Chapter 7/8 Review (Jan 2019)

Page 344

#3, 16 a i m

Page 372

4, 5 a b e f , 6 a-e

7.7 Review Answers Page 344

3\. a) $y = 21x^{10}$
$y' = 210x^9$

b) $y = 6\cos 3x$
$y' = -6\sin 3x \cdot 3$
$y' = -18\sin 3x$

c) $y = \frac{1}{5}\sin 20x$
$y' = \frac{1}{5}\cos 20x \cdot 20$
$y' = 4\cos 20x$

d) $y = 2e^{3x}$
$y' = 2e^{3x} \cdot 3$
$y' = 6e^{3x}$

e) $y = \ln(x^2+5x)$
$y' = \frac{1}{x^2+5x} \cdot (2x+5)$
$y' = \frac{2x+5}{x^2+5x}$

f) $y = \log_5(\sin x)$
$y' = \frac{1}{\sin x} \cdot \cos x \cdot \log_5 e$
$y' = \cot x \log_5 e$

g) $y = 10^x$
$y' = 10^x \cdot \ln 10$

h) $y = e^{\ln x^4}$
$y = x^4$
$y' = 4x^3$

i) $y = e^{-x^3}$
$y' = e^{-x^3} \cdot -3x^2$
$y' = -3x^2 e^{-x^3}$

j) $y = 5\ln 10x$
$y' = 5\left(\frac{1}{10x}\right) \cdot 10$
$y' = \frac{5}{x}$

k) $y = \cos(\ln 2x)$
$y' = -\sin(\ln 2x) \cdot \frac{1}{2x} \cdot 2$
$y' = \frac{-\sin(\ln 2x)}{x}$

l) $y = xe^{2x}$
$y' = xe^{2x} \cdot 2 + e^{2x}$
$y' = e^{2x}(2x+1)$

m) $y = x^2 \sin 2x$
$y' = x^2\cos 2x \cdot 2 + \sin 2x \cdot 2x$
$y' = 2x^2\cos 2x + 2x\sin 2x$
$y' = 2x(x\cos 2x + \sin 2x)$

n) $y = (e^x)^{20}$

$y' = 20(e^x)^{19} \cdot (e^x)'$

$y' = 20(e^x)^{20}$

o) $y = \ln(\cos 3x)$

$y' = \frac{1}{\cos 3x} \cdot \sin 3x \cdot 3$

$y' = \frac{3\sin 3x}{\cos 3x} = 3\tan 3x$

p) $y = -3\sin\left(\frac{x}{3}\right)$

$y' = -3\cos\left(\frac{x}{3}\right) \cdot \frac{1}{3}$

$y' = -\cos\frac{x}{3}$

q) $y = e^{-2\ln x}$

$y = e^{\ln x^{-2}}$

$y = x^{-2}$

$y' = -2x^{-3}$

r) $y = \frac{1}{15}(\cos x)^{30}$

$y' = 2(\cos x)^{29} \cdot (-\sin x)$

$y' = -2\sin x\cos^{29} x$

s) $y = 5^{x^3}$

$y' = 5^{x^3} \cdot 3x^2 \cdot \ln 5$

t) $y = \log e$

$y' = 0$

u) $y = \log x$

$y' = \frac{1}{x} \cdot \log e$

$y' = \frac{\log e}{x}$

v) $y = \ln\left(\frac{x}{x-1}\right)$

$y' = \frac{1}{\left(\frac{x}{x-1}\right)} \cdot \left[\frac{(x-1)(1) - x(1)}{(x-1)^2}\right]$

$y' = \frac{x-1}{x}\left[\frac{-1}{(x-1)^2}\right]$

$y' = \frac{-1}{x(x-1)}$

w) $y = \ln[\sin(2e^{6x})]$

$y' = \frac{1}{\sin(2e^{6x})} \cdot \cos(2e^{6x}) \cdot 2e^{6x} \cdot 6$

$y' = \frac{12e^{6x}\cos(2e^{6x})}{\sin(2e^{6x})}$

$y' = 12e^{6x}\cot(2e^{6x})$

x) $y = 4(e^{3x+1})^2$

$y' = 8(e^{3x+1}) \cdot e^{3x+1} \cdot 3$

$y' = 24(e^{3x+1})^2$

16. a) $y = \tan 2x$

$y' = \sec^2 2x \cdot 2$

$y' = 2\sec^2 2x$

i) $y = -6\tan \frac{1}{2}x$

$y' = -6\sec^2(\frac{1}{2}x) \cdot \frac{1}{2}$

$y' = -3\sec^2(\frac{1}{2}x)$

m) $y = x^2 \tan \frac{1}{3}x$

$y' = x^2\left[\sec^2 \frac{1}{3}x \cdot \frac{1}{3}\right] + \tan \frac{1}{3}x\ (2x)$

$y' = \frac{x^2}{3}\sec^2(\frac{1}{3}x) + 2x\tan(\frac{1}{3}x)$

$y' = x\left[\frac{x}{3}\sec^2(\frac{1}{3}x) + 2\tan(\frac{1}{3}x)\right]$

8.5 Review Exercises Page 372.

4 a) $\int (x^2+4x-5)\,dx$
$= \frac{x^3}{3} + 2x^2 - 5x + C$

b) $\int e^{9x}\,dx$
$= \frac{e^{9x}}{9} + C$

c) $\int 3\sin 6x\,dx$
$= \frac{3(-\cos 6x)}{6} + C$
$= -\frac{1}{2}\cos 6x + C$

d) $\int -\frac{2}{3}\cos 2x\,dx$
$= -\frac{2}{3}\left[\frac{\sin 2x}{2} + C\right]$
$= -\frac{1}{3}\sin 2x + C$

e) $\int \frac{8}{x}\,dx$
$= 8\left[\ln|x| + C\right]$
$= 8\ln|x| + C$

f) $\int x^5\,dx = \frac{x^6}{6} + C$

g) $\int \cos 3x\,dx$
$= \frac{\sin 3x}{3} + C = \frac{1}{3}\sin 3x + C$

h) $\int [(x-5)(x+4)]\,dx$
$= \int (x^2 - x - 20)\,dx$
$= \frac{x^3}{3} - \frac{x^2}{2} - 20x + C$

i) $\int \sin \frac{1}{2}x\,dx$
$= \frac{-\cos\frac{1}{2}x}{\frac{1}{2}} + C$
$= -2\cos\frac{1}{2}x + C$

j) $\int dx$
$= x + C$

K) $\int \frac{x^{10} - 9x^7}{x^5}\,dx = \int x^5 - 9x^2\,dx$
$= \frac{x^6}{6} - 3x^3 + C$

l) $\int \frac{1}{20}\,dx$

$= \frac{1}{20}x + C$

m) $\int \sin \pi x \, dx$

$= \frac{-\cos \pi x}{\pi} + C$

$= -\frac{1}{\pi}\cos \pi x + C$

n) $\int (1+\sqrt{x})^2 \, dx$

$= \int \left(1 + 2x^{\frac{1}{2}} + x\right) dx$

$= x + \frac{4}{3}x^{3/2} + \frac{x^2}{2} + C$

o) $\int 5^x \, dx$

$= \frac{5^x}{\ln 5} + C$

p) $\int -6\cos \frac{1}{2}x \, dx$

$= -6\left[\frac{\sin \frac{1}{2}x}{\frac{1}{2}} + C\right]$

$= -12 \sin \frac{1}{2}x + C$

q) $\int \left(x - \frac{1}{x}\right) dx$

$= \frac{x^2}{2} - \ln|x| + C$

r) $\int \sin \frac{1}{4}x \, dx$

$= \frac{-\cos \frac{1}{4}x}{\frac{1}{4}} + C$

$= -4\cos \frac{1}{4}x + C$

s) $\int e^{\frac{x}{2}} \, dx$

$= \frac{e^{\frac{x}{2}}}{\frac{1}{2}} + C$

$= 2e^{\frac{x}{2}} + C$

t) $\int 10^x \ln 10 \, dx$

$= \ln 10 \left[\frac{10^x}{\ln 10}\right] + C$

$= 10^x + C$

u) $\int \frac{x+2}{x}\,dx$

$= \int \left(1 + \frac{2}{x}\right)dx$

$= x + 2\ln|x| + C$

v) $\int \frac{12}{x^2}\,dx$

$= \int 12x^{-2}\,dx$

$= -12x^{-1} + C$

w) $\int \frac{x-10}{x^{1/3}}\,dx$

$\int \left(x^{2/3} - 10x^{-1/3}\right)dx$

$= \frac{3}{5}x^{5/3} - 15x^{2/3} + C$

x) $\int x^{-7/5}\,dx$

$= -\frac{5}{2}x^{-2/5} + C$

6.a) $\int_4^5 (4x-1)\,dx$

$= 2x^2 - x \Big|_4^5$

$= \left[2(5)^2 - 5\right] - \left[2(4)^2 - 4\right]$

$= 45 - 28$

$= 17$

b) $\int_0^{\pi/2} \sin \frac{1}{2}x\,dx$

$= \frac{-\cos \frac{1}{2}x}{\frac{1}{2}} \Big|_0^{\pi/2}$

$= -2\cos \frac{1}{2}x \Big|_0^{\pi/2}$

$= -2\left[\cos \frac{1}{2}(\pi/2) - \cos \frac{1}{2}(0)\right]$

$= -2\left[\cos \pi/4 - \cos 0\right]$

$= -2\left[\frac{\sqrt{2}}{2} - 1\right]$

$= -\sqrt{2} + 2$

c) $\int_{-4}^{-2} e^{-2x}$

$= \left. \frac{e^{-2x}}{-2} \right|_{-4}^{-2}$

$= -\frac{1}{2}\left[e^{-2x} \Big|_{-4}^{-2} \right]$

$= -\frac{1}{2}\left[e^{-2(-2)} - e^{-2(-4)} \right]$

$= -\frac{1}{2}\left[e^{4} - e^{8} \right]$

d) $\int_{e}^{e^4} \frac{1}{x}\, dx$

$= \ln|x| \Big|_{e}^{e^4}$

$= \ln e^4 - \ln e$

$= 4 - 1$

$= 3$

e) $\int_{1}^{9} x^{1/2}\, dx$

$= \left. \frac{2}{3} x^{3/2} \right|_{1}^{9}$

$= \frac{2}{3}(9)^{3/2} - \frac{2}{3}(1)^{3/2}$

$= \frac{2}{3}(27) - \frac{2}{3}$

$= 18 - \frac{2}{3}$

$= \frac{54}{3} - \frac{2}{3}$

$= \frac{52}{3}$

5 a) $\int x^2\sqrt{x^3-1}\,dx$

let $u = x^3 - 1$

$du = 3x^2\,dx$

$\frac{1}{3}\,du = x^2\,dx$

$= \frac{1}{3}\int \sqrt{u}\,du = \frac{1}{3}\int u^{1/2}\,du = \frac{1}{3}\left[\frac{2}{3}u^{3/2}\right] + C$

$= \frac{2}{9}u^{3/2} + C$

$= \frac{2}{9}(x^3-1)^{3/2} + C$

b) $\int (2x+3)(x^2+3x+1)^{11}\,dx$

let $u = x^2+3x+1$

$du = 2x+3\,dx$

$= \int u^{11}\,du$

$= \frac{u^{12}}{12} + C$

$= \frac{(x^2+3x+1)^{12}}{12} + C$

5e) $\int e^{2x}\cos\left(e^{2x}\right)dx$

let $u = e^{2x}$

$du = 2e^{2x}\,dx$

$\frac{1}{2}du = dx$

$= \frac{1}{2}\int \cos u \, du \quad = \frac{1}{2}\left[\sin u\right] + C$

$= \frac{1}{2}\sin e^{2x} + C$

f) $\int \frac{x}{\sqrt{x^2+1}}\,dx$

let $u = x^2 + 1$

$du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

$= \frac{1}{2}\int \frac{1}{\sqrt{u}}\,du$

$= \frac{1}{2}\int u^{-1/2}\,du \quad = \frac{1}{2}\left[2u^{1/2} + C\right]$

$= u^{1/2} + C$

$= \sqrt{x^2+1} + C$