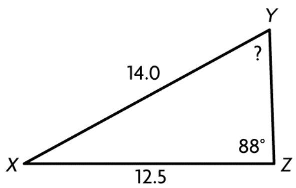
d) i)



ii)

$$\frac{\sin Y}{12.5} = \frac{\sin 88°}{14.0}$$

$$12.5\left(\frac{\sin Y}{12.5}\right) = 12.5\left(\frac{\sin 88°}{14.0}\right)$$

$$\sin Y = 12.5\left(\frac{\sin 88°}{14.0}\right)$$

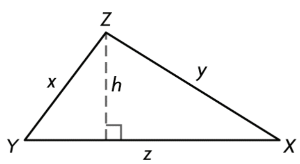
$$\sin Y = 0.8923...$$

$$\angle Y = \sin^{-1}(0.8923...)$$

$$\angle Y = 63.165...°$$

The measure of $\angle Y$ is 63.2°.

3. Agree.



In the large right triangle,

$$\sin X = \frac{h}{y}$$

$$y \sin X = y\left(\frac{h}{y}\right)$$

$$h = y \sin X$$

In the small right triangle,

$$\sin Y = \frac{h}{x}$$

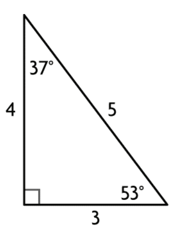
$$x \sin Y = x\left(\frac{h}{x}\right)$$

$$h = x \sin Y$$

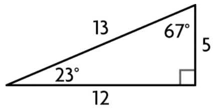
$\therefore y \sin X = x \sin Y$

4. e.g., You need the lengths of two sides and the measure of the angle opposite one of the sides or the measures of two angles and the length of any side.

5. e.g., Yes, the ratios are equivalent. Answers have been rounded to the nearest whole number.



$$\frac{3}{\sin 37°} = 5 \quad \frac{4}{\sin 53°} = 5 \quad \frac{5}{\sin 90°} = 5$$



$$\frac{5}{\sin 23°} = 13 \quad \frac{12}{\sin 67°} = 13 \quad \frac{13}{\sin 90°} = 13$$

Lesson 3.2: Proving and Applying the Sine Law, page 124

1. $\frac{q}{\sin Q} = \frac{r}{\sin R} = \frac{s}{\sin S}$ or

$$\frac{\sin Q}{q} = \frac{\sin R}{r} = \frac{\sin S}{s}$$

2. a)

$$\frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{b}{\sin 72°} = \frac{27.2}{\sin 43°}$$

$$\sin 72°\left(\frac{b}{\sin 72°}\right) = \sin 72°\left(\frac{27.2}{\sin 43°}\right)$$

$$b = \sin 72°\left(\frac{27.2}{\sin 43°}\right)$$

$$b = 37.930...$$

The length of b is 37.9 cm.

b)

$$\frac{\sin C}{c} = \frac{\sin B}{b}$$

$$\frac{\sin \theta}{37.1} = \frac{\sin 44°}{29.5}$$

$$37.1\left(\frac{\sin \theta}{37.1}\right) = 37.1\left(\frac{\sin 44°}{29.5}\right)$$

$$\sin \theta = 0.8736...$$

$$\theta = \sin^{-1}(0.8736...)$$

$$\theta = 60.882...°$$

The measure of θ is 61°.

3. a) $\angle F + 53° + 68° = 180°$

$$\angle F = 59°$$

$$\frac{d}{\sin D} = \frac{f}{\sin F}$$

$$\frac{d}{\sin 53°} = \frac{22.5}{\sin 59°}$$

$$\sin 53°\left(\frac{d}{\sin 53°}\right) = \sin 53°\left(\frac{22.5}{\sin 59°}\right)$$

$$d = \sin 53°\left(\frac{22.5}{\sin 59°}\right)$$

$$d = 20.963...$$

The length of d is 21.0 cm.

b) $\angle C + 40° + 60° = 180°$

$$\angle C = 80°$$

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$

$$\frac{a}{\sin 40°} = \frac{40.0}{\sin 80°}$$

$$\sin 40°\left(\frac{a}{\sin 40°}\right) = \sin 40°\left(\frac{40.0}{\sin 80°}\right)$$

$$a = \sin 40°\left(\frac{40.0}{\sin 80°}\right)$$

$$a = 26.108...$$

The length of a is 26.1 cm.

$$\frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{b}{\sin 60°} = \frac{40.0}{\sin 80°}$$

$$\sin 60°\left(\frac{b}{\sin 60°}\right) = \sin 60°\left(\frac{40.0}{\sin 80°}\right)$$

$$b = \sin 60°\left(\frac{40.0}{\sin 80°}\right)$$

$$b = 35.175...$$

The length of b is 35.2 cm.

c) $\angle Y + 88° + 25° = 180°$

$$\angle Y = 67°$$

$$\frac{y}{\sin Y} = \frac{z}{\sin Z}$$

$$\frac{y}{\sin 67°} = \frac{3.0}{\sin 25°}$$

$$\sin 67°\left(\frac{y}{\sin 67°}\right) = \sin 67°\left(\frac{3.0}{\sin 25°}\right)$$

$$y = \sin 67°\left(\frac{3.0}{\sin 25°}\right)$$

$$y = 6.534...$$

The length of y is 6.5 cm.

d)

$$\frac{\sin N}{n} = \frac{\sin L}{l}$$

$$\frac{\sin\theta}{45.2} = \frac{\sin 29°}{24.4}$$

$$45.2\left(\frac{\sin\theta}{45.2}\right) = 45.2\left(\frac{\sin 29°}{24.4}\right)$$

$$\sin\theta = 45.2\left(\frac{\sin 29°}{24.4}\right)$$

$$\theta = \sin^{-1}(0.8980...)$$

$$\theta = 63.908...°$$

The measure of θ is 64°.

e)

$$\frac{\sin S}{s} = \frac{\sin Q}{q}$$

$$\frac{\sin\alpha}{6.5} = \frac{\sin 50°}{5.0}$$

$$6.5\left(\frac{\sin\alpha}{6.5}\right) = 6.5\left(\frac{\sin 50°}{5.0}\right)$$

$$\sin\alpha = 6.5\left(\frac{\sin 50°}{5.0}\right)$$

$$\alpha = \sin^{-1}(0.9958...)$$

$$\alpha = 84.783...°$$

The measure of α is 85°.

$$\theta + \alpha + \angle Q = 180°$$

$$\theta + 85° + 50° = 180°$$

$$\theta = 45°$$

The measure of θ is 45°.

f)

$$\frac{\sin L}{l} = \frac{\sin K}{k}$$

$$\frac{\sin\theta}{2.9} = \frac{\sin 80°}{6.7}$$

$$2.9\left(\frac{\sin\theta}{2.9}\right) = 2.9\left(\frac{\sin 80°}{6.7}\right)$$

$$\sin\theta = 2.9\left(\frac{\sin 80°}{6.7}\right)$$

$$\theta = \sin^{-1}(0.4262...)$$

$$\theta = 25.230...°$$

The measure of θ is 25°.

$$\alpha + \theta + \angle Q = 180°$$

$$\alpha + 25° + 80° = 180°$$

$$\alpha = 75°$$

The measure of α is 75°.

$$\frac{j}{\sin J} = \frac{k}{\sin K}$$

$$\frac{j}{\sin\alpha} = \frac{6.7}{\sin 80°}$$

$$\sin 75°\left(\frac{j}{\sin 75°}\right) = \sin 75°\left(\frac{6.7}{\sin 80°}\right)$$

$$j = \sin 75°\left(\frac{6.7}{\sin 80°}\right)$$

$$j = 6.571...$$

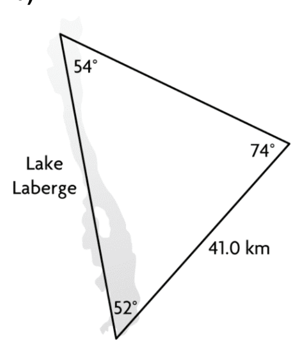
The length of j is 6.6 cm.

4. a) Determine the measure of the third angle:

$$180° - 74° - 52° = 54°$$

Scott's answer is incorrect because the longest side of a triangle is opposite the greatest interior angle. The length of the lake cannot be less than the given side length of 41.0 km.

b)



Let $\angle A = 54°$ represent the measure of the remaining unknown angle.

Let c represent the length of Lake Laberge in kilometres.

$$\frac{c}{\sin C} = \frac{a}{\sin A}$$

$$\frac{c}{\sin 74°} = \frac{41.0}{\sin 54°}$$

$$\sin 74°\left(\frac{c}{\sin 74°}\right) = \sin 74°\left(\frac{41.0}{\sin 54°}\right)$$

$$c = \sin 74°\left(\frac{41.0}{\sin 54°}\right)$$

$$c = 48.715...$$

The length of Lake Laberge is 48.7 km.

5. The rafters must be of equal length so the triangle formed by the shorter rafters and the base is isosceles. Therefore, the base angles of the triangle are equal. Let x represent the measure of a base angle of the triangle.

$$2x + 70° = 180°$$

$$2x = 110°$$

$$x = 55°$$

Let y represent the length of the shorter rafter.

$$\frac{y}{\sin 55°} = \frac{36}{\sin 70°}$$

$$\sin 55°\left(\frac{y}{\sin 55°}\right) = \sin 55°\left(\frac{36}{\sin 70°}\right)$$

$$y = \sin 55°\left(\frac{36}{\sin 70°}\right)$$

$$y = 31.3820...$$

Length of the rafter in total = y + 1

Length of the rafter in total = 31.382… + 1

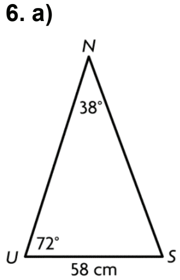
Length of the rafter in total = 32.382…

Convert the decimal to inches:

0.382… ft · 12 in./ft = 4.584… in.

The length of the rafter is 32 ft 5 in.

6. a)



$$\frac{u}{\sin U} = \frac{n}{\sin N}$$

$$\frac{u}{\sin 72°} = \frac{58}{\sin 38°}$$

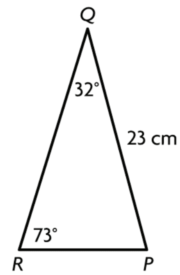
$$\sin 72°\left(\frac{u}{\sin 72°}\right) = \sin 72°\left(\frac{58}{\sin 38°}\right)$$

$$u = \sin 72°\left(\frac{58}{\sin 38°}\right)$$

$$u = 89.596...$$

The length of u is 90 cm.

b)



$$\frac{q}{\sin Q} = \frac{r}{\sin R}$$

$$\frac{q}{\sin 32°} = \frac{23}{\sin 73°}$$

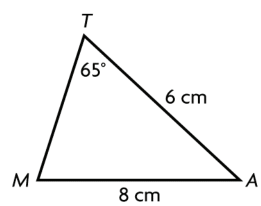
$$\sin 32°\left(\frac{q}{\sin 32°}\right) = \sin 32°\left(\frac{23}{\sin 73°}\right)$$

$$q = \sin 32°\left(\frac{23}{\sin 73°}\right)$$

$$q = 12.745...$$

The length of q is 13 cm.

c)



$$\frac{\sin M}{m} = \frac{\sin T}{t}$$

$$\frac{\sin M}{6} = \frac{\sin 65°}{8}$$

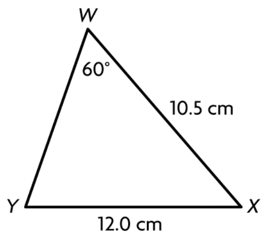
$$6\left(\frac{\sin M}{6}\right) = 6\left(\frac{\sin 65°}{8}\right)$$

$$\angle M = \sin^{-1}(0.6797...)$$

$$\angle M = 42.822...°$$

The measure of $\angle M$ is 43°.

d)



$$\frac{\sin Y}{y} = \frac{\sin W}{w}$$

$$\frac{\sin Y}{10.5} = \frac{\sin 60°}{12.0}$$

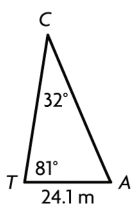
$$10.5\left(\frac{\sin Y}{10.5}\right) = 10.5\left(\frac{\sin 60°}{12.0}\right)$$

$$\angle Y = \sin^{-1}(0.7577...)$$

$$\angle Y = 49.268...°$$

The measure of $\angle Y$ is 49°.

7.



$$\angle A + \angle C + \angle T = 180°$$

$$\angle A + 32° + 81° = 180°$$

$$\angle A = 67°$$

$$\frac{t}{\sin T} = \frac{c}{\sin C}$$

$$\frac{t}{\sin 81°} = \frac{24.1}{\sin 32°}$$

$$\sin 81°\left(\frac{t}{\sin 81°}\right) = \sin 81°\left(\frac{24.1}{\sin 32°}\right)$$

$$t = 44.918...$$

The length of t is 44.9 m.

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$

$$\frac{a}{\sin 67°} = \frac{24.1}{\sin 32°}$$

$$\sin 67°\left(\frac{a}{\sin 67°}\right) = \sin 67°\left(\frac{24.1}{\sin 32°}\right)$$

$$a = \sin 67°\left(\frac{24.1}{\sin 32°}\right)$$

$$a = 41.863...$$

The length of a is 41.9 m.

8. a) i)

$$\sin N = \frac{n}{m}$$

$$\sin 36.9° = \frac{n}{10}$$

$$n = 10(\sin 36.9°)$$

$$n = 10(0.6004\ldots)$$

$$n = 6.004\ldots$$

The length of n is 6.0 m.

ii)

$$\frac{n}{\sin N} = \frac{m}{\sin M}$$

$$\frac{n}{\sin 36.9°} = \frac{10}{\sin 90°}$$

$$\sin 36.9°\left(\frac{n}{\sin 36.9°}\right) = \sin 36.9°\left(\frac{10}{\sin 90°}\right)$$

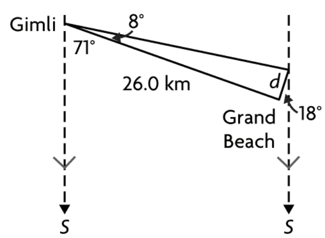
$$n = \sin 36.9°\left(\frac{10}{\sin 90°}\right)$$

$$n = 6.004...$$

The length of n is 6.0 m.

b) e.g., Since sin 90° = 1, you can rearrange the sine law formula to give the expression for the sine ratio.

9. a)



b) Let x represent the measure of the angle exterior to the east side of the triangle.

$x = 79°$ Alternate interior angles

Let y represent the angle opposite the side 26.0 km side.

$y + x + 18° = 180°$ Supplementary angles

$y + 79° + 18° = 180°$

$y = 83°$

Let g represent the distance between Gimli and Grand Beach.

Let d represent the distance from Janice to Grand Beach.

Let D represent the measure of the angle between and g.

$$\frac{d}{\sin D} = \frac{g}{\sin y}$$

$$\frac{d}{\sin 8°} = \frac{26.0}{\sin 83°}$$

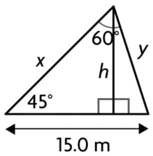
$$\sin 8°\left(\frac{d}{\sin 8°}\right) = \sin 8°\left(\frac{26.0}{\sin 83°}\right)$$

$$d = \sin 8°\left(\frac{26.0}{\sin 83°}\right)$$

$$d = 3.645...$$

The length of d is 3.6 km.

10. a)



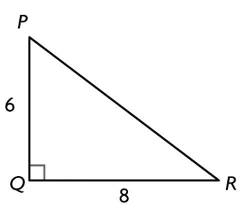
b) Let *A* represent the measure of remaining unknown angle in the largest triangle.

$$\angle A + 45° + 60° = 180°$$
$$\angle A = 75°$$

$$\frac{y}{\sin 45°} = \frac{15.0}{\sin 60°}$$
$$\sin 45°\left(\frac{y}{\sin 45°}\right) = \sin 45°\left(\frac{15.0}{\sin 60°}\right)$$
$$y = \sin 45°\left(\frac{15.0}{\sin 60°}\right)$$
$$y = 12.247...$$
$$\frac{x}{\sin A} = \frac{15.0}{\sin 60°}$$
$$\frac{x}{\sin 75°} = \frac{15.0}{\sin 60°}$$
$$\sin 75°\left(\frac{x}{\sin 75°}\right) = \sin 75°\left(\frac{15.0}{\sin 60°}\right)$$
$$x = \sin 75°\left(\frac{15.0}{\sin 60°}\right)$$
$$x = 16.730...$$
$$\sin 45° = \frac{h}{x}$$
$$h = x \sin 45°$$
$$h = 16.730\ldots(\sin 45°)$$
$$h = 11.830\ldots$$

The wires are 12.2 m and 16.7 m long. The pole is 11.8 m tall.

11.



Since there is a 90° angle, you could use the Pythagorean theorem to determine the length of *q*.

$$q^2 = r^2 + p^2$$
$$q^2 = 6^2 + 8^2$$
$$q^2 = 36 + 64$$
$$q^2 = 100$$
$$q = 10$$

Then use the sine ratio to determine the measure of $\angle P$.

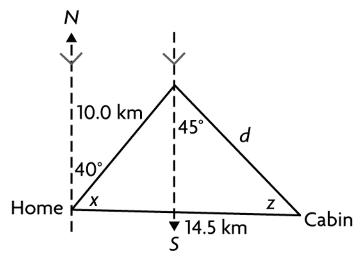
$$\sin P = \frac{8}{10}$$
$$\angle P = \sin^{-1}(0.8)$$
$$\angle P = 53.130\ldots°$$

You could also use the sine law formula to determine the measure of $\angle P$.

$$\frac{\sin P}{p} = \frac{\sin Q}{q}$$
$$\frac{\sin P}{8} = \frac{\sin 90°}{10}$$
$$8\left(\frac{\sin P}{8}\right) = 8\left(\frac{\sin 90°}{10}\right)$$
$$\angle P = \sin^{-1}(0.8)$$
$$\angle P = 53.130...°$$

Using both methods, angle *P* is 53°.

12.



Because the lines are parallel, the angle beside the 45° angle is also 40°. The entire angle is 85°.

Solve for *z*:

$$\frac{\sin z}{10.0} = \frac{\sin 85°}{14.5}$$
$$10.0\left(\frac{\sin z}{10.0}\right) = 10.0\left(\frac{\sin 85°}{14.5}\right)$$
$$\sin z = 10.0\left(\frac{\sin 85°}{14.5}\right)$$
$$z = \sin^{-1}(0.6870...)$$
$$z = 43.395...°$$

$x + 85° + z = 180°$ Sum of interior

$x + 85° + 43.395\ldots° = 180°$ angles of triangle

$x = 51.604\ldots°$

Let *d* represent the distance Stella travelled on the second leg of her trip.

$$\frac{d}{\sin x} = \frac{14.5}{\sin 85°}$$
$$\frac{d}{\sin 51.604...°} = \frac{14.5}{\sin 85°}$$
$$d = \sin 51.604...°\left(\frac{14.5}{\sin 85°}\right)$$
$$d = 11.407...$$

The distance travelled is 11.4 km.

13. Let A represent the measure of the remaining unknown angle in the largest triangle.

$$\angle A + 60° + 75° = 180°$$
$$\angle A = 45°$$

Let x represent the length of the side on the left side of the largest triangle.

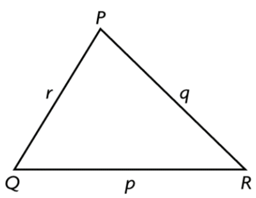
$$\frac{x}{\sin 75°} = \frac{21.0}{\sin 45°}$$
$$\frac{x}{\sin 75°} = \frac{21.0}{\sin 45°}$$
$$\sin 75°\left(\frac{x}{\sin 75°}\right) = \sin 75°\left(\frac{21.0}{\sin 45°}\right)$$
$$x = \sin 75°\left(\frac{21.0}{\sin 45°}\right)$$
$$x = 28.6865...$$

Let h represent the height of the triangle.

$$\sin 60° = \frac{h}{x}$$
$$h = x \sin 60°$$
$$h = 28.686\ldots(0.8660\ldots)$$
$$h = 24.843\ldots$$

The height of the gorge is 24.8 m.

14. a)



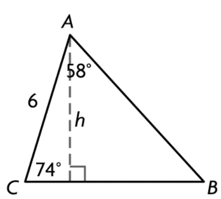
If you knew the measures of angle P, angle R, and the length of r, then you could solve the triangle.

b) If you only knew the measures of angle P, and the lengths of q and r, then you could not solve the triangle.

15. Agree. Jim needs to know the measure of an angle and the length of the side opposite the angle.

16. e.g., You could determine the measure of angle R since the sum of the interior angles of a triangle is 180°. Then you could use the sine law to determine the lengths of sides q and r.

17.



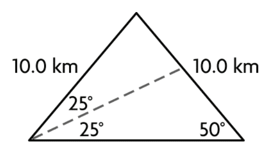
Let h represent the perpendicular height of the triangle.

$$\sin C = \frac{h}{6}$$
$$h = 6 \sin C$$
$$h = 6 \sin 74°$$
$$h = 5.7675\ldots$$
$$\angle B + \angle A + \angle C = 180°$$
$$\angle B + 58° + 74° = 180°$$
$$\angle B = 48°$$
$$\frac{BC}{\sin A} = \frac{6}{\sin B}$$
$$\frac{BC}{\sin 58°} = \frac{6}{\sin 48°}$$
$$\sin 58°\left(\frac{BC}{\sin 58°}\right) = \sin 58°\left(\frac{6}{\sin 48°}\right)$$
$$BC = \sin 58°\left(\frac{6}{\sin 48°}\right)$$
$$BC = 6.846...$$

$$\text{Area} = \frac{1}{2}(BC)h$$
$$\text{Area} = \frac{1}{2}(6.846\ldots)(5.767^*\ldots)$$
$$\text{Area} = 19.747\ldots$$

The area of the triangle is 19.7 square units.

18.



The angle at the top of the isosceles triangle is 180° – 2(50°) = 80°.

The remaining unknown angle in the top triangle is 180° – 25° – 80° = 75°.

Let d represent the length of the angle bisector.

$$\frac{d}{\sin 80°} = \frac{10}{\sin 75°}$$
$$\sin 80°\left(\frac{d}{\sin 80°}\right) = \sin 80°\left(\frac{10}{\sin 75°}\right)$$
$$d = \sin 80°\left(\frac{10}{\sin 75°}\right)$$
$$d = 10.195...$$

The length of the angle bisector is 10.2 cm.

19. e.g., **a)** $\frac{\sin A}{\sin B} = \frac{a}{b}$

b) $\frac{a}{c} = \frac{\sin A}{\sin C}$

c) $\frac{a \sin C}{c \sin A} = 1$