Chapter 2: Properties of Angles and Triangles

Lesson 2.1: Exploring Parallel Lines, page 72

1. a) e.g.,

Parallel Lines	Transversals
bottom rail lines rail ties supports top rail lines struts in top rail	diagonal struts rail ties

b) No. The photograph is a perspective image so the corresponding angles when measured or traced would not be equal and parallel lines on the bridge when traced will not be parallel.

2. The following are pairs of corresponding angles:
$\angle EGB = \angle GHD$, $\angle AGE = \angle CHG$,
$\angle AGH = \angle CHF$, $\angle BGH = \angle DHF$,
$\angle EGA = \angle HGB$, $\angle EGB = \angle HGA$,
$\angle GHD = \angle FHC$, $\angle GHC = \angle FHD$,
$\angle EGA = \angle FHD$, $\angle EGB = \angle FHC$,
$\angle GHD = \angle HGA$, $\angle GHC = \angle BGH$.
Yes. Pairs of angles that are not equal are supplementary angles.

3. Using a ruler, draw a horizontal line and then a transversal. Measure an angle made by the horizontal line and transversal. Create an angle with this measure using a protractor anywhere else but on the same side of the transversal. Use the particular angle to draw a parallel line.

4. The transversal is the top edge of the plank of wood. The bevel has a protractor on it. As long as the angle of the T-bevel is the same, then the lines will be parallel because corresponding angles will be equal. The plank must have a true straight edge for the T-bevel to rest on and angles to be drawn accurately.

5. a) No. The measures of corresponding angles $\angle BGE$ and $\angle DHG$ are not equal, so *AB* is not parallel to *CD*.
b) Yes. $\angle BGE$ and $\angle AGE$ are supplementary so $\angle AGE$ is 67°. $\angle CHF$ and $\angle CHG$ are supplementary so $\angle CHG$ is 67°. Corresponding angles $\angle AGE$ and $\angle CHE$ are equal, so *AB* is parallel to *CD*.
c) Yes. $\angle BGH$ and $\angle AGH$ are supplementary so $\angle AGH$ is 94°. Corresponding angles $\angle AGH$ and $\angle CHE$ are equal, so *AB* is parallel to *CD*.
d) No. $\angle CHG$ and $\angle DHG$ are supplementary, so $\angle CHE$ is 139°. Corresponding angles $\angle CHG$ and $\angle AGE$ are not equal, so *AB* is not parallel to *CD*.

6. Disagree. The perpendicular distances along pairs of lines are constant or equal. Therefore, the diagonal lines are parallel. The hatching across each diagonal creates an optical illusion that the diagonals are skewed.

Lesson 2.2: Angles Formed by Parallel Lines, page 78

1.

Statement	Justification
KP, *LQ*, *MR*, and *NS* are transversals for the parallel lines.	Given *WX* and *YZ* are parallel.
$\angle AWY = 90°$	Given
$\angle WYD + \angle AWY = 180°$ $\angle WYD = 90°$	Interior angles on the same side of a transversal are supplementary.
$\angle WAL = 115°$	Given
$\angle YDA = \angle WAL$ $\angle YDA = 115°$	Corresponding angles are equal.
$\angle CBE = 80°$	Given
$\angle DEB = \angle CBE$ $\angle DEB = 80°$	Alternate interior angles are equal.
$\angle XCN = 45°$	Given
$\angle EFS = \angle XCN$ $\angle EFS = 45°$	Alternate exterior angles are equal.

2. a) Yes. The lines are parallel because the two given corresponding angles are equal.
b) No. The lines are not parallel because the two given interior angles on the same side of the transversal are not supplementary.
c) Yes. The lines are parallel because the two given alternate exterior angles are equal.
d) Yes. The lines are parallel because the two given alternate exterior angles are equal.

3.

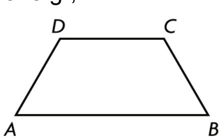
	Statement	Justification
a)	$k = p$	Alternate interior angles are equal.
b)	$a = j$	Corresponding angles are equal.
c)	$j = q$	Alternate exterior angles are equal.
d)	$g = d$	Vertically opposite angles are equal.
e)	$b = d$ $d = m$ $b = m$	Corresponding angles are equal. Corresponding angles equal. Apply the transitive property by substituting *m* for *d*.
f)	$e = g$ $g = p$ $e = p$	Corresponding angles are equal. Corresponding angles are equal. Apply the transitive property by substituting *p* for *g*.

	Statement	Justification
g)	$n = m$ $m = d$ $d = n$	Alternate exterior angles are equal. Corresponding angles are equal. Apply the transitive property by substituting *n* for *d*.
h)	$f + k = 180°$	Interior angles on the same side of a transversal are supplementary.

4.

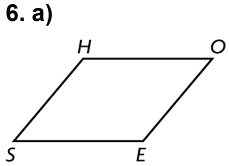
	Statement	Justification
a)	$w = 120°$	Vertically opposite angles
	$y + 120° = 180°$ $y = 60°$	Supplementary angles
	$x = y$ $x = 60°$	Corresponding angles
b)	$a = 112°$	Corresponding angles
	$e = a$ $e = 112°$	Vertically opposite angles
	$d = 55°$	Vertically opposite angles
	$f = d$ $f = 55°$	Alternate interior angles
	$b = f$ $b = 55°$	Vertically opposite angles
	$c + 112° = 180°$ $c = 68°$	Supplementary angles
c)	$d = 48°$	Corresponding angles
	$e + 48° = 180°$ $e = 132°$	Interior angles on same side of transversal
	$f = e$ $f = 132°$	Vertically opposite angles
	$a = 48°$	Corresponding angles
	$b = 48°$	Corresponding angles
	$c = d$ $c = 48°$	Alternate interior angles
	$g = f$ $g = 132°$	Alternate exterior angles

5. e.g.,



Draw *BC* and create a 120° angle at *C*, so that *CD* would be parallel to *AB*. Then draw *AD* to intersect *CB*.

6. a)



b)

Statement	Justification
$SH \parallel EO$ $SE \parallel HO$ $\angle S = 50°$	Given Given Given
$\angle E + \angle S = 180°$ $\angle E = 130°$	Interior angles on the same side of a transversal are supplementary.
$\angle H + \angle S = 180°$ $\angle H = 130°$	Interior angles on the same side of a transversal are supplementary.
$\angle O + \angle H = 180°$ $\angle O = 50°$	Interior angles on the same side of a transversal are supplementary.

$\angle S = \angle O$ and $\angle E = \angle H$

Opposite angles of a parallelogram are equal.

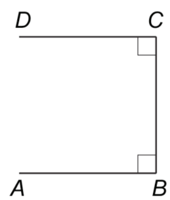
7. e.g.,

a)

Parallel Lines	Transversals
• black horizontal lines • diagonals of the same direction	• lines crossing parallel lines in middle portion of X's • black centre of middle X portion

b) To make sure lines are parallel, the pattern maker should make sure that alternate interior and exterior angles are equal and that the interior angles on the same side of a transversal are supplementary.

8. a) The transitive property cannot be applied to perpendicular lines, only to parallel lines. An example diagram can be drawn.



b) If $AB \perp BC$ and $BC \perp CD$, then $AB \parallel CD$.

9. If corresponding angles could be measured and are found to be equal, then it can be said that the trusses are parallel.

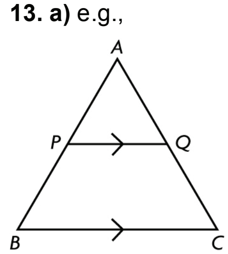
10. *Line 2 of Justification:* The interior angles on the same side of a transversal are supplementary, not equal. Since $\angle PQR + \angle QRS = 180°$, the statement that $QP \parallel RS$ is still valid.

11. e.g., The bottom edges of the windows are transversals for the vertical edges of the windows. The sloped roof also forms transversals for the vertical parts of the windows. The builders could ensure one window is vertical and then make all the corresponding angles equal so the rest of the windows are parallel.

12.

Statement	Justification
$\angle FXO = \angle FPQ$ $PQ \parallel XO$	Given Corresponding angles are equal.
$\angle FOX = \angle FRS$ $XO \parallel SR$	Given Corresponding angles are equal.
$PQ \parallel SR$	Apply the transitive property.

13. a) e.g.,



b)

Statement	Justification
$BC \parallel PQ$	Given
$\angle APQ = \angle ABC$	Corresponding angles are equal.
$\angle AQP = \angle ACB$	Corresponding angles are equal.
$\angle PAQ = \angle BAC$	They are the same angle.

The three pairs of corresponding angles between $\triangle BAC$ and $\triangle PAQ$ are equal. Therefore, the two triangles are similar.

14. a)

Statement	Justification
The top and bottom sides are parallel.	Given
$z + 120° = 180°$ $z = 60°$	Interior angles on the same side of a transversal are supplementary.
$y = z$ $y = 60°$	Angles on the base of an isosceles trapezoid are equal.
$x + y = 180°$ $x = 60°$	Interior angles on the same side of a transversal are supplementary.

b) Isosceles triangles have two pairs of congruent, adjacent angles.

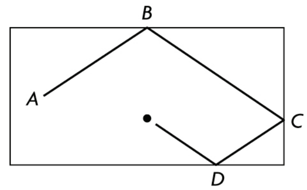
15.

Statement	Justification
$PQ \parallel SR$ $\angle RST = 48°$	Given Given
$\angle PQT = \angle RST$ $\angle PQT = 48°$	Alternate interior angles
$\angle QPT = 54°$	Given
$\angle SRT = \angle QPT$ $\angle SRT = 54°$	Alternate interior angles
$\angle PTQ + \angle QPT + \angle PQT = 180°$ $\angle PTQ = 78°$	Sum of interior angles in a triangle
$\angle QTR + \angle PTQ = 180°$ $\angle QTR = 102°$	Supplementary angles
$\angle QRT = 29°$	Given
$\angle RQT + \angle QRT + \angle QTR = 180°$ $\angle RQT = 49°$	Sum of interior angles in a triangle
$\angle RTS + \angle QTP = 180°$ $\angle RTS = 78°$	Supplementary angles

16.

Statement	Justification
$AB \parallel DE$ and $DE \parallel FG$	Given
$AB \parallel FG$	Transitive property
$\angle BAC = \angle ACF$	Alternate interior angles
$\angle FCD = \angle CDE$	Alternate interior angles
$\angle ACD = \angle ACF + \angle FCD$ $\angle ACD = \angle BAC + \angle CDE$	Property of equality Substitution

17. a) Alternate straight paths will be parallel.
b) e.g.,



c) $AB \parallel FG$; $CB \parallel DE$; My predication was correct.
d) Yes. The pattern will continue until the ball stops.

18.

Statement	Justification
$QP \parallel SR$	Given
$\angle PQR = \angle QRS$	Alternate interior angles
RT bisects $\angle QRS$	Given
$\angle TRQ = \frac{1}{2}\angle QRS$	Property of angle bisector
QU bisects $\angle PQR$	Given
$\angle RQU = \frac{1}{2}\angle PQR$	Property of angle bisector
$\angle TRQ = \angle RQU$	Transitive property
$QU \parallel RT$	Alternate interior angles

19. a) Disagree. You only need to show that any one of the statements is true.
b) Yes, there are other ways.

Statement	Justification
$\angle MCD = \angle CDQ$	Alternate interior angles
$\angle XCL = \angle CDQ$	Corresponding angles
$\angle LCD + \angle CDQ = 180°$	Interior angles on same side of transversal
$\angle LCD = \angle QDY$	Corresponding angles
$\angle MCD = \angle RDY$	Corresponding angles
$\angle XCM = \angle QDY$	Alternate exterior angles
$\angle XCL = \angle RDY$	Alternate exterior angles

20. a) $(3x + 10) = (6x - 14)$ Alternate exterior angles

$3x = 24$
$x = 8$

b) $(9x + 32) + (11x + 8) = 180$ Interior angles on same side of transversal

$20x + 40 = 180$
$20x = 140$
$x = 7$

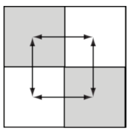
21. e.g.,
a) Measure the top angle of the rhombus at the left end of the bottom row; it will have the same measure as the angle at the peak.
b) Opposite sides of a rhombus are parallel, so the top right sides of all the rhombuses form parallel lines. The top right side of the peak rhombus and the top right side of the bottom left rhombus are parallel. The left edge of the pyramid is a transversal, so the angle at the peak and the top angle of the bottom left rhombus are equal corresponding angles.

Applying Problem-Solving Strategies, page 83

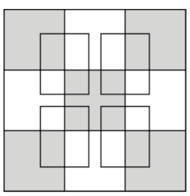
e.g.,

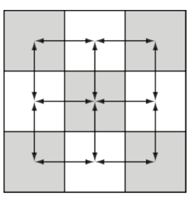
Checkerboard	Number of Quadrilaterals
1-by-1	1 (1^2)
2-by-2	9 (3^2)
3-by-3	36 (6^2)
4-by-4	100 (10^2)
5-by-5	225 (15^2)

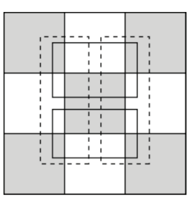
A. There is only **one** square on a 1-by-1 checkerboard.
B. On a 2-by-2 checkerboard, there are five squares, four of them 1-by-1 and one large square, as well as four rectangles (each rectangle is shown by an arrow in the diagram). There are **nine** quadrilaterals in total.



C. There are nine small squares, four 2-by-2 squares, and one 3-by-3 square, for a total of **14** squares. There are two 1-by-2 rectangles in each row and each column, for a total of 12 1-by-2 rectangles. There is one 1-by-3 rectangle in each row and each column, for a total of six 1-by-3 rectangles. There are four 3-by-2 rectangles. There are **22** rectangles in all, making a total of **36** quadrilaterals.


2 by 2 squares


1 by 2 rectangles


3 by 2 rectangles

D. e.g., I decided to count the squares first and then the rectangles. On a 4-by-4 checkerboard, there are 16 1-by-1 squares, nine 2-by-2 squares, four 3-by-3 squares, and one 4-by-4 square, for a total of **30** squares. Each row has three 2-by-1 rectangles. There are four rows, so there are 12 2-by-1 rectangles. Also, each column has three 1-by-2 rectangles. There are four columns, so there are 12 1-by-2 rectangles. Each row has two 3-by-1 rectangles. There are four rows, so there are eight 3-by-1 rectangles. Also, each column has two 1-by-3 rectangles. There are four columns, so there are eight 1-by-3 rectangles. There are four 4-by-1 rectangles, and four 1-by-4 rectangles.
There are six 3-by-2 rectangles, six 2-by-3 rectangles, three 4-by-2 rectangles, and three 2-by-4 rectangles.
There are two 4-by-3 rectangles and two 3-by-4 rectangles. There are **70** rectangles in total. The total number of quadrilaterals on a 4-by-4 checkerboard is **100.**
E. e.g., My strategy worked, but it took a long time and it would be difficult for larger checkerboards. I decided that a square is a rectangle, so I could include squares in my rectangle count, rather than count them separately. I also noticed that the number of rectangles in each row is the same as the number of rectangles in each column.
I created a table indicating the possible lengths of a rectangle and the number of ways to lay it out across the checkerboard. I tried this for a 4-by-4 checkerboard.
For this strategy, I assumed that the lengths would be horizontal and the widths would be vertical.

Length of Rectangles	Number of Ways
4	1
3	2
2	3
1	4
TOTAL	**10**

Since the checkerboard is a square, there are also 10 ways for the width of the rectangle. This means that there are 10 times 10 or 100 possible quadrilaterals. I got the same answer as I did by counting. I checked my strategy by trying it on the 2-by-2 and the 3-by-3 checkerboards.

2-by-2 Checkerboard	
Length of Rectangle	**Number of Ways**
1	2
2	1
TOTAL	**3**

There are 3 times 3 or 9 quadrilaterals on a 2-by-2 checkerboard.

3-by-3 Checkerboard	
Length of Rectangle	**Number of Ways**
1	3
2	2
3	1
TOTAL	**6**

There are 6 times 6 or 36 quadrilaterals on a 3-by-3 checkerboard.

My answers matched the answers I got in prompts B and C.
F. e.g., I used the strategy I figured out in prompt E. For an 8-by-8 checkerboard, I determined the various possible lengths of the rectangle and the number of ways the rectangles could be laid across the checkerboard.

Length of Rectangles	Number of Ways
8	1
7	2
6	3
5	4
4	5
3	6
2	7
1	8
TOTAL	**36**

Since there are also 36 ways to have the width, there are 36 by 36 or 1296 quadrilaterals on an 8-by-8 checkerboard. I would have achieved the same answer if I extended my table from prompt C using the pattern I observed.

Size	Number of Ways
1-by-1	$1 = 1^2$
2-by-2	$9 = (1 + 2)^2$
3-by-3	$36 = (1 + 2 + 3)^2$
4-by-4	$100 = (1 + 2 + 3 + 4)^2$
5-by-5	$225 = (1 + 2 + 3 + 4 + 5)^2$
6-by-6	$441 = (1 + 2 + 3 + 4 + 5 + 6)^2$
7-by-7	$784 = (1 + 2 + 3 + 4 + 5 + 6 + 7)^2$
8-by-8	$1296 = (1 + 2 + 3 + 4 + 5 + 6 + 7 + 8)^2$

G. e.g., There were several strategies; for example one person tried to do this counting the total number of different quadrilaterals in each checkerboard. We did get the same results.
H. I like using the pattern because it is more efficient and less time-consuming than counting the total number of different quadrilaterals in each checkerboard.

Mid-Chapter Review, page 85

1. a) Yes.

Statement	Justification
$\angle BXY = 105°$ $\angle CYZ = 105°$	Given Given
$\angle BXY = \angle CYZ$	
$AB \parallel CD$	Alternate interior angles are equal.

b) No.

Statement	Justification
$\angle AXY = 95°$ $\angle CYX = 95°$	Given Given
$\angle AXY + \angle CYX = 190°$	Substitute
$AB \neq CD$	Interior angles on same side of transversal are not supplementary.

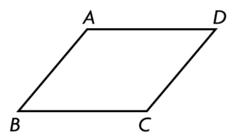
c) Yes.

Statement	Justification
$\angle BXW = 63°$ $\angle CYZ = 63°$	Given Given
$AB \parallel CD$	Alternate exterior angles are equal.

d) Yes.

Statement	Justification
$\angle BXY = 73°$ $\angle CYZ = 107°$	Given Given
$\angle AXY + \angle BXY = 180°$ $\angle AXY = 107°$	Supplementary angles
$\angle CYZ = \angle AXY$	Property of equality
$AB \parallel CD$	Corresponding angles are equal.

2. The sum of the interior angles of a quadrilateral is 360°. The measure of angle *P* is 125°. In a parallelogram, the measures of opposite angles are equal or the interior angles on the same side of a transversal are supplementary. Therefore, *PQRS* is a parallelogram.
3. e.g., The red lines are parallel since any of the black lines can be used to prove that the corresponding angles are equal.
4. e.g.,



I drew $\angle ABC$. I measured it and drew $\angle BCD$ supplementary to it. Then I measured *AB*, made *CD* the same length, and connected *A* to *D*.
5. a)

Statement	Justification
$CF \parallel DE$ $\angle ABC = 105°$	Given Given
$\angle ABF + \angle ABC = 180°$ $\angle ABF = 75°$	Supplementary angles
$\angle CBD = \angle ABF$ $\angle CBD = 75°$	Vertically opposite angles
$\angle BDE = \angle ABF$ $\angle BDE = 75°$	Corresponding angles
$\angle DEB = 36°$	Given
$\angle FBE = \angle DEB$ $\angle FBE = 36°$	Alternate interior angles
$\angle FEB + \angle FBE + \angle EFB = 180°$ $\angle FEB = 69°$	Sum of interior angles in triangle
$\angle EBD + \angle BDE + \angle DEB = 180°$ $\angle EBD = 69°$	Sum of interior angles in triangle

b) e.g., Yes. *BD* is parallel to *EF* because $\angle FEB$ and $\angle EBD$ are equal alternate interior angles.
6. a)

Statement	Justification
$\angle ABE = 55°$ $\angle BED = 55°$	Given Given
$\angle ABE = \angle BED$	Property of equality
$AC \parallel ED$	Alternate interior angles are equal.