b)

| Statement | Justification |
| :---: | :--- |
| $\angle B E D=55^{\circ}$ | Given |
| $A C \\| E D$ |  | Proven | $\angle B F G=\angle B E D$ |
| :---: | :--- |
| $\angle B F G=55^{\circ}$ | Property of similar triangles $\quad$| $F G \\| E D$ | For $F G$ and $E D$, corresponding <br> angles are equal. |
| :---: | :--- |

c)

| Statement | Justification |
| :---: | :--- |
| $\angle A B F=55^{\circ}$ | Given |
| $\angle B F G=55^{\circ}$ | Proven |
| $\angle B F G=\angle B E D$ | Property of equality |
| $A C \\| F G$ | Alternate interior angles are <br> equal. |

7. e.g., In each row of parking spots, the lines separating each spot are parallel. The line down the centre is the transversal to the two sets of parallel lines.
8. e.g., Yes, the sides are parallel. The interior angles are supplementary and so the lines are always the same distance apart.

## Lesson 2.3: Angle Properties in Triangles, page 90

1. No. It only proves the sum is $180^{\circ}$ in that one triangle.
2. Disagree. The sum of the three interior angles in a triangle is $180^{\circ}$.
3. a)

| Statement | Justification |
| :---: | :---: |
| $\angle W X Y=101^{\circ}$ | Given |
| $\begin{aligned} \angle Y X Z+\angle W X Y & =180^{\circ} \\ \angle Y X Z & =79^{\circ} \end{aligned}$ | Supplementary angles |
| $\begin{aligned} & \angle X Y Z=64^{\circ} \\ & \angle Y X Z=79^{\circ} \end{aligned}$ | Given Proven |
| $\begin{aligned} \angle Z+\angle Y X Z+\angle W X Y & =180^{\circ} \\ \angle Z & =37^{\circ} \end{aligned}$ | Sum of interior angles in triangle |

b)

| Statement | Justification |
| ---: | :--- |
| $\angle B C E=134^{\circ}$ | Given |
| $\angle A C B+\angle B C E=180^{\circ}$ | Supplementary |
| $\angle A C B=46^{\circ}$ | angles |
| $\angle D C E=\angle A C B$ | Vertically opposite |
| $\angle D C E=46^{\circ}$ | angles |
| $\angle B=49^{\circ}$ | Given |
| $\angle A+\angle B+\angle A C B=180^{\circ}$ | Sum of interior |
| $\angle A=85^{\circ}$ | angles in triangle |

4. The lengths of $Q R$ and $Q S$ are equal, so $\triangle Q R S$ is isosceles. So, the measures of $\angle R$ and $\angle S$ are equal by definition.

| Statement | Justification |
| :--- | :--- |
| Let the measure of $\angle Q$ be $n$, in <br> degrees.$\angle R=\angle S$ | Property of <br> isosceles <br> triangle |
| $\angle Q+\angle R+\angle S=180^{\circ}$ | Sum of interior <br> angles of <br> $n^{\circ}+\angle R+\angle R=180^{\circ}$ |
| $2 \angle R=(180-n)^{\circ}$ | triangle <br> Substitute the <br> known <br> quantities. |
| $\qquad R=\frac{1}{2}(180-n)^{\circ}$ |  |

5. 

| Statement | Justification |
| :---: | :---: |
| $B C, B C, C D$, and $A D$ are equal in length. | Given |
| In $\triangle B C D$, since the three sides are equal, $\triangle B C D$ is equilateral. Therefore, $\begin{aligned} \angle C B D & =60^{\circ} \\ \angle C & =60^{\circ} \\ \angle B D C & =60^{\circ} \end{aligned}$ | Property of equilateral triangle |
| $\begin{aligned} \angle B D A+\angle B D C & =180^{\circ} \\ \angle B D A & =120^{\circ} \end{aligned}$ | Supplementary angles |
| In $\triangle A B D$, since two sides are equal, $\triangle A B D$ is isosceles. Therefore, $\angle D B A=\angle A$ | Property of isosceles triangle |
| $\begin{aligned} \angle A+\angle D B A+\angle B D A & =180^{\circ} \\ \angle A+\angle A+120^{\circ} & =180^{\circ} \\ \angle A & =60^{\circ} \\ \angle A & =30^{\circ} \end{aligned}$ | Sum of interior angles of a triangle Substitute known quantities. |

6. e.g., Draw an equilateral triangle to help you.


| Statement | Justification |
| :--- | :--- |
| For equilateral $\triangle A B C$, the <br> measures of the three angles are <br> equal. So,Property of <br> equilateral <br> $\angle A=60^{\circ}$ | triangle |
| $\angle B=60^{\circ}$ |  |
| $\angle A C B=60^{\circ}$ |  |
| $\angle A C D$ is the exterior angle to $\angle A$ Given <br> and $\angle B$. Supplementary <br> angles <br> $\angle A C D+\angle A C B=180^{\circ}$  <br> $\angle A C D=120^{\circ}$  |  |

7. 

| Statement | Justification |
| ---: | :--- |
| $\angle A N D=98^{\circ}$ | Given |
| $\angle D Y S=29^{\circ}$ | Given |
| For $\triangle N S Y$, <br> $\angle A S Y+\angle A N D+\angle D Y S=180^{\circ}$ <br> $\angle A S Y=53^{\circ}$ | Sum of interior <br> angles in <br> triangle |
| $\angle S A D=127^{\circ}$ | Given |
| $\angle A S Y+\angle S A D=180^{\circ}$ | Property of <br> equality |
| $S Y \\| A D$ | Interior angles <br> on same side of <br> transversal are <br> supplementary. |

8. a) The sum of $a, c$, and $e$ is $360^{\circ}$.
b) Yes. Pairs of vertically opposite angles, so $b=a$, $d=c, f=e$. So, the sum of $b, d$, and $f$ is also $360^{\circ}$.
c)

| Statement | Justification |
| :---: | :---: |
| $\begin{aligned} x+a & =180^{\circ} \\ a & =180^{\circ}-x \end{aligned}$ | Supplementary angles |
| $\begin{aligned} y+c & =180^{\circ} \\ c & =180^{\circ}-y \end{aligned}$ | Supplementary angles |
| $\begin{aligned} z+e & =180^{\circ} \\ e & =180^{\circ}-z \end{aligned}$ | Supplementary angles |
| $x+y+z=180^{\circ}$ | Sum of interior angles in triangle |
| $\begin{aligned} \text { Let } S & =a+c+e . \\ S= & \left(180^{\circ}-x\right)+\left(180^{\circ}-y\right) \\ & +\left(180^{\circ}-z\right) \\ S= & 540^{\circ}-(x+y+z) \\ S= & 540^{\circ}-180^{\circ} \\ S= & 360^{\circ} \end{aligned}$ | Substitute for the known quantities. |

9. a) If $D U C K$ is a parallelogram, the measures of opposite pairs of angles are equal. In Benji's solution, $\angle D$ should equal $\angle C$, but does not.
b)

| Statement | Justification |
| :--- | :--- |
| $D K \\| U C$ <br> $\angle K U C=35^{\circ}$ | Property of <br> parallelogram <br> Given |
| $\angle D K U=\angle K U C$ | Alternate interior <br> angles |
| $\angle D K U=35^{\circ}$ | Given |
| $\angle K D U=100^{\circ}$ | $180^{\circ}=\angle D U K+\angle D K U+\angle K D U$ <br> $\angle D U K=180^{\circ}-\left(35^{\circ}+100^{\circ}\right)$ <br> $\angle D U K=45^{\circ}$ |
| Sum of interior <br> angles of <br> triangle <br> Substitute for <br> the known <br> quantities |  |
| $D U \\| K C$ | Property of <br> parallelogram |
| $\angle U K C=\angle D U K$ | Alternate interior <br> angles |
| $\angle U K C=45^{\circ}$ | Opposite angles <br> in parallelogram |
| $\angle U C K=\angle K D U$ |  |
| $\angle U C K=100^{\circ}$ |  |

10. e.g.,

| Statement | Justification |
| :---: | :--- |
| $\angle M T H=45^{\circ}$ | Given <br> Given |
| $\angle A M T=45^{\circ}$ |  |$\quad$| $\angle M T H=\angle A M T$ | Property of equality |
| :--- | :--- |
| $M A \\| H T$ | Alternate interior angles <br> are equal. |
| $\angle H T A=110^{\circ}$ | Given <br> Given |
| $\angle M H T=70^{\circ}$ | Property of equality |
| $\angle H T A+\angle M H T=180^{\circ}$ | Interior angles on same <br> side of transversal are <br> supplementary |
| $M H \\| A T$ |  |

11. 

| Statement | Justification |
| ---: | :--- |
| $a=30^{\circ}$ | Vertically opposite <br> angles are equal. |
| $b+30^{\circ}=180^{\circ}$ | Supplementary angles |
| $b=150^{\circ}$ |  |
| $d+115^{\circ}=180^{\circ}$ | Supplementary angles |
| $d=65^{\circ}$ |  |
| $c+d+30^{\circ}=180^{\circ}$ | Sum of interior angles in <br> triangle |
| $c=85^{\circ}$ |  |

12. e.g.,
a) Disagree. $\angle F G H$ and $\angle I H J$ are not corresponding angles, alternate interior angles, or alternate exterior angles.
b)

| Statement | Justification |
| :---: | :--- |
| $\angle G F H=180^{\circ}-\left(55^{\circ}+75^{\circ}\right)$ | The sum of the angles <br> of $\triangle F G H$ is $180^{\circ}$ |
| $\angle G F H=50^{\circ}$ | $\angle G F H$ and $\angle I H J$ are <br> equal corresponding <br> angles. |
| $F G \\| H I$ |  |

13. 

| Statement | Justification |
| :---: | :---: |
| $\angle N O P=110^{\circ}$ | Given |
| $\angle J=110^{\circ}$ | Corresponding angles |
| $\angle L K O=140^{\circ}$ | Given |
| $\begin{aligned} \angle J K O+\angle L K O & =180^{\circ} \\ \angle J K O & =40^{\circ} \end{aligned}$ | Supplementary angles |
| $\begin{aligned} \angle J O K+\angle J+\angle J K O & =180^{\circ} \\ \angle J O K & =30^{\circ} \end{aligned}$ | Sum of interior angles in triangle |
| $\angle L N O=140^{\circ}$ | Opposite angles in parallelogram are equal. |
| $\begin{aligned} \angle N O K+\angle L N O & =180^{\circ} \\ \angle N O K & =40^{\circ} \end{aligned}$ | Interior angles on same side of transversal |
| $\angle K L N=40^{\circ}$ | Opposite angles in parallelogram are equal. |
| $\begin{aligned} \angle L N M+\angle L N O & =180^{\circ} \\ \angle L N M & =40^{\circ} \end{aligned}$ | Supplementary angles |
| $\begin{aligned} & \angle J O N=\angle K O N+\angle J O K \\ & \angle J O N=70^{\circ} \end{aligned}$ | Property of equality |


| $\angle M+\angle J O N=180^{\circ}$ | Interior angles on <br> same side of <br> transversal |
| :--- | :--- |
| $\angle M=110^{\circ}$ |  |
| $\angle M L N+\angle L N M+\angle M=180^{\circ}$ | Sum of interior angles <br> in triangle |
| $\angle M L N=30^{\circ}$ |  |
| $\angle K L M=\angle K L N+\angle M L N$ | Property of equality |
| $\angle K L M=70^{\circ}$ |  |

14. 

| Statement | Justification |
| ---: | :--- |
| $\angle A F N=115^{\circ}$ | Given |
| $\angle N F U+\angle A F N=180^{\circ}$ | Supplementary |
| $\angle N F U=65^{\circ}$ | angles |
| $\angle B N U=149^{\circ}$ | Given |
| $\angle U N F+\angle B N U=180^{\circ}$ | Supplementary |
| $\angle U N F=31^{\circ}$ | angles |
| $\angle F U N+\angle N F U+\angle U N F=180^{\circ}$ | Sum of interior |
| $\angle F U N=84^{\circ}$ | angles in triangle |

15. a)

| Statement | Justification |
| :---: | :---: |
| $\angle Y X Z=35^{\circ}$ | Given |
| $\begin{aligned} \angle A X Z+\angle Y X Z & =180^{\circ} \\ \angle A X Z & =145^{\circ} \end{aligned}$ | Supplementary angles |
| $\angle X Z Y=50^{\circ}$ | Given |
| $\begin{aligned} \angle E Z Y+\angle X Z Y & =180^{\circ} \\ \angle E Z Y & =130^{\circ} \end{aligned}$ | Supplementary angles |
| $\begin{aligned} \angle X Y Z+\angle Y X Z+\angle X Z Y & =180^{\circ} \\ \angle X Y Z & =95^{\circ} \end{aligned}$ | Sum of interior angles in triangle |
| $\begin{aligned} \angle X Y C+\angle X Y Z & =180^{\circ} \\ \angle X Y C & =85^{\circ} \end{aligned}$ | Supplementary angles |

b) $\angle A X Z+\angle X Y C+\angle E Z Y=145^{\circ}+85^{\circ}+130^{\circ}$ $\angle A X Z+\angle X Y C+\angle E Z Y=360^{\circ}$
16.

| Statement | Justification |
| :---: | :---: |
| $M O$ and $N O$ are angle bisectors. | Given |
| $\angle L N P$ is an exterior angle for $\triangle L M N$. | Given |
| $\begin{aligned} \angle L+2 a & =2 b \\ \angle L & =2 b-2 a \\ \angle L & =2(b-a) \end{aligned}$ | Exterior angle is equal to sum of the two nonadjacent angles. |
| $\angle O N P$ is an exterior angle for $\triangle M N O$. | Given |
| $\begin{aligned} \angle O+a & =b \\ \angle O & =b-a \end{aligned}$ | Exterior angle is equal to sum of the two nonadjacent angles. |
| $\begin{aligned} & \angle L=2(b-a) \\ & \angle L=2(\angle O) \end{aligned}$ | Substitute for the known quantity. |

17. e.g., Drawing a parallel line through one of the vertices and parallel to one of the sides creates three angles whose sum is $180^{\circ}$. The two outside angles are equal to the alternate interior angles in the triangle. The middle angle is the third angle in the triangle. Therefore, the three angles in the triangle add up to $180^{\circ}$.
$\angle P A B=\angle A B C$ and $\angle Q A C=\angle A C B$

18. 

| Statement | Justification |
| :---: | :--- |
| $\angle B A E=\angle C A E$ | Property of <br> angle bisector |
| Property of <br> isosceles <br> triangle |  |
| $\angle A B D=90^{\circ}+y$ | Property of <br> equality |
| $\angle D A B+\angle A B D+\angle B D A=180^{\circ}$ | Sum of interior <br> angles in <br> $2 x+\left(90^{\circ}+y\right)+y=180^{\circ}$ <br> $2 x+2 y=90^{\circ}$ <br> $x+y=45^{\circ}$ |
| $\triangle A B D$ <br> Substitute. |  |
| $\angle A E B=x+y$ | $\angle A E B$ is an exterior <br> angle for $\triangle A E D . ~ A n ~$ <br> exterior angle is equal <br> to sum of the two non- <br> adjacent angles. <br> Substitute. |

19. e.g.,


| Statement | Justification |
| :---: | :--- |
| $\angle M=L N$ | Property of isosceles triangle |
| $\angle R \\| M N$ | Given |
| $\angle D L R=\angle L M N$ | Corresponding angles |
| $\angle R L N=\angle L N M$ | Alternate interior angles |
| $\angle L M N=\angle L N M$ | Property of isosceles triangle |
| $\angle D L R=\angle R L N$ | Transitive property |

