2022

Calculus 30 – Practice Final Exam

1. Write the following as an inequality:

   (a) (-1, 5] (b) $[-5,9]$ (c) $(-\infty, 6)$

2. Factor $x^2 - 7 = 0$ as a difference of squares over the real numbers.

3. Factor each of the following so that the second factor contains no negative exponents and no fractional coefficients.

   (a) $16x^{-3} + \frac{8}{3}x^{-2}$ (b) $2x^{\frac{-1}{2}} - 6x^{\frac{-3}{2}} + 4x^{\frac{-5}{2}}$

4. Find the value(s) of x, if any, for which the following function is (i) zero, (ii) undefined, (iii) indeterminate: $f(x) = \dfrac{(x^3-1)(x+4)}{2x(x^2-2x+1)}$

5. Solve the following:

   (a) $-3 \le 5x - 8 < -1$ (b) $\dfrac{x-7}{x^2-16} < 0$

   (c) $|-7x+6| = 8 - 5x$ (d) $|3x+11| > 5$ e) $\left|\frac{x}{2-x}\right| \le 3$

6. If $f(x) = \begin{cases} 3x^2+1, (-\infty, -2] \\ 10 - x, (-2, 2) \\ 2x - 5, [2, \infty) \end{cases}$ calculate the following:

   (a) $f(-2)$ (b) $f(5)$ (c) $f(0)$ (d) $f(2)$

7. Determine the domain and range for the following functions:

   (a) $f(x) = \sqrt{x+5}$ (b) $p(x) = 4x^3 - 6x + 1$

   (c) $h(x) = 5(x-2)^2 - 9$ (d) $g(x) = e^x$

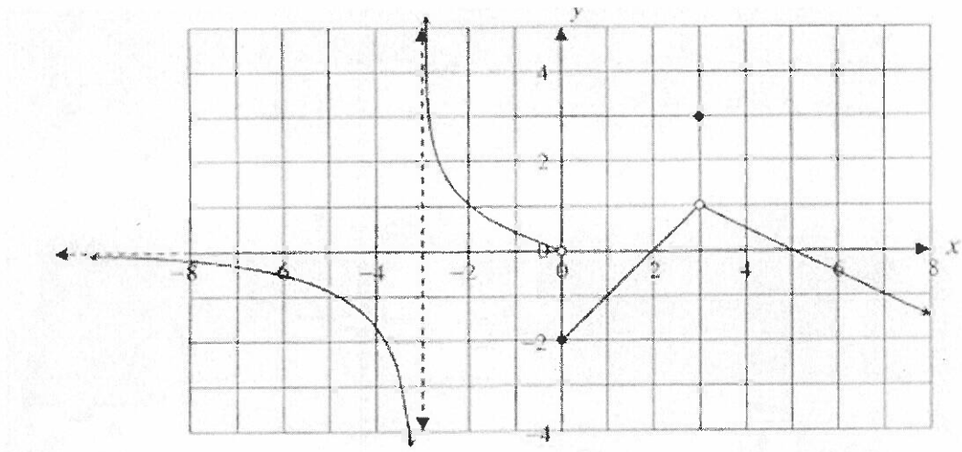
8. Sketch the following piecewise function: $g(x) = \begin{cases} 3, (-\infty, -1) \\ x^2, [-1, 3) \\ 2x - 1, [3, \infty) \end{cases}$

9. If $f(x) = 4x - 7$ and $g(x) = x^2$, find $g(f(x))$.

10. Calculate the following limits:

    (a) $\lim\limits_{x \to 0} \dfrac{\frac{1}{(x+2)^2} - \frac{1}{4}}{x}$ (b) $\lim\limits_{x \to 5} \dfrac{x-5}{\sqrt{x+4} - 3}$

    (c) $\lim\limits_{x \to 5} \sqrt{\dfrac{x^2+15}{x-1}}$ (d) $\lim\limits_{x \to \infty} \dfrac{6x^2-1}{2x^2+3x}$



$y = f(x)$

11. Using the graph above

(a) find all x-values at which the graph is discontinuous
(b) classify the type of discontinuity

12. Use the test for continuity to determine if $g(x) = \begin{cases} 2x, (-\infty, 2) \\ x^2, [2, \infty) \end{cases}$ is continuous at $x = 2$.

13. Where is $f(x) = \dfrac{x+3}{x^2-9}$ discontinuous and what type of discontinuity exists there?

14. Find the first derivatives of the following functions, and simplify:

(a) $f(x) = (2x+3)^2$ (b) $f(x) = (x^3 + 3x)(8 - x)$

(c) $f(x) = \left(\dfrac{2x}{x+2}\right)^{-2}$ (d) $y = \dfrac{x\sqrt{x+3}}{x-2}$

(e) $f(x) = (x+3)e^{3x}$ (f) $y = -6\sin(3x^2)$

(g) $f(x) = x\ln(xe^x)$ (h) $y = x^2 \tan 3x$

15. Use implicit differentiation to find the first and second derivative of $y^2 + 3y - x = 4$.

16. Find the equation of the tangent line to the function $f(x) = x^3 + 2x - 7$ at $x = 2$.

17. Find the coordinates of the point on $f(x) = x^2 - 5x + 1$ where the tangent line is parallel to $5x - y = 10$.

18. Given: $f(x) = (x-2)^3(x+2)$

(a) Find the first derivative.
(b) Determine the intervals of increase and decrease.
(c) Determine the coordinates of any relative extrema.
(d) Find the second derivative.
(e) Determine the intervals of concavity.
(f) Determine the coordinates of any inflection points.
(g) Determine the x and y intercepts.
(h) Sketch the graph.

19. An open topped box is to be made from 300 cm^2 of material. If the box has a square base, find its dimensions in order to maximize the volume.

20. A particle moves along the x-axis so that its position in metres after t seconds is given by the function: $s(t) = 2t^3 - 7t^2 + 3t$

(a) Where is the ball after 3 seconds?
(b) Find the velocity function.
(c) Find the instantaneous velocity of the ball after 2 seconds.
(d) Find the average velocity of the ball from 4 seconds to 6 seconds.
(e) Find the acceleration function.
(f) What is the instantaneous acceleration at 10 seconds?
(g) What is the instantaneous velocity of the ball when the acceleration is 70 m/s^2?

21. Tamara boarded a "see-through" elevator, which climbed the outside of a building at a rate of 3 m/s. Her friend, Candice, stood on the sidewalk 24 m from the base of the elevator and watched Tamara ascend. At what rate was the distance between them changing when Tamara was 45 m above the sidewalk?

22. A beach ball is being inflated with air so that its circumference is increasing at a rate of 2 cm/s. When the diameter is 30 cm, find the rate at which the volume is increasing.

23. Find the following indefinite integrals:

(a) $\int(-2x^2 + 5x - 3)dx$ (b) $\int x^4(2x-3)dx$

(c) $\int(x^{\frac{3}{4}} + 6x^{\frac{1}{3}})dx$ (d) $\int(e^{3x} + 25)dx$

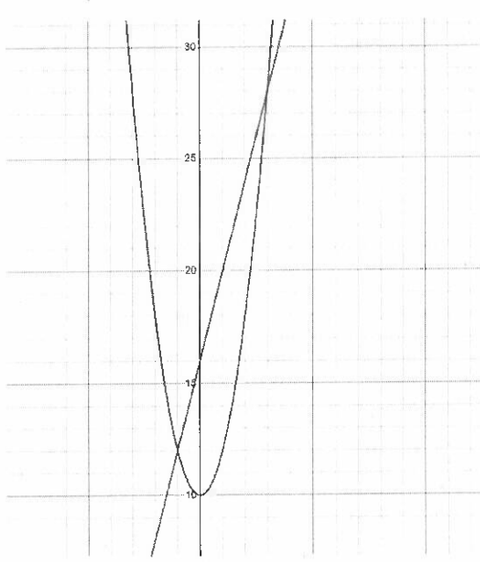
24. Find the following definite integrals:

(a) $\int_{-5}^{5}(5x^4 + 6x)dx$ (b) $\int_{1}^{9}\sqrt{x}\,dx$

25. Find the following integrals, U can do it!

a) $\int(2x^2 + 7)^5 x dx$ b) $\int e^{sinx} cosx dx$

26. Find the area between the two curves $y = 4x + 16$ and $y = 2x^2 + 10$



27. A cone has a height of 20 cm and a radius of 30 cm. Water is being poured into the cone at a rate of 16 cm^3/s. Find the rate change of the height of the water in the cone when the height of the water is 12 cm.