Lesson 2.4: Angle Properties in Polygons, page 99

1. a)

Statement	Justification
$S(n) = 180°(n – 2)$ $S(12) = 180°(12 – 2)$ $S(12) = 180°(10)$ $S(12) = 1800°$	A dodecagon has 12 sides, so n is 12.

The sum of the interior angles in a regular dodecagon is 1800°.

b)

Statement	Justification
$S(12) = 1800°$	Shown in part a).
$\frac{1800°}{12} = 150°$	Each interior angle in a regular dodecagon is equal, so each angle must measure $\frac{1}{12}$ of the sum of the angles.

The measure of each interior angle of a regular dodecagon is 150°.

2.

Statement	Justification
$S(n) = 180°(n – 2)$ $S(20) = 180°(20 – 2)$ $S(20) = 180°(18)$ $S(20) = 3240°$	A 20-sided convex polygon has 20 sides, so n is 20.

The sum of the interior angles in a 20-sided convex polygon is 3240°.

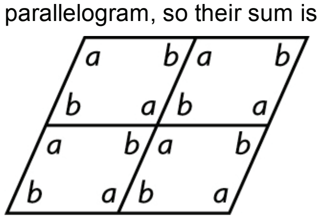
3.

Statement	Justification
$S(n) = 180°(n – 2)$	
$3060° = 180°(n – 2)$	Substitute the known quantity.
$17 = n – 2$	Divide both sides by 180°.
$19 = n$	Add 2 to both sides.

A 19-sided regular convex polygon has a sum of 3060° for its interior angles.

4. e.g., The interior angles of a hexagon equal 120°. Three hexagons will fit together since the sum is 360° at the vertex where they are joined.

5. e.g., Yes. You can align parallel sides to create a tiling pattern; the angles that meet are the four angles of the parallelogram, so their sum is 360°.



6. A loonie is a regular 11-sided convex polygon.

Statement	Justification
$S(n) = 180°(n – 2)$ $S(11) = 180°(11 – 2)$	A loonie has 11 sides, so n is 11.
$S(11) = 180°(9)$ $S(11) = 1620°$	Sum of interior angles
$\frac{1620°}{11} = 147.272…°$	Each interior angle in a regular 11-sided polygon is equal, so each angle must measure $\frac{1}{11}$ of the sum of the angles.

The measure of each interior angle of a regular 11-sided polygon is about 147.27°.

7. a)

Statement	Justification
$S(n) = 180°(n – 2)$	Sum of interior angles
One angle $= \frac{S(n)}{n}$ One angle $= \frac{180°(n-2)}{n}$	Measure of one interior angle
$\frac{180°(n-2)}{n} = 140°$ $180°(n-2) = 140°n$ $180°n - 360° = 140°n$ $40°n = 360°$ $n = 9$	Substitute the known quantity.

A 9-sided regular convex polygon has an interior angle measure of 140°.

b)

Statement	Justification
180° = exterior angle + interior angle 180° = exterior angle + 140° 40° = exterior angle	Supplementary angles
Let S represent the sum of the exterior angles. S = number of exterior angles • measure of one exterior angle $S = 9(40°)$ $S = 360°$	Determine the sum of the measures of the exterior angles.

The sum of the measures of the exterior angles of a regular 9-sided convex polygon is 360°.

8. a) The sum of the measures of the exterior angles of a any convex polygon is 360°. Therefore, the sum of the measures of the exterior angles of a regular octagon is 360°. Let E represent the measure of an exterior angle.

$$E = \frac{\text{sum of exterior angles}}{\text{number of sides}}$$

$$E = \frac{360°}{8}$$

$$E = 45°$$

The measure of an exterior angle of a regular octagon is 45°.

b)

Statement	Justification
180° = interior angle + exterior angle	Supplementary angles
180° = interior angle + 45° 135° = interior angle	Substitute the known quantity.

The measure of an interior angle of a regular octagon is 135°.

c) Let S represent the sum of the interior angles.

S = number of sides • measure of one interior angle
$S = 8(135°)$
$S = 1080°$

The sum of the measures of the interior angles of a regular octagon is 1080°.

d)

Statement	Justification
$S(n) = 180°(n – 2)$ $S(8) = 180°(8 – 2)$ $S(8) = 180°(6)$ $S(8) = 1080°$	An octagon has 8 sides, so n is 8.

The answers for parts c) and d) are the same.

9. a) e.g., Agree. If you draw a regular hexagon, you can draw three rectangles using opposite sides. The rectangles have opposite sides that are parallel. You cannot do this for a regular polygon with an odd number of sides.

b) e.g., Opposite sides are parallel in a regular polygon that has an even number of sides.

10. a)

Statement	Justification
$\angle LPO = 108°$	$LMNOP$ is a regular pentagon, so n is 5. $S(n) = 180°(n – 2)$ $S(5) = 180°(5 – 2)$ $S(5) = 180°(3)$ $S(5) = 540°$ $\frac{540°}{5} = 108°$
$PL = OP$ $\triangle OLP$ is isosceles. $\angle LOP = \angle OLP$	Given Definition of isosceles triangle Property of isosceles triangle
$\angle LOP + \angle OLP + \angle LPO = 180°$	Sum of interior angles in triangle
$\angle LOP + \angle OLP + 108° = 180°$ $\angle LOP + \angle OLP = 72°$ $2\angle LOP = 72°$ $\angle LOP = 36°$ $\angle OLP = 36°$	Property of equality
$LM = NM$ $\triangle LMN$ is isosceles. $\angle LMN = \angle NLM$ $\angle LMN = 108°$	Given Definition of isosceles triangle Property of isosceles triangle Measure of interior angle of regular pentagon
$\angle LNM + \angle NLM + \angle LMN = 180°$	Sum of interior angles in triangle
$\angle LNM + \angle NLM + 108° = 180°$ $\angle LNM + \angle NLM = 72°$ $2\angle NLM = 72°$ $\angle NLM = 36°$ $\angle LNM = 36°$	Property of equality
$\angle MLP = 108°$	Measure of interior angle of regular pentagon
$\angle MLP = \angle OLN + \angle OLP + \angle MLN$ $108° = \angle OLN + 36° + 36°$ $36° = \angle OLN$	Property of equality Substitution

b)

Statement	Justification
$\angle NOP = \angle ONM$	Measures of interior angles in a regular polygon are equal.
$\angle LOP = 36°$ and $\angle LMN = 36°$ $\angle LOP = \angle LMN$	Determined in part a) Transitive property
$\angle NOP = \angle LOP + \angle LON$ $108° = 36° + \angle LON$ $72° = \angle LON$	Property of equality Substitution
$\angle LON = 72°$ and $\angle LNO = 72°$ $\angle LON = \angle LNO$ $\triangle OLN$ is isosceles.	Determined above Transitive property Definition of isosceles triangle

11. The formula for the measure of an interior angle of a regular polygon is $S(n) = \frac{180°(n – 2)}{n}$.

In the formula, Sandy wrote 1 instead of 2.

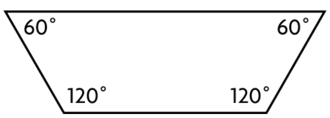
$S(10) = \frac{180°(10 – 2)}{10}$
$S(10) = \frac{180°(8)}{10}$
$S(10) = 144°$

12. a) e.g., A test could be a single line drawn anywhere through the polygon. For convex polygons, it intersects two sides only. For non-convex polygons, it can intersect in more than two sides.

b) If any diagonal is exterior to the polygon, the polygon is non-convex.

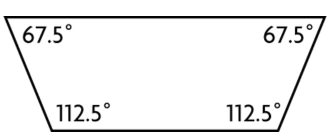
13. a) Assume the hexagonal table top is in the shape of a regular hexagon. Each trapezoidal piece of wood in a section forms a triangle with the angle at the centre vertex. Each triangle in a section is similar to each other so their corresponding angles are equal. The corresponding angles in each trapezoid are equal.

Statement	Justification
Each interior angle of the hexagon is 120°.	The table is a regular hexagon, so n is 6. $S(n) = 180°(n - 2)$ $S(6) = 180°(6 - 2)$ $S(6) = 180°(4)$ $S(6) = 720°$ $\frac{720°}{6} = 120°$
Each base angle of a sector triangle is 60°.	Each diameter is an angle bisector of an interior angle. $\frac{120°}{2} = 60°$
$x + 60° = 180°$ $x = 120°$	In each trapezoid, the sum of the interior angles on the same side of a transversal (diameter), are supplementary. Let the other interior angle be x.

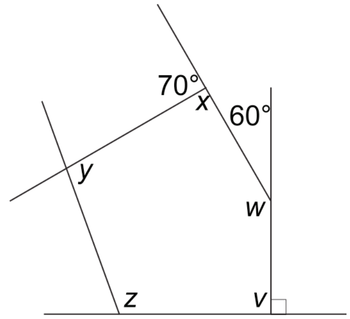


b)

Statement	Justification
Each interior angle of the octagon is 135°.	The table is a regular octagon, so n is 8. $S(n) = 180°(n - 2)$ $S(8) = 180°(8 - 2)$ $S(8) = 180°(6)$ $S(8) = 1080°$ $\frac{1080°}{8} = 135°$
Each base angle of a sector triangle is 67.5°.	Each diameter is an angle bisector of an interior angle. $\frac{135°}{2} = 67.5°$
$x + 67.5° = 180°$ $x = 112.5°$	In each trapezoid, the sum of the interior angles on the same side of a transversal (diameter), are supplementary. Let the other interior angle be x.



14. I drew a diagram to help me.

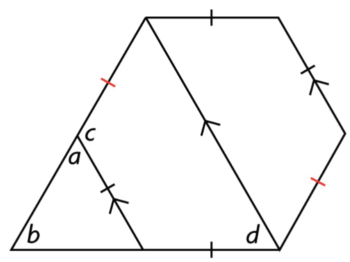


Statement	Justification
$v + 90° = 180°$ $v = 90°$	Supplementary angles
$w + 60° = 180°$ $w = 120°$	Supplementary angles
$x + 70° = 180°$ $x = 110°$	Supplementary angles
$y = z$	Given
$S(n) = 180°(n - 2)$ $S(5) = 180°(5 - 2)$ $S(5) = 180°(3)$ $S(5) = 540°$	The polygon is a pentagon, so n is 5. Sum of interior angles
Let S represent the sum of the angles in the pentagon. $S = y + z + v + w + x$ $S = 2y + 90° + 120° + 110°$ $S = 2y + 320°$ Therefore, $2y + 320° = 540°$ $2y = 220°$ $y = 110°$	Property of equality Substitution
$z = 110°$	Transitive property

The measures of the interior angles are 110°, 120°, 90°, 110°, and 110°.

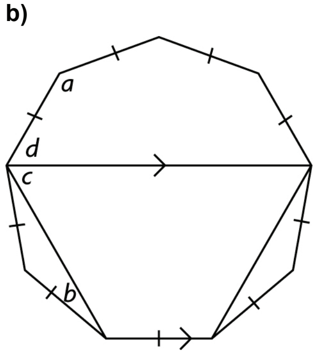
15. The angles are exterior angles of a pentagon, so the sum of the measures of the exterior angles is 360°.

16. a) Angle c is an interior angle of a regular hexagon.



Statement	Justification
$S(n) = 180°(n - 2)$ $S(6) = 180°(6 - 2)$ $S(6) = 180°(4)$ $S(6) = 720°$ $\frac{720°}{6} = 120°$ $c = 120°$	The larger polygon is a hexagon, so n is 6. The measure of one interior angle
$d = \frac{1}{2}c$ $d = 60°$	The two trapezoids are congruent so, the bisector of the hexagon is an angle bisector of an interior angle.
$a + c = 180°$ $a = 60°$	Supplementary angles
$x + d = 180°$ $x = 120°$	Let x be the measure of the unknown angle that is an interior angle on the same side of the transversal as d.
$z + x = 180°$ $z = 60°$	Let z be the measure of the unknown angle in the triangle.
$b + z + a = 180°$ $b + 60° + 60° = 180°$ $b = 60°$	Sum of interior angles in triangle

b)



Statement	Justification
$S(n) = 180°(n - 2)$ $S(9) = 180°(9 - 2)$ $S(9) = 180°(7)$ $S(9) = 1260°$ $\frac{1260°}{9} = 140°$ $a = 140°$	The polygon is a regular nonagon, so n is 9. Measure of one interior angle
$z = 140°$ $2b + z = 180°$ $2b = 40°$ $b = 20°$	Let z be the measure of the nonagon angle in the isosceles triangle containing b.
$x + b = 140°$ $x = 120°$	Let x be the measure of the unknown angle adjacent to b. Property of equality
$c + x = 180°$ $c = 60°$	Interior angles on the same side of a transversal
$d + c + b = 140°$ $d + 60° + 20° = 140°$ $d = 60°$	Property of equality

17. There are two quadrilaterals.
Quadrilateral 1 contains angles: a, c, e, g
Quadrilateral 2 contains angles: b, d, f, h

Statement	Justification
$S(n) = 180°(n - 2)$ $S(4) = 180°(4 - 2)$ $S(4) = 180°(2)$ $S(4) = 360°$ $2(360°) = 720°$	The polygon is a quadrilateral, so n is 4. Measure of all interior angles of one quadrilateral Measure of all interior angles of two quadrilaterals

18.

Statement	Justification
AB, BC, CD, DE, and EA are equal. $EO = DO$ $DO = CO$ $\triangle EOD \cong \triangle DOC$	Property of regular pentagon Given Given Three pairs of corresponding sides are equal.
$\angle ODE = \angle ODC$ $\angle ODE = \angle OED$	$\triangle EOD$ and $\triangle DOC$ are congruent, isosceles triangles.
$S(n) = 180°(n - 2)$ $S(5) = 180°(5 - 2)$ $S(5) = 180°(3)$ $S(5) = 540°$ $\frac{540°}{5} = 108°$	The larger polygon is a regular pentagon, so n is 5. Measure of one interior angle
$\angle ODE + \angle ODC = 108°$ $2\angle ODE = 108°$ $\angle ODE = 54°$	Property of equality Transitive property
$\triangle ADE$ is isosceles. $\angle EAD = \angle EDA$	$\triangle ADE$ is isosceles because AE and DE are equal. Property of isosceles triangle.
$\angle ADE + \angle EAD + \angle DEA = 180°$ $2\angle ADE + 108° = 180°$ $2\angle ADE = 72°$ $\angle ADE = 36°$	Sum of interior angles in triangle ($\angle ADE$ and $\angle EDA$ are the same angle.)
$\angle EFD + \angle EDF + \angle FED = 180°$ $\angle EFD + 36° + 54° = 180°$ $\angle EFD = 90°$	$\angle ADE$ and $\angle EDF$ are the same angle. $\angle FED$ and $\angle OED$ are the same angle. Sum of interior angles in triangle

$\triangle EFD$ is a right triangle.

19. e.g., If a polygon is divided into triangles by joining one vertex to each of the other vertices, there are always two fewer triangles than the original number of sides. Every triangle has an angle sum of 180°.

20. e.g.,

Statement	Justification
$S(n) = 180°(n - 2)$ $S(5) = 180°(5 - 2)$ $S(5) = 180°(3)$ $S(5) = 540°$	The polygon is a pentagon, so n is 5.
$3a + 2(90°) = 540°$ $3a + 180° = 540°$ $3a = 360°$ $a = 120°$	Sum of interior angles

Yes. A tiling pattern can be created by putting four 90° angles together or three 120° angles together for a sum of 360° at the common vertex.

21.

Statement	Justification
$5x + x = 180°$ $6x = 180°$ $x = 30°$	Let x be the measure of an exterior angle. Then $5x$ is the measure of a corresponding interior angle. Supplementary angles
$150° = \dfrac{180°(n - 2)}{n}$ $150°n = 180°n - 360°$ $30°n = 360°$ $n = 12$	Measure of one interior angle

The regular polygon has 12 sides. It is called a regular dodecagon.

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e.g.,

Number of Sides	Sum of Measures of Angles: $180°(n - 2)$	Measure of Each Angle
12	1800°	150°
18	2880°	160°
24	3960°	165°

I used software to draw several regular polygons. I noticed that the measure of each interior angle gets closer and closer to 180° as the number of sides increases, so there is less of a "bend" going from one side to another. In other words, the angle is not obvious and it seems to be "smoothed out."

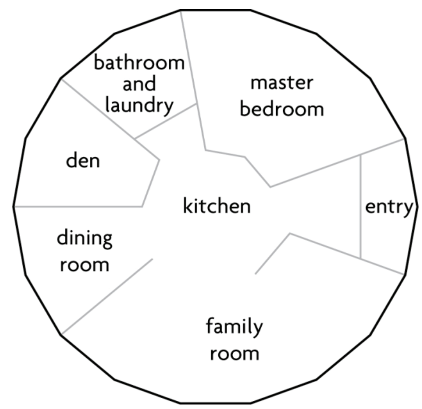
Practical limitations on the number of sides of a building could include the following:

- The availability of materials in a convenient size to build the walls.
- If the wall is too narrow, the framing for the wall would be nearly solid. Insulation could not be placed between the framing.
- It would be difficult to finish the insides of the walls. There would be too many seams in the drywall.
- If the sides were very short, there would not be enough space for electrical outlets to be installed.

The optimal number of sides for a home depends on the square footage of the home.

I decided to make my home about 1000 square feet. Using the formula $A = \pi r^2$, I determined that the radius should be about 17.8 feet. I approximated the perimeter of the home by using the formula for the circumference of a circle: $C = 2\pi r$. The perimeter should be approximately 112 feet. If I built an 18-sided house, I could use 6 foot panels, giving a perimeter of 108 feet.

Drywall for interior walls can be purchased in 12 foot lengths, so I could cut the drywall in half and not have any waste. This seems reasonable.



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e.g.,

A. The buckyball was created with regular pentagons and regular hexagons.

B. First, I determined the measures of the interior angles in both shapes.

Shape	Sum of Measures of Angles	Measure of One Angle
regular pentagon	$180°(5 - 2) = 540°$	108°
regular hexagon	$180°(6 - 2) = 720°$	120°

At each vertex, there are two angles from the hexagon and one angle from the pentagon. The sum of the measures of these angles is:

$2(120°) + 108° = 348°$

C. This value makes sense. If the sum were 360°, then the three shapes would lie flat.

To get them to form a convex shape, the angle must be less than 360°. In the diagram I drew, you can see that the pentagons must be bent to be sewn to the hexagons.