Key

A: Measurement

1. Convert each of the following

(a) convert 1458 m to miles

$1458\ m = 1.458\ km$

$1.458\ (0.6) = 0.8748\ mi$

(b) convert 3 yd. 2 ft. to cm

$\left.\begin{array}{l} 3yd = 36\ in\ (3) = 108 \\ 2ft = 24\ in \end{array}\right\}\ 132\ in$

$132\ (2.5) = 330\ cm$

2. A hemisphere has a diameter of 10 feet.

(a) What is the surface area of this hemisphere, to the nearest square foot?

$d = 10\ ft$

$r = 5\ ft$

$SA = 3\pi r^2$

$= 3\pi (5)^2$

$= 236\ ft^2$

(b) What is the volume of the hemisphere, the nearest cubic foot?

$V = \frac{2}{3}\pi r^3$

$V = \frac{2}{3}\pi (5)^3$

$V = 261.7993878..$

$V \doteq 262\ ft^3$

3. A right ~~rectangular~~ pyramid has a square base with ~~dimensions 8m by 7m~~ sides of 7.5m, and slant height of 6m. Determine the surface area of this pyramid to the nearest square metre.

told class to change this question's wording

$SA = \frac{1}{2}s\,(\text{perimeter}) + (\text{base area})$

$SA = \frac{1}{2}(6)(30) + 56.25$

$SA = 90 + 56.25 = 146.25$

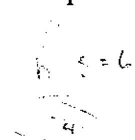
$SA \doteq 146\ m^2$

4. A bowl of sugar was knocked over. The spilled sugar formed a cone with a radius of 4 cm and a slant height of 6 cm. How much sugar was in the pile?

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi (4)^2 (4.211)$

$V = 120.8\ cm^3$



$h^2 = 6^2 - 4^2$

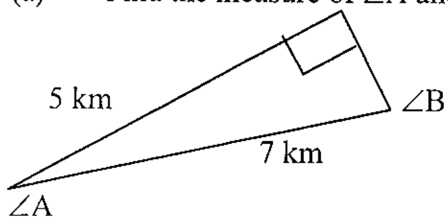
$h = \sqrt{36 - 16}$

$h = \sqrt{20} \doteq 4.211$

B: Trigonometry

1. Refer to the right triangle below.

(a) Find the measure of ∠A and ∠B to the nearest degree.



$\cos A = \frac{5}{7}$

$\angle A = \cos^{-1}(5/7)$

$\angle A = 44.4^\circ$

$\sin B = \frac{5}{7}$

$\angle B = \sin^{-1}(5/7)$

$\angle B = 45.6^\circ$

(b) Now find the length of the missing side.

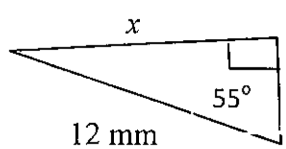
$7^2 = 5^2 + a^2$

$49 = 25 + a^2$

$24 = a^2$

$a = \sqrt{24} \doteq 4.9$ km

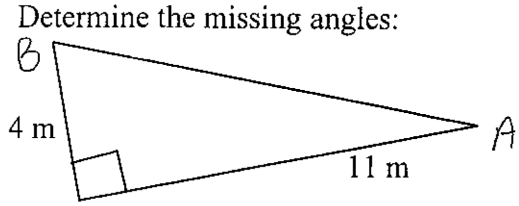
2. Determine the length of x:



$\sin 55^\circ = \frac{x}{12}$

$x = 12 \cdot \sin 55^\circ = 9.8$ mm

3. Determine the missing angles:



$\tan A = \frac{4}{11}$

$A = \tan^{-1}(\frac{4}{11})$

$A = 19.983...$

$A \doteq 20^\circ$

$\tan B = \frac{11}{4}$

$B = \tan^{-1}(11/4)$

$B = 70.016...$

$B \doteq 70^\circ$

4. Find the values of the following to 4 decimal places using your calculator:

(a) sin 34° = 0.5592

(b) cos 58° = 0.5299

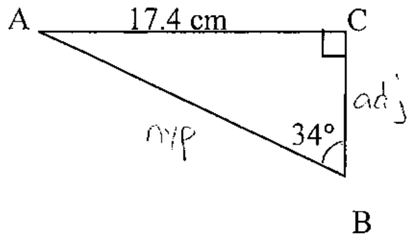
(c) tan 46° = 1.0355

5. If cos θ = 0.5402, find the measure of angle θ rounded to one decimal place.

$\theta = \cos^{-1}(0.5402)$

$\theta = 57.3^\circ$

6. Solve the triangle found below.



m∠A = $90^\circ - 34^\circ = 56^\circ$

m ∠C = 90°

$\overline{AB}$ = 31.1 cm

$\overline{BC}$ = 25.8 cm

$\sin 34^\circ = \frac{17.4}{AB}$

$AB = \frac{17.4}{\sin 34^\circ}$

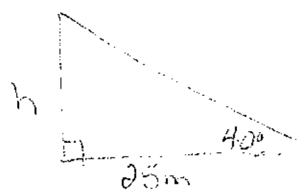
$AB = 31.116\ldots$

$\tan 34^\circ = \frac{17.4}{CB}$

$BC = \frac{17.4}{\tan 34^\circ}$

$BC = 25.796\ldots$

7. A flagpole casts a shadow that is 25 m long when the angle of between the sun's ray and the ground is 40 degrees. What is the height of the flagpole to the nearest meter?



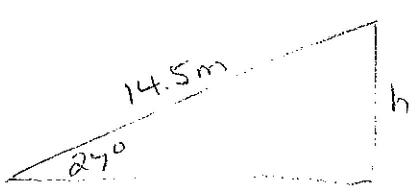
$\tan 40^\circ = \frac{h}{25}$

$h = 25 \tan 40^\circ$

$h = 20.977..$

$h \doteq 21\text{m}$

8. An escalator is 14.5 m long. The escalator makes an angle of 27 degrees with the ground. What is the height of the escalator? Give your answer to the nearest tenth of a meter.



$\sin 27^\circ = \frac{h}{14.5}$

$h = 14.5 \sin 27^\circ$

$h = 6.58286\ldots$

$h \doteq 6.6\text{m}$

C: Factors and Products

1. Expand and simplify

(a) $(2x+8)(3x-2)$

$= 2x(3x-2)+8(3x-2)$

$= 6x^2-4x+24x-16$

$= 6x^2+20x-16$

(b) $5(x-3)^2$

$= 5(x-3)(x-3)$

$= 5(x^2-3x-3x+9)$

$= 5(x^2-6x+9)$

$= 5x^2-30x-45$

(c) $(4x+3)^2$

$= (4x+3)(4x+3)$

$= 4x(4x+3)+3(4x+3)$

$= 16x^2+12x+12x+9$

$= 16x^2+24x+9$

2. Factor *(use prime factorization)*

(a) $324 = 2\cdot 2\cdot 3\cdot 3\cdot 3\cdot 3$

$= 2^2\cdot 3^4$

(b) $120 = 2\cdot 2\cdot 2\cdot 3\cdot 5$

$= 2^3\cdot 3\cdot 5$

3. Find the Greatest Common Factor

(a) 12 and 18

$12 = 2\cdot 2\cdot 3$

$18 = 2\cdot 3\cdot 3$

gcf $= 2\cdot 3 = 6$

(b) 24 and 60

$24 = 2\cdot 2\cdot 2\cdot 3$

$60 = 2\cdot 2\cdot 3\cdot 5$

gcf $= 2\cdot 2\cdot 3 = 12$

4. Find the Least Common Multiple

(a) 12 and 15

$12 = 2\cdot 2\cdot 3$

$15 = 3\cdot 5$

LCM $= 2\cdot 2\cdot 3\cdot 5$

$= 60$

(b) 16 and 20

$16 = 2\cdot 2\cdot 2\cdot 2$

$20 = 2\cdot 2\cdot 5$

LCM $= 2\cdot 2\cdot 2\cdot 2\cdot 5$

$= 80$

5. Perfect Squares and Cube Roots

(a) Using Prime Factorization find the square and cube root of 64

$64 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$

$= (2 \cdot 2)(2 \cdot 2)(2 \cdot 2)$

$= 4 \cdot 4 \cdot 4$

$\therefore \sqrt[3]{64} = 4$

6. Factor polynomials using a common factor

(a) $6n^2 - 18n$

$= 6n(n-3)$

(b) $6x^3y^2 + 2xy^5$

$= 2xy^2(3x^2 + y^3)$

7. Factor Trinomials

(a) $(a^2 + 7a - 18)$

$= (a+9)(a-2)$

(b) $x^2 - 8x + 7$

$= (x-7)(x-1)$

(c) $3v^2 - 8v + 4$ $\quad 12$ $\quad -6, -2$

$3v^2 - 6v - 2v + 4$

$= 3v(v-2) - 2(v-2)$

$= (3v-2)(v-2)$

(d) $w^2 - 14w + 49$

$= (w-7)(w-7)$

$= (w-7)^2$

8. Factor Difference of Squares

(a) $d^2 - 16$

$= (d+4)(d-4)$

(b) $x^2 - 81$

$= (x-9)(x+9)$

9. Factor out a common factor then use difference of squares: $8x^2 - 72y^2$

$8(x^2 - 9y^2) = 8(x-3y)(x+3y)$

D: Roots and Powers

1. Use prime factorization to simplify:

(a) $\sqrt{72}$

$= \sqrt{2 \cdot 3 \cdot 2 \cdot 2 \cdot 3}$

$= 2 \cdot 3\sqrt{2} = 6\sqrt{2}$

(b) $\sqrt[4]{256}$

$= \sqrt[4]{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2}$

$= 2 \cdot 2 = 4$

(c) $\sqrt{147}$

$= \sqrt{3 \cdot 7 \cdot 7}$

$= 7\sqrt{3}$

2. Write as an Entire radical:

(a) $2\sqrt{7}$

$= \sqrt{2 \cdot 2 \cdot 7}$

$= \sqrt{28}$

(b) $5\sqrt[3]{3}$

$= \sqrt[3]{5 \cdot 5 \cdot 5 \cdot 3}$

$= \sqrt[3]{375}$

3. Estimate to 1 decimal place:

(a) $\sqrt{52}$

$49 \rightarrow 52 \rightarrow 64$

$\sqrt{49} \rightarrow \sqrt{52} \rightarrow \sqrt{64}$

52 is closer to 49 than to 64

$7 \rightarrow \sqrt{52} \rightarrow 8$

$\therefore \doteq 7.2$

(b) $\sqrt[3]{30}$

$27 \rightarrow 30 \rightarrow 64$

$\sqrt[3]{27} \rightarrow \sqrt[3]{30} \rightarrow \sqrt[3]{64}$

30 is much closer to 27 than to 64

$3 \rightarrow \sqrt[3]{30} \rightarrow 4$

$\therefore \doteq 3.1$

4. Are the following Rational or Irrational Numbers?

(a) $\sqrt{\frac{4}{9}} = \frac{\sqrt{4}}{\sqrt{9}} = \frac{2}{3}$

$\therefore$ rational

(b) $\sqrt[2]{17}$ 17 is not a perfect sq

$\therefore$ irrational

5. Evaluate:

(a) $36^{\frac{-1}{2}}$

$= \left(\frac{1}{36}\right)^{\frac{1}{2}}$

$= \sqrt{\frac{1}{36}} = \frac{\sqrt{1}}{\sqrt{36}}$

$= \frac{1}{6}$

(b) $27^{\frac{2}{3}}$

$= \left(\sqrt[3]{27}\right)^{2}$

$= 3^{2}$

$= 9$

(c) $\left(\sqrt{\frac{4}{9}}\right)^{-3}$

$= \left(\sqrt{\frac{9}{4}}\right)^{3}$

$= \left(\frac{3}{2}\right)^{3} = \frac{3^3}{2^3} = \frac{27}{8}$

6. Write Power as a radical: $15^{\frac{2}{3}} = \left(\sqrt[3]{15}\right)^{2}$

7. Write as a power with a fractional exponent: $\sqrt[3]{5^2} = 5^{2/3}$

8. Multiply and Divide (do not leave negative exponents in your final answer)

(a) $(2^5)(2^3)$

$= 2^{5+3} = 2^8 = 256$

(b) $x^{\frac{1}{2}} \cdot x^{-2}$

$= x^{\frac{1}{2}+(-2)} = x^{\frac{1}{2}-\frac{4}{2}}$

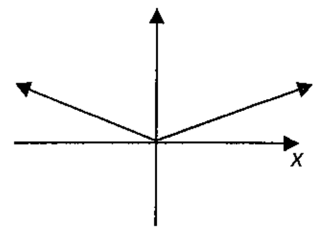
$= x^{-3/2} = \frac{1}{x^{3/2}}$

(c) $\frac{(4xy)^2}{2y} = \frac{16x^2y^2}{2y}$

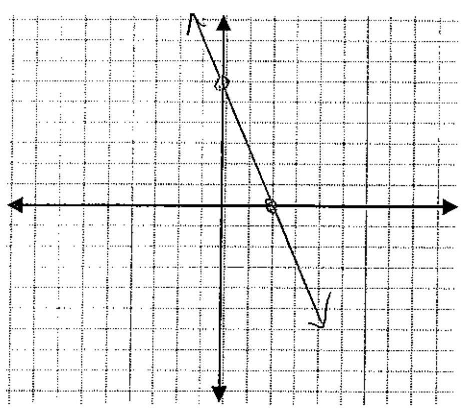
$= \frac{16}{2} \cdot \frac{x^2}{1} \cdot \frac{y^2}{y} = 8x^2y$

E: Functions and Relations

1. Refer to the graph to the right. Is this relation a function? Why or why not? Yes. It passes the vertical line test.



2. Graph $6x + 2y = 12$ using x and y intercepts



x-int

$6x + 2(0) = 12$

$\frac{6x}{6} = \frac{12}{6}$

$x = 2$

y-int

$6(0) + 2y = 12$

$\frac{2y}{2} = \frac{12}{2}$

$y = 6$

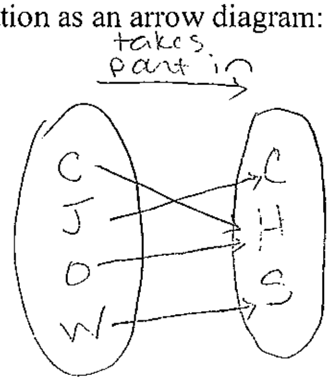
3. Answer the questions about the relations in the following 2 tables:

Athlete	Sport
Crosby	Hockey
Jones	Curling
Wotherspoon	Speed Skating
Ovechkin	Hockey

(a) Why is this relation a function?

No athlete has been paired with more than one sport.

(b) Represent this relation as an arrow diagram:



(c) Write the domain and range:

D = {Crosby, Jones, Ovechkin, Wotherspoon}

R = {Curling, Hockey, Speed Skating}

Number of Minutes n	Cost, C ($)
10	2
20	4
30	6
40	8
50	10

+10 each

+2 each

goes up by 10 each

goes up by 2 each

∴ linear

(d) Why is this relation a function?

None of the n values is paired with more than one C value.

(e) Write the domain and range.

$D = \{10, 20, 30, 40, 50\}$

$R = \{2, 4, 6, 8, 10\}$

(f) Identify the dependent and independent variables.

Ind var = n

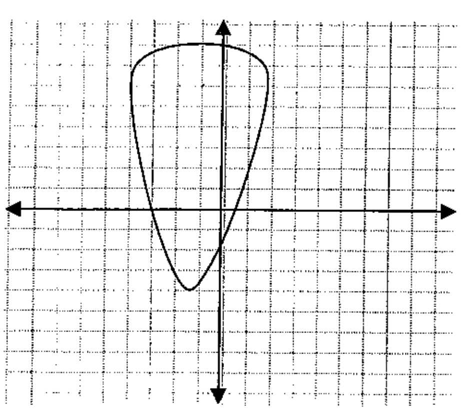
dep var = C

(g) Does this table represent a linear relation? How could you find out mathematically? How could you find out from a graph?

- it is linear
- the pts would lie on a line

4. Find the domain and range for each graph. Are the following Functions?

(a)

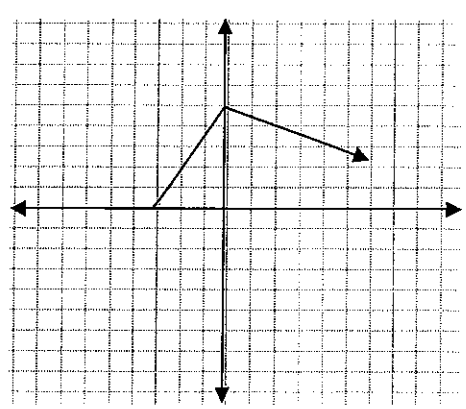


D = $-4 \leq x \leq 1.8$

R = $-4 \leq y \leq 8$

fails VLT → not a function

(b)



D = $x \geq -3$

R = $y \leq 5$

passes VLT → is a function

5. Carmen works for a research company. The equation $P = 5n + 30$ represents her daily pay, P dollars, when she conducts n surveys.

(a) Write the equations using function notation.

$P(n) = 5n + 30$

(b) Find the value of $P(8)$. Explain what this number represents.

$P(8) = 5(8) + 30$
$= 40 + 30$
$= 70$

"70" represents Carmen's pay for conducting 8 surveys."

(c) Find the value of n when $P(n) = 90$. Explain what this number represents.

$90 = 5n + 30$
$-30 \quad -30$
$\frac{60}{5} = \frac{5n}{5}$
$12 = n$

"12" represents how many surveys Carmen conducted when she got paid \$90.

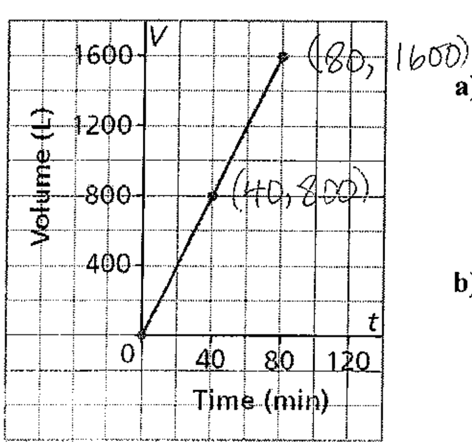
6. Create a table of values for $y = x + 2$ choosing two negative x values, two positive values, and 0 for x. Does this table represent a linear relation?

x	y
-2	0
-1	1
0	2
1	3
2	4

Yes it is linear.

7. Rate of Change

Filling a Hot Tub



a) Identify the dependent and independent variables.

ind var = t
dep var = V

b) Determine the rate of change of this relation, then describe what it represents.

rate of change = $\frac{1600-800}{80-40} = \frac{800}{40} = 20$

the tub fills at a rate of 20 L/min

F: Linear Functions

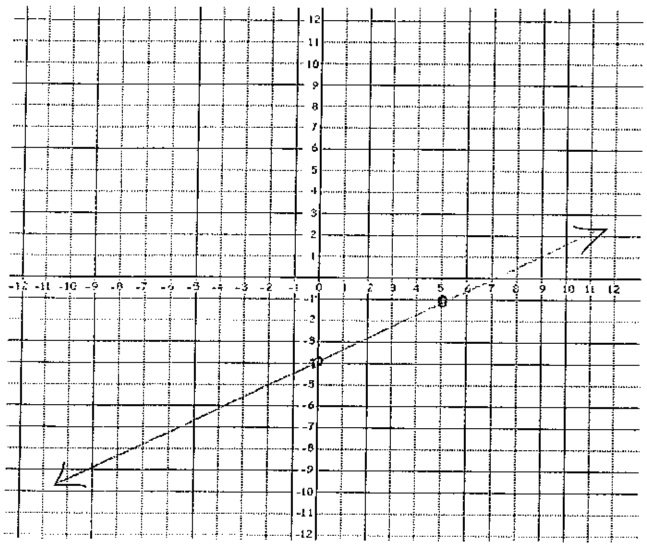
1. Graph the following linear function in the space provided using the slope and y-intercept method only. State the slope and y-intercept first.

$y = mx + b$

$$y = \frac{3}{5}x - 4$$

slope $= m = \frac{3}{5}$ - rise / - run

y-intercept $= b = -4$



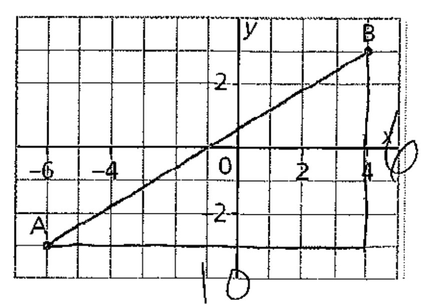
2. Find the slope:

(a) of a line that passes through (2, -3) and (-4, 3)

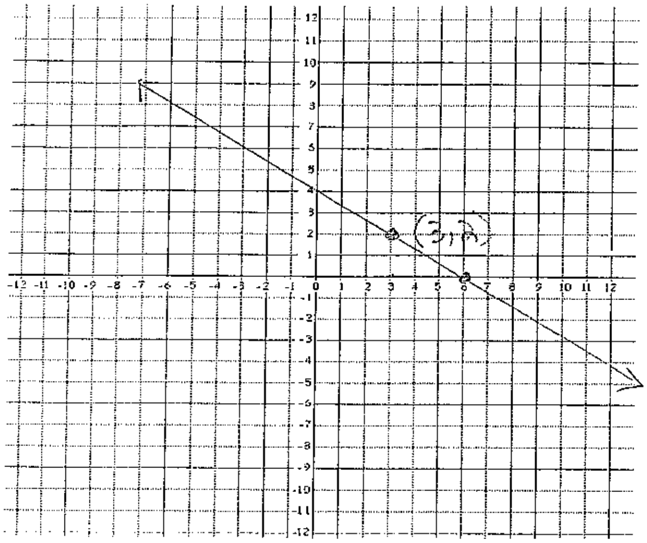
$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3-(-3)}{-4-2} = \frac{6}{-6} = -1$$

(b) Find the slope from the graph to the right.

$$\text{slope} = m = \frac{\text{rise}}{\text{run}} = \frac{6}{10} = \frac{3}{5}$$



3. Graph a line whose slope is $\frac{-2}{3}$ and goes through (3, 2), using the slope-point method.



- draw a pt. at (3,2)
- then go down 2, and 3 to the right

4. Are $y = 2x - 3$ and $y = \frac{-1}{2}x - 4$ parallel, perpendicular or neither? Explain how you know.

$y = 2x - 3 \quad m = 2$ $\qquad 2(-\frac{1}{2}) = -1 \quad \therefore$ perpendicular

$y = -\frac{1}{2}x - 4 \quad m = -\frac{1}{2}$

5. Write the equation of the line in the specified form:

(a) Write the equation of the line in point-slope form whose slope is 3 and passes through the point (2, 5)

$m = 3$ $\qquad y - y_1 = m(x - x_1)$

$x_1 = 2$

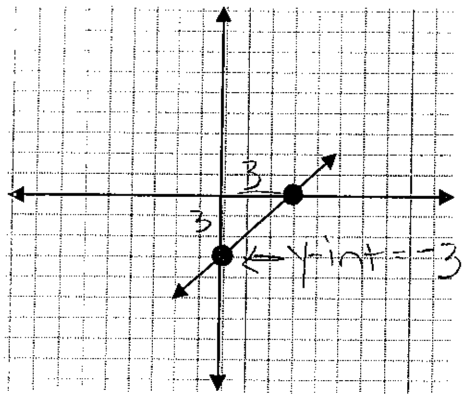
$y_1 = 5$ $\qquad y - 5 = 3(x - 2)$

(b) Write the equation of the line in general form that passes through (0, 3) and (2, 3).

$m = \frac{3-3}{2-0} = \frac{0}{2} = 0 \quad \therefore$ horizontal line

$y = 3 \quad \Rightarrow \quad y - 3 = 0$ is general form

(c) Write an equation in slope-intercept form for the line in the graph below:



slope $= \frac{3}{3} = 1$

$\therefore \quad y = x - 3$

(d) Now write that same equations from questions (a), (b), and (c) in standard form and general form.

(a) $y - 5 = 3(x-2)$

$y - 5 = 3x - 6$

$1 = 3x - y$

$3x - y = 1$ (standard) $\quad 3x - y - 1 = 0$ (general)

(b) $y = 3$ (standard) $\quad y - 3 = 0$ (general)

(c) $y = x - 3$

$3 = x - y$

$x - y = 3$ (standard) $\quad x - y - 3 = 0$ (general)

(e) Write the equation of a line in slope-point form that has a slope of $\frac{-3}{4}$ and passes through (2, -3).

$$y+3=-\frac{3}{4}(x-2)$$

(f) Write the equation of a line in slope-point form that passes through (-3,5) and (3,1)

$$m=\frac{5-1}{-3-3}=\frac{4}{-6}=-\frac{2}{3}$$

$$y-5=-\frac{2}{3}(x+3) \quad \text{or} \quad y-1=-\frac{2}{3}(x-3)$$

(g) Write the equation $y=\frac{1}{2}x-2$ in standard form and then into general form.

$$2\left[y=\frac{1}{2}x-2\right]$$

$$2y=x-4$$

$$-x+2y=-4$$

$$x-2y=4 \Rightarrow \text{standard}$$

$$x-2y-4=0 \Rightarrow \text{general}$$

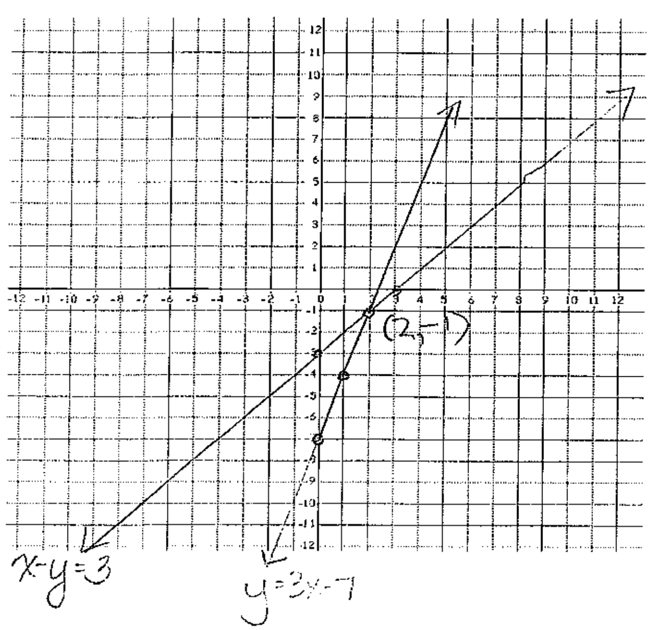
G: Linear Systems

1. Solve (and verify) this system graphically:

$y=3x-7 \rightarrow m=3$
$b=-7$

$x-y=3$

intercepts: $x=3$
$y=-3$



Verify

$y=3x-7$	$x-y=3$
$LS=-1$	$LS=2-(-1)$
$RS=3(2)-7$	$=2+1$
$=6-7$	$=3$
$=-1$	$RS=3$

$\therefore$ Sol'n is $x=2$
$y=-1$

2. Solve (and verify) the system by substitution:

$4x - 3y = 7$

$x + 2y = 10 \rightarrow x = 10 - 2y$

$-2y \quad -2y$

$4(10-2y) - 3y = 7$

$-40 \quad 40 - 8y - 3y = 7 - 40$

$\frac{-11y}{-11} = \frac{-33}{-11}$

$y = 3$

$x + 2(3) = 10$

$x + 6 = 10$

$x = 4$

Verify

$4x - 3y = 7$

LS: $4(4) - 3(3)$

$= 16 - 9$

$= 7$

RS $= 7$

$x + 2y = 10$

LS $= 4 + 2(3)$

$= 4 + 6$

$= 10$

RS $= 10$

$\therefore$ The sol'n is $x = 4$, $y = 3$.

3. Solve (and verify) the system by elimination:

$5x + 2y = -5 \xrightarrow{\times 2} 10x + 4y = -10$ subtract

$3x + 4y = 11 \longrightarrow 3x + 4y = 11$

$\frac{7x}{7} = \frac{-21}{7}$

$x = -3$

$5(-3) + 2y = -5$

$-15 + 2y = -5 + 15$

$+15$

$\frac{2y}{2} = \frac{10}{2}$

$y = 5$

Verify:

LS $= 5(-3) + 2(5)$

$= -15 + 10$

$= -5$

RS $= -5$

LS $= 3(-3) + 4(5)$

$= -9 + 20$

$= 11$

RS $= 11$

The sol'n is $x = -3$, $y = 5$

4. Without finding the solution, how many solutions does this system have? Justify your answer.

$y = -\frac{5}{2}x + 4 \rightarrow m = -5/2$, $b = 4$

$50x + 20y = 100$

$\frac{20y}{20} = \frac{-50x + 100}{20}$

$y = -\frac{5}{2}x + 5 \quad m = -\frac{5}{2}$, $b = 5$

same m, diff b

$\Rightarrow$ parallel lines

$\therefore$ No solution.

5. Model this situation with a system of linear equations. Solve your linear system to solve the problem.

"Jack and Jill fetched a total of 27 pails of water between the two of them. Jill fetched 5 less pails than Jack did. How many pails did each of them fetch?"

Let x = # pails for Jack
y = # " " Jill

① $x+y=27 \rightarrow x+y=27$
② $x-5=y \rightarrow x-y=5$ add

$$\frac{2x}{2}=\frac{32}{2}$$

$x=16$

$16+y=27$
$y=11$

Jack fetched 16 pails and Jill fetched 11 pails.