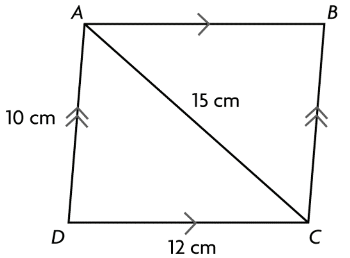
Determine the length of e:

$e^2 = f^2 + g^2 - 2fg\cos E$
$e^2 = 160^2 + 100^2 - 2(160)(100)\cos 70°$
$e^2 = 25\,600 + 10\,000 - 32\,000(0.3420\ldots)$
$e^2 = 24\,655.355\ldots$
$e = 157.020\ldots$

The distance between the planes is 157.0 km.

b) The airplane that is closest to the airport (100 km) will arrive first.

13. Draw a parallelogram *ABCD*.



$$\begin{aligned} AC^2 &= DA^2 + CD^2 - 2(DA)(CD)\cos D \\ 15^2 &= 10^2 + 12^2 - 2(10)(12)\cos D \\ 225 &= 100 + 144 - 240\cos D \\ -19 &= -240\cos D \\ \frac{-19}{-240} &= \cos D \\ \angle D &= \cos^{-1}\left(\frac{-19}{-240}\right) \\ \angle D &= \cos^{-1}(0.079\ldots) \\ \angle D &= 85.459\ldots° \end{aligned}$$

The measure of $\angle D$ is 85°.

$\angle D$ is opposite $\angle B$. Therefore, they are equal because opposite angles of a parallelogram are equal.

Since each of the remaining angles, $\angle DCB$ and $\angle BAD$, is an interior angle on the same side of a transversal as $\angle D$, they most both equal

180° – 85° = 95°.

The angles are 85°, 95°, 85°, and 95°.

14. e.g.,

Step 1: Determine the measure of $\angle BDC$.

Step 2: Use the sine law to determine the length of *CD*.

Step 3: In $\triangle ADC$, use the tangent ratio to determine *h*.

$$\begin{aligned} \angle BDC + \angle DCB + \angle CBD &= 180° \\ \angle BDC + 66° + 50° &= 180° \\ \angle BDC &= 64° \end{aligned}$$

$$\begin{aligned} \frac{CD}{\sin CBD} &= \frac{BC}{\sin BDC} \\ \frac{CD}{\sin 50°} &= \frac{175.0}{\sin 64°} \\ \sin 50°\left(\frac{CD}{\sin 50°}\right) &= \sin 50°\left(\frac{175.0}{\sin 64°}\right) \\ CD &= \sin 50°\left(\frac{175.0}{\sin 64°}\right) \\ CD &= 149.152... \\ \tan ACD &= \frac{h}{CD} \\ \text{CD}(\tan ACD) &= CD\left(\frac{h}{CD}\right) \\ 149.152\ldots(\tan 74°) &= h \\ 520.158\ldots &= h \end{aligned}$$

The altitude is 520.2 m.

15. e.g., Starr and Davis leave school from the same spot. Starr walks N65°E at 3 km/h while Davis walks S30°E at 4 km/h. How far apart are they after 20 min? The problem can be solved using the cosine law.

16. a)

$$\begin{aligned} AB^2 &= BE^2 + EA^2 - 2(BE)(EA)\cos AEB \\ 232.6^2 &= 221.2^2 + 221.2^2 \\ &\quad - 2(221.2)(221.2)\cos AEB \\ 54\,102.76 &= 48\,929.44 + 48\,929.44 \\ &\quad - 97\,858.88\cos AEB \\ -43\,756.12 &= -97\,858.88\cos AEB \\ \frac{-43\,756.12}{-97\,858.88} &= \cos AEB \\ 0.4471\ldots &= \cos AEB \\ \angle AEB &= \cos^{-1}(0.447\ldots \\ \angle AEB &= 63.439\ldots° \end{aligned}$$

The measure of an apex angle is 63°.

b) Each face of the pyramid is an isosceles triangle. The base angles of each triangle are equal.

In $\triangle EBC$,

$$\begin{aligned} \angle EBC + \angle BCE + \angle BEC &= 180° \\ 2\angle BCE + 63.439\ldots° &= 180° \\ 2\angle BCE &= 116.560\ldots° \\ \angle BCE &= 58.280\ldots° \end{aligned}$$

EG is a perpendicular bisector of isosceles $\triangle EBC$.

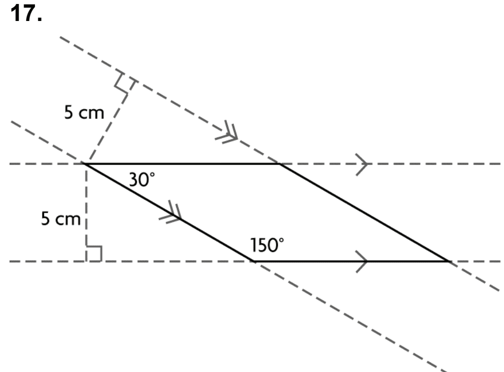
$$\begin{aligned} \sin BCE &= \frac{EG}{CE} \\ \sin 58.280\ldots° &= \frac{EG}{221.2} \\ 221.2(\sin 58.280\ldots°) &= 221.2\left(\frac{EG}{221.2}\right) \\ 188.158\ldots &= EG \end{aligned}$$

F is at the centre of the base, so *F* is a midpoint of a diagonal of the base. Therefore, *FG* is half the length of a base side or 116.3 cm.

$$\begin{aligned} \cos EGF &= \frac{FG}{EG} \\ \cos EGF &= \frac{116.3}{188.158...} \\ \cos EGF &= 0.618\ldots \\ \angle EGF &= \cos^{-1}(0.618\ldots) \\ \angle EGF &= 51.822\ldots° \end{aligned}$$

The measure of an angle that a face makes with the base is 52°.

17.



The overlapping region forms a parallelogram because opposite sides are parallel edges of the strips of paper. The lengths of the sides of the parallelogram, s, can be determined using the sine ratio:

$$\sin 30° = \frac{5}{s}$$
$$s = \frac{5}{\sin 30°}$$
$$s = 10$$

I can verify this by measuring the length, which is 10 cm.

Area of parallelogram = sh
Area of parallelogram = (10)(5)
Area of parallelogram = 50

The area of the parallelogram is 50 cm^2.

Math in Action, page 149

e.g., **Plan to Determine Peripheral Vision**

Right eye:

- The person being measured (the subject) will stand in front of a wall with hips parallel to the wall and the right arm fully extended so that the hand touches the wall.
- The subject will hold a pencil by its point, with the pencil extending vertically upward from the hand.
- The subject will close the left eye. The assistant will mark the point on the wall directly behind the pencil's eraser. The assistant will measure the distance from the eraser to the right eye of the subject.
- While staring at the mark on the wall, the subject will move the right arm outward, keeping the hand at the same height from the floor until the subject cannot see the eraser any more.

Using string, the assistant will then measure the distance from the mark on the wall to the eraser and from the eraser to the right eye.

We will have a triangle with three sides known, so the cosine law can be used to determine the peripheral vision angle.

Left eye:

Repeat the procedure using the left eye.

Evaluation

The plan seemed to work because it resulted in angles for the right and left eyes that were about the same for all the subjects, which is what we expected.

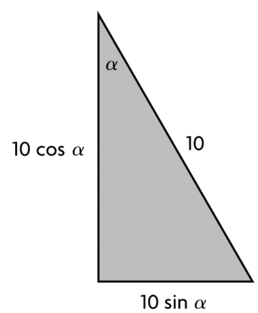
History Connection, page 150

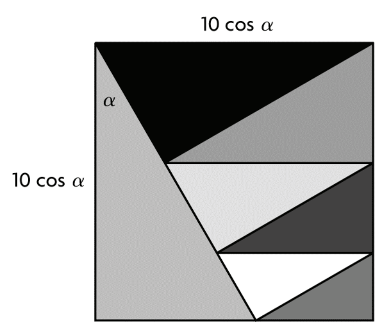
A. e.g., A Fire Centre could use triangulation on a map (the towers would already be on the map, and the angles relayed from the observers would allow construction of lines that intersect at the fire) or the known distance between the towers, the angles provided by the observers, and the sine law to calculate the distance from each tower to the fire.

B. e.g., Once the Fire Centre has told the observer the distance to the fire (using information from the observer and from an observer at another tower), the observer could measure the direction angles to each "end" of the fire, assume that each end was about equal in distance from the tower, and use the cosine law to establish the "width" of the fire. If the fire has been observed by multiple observers, the Fire Centre may be able to construct an accurate picture of the size of the fire.

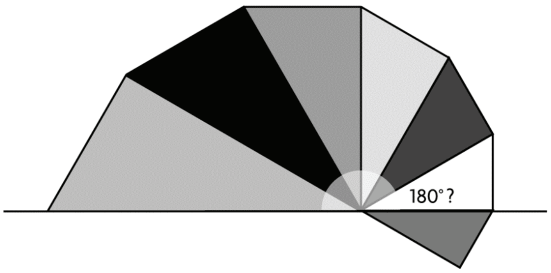
Applying Problem-Solving Strategies, page 165

B. See the square in the answer to part E.
C. 75 square units
D. I started with the largest triangle. I assumed that its right angle would be one corner of the square. I tried matching sides to figure out if the triangles would fit together without any parts sticking out. I noticed that, for each triangle, the longest side that is not the hypotenuse is the same length as the hypotenuse of the next larger triangle. When I arranged the six smaller triangles with their equal sides together (as shown in my answer to part E), I noticed that the triangles formed a figure with parallel sides. I placed this figure along the hypotenuse of the largest triangle to make a square.
E. e.g., I knew that the hypotenuse of the largest triangle is 10 units. I drew a diagram and labelled the sides. I chose the smaller angle to be α.





To calculate the area of the square, I needed to determine α. I noticed that all the triangles are similar—they share two angles that are the same and the right angle. I noticed that if I fanned out the seven triangles, with the longest non-hypotenuse side touching the hypotenuse of the next largest triangle, the largest six triangles formed an angle that looked like 180°. I measured with a protractor, but it was difficult to be sure that the angle was exactly 180°.



I could use the measurement to make an estimate.

$\alpha \doteq \dfrac{180°}{6}$

$\alpha \doteq 30°$

Area $\doteq$ (10 cos 30°)(10 cos 30°)

Area $\doteq$ (10 · 0.8660)(10 · 0.8660)

Area $\doteq$ 75

The area of the square is about 75 units.

Chapter Self-Test, page 152

1. a)

$$\frac{\sin B}{AC} = \frac{\sin A}{BC}$$
$$\frac{\sin B}{4.1} = \frac{\sin 82°}{6.0}$$
$$4.1\left(\frac{\sin B}{4.1}\right) = 4.1\left(\frac{\sin 82°}{6.0}\right)$$
$$\sin B = 4.1\left(\frac{\sin 82°}{6.0}\right)$$
$$\sin B = 0.6766...$$
$$\angle B = \sin^{-1}(0.6766...)$$
$$\angle B = 42.584...°$$

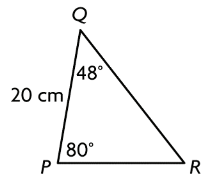
The measure of $\angle B$ is 42.6°.

b)

$$\angle C + \angle A + \angle B = 180°$$
$$\angle C + 78° + 69° = 180°$$
$$\angle C = 33°$$
$$\frac{c}{\sin C} = \frac{b}{\sin B}$$
$$\frac{c}{\sin 33°} = \frac{4.1}{\sin 69°}$$
$$\sin 33°\left(\frac{c}{\sin 33°}\right) = \sin 33°\left(\frac{4.1}{\sin 69°}\right)$$
$$c = \sin 33°\left(\frac{4.1}{\sin 69°}\right)$$
$$c = 2.391...$$

The length of c is 2.4 cm.

2. Here is the diagram.



$$\angle R + \angle P + \angle Q = 180°$$
$$\angle R + 80° + 48° = 180°$$
$$\angle R = 52°$$

The measure of $\angle R$ is 52°.

$$\frac{p}{\sin P} = \frac{r}{\sin R}$$
$$\frac{p}{\sin 80°} = \frac{20}{\sin 52°}$$
$$\sin 80°\left(\frac{p}{\sin 80°}\right) = \sin 80°\left(\frac{20}{\sin 52°}\right)$$
$$p = \sin 80°\left(\frac{20}{\sin 52°}\right)$$
$$p = 24.994...$$

The length of p is 25.0 cm.

$$\frac{q}{\sin Q} = \frac{r}{\sin R}$$
$$\frac{q}{\sin 48°} = \frac{20}{\sin 52°}$$
$$\sin 48°\left(\frac{q}{\sin 48°}\right) = \sin 48°\left(\frac{20}{\sin 52°}\right)$$
$$q = \sin 48°\left(\frac{20}{\sin 52°}\right)$$
$$q = 18.861...$$

The length of q is 18.9 cm.

3. Let x represent the measure of the angle between the two known sides.

$x + 45° + 50° = 180°$

$x = 85°$

Let y represent the distance the boats are apart.

$y^2 = 70^2 + 100^2 - 2(70)(100) \cos 85°$

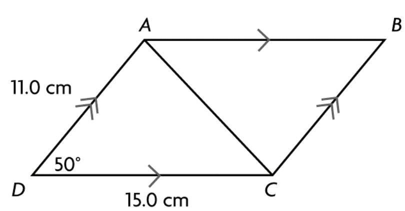
$y^2 = 4900 + 10\,000 - 14\,000(0.0871\ldots)$

$y^2 = 13\,679.819\ldots$

$y = 116.960\ldots$

The boats are 117.0 km apart.

4.



$AC^2 = 11.0^2 + 15.0^2 + - 2(11.0)(15.0) \cos 50°$

$AC^2 = 121.0 + 225.0 - 330(0.642\ldots)$

$AC^2 = 133.880\ldots$

$AC = 11.570\ldots$

The length of the shorter diagonal is 11.6 cm.