Lesson 3.3: Proving and Applying the Cosine Law, page 136

1. a) No. To determine the measure of side *c*, you also need the measure of side *b*.
b) Yes, you can determine the measure of side *c* with the given information.

2.
$$\begin{aligned} x^2 &= y^2 + z^2 - 2yz \cos X \\ x^2 &= 15^2 + 18^2 - 2(15)(18) \cos 46° \\ x^2 &= 225 + 324 - 540(0.6946\ldots) \\ x^2 &= 173.884\ldots \\ x &= 13.186\ldots \end{aligned}$$
The length of x is 13 cm.

3.
$$\begin{aligned} p^2 &= q^2 + r^2 - 2qr \cos P \\ 5.9^2 &= 6.2^2 + 2.3^2 - 2(6.2)(2.3) \cos P \\ 34.81 &= 38.44 + 5.29 - 28.52 \cos P \\ -8.92 &= -28.52 \cos P \\ 0.3127\ldots &= \cos P \\ \angle P &= \cos^{-1}(0.3127\ldots) \\ \angle P &= 71.774\ldots° \end{aligned}$$
The measure of $\angle P$ is 72°.

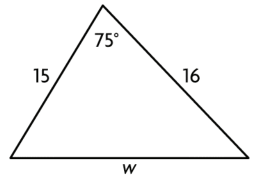
4. a)
$$\begin{aligned} b^2 &= a^2 + c^2 - 2ac \cos B \\ b^2 &= 9.5^2 + 10.5^2 - 2(9.5)(10.5) \cos 40° \\ b^2 &= 90.25 + 110.25 - 199.95(0.7660\ldots) \\ b^2 &= 47.674\ldots \\ b &= 6.904\ldots \end{aligned}$$
The length of b is 6.9 cm.

b)
$$\begin{aligned} e^2 &= d^2 + f^2 - 2df \cos E \\ e^2 &= 11.0^2 + 13.0^2 - 2(11.0)(13.0) \cos 75° \\ e^2 &= 121.0 + 169.0 - 286.0(0.2588\ldots) \\ e^2 &= 215.977\ldots \\ e &= 14.696\ldots \end{aligned}$$
The length of e is 14.7 cm.

5. a)
$$\begin{aligned} p^2 &= q^2 + r^2 - 2qr \cos P \\ 2.2^2 &= 3.9^2 + 3.5^2 - 2(3.9)(3.5) \cos P \\ 4.84 &= 15.21 + 12.25 - 27.3 \cos P \\ -22.62 &= -27.3 \cos P \\ 0.8285\ldots &= \cos P \\ \angle P &= \cos^{-1}(0.8285\ldots) \\ \angle P &= 34.047\ldots° \end{aligned}$$
The measure of $\angle P$ is 34°.

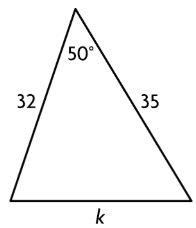
b)
$$\begin{aligned} z^2 &= x^2 + y^2 - 2xy \cos Z \\ 2.9^2 &= 2.2^2 + 2.6^2 - 2(2.2)(2.6) \cos Z \\ 8.41 &= 4.84 + 6.76 - 11.44 \cos Z \\ -3.19 &= -11.44 \cos Z \\ 0.2788\ldots &= \cos Z \\ \angle Z &= \cos^{-1}(0.2788\ldots) \\ \angle Z &= 73.808\ldots° \end{aligned}$$
The measure of $\angle Z$ is 74°.

6. a)



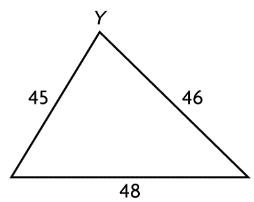
$$\begin{aligned} w^2 &= 15^2 + 16^2 - 2(15)(16) \cos 75° \\ w^2 &= 225 + 256 - 480(0.2588\ldots) \\ w^2 &= 356.766\ldots \\ w &= 18.888\ldots \end{aligned}$$
The length of w is 18.9 units.

b)



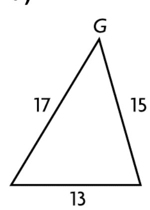
$$\begin{aligned} k^2 &= 32^2 + 35^2 - 2(32)(35) \cos 50° \\ k^2 &= 1024 + 1225 - 2240(0.6427\ldots) \\ k^2 &= 809.155\ldots \\ k &= 28.445\ldots \end{aligned}$$
The length of k is 28.4 units.

c)



$$\begin{aligned} 48^2 &= 46^2 + 45^2 - 2(46)(45) \cos Y \\ 2304 &= 2116 + 2025 - 4140 \cos Y \\ -1837 &= -4140 \cos Y \\ 0.4437\ldots &= \cos Y \\ \angle Y &= \cos^{-1}(0.4437\ldots) \\ \angle Y &= 63.658\ldots° \end{aligned}$$
The measure of $\angle Y$ is 64°.

d)



$$\begin{aligned} 13^2 &= 17^2 + 15^2 - 2(17)(15) \cos G \\ 169 &= 289 + 225 - 510 \cos G \\ -345 &= -510 \cos G \\ 0.6764\ldots &= \cos G \\ \angle G &= \cos^{-1}(0.6764\ldots) \\ \angle G &= 47.431\ldots° \end{aligned}$$
The measure of $\angle G$ is 47°.

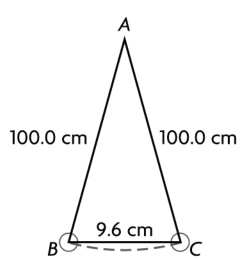
7. a)
$$\begin{aligned}
f^2 &= d^2 + e^2 - 2de\cos F\\
f^2 &= 5.0^2 + 6.5^2 - 2(5.0)(6.5)\cos 65°\\
f^2 &= 25.0 + 42.25 - 65.0(0.4226\ldots)\\
f^2 &= 39.779\ldots\\
f &= 6.307\ldots
\end{aligned}$$
The length of f is 6.3 cm.
$$\begin{aligned}
d^2 &= e^2 + f^2 - 2ef\cos D\\
5.0^2 &= 6.5^2 + 6.307\ldots^2\\
&\quad - 2(6.5)(6.307\ldots)\cos D\\
25.0 &= 42.25 + 39.779\ldots\\
&\quad - 81.992\ldots\cos D\\
-57.029\ldots &= -81.992\ldots\cos D\\
0.6955\ldots &= \cos D\\
\angle D &= \cos^{-1}(0.6955\ldots)\\
\angle D &= 45.929\ldots°
\end{aligned}$$
The measure of $\angle D$ is 45.9°.
$$\begin{aligned}
e^2 &= d^2 + f^2 - 2df\cos E\\
6.5^2 &= 5.0^2 + 6.307\ldots^2\\
&\quad - 2(5.0)(6.307\ldots)\cos E\\
42.25 &= 25.0 + 39.779\ldots - 63.071\ldots\cos E\\
-22.529\ldots &= -63.071\ldots\cos E\\
0.3572\ldots &= \cos E\\
\angle E &= \cos^{-1}(0.3552\ldots)\\
\angle E &= 69.070\ldots°
\end{aligned}$$
The measure of $\angle E$ is 69.1°.

b)
$$\begin{aligned}
r^2 &= p^2 + q^2 - 2pq\cos R\\
r^2 &= 6.4^2 + 9.0^2 - 2(6.4)(9.0)\cos 80°\\
r^2 &= 40.96 + 81.0 - 115.2(0.1736\ldots)\\
r^2 &= 101.955\ldots\\
r &= 10.097\ldots
\end{aligned}$$
The length of r is 10.1 m.
$$\begin{aligned}
p^2 &= q^2 + r^2 - qr\cos P\\
6.4^2 &= 9.0^2 + 10.1^2 - 2(9.0)(10.1)\cos P\\
40.96 &= 81.0 + 102.01 - 181.8\cos P\\
-142.05 &= -181.8\cos P\\
0.7813\ldots &= \cos P\\
\angle P &= \cos^{-1}(0.7813\ldots)\\
\angle P &= 38.615\ldots°
\end{aligned}$$
The measure of $\angle P$ is 38.6°.
$$\begin{aligned}
q^2 &= p^2 + r^2 - pr\cos Q\\
9.0^2 &= 6.4^2 + 10.1^2 - 2(6.4)(10.1)\cos Q\\
81.0 &= 40.96 + 102.01 - 129.28\cos Q\\
-61.97 &= -129.28\cos Q\\
0.4793\ldots &= \cos Q\\
\angle Q &= \cos^{-1}(0.4793\ldots)\\
\angle Q &= 61.357\ldots°
\end{aligned}$$
The measure of $\angle Q$ is 61.4°.

c)
$$\begin{aligned}
l^2 &= m^2 + n^2 - mn\cos L\\
5.5^2 &= 4.6^2 + 3.3^2 - 2(4.6)(3.3)\cos L\\
30.25 &= 21.16 + 10.89 - 30.36\cos L\\
-1.8 &= -30.36\cos L\\
0.0592\ldots &= \cos L\\
\angle L &= \cos^{-1}(0.0592\ldots)\\
\angle L &= 86.601\ldots°
\end{aligned}$$
The measure of $\angle L$ is 86.6°.
$$\begin{aligned}
m^2 &= l^2 + n^2 - ln\cos M\\
4.6^2 &= 5.5^2 + 3.3^2 - 2(5.5)(3.3)\cos M\\
21.16 &= 30.25 + 10.89 - 36.3\cos M\\
-19.98 &= -36.3\cos M\\
0.5504\ldots &= \cos M\\
\angle M &= \cos^{-1}(0.5504\ldots)\\
\angle M &= 56.604\ldots°
\end{aligned}$$
The measure of $\angle M$ is 56.6°.
$$\begin{aligned}
n^2 &= l^2 + m^2 - lm\cos N\\
3.3^2 &= 5.5^2 + 4.6^2 - 2(5.5)(4.6)\cos N\\
10.89 &= 30.25 + 21.16 - 50.6\cos N\\
-40.52 &= -50.6\cos N\\
0.8007\ldots &= \cos N\\
\angle N &= \cos^{-1}(0.8007\ldots)\\
\angle N &= 36.794\ldots°
\end{aligned}$$
The measure of $\angle N$ is 36.8°.

d)
$$\begin{aligned}
x^2 &= y^2 + z^2 - yz\cos X\\
5.2^2 &= 4.0^2 + 4.5^2 - 2(4.0)(4.5)\cos X\\
27.04 &= 16.00 + 20.25 - 36.0\cos X\\
-9.21 &= -36.0\cos X\\
0.2558\ldots &= \cos X\\
\angle X &= \cos^{-1}(0.2558\ldots)\\
\angle X &= 75.177\ldots°
\end{aligned}$$
The measure of $\angle X$ is 75.2°.
$$\begin{aligned}
y^2 &= x^2 + z^2 - xz\cos Y\\
4.0^2 &= 5.2^2 + 4.5^2 - 2(5.2)(4.5)\cos Y\\
16.00 &= 27.04 + 20.25 - 46.8\cos Y\\
-31.29 &= -46.8\cos Y\\
0.6685\ldots &= \cos Y\\
\angle Y &= \cos^{-1}(0.6685\ldots)\\
\angle Y &= 48.041\ldots°
\end{aligned}$$
The measure of $\angle Y$ is 48.0°.
$$\begin{aligned}
z^2 &= x^2 + y^2 - xy\cos Z\\
4.5^2 &= 5.2^2 + 4.0^2 - 2(5.2)(4.0)\cos Z\\
20.25 &= 27.04 + 16.0 - 41.6\cos Z\\
-22.79 &= -41.6\cos Z\\
0.5478\ldots &= \cos Z\\
\angle Z &= \cos^{-1}(0.5478\ldots)\\
\angle Z &= 56.781\ldots°
\end{aligned}$$
The measure of $\angle Z$ is 56.8°.

8. a) I assumed that the length of the pendulum is measured from the fixed point to the centre of mass of the bob.



b) $a^2 = b^2 + c^2 - bc \cos A$

$9.6^2 = 100.0^2 + 100.0^2 - 2(100.0)(100.0) \cos A$

$92.16 = 10\,000.00 + 10\,000.00 - 20\,000.00 \cos A$

$-19\,907.84 = -20\,000.00 \cos A$

$\frac{-19\,907.84}{-20\,000.00} = \cos A$

$0.9953\ldots = \cos A$

$\angle A = \cos^{-1}(0.9953\ldots)$

$\angle A = 5.502\ldots°$

The measure pendulum swing is 5.5°.

9. $s^2 = r^2 + t^2 - rt \cos S$

$s^2 = 15^2 + 20^2 - 2(15)(20) \cos 60°$

$s^2 = 225 + 400 - 600(0.5)$

$s^2 = 325$

$s = 18.0277\ldots$

Perimeter = $s + r + t$

Perimeter = 18.0 + 15 + 20

Perimeter = 53.0

The perimeter is 53.0 cm.

10. e.g., You can use the cosine law; the 70° angle is one of the acute angles across from the shorter diagonal. It is contained between an 8 cm side and a 15 cm side.

11. a) i) At 2:00, the angle of the hour is $\frac{2}{12}$ or $\frac{1}{6}$ of 360° or 60°.

Let d represent the distance between the two arms.

$d^2 = 20^2 + 12^2 - 2(20)(12) \cos 60°$

$d^2 = 400 + 144 - 480(0.5)$

$d^2 = 304$

$d = 17.435\ldots$

The distance between the two arms is approximately 17.4 cm.

ii) At 10:00, the angle of the hour is $\frac{10}{12}$ or $\frac{5}{6}$ of 360° or 300°. However, this shortest distance between the two arms is through a 60° angle. Therefore, the distance between the two arms is approximately 17.4 cm.

b) e.g., The hour and minute hands are the same distance apart at both times, so the triangles are congruent. The distance between the hands should be the same.

12. e.g., No. When you substitute the side lengths into the expression for the cosine law, the following restriction is not true, $-1 \le \cos \theta \le 1$.

Also, the 60 m side is longer than the sum of the other two sides (15 + 36 = 51).

13. Let a represent the distance driver 1 travelled.

Driver 1 distance = speed · time

Driver 1 distance = $33.0\left(\frac{45}{60}\right)$

Driver 1 distance = 24.75

Let b represent the distance driver 2 travelled.

b = speed · time

$b = 45.0\left(\frac{45}{60}\right)$

$b = 33.75$

Let c represent the distance the two cars are apart.

$c^2 = a^2 + b^2 - 2ab \cos 70°$

$c^2 = 24.75^2 + 33.75^2 - 2(24.75)(33.75)\cos 70°$

$c^2 = 612.562 + 1139.062 - 1670.625(0.3420\ldots)$

$c^2 = 1180.237\ldots$

$c = 34.354\ldots$

The drivers will be 34.4 km apart.

14. e.g., Kathryn wants to determine the length of a pond. From where she is standing, one end of the pond is 35 m away. If she turns 35° to the left, the distance to the other end of the pond is 30 m. How long is the pond? Use the cosine law to determine the unknown side length.

15. Ten isosceles triangles can be created in the decagon. The vertex angle of each triangle is $\frac{1}{10}$ of 360° or 36°. The base angle of each triangle is 72°.

Let a represent the length of a side of the decagon.

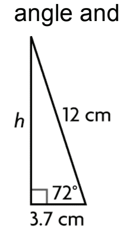
$a^2 = 12^2 + 12^2 - 2(12)(12) \cos 36°$

$a^2 = 144 + 144 - 288(0.8090\ldots)$

$a^2 = 55.003\ldots$

$a = 7.416\ldots$

The angle bisector of the vertex angle of an isosceles triangle bisects the side opposite the angle and is also perpendicular to the side.



$\sin 72° = \frac{h}{12}$

$h = 12 \sin 72°$

$h = 12(0.9510\ldots)$

$h = 11.412\ldots$

Area of one triangle = $\frac{1}{2}ah$

Area of one triangle = $\frac{1}{2}(7.416\ldots)(11.412\ldots)$

Area of one triangle = 42.320…

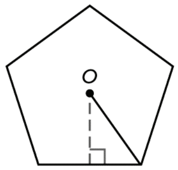
Area of 10 triangles = 10(area of one triangle)

Area of 10 triangles = 10(42.320…)

Area of 10 triangles = 423.205…

The area of the decagon is 423 cm^2.

16.

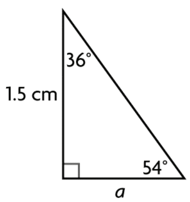


Each of the five triangles in the pentagon is isosceles.

The angle at the centre of the pentagon is $\frac{1}{5}$ of 360° or 72°.

The base angle of each triangle is 54°.

The perpendicular bisector bisects the vertex angle, so the angle in a right triangle is 36°.



Let a represent the base of the triangle.

$$\frac{a}{\sin 36°} = \frac{1.5}{\sin 54°}$$
$$\sin 36°\left(\frac{a}{\sin 36°}\right) = \sin 36°\left(\frac{1.5}{\sin 54°}\right)$$
$$a = \sin 36°\left(\frac{1.5}{\sin 54°}\right)$$
$$a = 1.089...$$

The length of a side of a pentagon is $2a$ or 2.179… cm.

Perimeter = 5(side length of pentagon)
Perimeter = 5(2.179…)
Perimeter = 10.898…
The perimeter of the pentagon is 10.9 cm.

Area of one triangle = $\frac{1}{2}a(1.5)$

Area of one triangle = $\frac{1}{2}(2.179\ldots)(1.5)$

Area of one triangle = 1.634…
Area of five triangles = 5(area of one triangle)
Area of five triangles = 5(1.634…)
Area of five triangles = 8.173…
The area of the pentagon is 8.2 cm^2.

17. e.g., The vertex angle at the handle of the knife is about 110°. Each of the sides of the knife is about 8.5 cm in length.

Lesson 3.4: Solving Problems Using Acute Triangles, page 147

1. a) The measures of two sides and an angle opposite one of the sides are given, so use the sine law to solve for θ.
b) The triangle is a right triangle. The length of the side adjacent to the right angle is given along with the measure of one acute angle. You can use the tangent ratio to solve for c. You could also use the sine ratio to solve for c by finding the measure of $\angle A$ first.
c) The lengths of three sides are given, so you can use the cosine law to solve for θ.

2. a) part a:
$$\frac{\sin \theta}{3.1} = \frac{\sin 60°}{2.7}$$
$$3.1\left(\frac{\sin \theta}{3.1}\right) = 3.1\left(\frac{\sin 60°}{2.7}\right)$$
$$\sin \theta = 3.1\left(\frac{\sin 60°}{2.7}\right)$$
$$\theta = \sin^{-1}(0.9943...)$$
$$\theta = 83.893...°$$

The measure of θ is 83.9°.

part b: $\tan 47° = \frac{c}{1.8}$
$$1.8(\tan 47°) = c$$
$$1.8(1.0723\ldots) = c$$
$$1.930\ldots = c$$

The length of c is 1.9 cm.

part c: $a^2 = b^2 + c^2 - 2bc \cos A$
$$2.0^2 = 2.9^2 + 3.0^2 - 2(2.9)(3.0)\cos \theta$$
$$4.0 = 8.41 + 9.0 - 17.4 \cos \theta$$
$$-13.41 = -17.4 \cos \theta$$
$$0.7706\ldots = \cos \theta$$
$$\theta = \cos^{-1}(0.7706\ldots)$$
$$\theta = 39.584\ldots°$$

The measure of θ is 39.6°.

b) e.g., Using a trigonometric ratio is more efficient because you have fewer calculations to do.

3. a) Since the measures of two sides and a contained angle are given, I would use the cosine law.
b) Let d represent the distance between the kayaks.
$$d^2 = 4.0^2 + 5.0^2 - 2(4.0)(5.0)\cos 30°$$
$$d^2 = 16.0 + 25.0 - 40.0(0.8660\ldots)$$
$$d^2 = 6.358\ldots$$
$$d = 2.521\ldots$$

The distance the kayaks are apart is 2.5 km.