1.

a) $\int_0^{40} R(t)\,dt = 10\left[9 + 17 + 13 + 9\right]$

$= 480$ gallons

b) $\int_0^{40} R(t)\,dt = 5\left[12 + 9 + 8 + 17 + 14 + 13 + 10 + 9\right]$

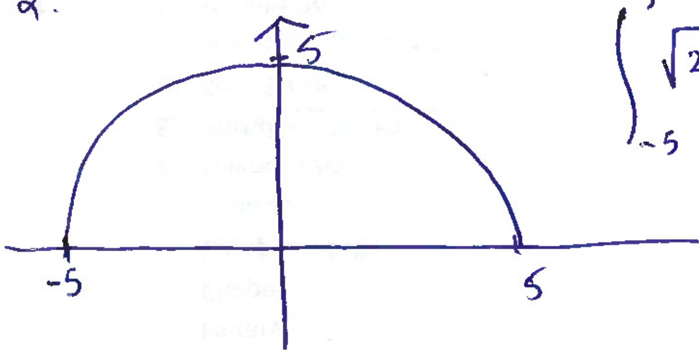
$= 460$ gallons

c) $\int_0^{40} R(t)\,dt = \frac{40-0}{2(8)}\left[12 + 2(9) + 2(8) + 2(17) + 2(14) + 2(13) + 2(10) + 2(9) + 7\right]$

$= \frac{5}{2}\left[179\right] = 447.5$ gallons

d) $\int_0^{40} R(t)\,dt \approx$ the number of gallons that flowed through the pipe from $t=0$ to $t=40$ minutes.

2.



$\int_{-5}^{5} \sqrt{25 - x^2}\,dx = \frac{1}{2}\pi(5)^2 = \frac{25\pi}{2}$

3. a) $\frac{1}{2}(2)(2) + (2)(2)$
$= 2 + 4 = 6$

b) $-\left[2(2) + \frac{1}{2}(1)(2)\right] = -5$

c) $\frac{1}{2}(2)(2) + (2)(2) + \frac{1}{2}(1)(2) - \frac{1}{2}(2)(2)$
$= 4 + 1 = 5$

d) $= 4[$

3. a) $\frac{1}{2}(2)(1+2) + (2)(2)$

$= 3 + 4 = 7$

b) $-\left[2(2) + \frac{1}{2}(1)(2)\right] = -5$

c) $\frac{1}{2}(2)(1+2) + (2)(2) + \frac{1}{2}(1)(2) - \frac{1}{2}(2)(2)$

$= 3 + 4 + 1 - 2 = 6$

d) $= 4\left[\frac{1}{2}(2)(1+2)\right] - \frac{1}{2}\left[\frac{1}{2}(1)(2)\right]$

$= 12 - \frac{1}{2} = \frac{23}{2}$

4. a) $\frac{2}{3}x^3 - 2x^2 + 2x \Big|_1^3$

$= \left[\frac{2}{3}(3)^3 - 2(3)^2 + 2(3)\right) - \left[\frac{2}{3}(1)^3 - 2(1)^2 + 2(1)\right]$

$= \frac{16}{3}$

b) $\frac{4}{3}x^{3/2} - \frac{2}{5}x^{5/2} \Big|_1^9$

$= \left[\frac{4}{3}(9)^{3/2} - \frac{2}{5}(9)^{5/2}\right] - \left[\frac{4}{3} - \frac{2}{5}\right]$

$= -\frac{932}{15}$

c) $\int_1^4 \left(\frac{x-2}{\sqrt{x}}\right) dx = \int_1^4 \left(x^{1/2} - 2x^{-1/2}\right) dx = \frac{2}{3}x^{3/2} - 4x^{1/2} \Big|_1^4$

$= \left(\frac{2}{3}(4)^{3/2} - 4(4)^{1/2}\right) - \left(\frac{2}{3} - 4\right)$

$= \frac{2}{3}$

d) $\displaystyle\int_{\pi/8}^{\pi/6} \cos(2x)\,dx = \frac{1}{2}\sin 2x \Big|_{\pi/8}^{\pi/6} = \frac{1}{2}\left[\sin\left(2\left(\frac{\pi}{6}\right)\right) - \sin\left(2\left(\frac{\pi}{8}\right)\right)\right]$

$= \frac{1}{2}\left[\sin\frac{\pi}{3} - \sin\frac{\pi}{4}\right]$

$= \frac{1}{2}\left[\frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2}\right]$

$= \frac{\sqrt{3}}{4} - \frac{\sqrt{2}}{4} = \frac{\sqrt{3}-\sqrt{2}}{4}$

e) $\displaystyle\int_{3}^{7} \left(3x^2 + \frac{1}{x}\right) dx$

$= x^3 + \ln|x| \Big|_3^7$

$= \left((7)^3 + \ln|7|\right) - \left((3)^3 + \ln|3|\right)$

$= 343 + \ln 7 - 27 - \ln 3$

$= 316 + \ln\left(\frac{7}{3}\right)$

f) $\displaystyle\int_0^{\pi} \sin\left(\frac{1}{2}x\right) dx = -2\cos\left(\frac{1}{2}x\right) \Big|_0^{\pi}$

$= -2\left[\cos\frac{1}{2}(\pi) - \cos 0\right]$

$= -2\left[0 - 1\right] = 2.$

5. $\frac{1}{3-1}\int_1^3 (4x^2-2x)\,dx$

$= \frac{1}{2}\left[\frac{80}{3}\right] = \frac{40}{3}$

6. a) $3x^3$

b) $-\left[2(2x)^2 - 2x\right]\cdot 2$

$= -2\left[2(4x^2) - 2x\right]$

$= -2\left[8x^2 - 2x\right]$

c) $\int_3^{x^2} (64b^3)\,db$

$= \left[64(x^2)^3\right]\cdot 2x = \left[64(x^6)\right] 2x$

$= 128x^7$

d) $-\left[\sin 4(3x^2)\right]\cdot 6x$

$= -6x\left[\sin 12x^2\right]$

7. a) $g(4) = \int_0^4 f(t)\,dt = -\left[(3)(1) + \frac{1}{2}(1)(1)\right] = -\left[3 + \frac{1}{2}\right] = -\frac{7}{2}$

b) $g(6) = \int_0^6 f(t)\,dt = -\left[(3)(1) + \frac{1}{2}(1)(1)\right] + \frac{1}{2}(2)(2)$

$= -\frac{7}{2} + 2 = -\frac{3}{2}$

c) 2 d) $-\frac{1}{2}$

e) $x=6$ b/c g' changes from inc to dec.

f) $g(6) = -\frac{3}{2}$ $g'(6) = 2$

$y + \frac{3}{2} = 2(x-6)$

$y + \frac{3}{2} = 2x - 12$

$y = 2x - 27/2$

8.

a) $\int_{-4}^{-2} f'\,dx = f(-2) - f(-4)$

$-\frac{1}{2}(2)(1) = 4 - f(-4)$

$-1 = 4 - f(-4)$

$f(-4) = 5$

b) $\int_{-2}^{4} f' = f(4) - f(-2)$

$-\frac{1}{2}(\pi)(2)^2 + \frac{1}{2}(2)(2) = f(4) - 4$

$-2\pi + 2 = f(4) - 4$

$6 - 2\pi = f(4)$

c) $\int_{-2}^{6} f' = f(6) - f(-2)$

$-\frac{\pi(2)^2}{2} + \frac{1}{2}(4)(2) = f(6) - 4$

$-2\pi + 4 = f(6) - 4$

$8 - 2\pi = f(6)$

d) $f(-4) = 5$

$f(6) = 8 - 2\pi$

$\int_{-2}^{2} f' = f(2) - f(-2)$

$-\frac{1}{2}\pi(2)^2 = f(2) - 4$

$-2\pi + 4 = f(2)$

$4 - 2\pi = f(2)$ abs min

e) $x = -2$ b/c f' changes from inc to dec and

$x = 4$ b/c f' changes from inc to dec.

$x = 0$ b/c f' " " dec to inc

$(-2, 0)$ b/c $f' < 0$ and f' decreasing

9.

$$\text{Area} = \frac{21-0}{2(7)}\left[3.2 + 2(2.9) + 2(3.1) + 2(3.5) + 2(3.3) + 2(3.7) + 2(3.6) + 2.1\right]$$

$$= \frac{3}{2}\left[45.5\right]$$

$$= 68.25 \text{ Km}^2$$

10.

a) $$\int_1^6 (3x-4)\,dx$$

$$\Delta x = \frac{6-1}{n} = \frac{5}{n}$$

$$c_i = 1 + \frac{5}{n}i$$

$$\lim_{n\to\infty} \sum_{i=1}^{n} \left(3\left(1+\frac{5}{n}i\right)-4\right)\cdot\frac{5}{n}$$

b) $$\int_{-2}^{0} \sqrt{x^2+1}\;dx$$

$$\Delta x = \frac{0-(-2)}{n} = \frac{2}{n}$$

$$c_i = -2 + \frac{2}{n}i$$

$$\lim_{n\to\infty} \sum_{i=1}^{n} \sqrt{\left(-2+\frac{2}{n}i\right)^2 + 1}\cdot\frac{2}{n}$$