Practice Final Exam

1a) $(-1, 5]$ b) $[-5, 9]$ c) $(-\infty, 6)$

$-1 < x \leq 5$ $-5 \leq x \leq 9$ $x < 6$

2. $x^2 - 7 = (x - \sqrt{7})(x + \sqrt{7})$

3a) $16x^{-3} + \frac{8}{3}x^{-2}$

$= \frac{1}{3}x^{-3}(48 + 8x)$

$= \frac{8}{3}x^{-3}(6 + x)$

b) $2x^{-1/2} - 6x^{-3/2} + 4x^{-5/2}$

$= 2x^{-5/2}(x^2 - 3x + 2)$

$= 2x^{-5/2}(x-2)(x-1)$

4. $f(x) = \dfrac{(x^3-1)(x+4)}{2x(x^2-2x+1)} = \dfrac{(x-1)(x^2+x+1)(x+4)}{2x(x-1)(x-1)}$

$N = \{1, -4\}$ $D = \{0, 1\}$

i) $x = -4$ ii) $x = 0$ iii) $x = 1$

5. a) $-3 \le 5x - 8 < -1$

$-3 \le 5x - 8$

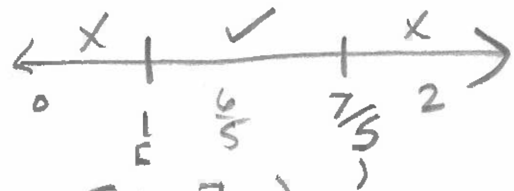
$5 \le 5x$

$1 \le x$

$5x - 8 < -1$

$5x < 7$

$x < 7/5$



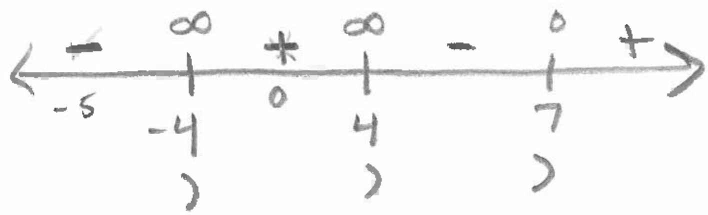
$[1, 7/5)$

b) $\dfrac{x-7}{(x-4)(x+4)} < 0$

$x \ne \pm 4$

$x - 7 < 0$

$x < 7$



$(-\infty, -4) \cup (4, 7)$

c) $|-7x+6| = 8 - 5x$

$-7x + 6 = 8 - 5x$ or $-(-7x+6) = 8 - 5x$

$-2x = 2$

$x = -1$

$7x - 6 = 8 - 5x$

$12x = 14$

$x = 7/6$

Verify

$|-7(-1)+6| = 8 - 5(-1)$

$13 = 13$

✓

Verify

$|-7(7/6)+6| = 8 - 5(7/6)$

$|-14/6 + 6| = 8 - 35/6$

$\left|\frac{13}{6}\right| = \left|\frac{13}{6}\right|$

✓

$\{-1, 7/6\}$

d) $|3x+11| > 5$

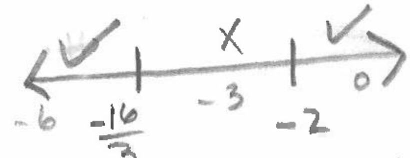
$3x + 11 > 5$ or $3x + 11 > -5$

$3x > -6$

$x > -2$

$3x > -16$

$x > -16/3$



$(-\infty, -16/3) \cup (-2, \infty)$

e) $\left|\dfrac{x}{2-x}\right| \leq 3$ $\qquad x \neq 2$

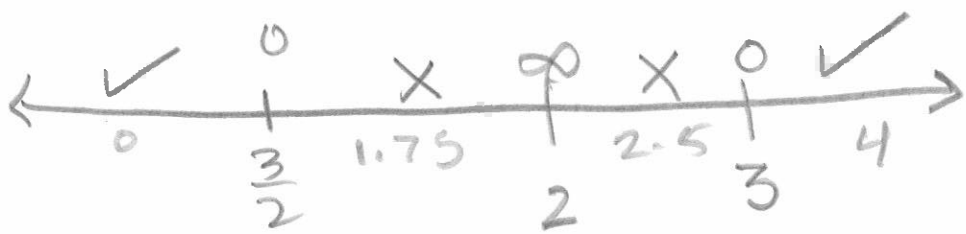
$\dfrac{x}{2-x} \leq 3$ $\qquad\qquad$ $\dfrac{-x}{2-x} \leq 3$

$x \leq 3(2-x)$ $\qquad\qquad$ $-x \leq 3(2-x)$

$x \leq 6-3x$ $\qquad\qquad$ $-x \leq 6-3x$

$4x \leq 6$ $\qquad\qquad$ $2x \leq 6$

$x \leq \frac{3}{2}$ $\qquad\qquad$ $x \leq 3$



test points in
original question to
see if valid.

$(-\infty, \frac{3}{2}] \cup [3, \infty)$

6. a) $f(-2) = 3(-2)^2 + 1$
$= 13$

b) $f(5) = 2(5) - 5$
$= 5$

c) $f(0) = 10 - 0$
$= 10$

d) $f(2) = 2(2) - 5$
$= -1$

7. a) $f(x) = \sqrt{x+5}$

Domain: $x + 5 \geq 0$
$x \geq -5$

Range: $y \geq 0$

b) $p(x) = 4x^3 - 6x + 1$

Domain: $x \in \mathbb{R}$
Range: $y \in \mathbb{R}$

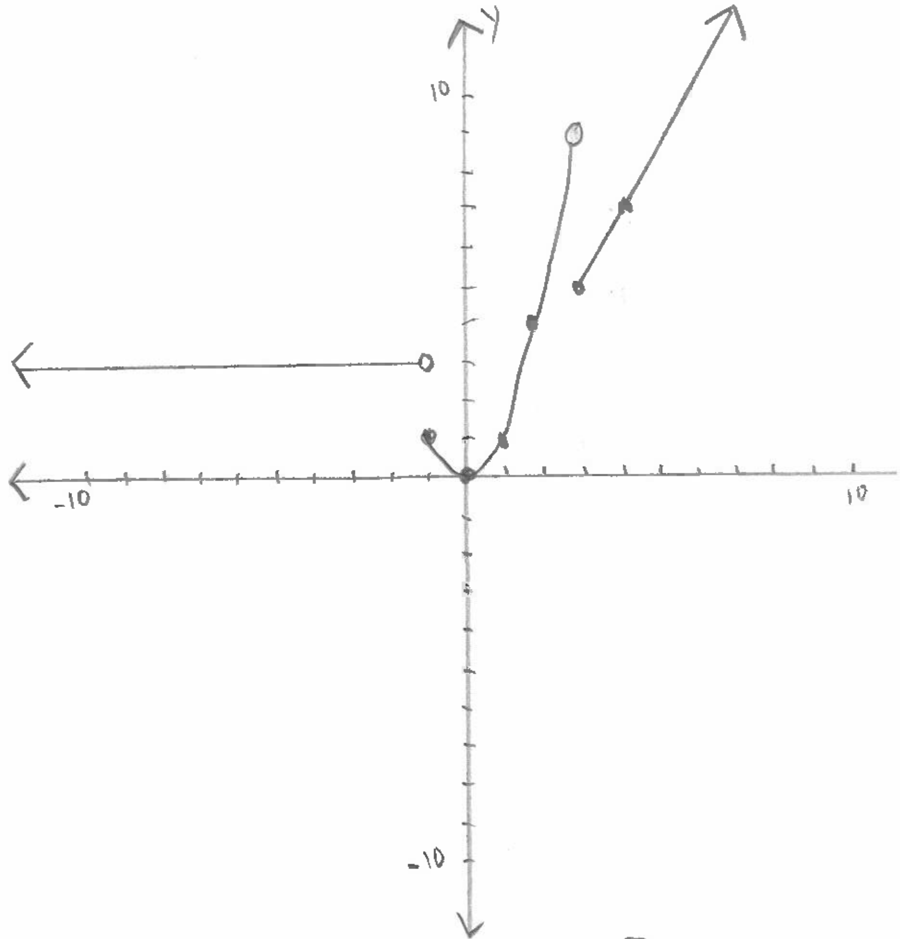
c) $h(x) = 5(x-2)^2 - 9$

Domain: $x \in \mathbb{R}$
Range: $y \geq -9$

d) $g(x) = e^x$

Domain: $x \in \mathbb{R}$
Range $y > 0$

8. $$g(x) = \begin{cases} 3 & (-\infty,-1) \\ x^2 & [-1,3) \\ 2x-1 & [3,\infty) \end{cases}$$



9. $f(x) = 4x - 7 \qquad g(x) = x^2$

$$g(f(x)) = g(4x-7)$$
$$= (4x-7)^2$$
$$= 16x^2 - 56x + 49$$

10.

a) $\lim_{x \to 0} \dfrac{\frac{1}{(x+2)^2} - \frac{1}{4}}{x} \cdot \dfrac{4(x+2)^2}{4(x+2)^2}$

$= \lim_{x \to 0} \dfrac{4 - (x+2)^2}{4x(x+2)^2}$

$= \lim_{x \to 0} \dfrac{4 - (x^2+4x+4)}{4x(x+2)^2}$

$= \lim_{x \to 0} \dfrac{4 - x^2 - 4x - 4}{4x(x+2)^2}$

$= \lim_{x \to 0} \dfrac{-\cancel{x}(x+4)}{4\cancel{x}(x+2)^2}$

$= \dfrac{-(0+4)}{4(0+2)^2} = \dfrac{-4}{16} = -\dfrac{1}{4}$

b) $\lim_{x \to 5} \dfrac{x-5}{\sqrt{x+4} - 3} \cdot \dfrac{(\sqrt{x+4} + 3)}{(\sqrt{x+4} + 3)}$

$= \lim_{x \to 5} \dfrac{(x-5)(\sqrt{x+4} + 3)}{x+4-9}$

$= \lim_{x \to 5} \dfrac{\cancel{(x-5)}(\sqrt{x+4} + 3)}{\cancel{(x-5)}}$

$= \sqrt{5+4} + 3$

$= 6$

c) $\lim_{x \to 5} \sqrt{\dfrac{x^2+15}{x-1}}$

$= \lim_{x \to 5} \sqrt{\dfrac{(5)^2+15}{5-1}}$

$= \dfrac{\sqrt{40}}{2}$

$= \dfrac{2\sqrt{10}}{2} = \sqrt{10}$

d) $\lim_{x \to \infty} \dfrac{6x^2 - 1}{2x^2 + 3x} = \dfrac{6}{2} = 3$

11. a) $x=-3$ $\quad x=0$ $\quad x=3$

b) infinite $\quad$ jump $\quad$ remarable

12. $g(x) = \begin{cases} 2x & (-\infty,2) \\ x^2 & [2,\infty) \end{cases}$

i) $g(2) = 2^2 = 4$ exists

ii) $\lim_{x\to 2^+} (2)^2 = 4$

$\lim_{x\to 2^-} 2(2) = 4$ $\quad \therefore \lim_{x\to 2} g(x) = 4$

iii) $g(2) = \lim_{x\to 2} g(x)$ $\therefore$ continuous at $x=2$.

13. $f(x) = \dfrac{x+3}{(x-3)(x+3)}$

discontinuous at $x=-3$ remarable

discontinuous at $x=3$ infinite.

14. a) $f(x) = (2x+3)^2$

$f'(x) = 2(2x+3)^1(2)$

$= 4(2x+3)$

b) $f(x) = (x^3+3x)(8-x)$

$f'(x) = (x^3+3x)(-1) + (8-x)(3x^2+3)$

$= -x^3 - 3x + 24x^2 + 24 - 3x^3 - 3x$

$= -4x^3 + 24x^2 - 6x + 24$

c) $f(x) = \left(\dfrac{2x}{x+2}\right)^{-2}$

$f'(x) = -2\left(\dfrac{2x}{x+2}\right)^{-3}\left[\dfrac{(x+2)(2) - 2x(1)}{(x+2)^2}\right]$

$= -2\left(\dfrac{(x+2)^3}{(2x)^3}\right)\left(\dfrac{2x+4-2x}{(x+2)^2}\right]$

$= \dfrac{-2(x+2)(4)}{8x^3} = -\dfrac{8(x+2)}{8x^3} = \dfrac{-(x+2)}{x^3}$

d) $y = \dfrac{x\sqrt{x+3}}{x-2}$

$$y' = \frac{(x-2)\left(x\left(\frac{1}{2}(x+3)^{-1/2}\right) + (x+3)^{1/2}\right) - x(x+3)^{1/2}(1)}{(x-2)^2}$$

$$y' = \frac{(x-2)\left(\frac{x}{2(x+3)^{1/2}} + (x+3)^{1/2}\right) - x(x+3)^{1/2}}{(x-2)^2} \cdot \frac{2(x+3)^{1/2}}{2(x+3)^{1/2}}$$

$$= \frac{(x-2)(x + 2(x+3)) - 2x(x+3)}{2(x+3)^{1/2}(x-2)^2}$$

$$= \frac{(x-2)(3x+6) - 2x^2 - 6x}{2(x+3)^{1/2}(x-2)^2}$$

$$= \frac{3x^2 + 6x - 6x - 12 - 2x^2 - 6x}{2(x+3)^{1/2}(x-2)^2}$$

$$= \frac{x^2 - 6x - 12}{2(x+3)^{1/2}(x-2)^2}$$

e) $f(x) = (x+3)(e^{3x})$

$$f'(x) = (x+3)(e^{3x} \cdot 3) + e^{3x}(1)$$

$$= e^{3x}(3(x+3) + 1)$$

$$= e^{3x}(3x + 9 + 1)$$

$$= e^{3x}(3x+10)$$

f) $y = -6\sin(3x^2)$

$y' = -6\cos(3x^2)(6x)$

$y' = -36x\cos(3x^2)$

g) $f(x) = x\ln(xe^x)$

$f' = x\left(\frac{1}{xe^x} \cdot (xe^x + e^x)\right) + \ln(xe^x)(1$

$f' = \frac{xe^x + e^x}{e^x} + \ln(xe^x)$

$f' = x + 1 + \ln(xe^x)$

h) $y = x^2 \tan 3x$

$y' = x^2 \sec^2 3x \cdot 3 + \tan 3x\,(2x)$

$= 3x^2 \sec^2 3x + 2x\tan 3x$

$= x(3x\sec^2 3x + 2\tan 3x)$

15. $y^2 + 3y - x = 4$

$2y\frac{dy}{dx} + 3\frac{dy}{dx} - 1 = 0$

$(2y+3)\frac{dy}{dx} = 1$

$\frac{dy}{dx} = \frac{1}{2y+3}$

$\frac{dy}{dx} = (2y+3)^{-1}$

$\frac{d^2y}{dx^2} = -1(2y+3)^{-2}\left(2\frac{dy}{dx}\right)$

$= \frac{-2\frac{dy}{dx}}{(2y+3)^2}$

$= \frac{-2\left(\frac{1}{2y+3}\right)}{(2y+3)^2}$

$= \frac{-2}{(2y+3)^3}$

16. $f'(x) = 3x^2 + 2$

$f'(2) = 3(2)^2 + 2$

$= 14$

$m = 4$

$f(2) = (2)^3 + 2(2) - 7$

$= 8 + 4 - 7$

$= 5$

$(2, 5)$

$y - 5 = 14(x-2)$

$y - 5 = 14x - 28$

$y = 14x - 23$

17. $f'(x) = 2x - 5$ $\quad /\!/ \quad$ $y = 5x - 10$

$m = 5$

$2x - 5 = 5$

$2x = 10$

$x = 5$

$f(5) = (5)^2 - 5(5) + 1$

$= 1$

$(5, 1)$

18. $f(x) = (x-2)^3(x+2)$

a) $f'(x) = (x-2)^3(1) + (x+2)(3(x-2)^2)$

$= (x-2)^2(x - 2 + 3(x+2))$

$= (x-2)^2(x - 2 + 3x + 6)$

$= (x-2)^2(4x+4)$

$= 4(x-2)^2(x+1)$

b) sign of f': $-$ (test -3) | 0 at -1 | $+$ (test 0) | 0 at 2 | $+$ (test 3)

inc $(-1, \infty)$

dec $(-\infty, -1)$

c) $x = -1$ rel min

$f(-1) = (-1-2)^3(-1+2)$

$= -27$

$(-1, -27)$

d) $f''(x) = 4\left[(x-2)^2 + (x+1)(2(x-2))\right]$

$= 4\left[(x-2)(x - 2 + 2(x+1))\right]$

$= 4(x-2)(x - 2 + 2x + 2)$

$= 4(x-2)(3x)$

$= 12x(x-2)$

e) sign of f'': $+$ (test -1) | 0 | $-$ (test 1) | 2 | $+$ (test 3)

CU $(-\infty, 0) \cup (2, \infty)$

CD $(0, 2)$

f) IP

$x = 0$

$x = 2$

$f(0) = (0-2)^3(0+2)$

$= -16$

$(0, -16)$

$f(2) = 0$

$(2, 0)$

g) $\underline{x \text{ int}}$

$(x-2)^3(x+2)=0$

$x=2$ or $x=-2$

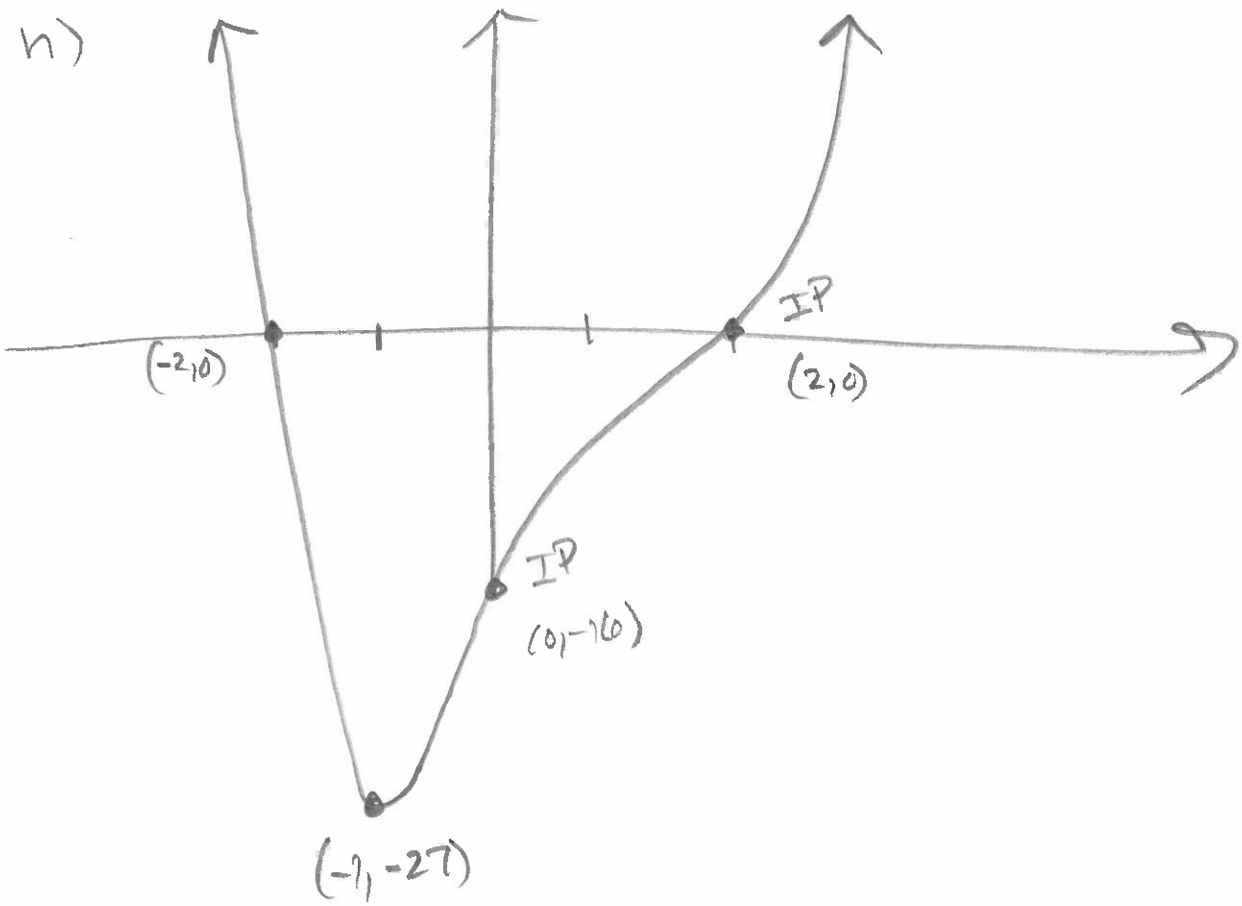
$\underline{y \text{ int}}$

$(0-2)^3(0+2)$

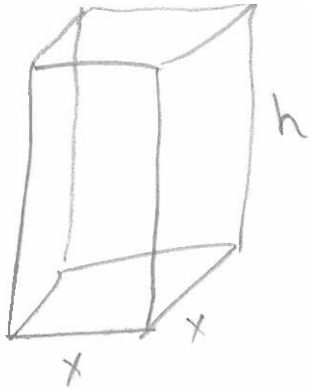
-16

$(0,-16)$

h)



19.



$V = x^2 h$

$300 = x^2 + 4xh$

$\frac{300 - x^2}{4x} = h$

$V = x^2\left(\frac{300 - x^2}{4\cancel{x}}\right)$

$V = \frac{300x}{4} - \frac{x^3}{4}$

$V = \frac{1}{4}(300x - x^3)$

$V' = \frac{1}{4}(300 - 3x^2)$

$300 - 3x^2 = 0$

$300 = 3x^2$

$100 = x^2$

$\pm 10 = x$

$x = 10$

$V''(x) = \frac{1}{4}(-6x)$

$V''(10) = \frac{1}{4}(-6(10)) < 0$

$\therefore x = 10$ max

$h = \frac{300 - (10)^2}{4(10)} = \frac{200}{40} = 5$

$\therefore$ dimensions are 10cm x 10cm x 5cm

20. $s(t) = 2t^3 - 7t^2 + 3t$

a) $s(3) = 2(3)^3 - 7(3)^2 + 3(3)$
$= 54 - 63 + 9$
$= 0$ m

b) $s'(t) = v(t) = 6t^2 - 14t + 3$

c) $v(2) = 6(2)^2 - 14(2) + 3$
$= 24 - 28 + 3$
$= -1$ m/s

d) ave vel $= \frac{s(6) - s(4)}{6 - 4}$
$= \frac{198 - 28}{2}$
$= 85$ m/s

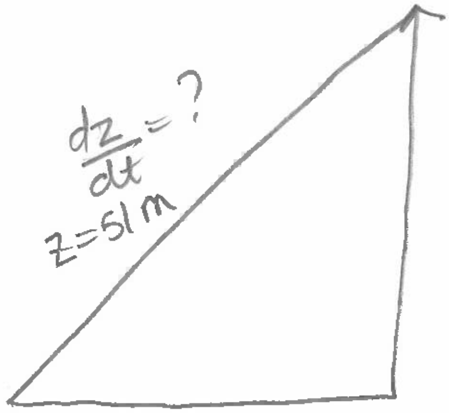
e) $a(t) = v'(t) = 12t - 14$

f) $a(10) = 12(10) - 14$
$= 106$ m/s^2

g) $12t - 14 = 70$
$12t = 84$
$t = 7$ s

$v(7) = 6(7)^2 - 14(7) + 3$
$= 199$ m/s

21



$\frac{dz}{dt} = ?$

$z = 51\text{ m}$

$\frac{dy}{dt} = 3\text{ m/s}$

$y = 45\text{ m}$

$x = 24\text{ m}$

$\frac{dx}{dt} = 0$

Find z

$x^2 + y^2 = z^2$

$(24)^2 + (45)^2 = z^2$

$\pm 51 = z$

$51 = z$

$$x^2 + y^2 = z^2$$

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2z\frac{dz}{dt}$$

$$\frac{y}{z}\,dy/dt = dz/dt$$

$$\frac{45}{51}(3) = \frac{dz}{dt} = 2.65\text{ m/s}$$

The distance between Candice and Tamara is changing at a rate of 2.65 m/s.

22. $\frac{dC}{dt} = 2\text{ cm/s}$

$d = 30\text{ cm}$

$\frac{dV}{dt} = ?$

$r = 15\text{ cm}$

$C = 2\pi r$

$\frac{dc}{dt} = 2\pi\frac{dr}{dt}$

$2 = 2\pi\frac{dr}{dt}$

$\frac{1}{\pi}\text{ cm/s} = \frac{dr}{dt}$

$$V = \frac{4}{3}\pi r^3$$

$$\frac{dV}{dt} = 4\pi r^2\frac{dr}{dt}$$

$$\frac{dV}{dt} = 4\pi(15)^2\left(\frac{1}{\pi}\right)$$

$$\frac{dV}{dt} = 900\text{ cm}^3\text{/s}.$$

The volume is increasing at a rate of 900 cm^3/s.

23. a) $\int(-2x^2+5x-3)\,dx$

$= -\frac{2}{3}x^2 + \frac{5}{2}x^2 - 3x + C$

b) $\int x^4(2x-3)\,dx$

$= \int(2x^5 - 3x^4)\,dx$

$= \frac{1}{3}x^6 - \frac{3}{5}x^5 + C$

c) $\int (x^{3/4} + 6x^{1/3})\,dx$

$= \frac{4}{7}x^{7/4} + 6 \cdot \frac{3}{4}x^{4/3} + C$

$= \frac{4}{7}x^{7/4} + \frac{9}{2}x^{4/3} + C$

d) $\int (e^{3x} + 25)\,dx$

$= \frac{1}{3}e^{3x} + 25x + C$

24. a) $\int_{-5}^{5} (5x^4 + 6x)\,dx$

$= x^5 + 3x^2 \Big|_{-5}^{5}$

$= \left[(5)^5 + 3(5)^2\right] - \left[(-5)^5 + 3(-5)^2\right]$

$= (3125 + 75) - (-3125 + 75)$

$= 6250$

b) $\int_1^9 \sqrt{x}\,dx$

$= \frac{2}{3}x^{3/2} \Big|_1^9 = \frac{2}{3}(9)^{3/2} - \frac{2}{3}(1)^{3/2}$

$= 18 - \frac{2}{3}$

$= \frac{54}{3} - \frac{2}{3} = \frac{52}{3}$

(25) a) $\int (2x^2+7)^5 x\,dx$

let $u = 2x^2+7$

$du = 4x\,dx$

$\frac{1}{4}du = x\,dx$

$= \frac{1}{4}\int u^5\,du$

$= \frac{1}{4}\left[\frac{u^6}{6}\right] + C$

$= \frac{1}{24}\left[2x^2+7\right]^6 + C$

b) $\int e^{\sin x}\cos x\,dx$

let $u = \sin x$

$du = \cos x\,dx$

$= \int e^u\,du = e^u + C$

$= e^{\sin x} + C$

(26) Intersection Points

$4x+16 = 2x^2+10$

$0 = 2x^2-4x-6$

$0 = 2(x^2-2x-3)$

$0 = 2(x-3)(x+1)$

$x = 3$ or $x = -1$

$\text{Area} = \int_{-1}^{3} (4x+16) - (2x^2+10)\,dx$

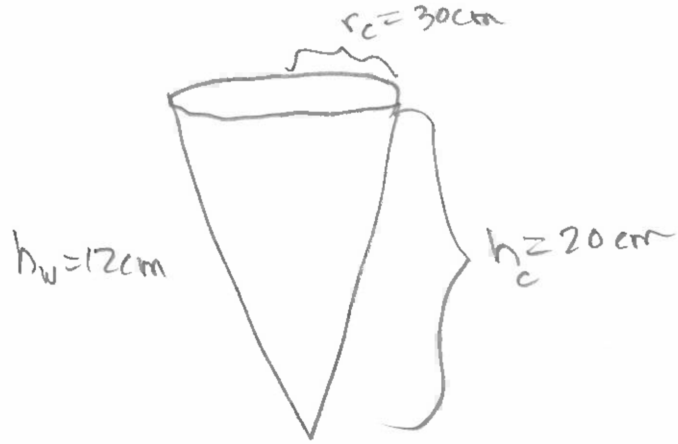
$= \int_{-1}^{3} (4x-2x^2+6)\,dx$

$= 2x^2 - \frac{2}{3}x^3 + 6x \Big|_{-1}^{3}$

$= \left(2(3)^2 - \frac{2}{3}(3)^3 + 6(3)\right) - \left(2(-1)^2 - \frac{2}{3}(-1)^3 + 6(-1)\right)$

$= 64/3$

27. $\frac{dV}{dt} = 16\,cm^3/s$



$\frac{r}{h} = \frac{30}{20}$

$r = \frac{3}{2}h$

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi\left(\frac{3}{2}h\right)^2 h$

$V = \frac{1}{3}\pi\left(\frac{9h^2}{4}\right)h$

$V = \frac{9\pi h^3}{12}$

$\frac{dV}{dt} = \frac{27\pi}{12}h^2\frac{dh}{dt}$

$16 = \frac{27\pi}{12}(12)^2\frac{dh}{dt}$

$0.016\,cm/s = \frac{dh}{dt}$

26. $\int_0^4 \left(\sqrt{x} - \frac{1}{2}x\right) dx$

$= \int_0^4 \left(x^{1/2} - \frac{1}{2}x\right) dx$

$= \frac{2}{3}x^{3/2} - \frac{1}{4}x^2 \Big|_0^4$

$= \frac{2}{3}(4)^{3/2} - \frac{1}{4}(4)^2 - 0$

$= \frac{16}{3} - 4$

$= \frac{16}{3} - \frac{12}{3} = 4/3$