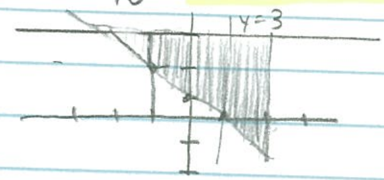
9.3 Area Between Two Curves P. 384 1-18



1. $y=3,\ y=1-x,\ x=-1,\ x=2$

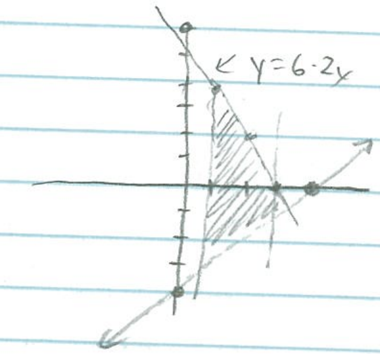
$$\int_{-1}^{2}\left[\overset{\text{high}}{(3)} - \overset{\text{(low)}}{(1-x)}\right]dx$$

$$= \int_{-1}^{2}(2+x)\,dx$$

$$= 2x + \frac{1}{2}x^2\Big|_{-1}^{2}$$

$$= \left(2(2)+\frac{1}{2}(4)\right) - \left(2(-1)+\frac{1}{2}(-1)^2\right)$$

$$= 6 - \left(\frac{-3}{2}\right) = \frac{15}{2}u^2$$



2. $y=x-4,\ y=6-2x,\ x=1,\ x=3$

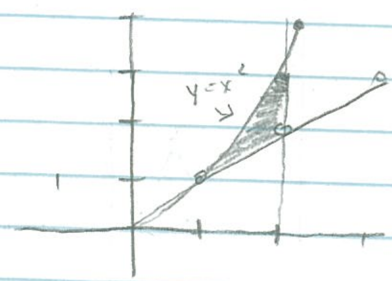
$$\int_{1}^{3}\left[\overset{\text{high}}{(6-2x)} - \overset{\text{low}}{(x-4)}\right]dx$$

$$= \int_{1}^{3}\left[10-3x\right]dx$$

$$= 10x - \frac{3}{2}x^2\Big|_{1}^{3}$$

$$= \left[10(3)-\frac{3}{2}(3)^2\right] - \left[10(1)-\frac{3}{2}(1)^2\right]$$

$$= \left[30-\frac{27}{2}\right] - \left[10-\frac{3}{2}\right] = 8u^2$$



3. $y=x,\ y=x^2,\ x=1,\ x=2$

$$\int_{1}^{2}\left[\overset{\text{high}}{(x^2)} - \overset{\text{low}}{(x)}\right]dx$$

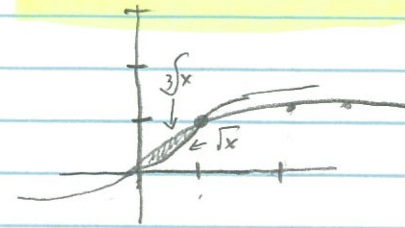
$$\frac{1}{3}x^3 - \frac{1}{2}x^2\Big|_{1}^{2}$$

$$= \left[\frac{1}{3}(2)^3 - \frac{1}{2}(2)^2\right] - \left[\frac{1}{3}(1)^3 - \frac{1}{2}(1)^2\right]$$

$$= \left[\frac{8}{3}-2\right] - \left[\frac{1}{3}-\frac{1}{2}\right] = \frac{5}{6}u^2$$

9.3 - Continued

4 $y=\sqrt{x}$, $y=\sqrt[3]{x}$, $x=0$, $x=1$

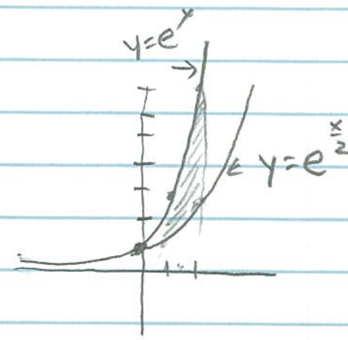


$$\int_0^1 \Big[\underset{\text{high}}{\sqrt[3]{x}} - \underset{\text{low}}{\sqrt{x}}\Big]\,dx$$

$$\left(\frac{3}{4}\right)x^{4/3} - \left(\frac{2}{3}\right)x^{\frac{3}{2}}\Big|_0^1$$

$$= \frac{3}{4}(1)^{4/3} - \frac{2}{3}(1)^{3/2} = \frac{3}{4} - \frac{2}{3} = \frac{1}{12}\,u^2$$

5. $y=e^x$, $y=e^{\frac{x}{2}}$, $x=0$, $x=2$



$$\int_0^2 \Big(\underset{\text{high}}{e^x} - \underset{\text{low}}{e^{\frac{x}{2}}}\Big)\,dx$$

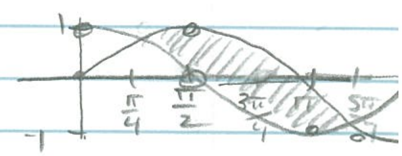
Let $u=\frac{1}{2}x$
$du=\frac{1}{2}dx$

$$e^x - 2e^{\frac{x}{2}}\Big|_0^2$$

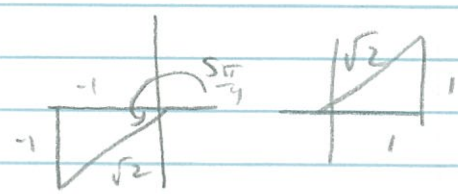
$$=(e^2-2e^1)-(e^0-2e^0)$$

$$= e^2-2e^1+1 \quad u^2$$

6. $y=\sin x$, $y=\cos x$, $x=\frac{\pi}{4}$, $x=\frac{5\pi}{4}$



$$\int_{\frac{\pi}{4}}^{\frac{5\pi}{4}} \Big[\underset{\text{high}}{(\sin x)} - \underset{\text{low}}{(\cos x)}\Big]\,dx$$



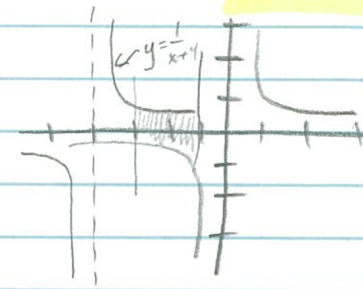
$$-\cos x - \sin x\Big|_{\frac{\pi}{4}}^{\frac{5\pi}{4}}$$

$$=\left[-\cos\frac{5\pi}{4} - \sin\frac{5\pi}{4}\right] - \left[-\cos\frac{\pi}{4} - \sin\frac{\pi}{4}\right]$$

$$=\left[-\left(-\frac{\sqrt{2}}{2}\right) - \left(-\frac{\sqrt{2}}{2}\right)\right] - \left[-\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}\right]$$

$$= \sqrt{2} + \sqrt{2} = 2\sqrt{2}\ u^2$$

9.3 Continued



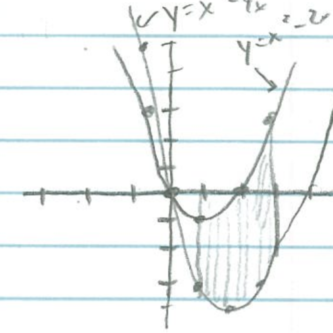
7. $y=\frac{1}{x},\ y=\frac{1}{x+4},\ x=-3,\ x=-1$

$$\int_{-1}^{-3}\Big[\underbrace{\tfrac{1}{x+4}}_{\text{high}}-\underbrace{\tfrac{1}{x}}_{\text{low}}\Big]dx$$

$$\ln|x+4|-\ln|x|\Big|_{-3}^{-1}$$

$$=\big[\ln|3|-\ln|-1|\big]-\big[\ln|1|-\ln|-3|\big]=\big[\ln|3|-\ln|1|\big]-\big[\ln|1|-\ln|3|\big]$$

$$=\ln 3-\ln\tfrac{1}{3}=\ln 9=\ln 3^2=2\ln 3$$

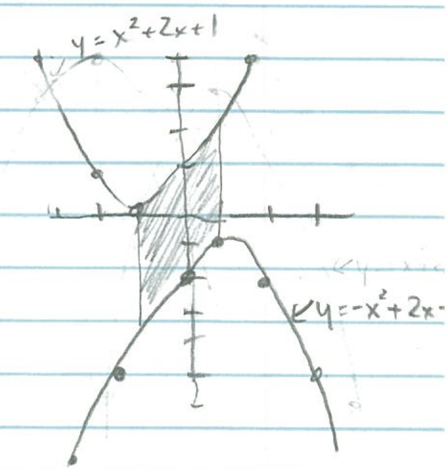


8. $y=x^2-4x,\ y=x^2-2x,\ x=1,\ x=3$

$$\int_{1}^{3}\Big[\underbrace{(x^2-2x)}_{\text{high}}-\underbrace{(x^2-4x)}_{\text{low}}\Big]dx$$

$$=\int_{1}^{3}(2x)\,dx$$ ← simplify first

$$=x^2\Big|_{1}^{3}=(3)^2-(1)^2=9-1=8u^2$$



9. $y=x^2+2x+1\quad y=-x^2+2x-2,\ x=-1,\ x=1$

$$\int_{-1}^{1}\Big[\underbrace{(x^2+2x+1)}_{\text{high}}-\underbrace{(-x^2+2x-2)}_{\text{low}}\Big]dx$$

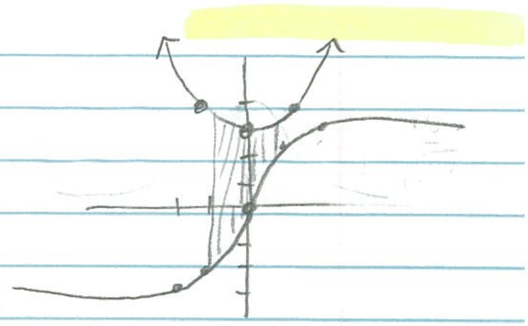
$$=\int_{-1}^{1}(2x^2+3)\,dx$$

$$=+\frac{2}{3}x^3+3x\Big|_{-1}^{1}$$

$$=\Big[+\tfrac{2}{3}(1)^3+3(1)\Big]-\Big[+\tfrac{2}{3}(-1)^3+3(-1)\Big]$$

$$=\Big(\tfrac{11}{3}\Big)-\Big(-\tfrac{11}{3}\Big)=\frac{22}{3}u^2$$

9.3 - Continued



10. $y = \frac{4x}{\sqrt{x^2+1}}$, $y = x^2+3$, $x=-1$, $x=1$

$$\int_{-1}^{1} \left[(x^2+3) - \left(\frac{4x}{\sqrt{x^2+1}} \right) \right] dx$$

$$= \frac{1}{3}x^3 + 3x - 4(x^2+1)^{1/2} \Big|_{-1}^{1}$$

$$= \left[\tfrac{1}{3}(1)^3 + 3(1) - 4(1^2+1)^{1/2} \right] - \left[\tfrac{1}{3}(-1)^3 + 3(-1) - 4((-1)^2+1)^{1/2} \right]$$

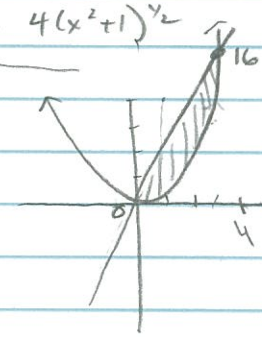
$$= \left[\frac{10}{3} - 4\sqrt{2} \right] - \left[\frac{-10}{3} - 4\sqrt{2} \right] = \frac{20}{3} u^2$$

$\int \frac{4x}{\sqrt{x^2+1}} dx$ Let $u = x^2+1$ $du = 2x dx$

$= 4\left(\frac{1}{2}\right)\int (x^2+1)^{-1/2} 2x\, dx$

$= 2\,(2)(x^2+1)^{1/2} + C$

$= 4(x^2+1)^{1/2}$



11. $y = 4x$ $\quad y = x^2$

$$\int_0^4 \underset{\text{high}}{[(4x)} - \underset{\text{low}}{(x^2)]}\, dx$$

$$= 2x^2 - \frac{1}{3}x^3 \Big|_0^4$$

$$= \left[2(4)^2 - \tfrac{1}{3}(4)^3 \right] - 0$$

$$= \left[32 - \frac{64}{3} \right] = \frac{32}{3} u^2$$

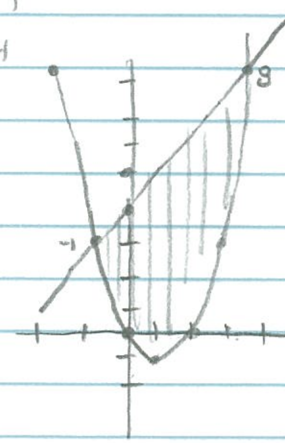
$y = y$

$4x = x^2$

$0 = x^2 - 4x$

$0 = x(x-4)$

$x=0 \quad x=4$



12. $y = x^2 - 2x$, $\quad y = x+4$

$$\int_{-1}^{4} \left[(x+4) - (x^2-2x) \right] dx$$

$$= \int_{-1}^{4} \left[-x^2 + 3x + 4 \right] dx$$

$$-\frac{1}{3}x^3 + \frac{3}{2}x^2 + 4x \Big|_{-1}^{4}$$

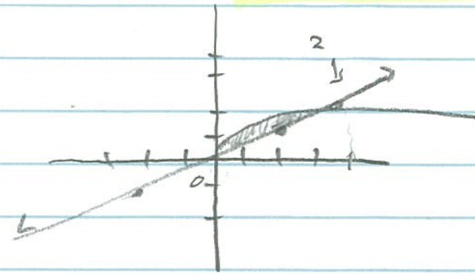
$$= \left[-\tfrac{1}{3}(4)^3 + \tfrac{3}{2}(4)^2 + 4(4) \right] - \left[-\tfrac{1}{3}(-1)^3 + \tfrac{3}{2}(-1)^2 + 4(-1) \right]$$

$$\left[-\frac{64}{3} + 24 + 16 \right] - \left[\frac{1}{3} + \frac{3}{2} - 4 \right]$$

$$\left[\frac{56}{3} \right] - \left[\frac{-13}{6} \right] = \frac{125}{6} u^2$$

9.3 - Continued

13. $y = \sqrt{x}$, $y = \frac{1}{2}x$



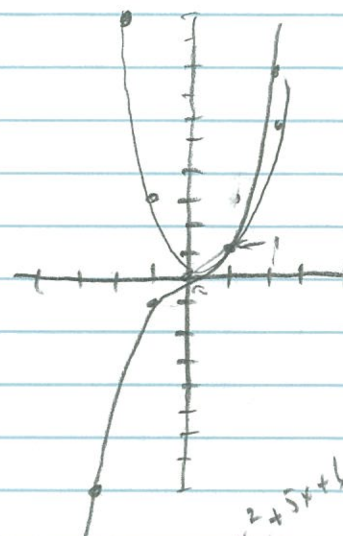
$$\int_0^4 \left(\sqrt{x} - \frac{1}{2}x\right)$$

$$= \left[\frac{2}{3}x^{3/2} - \frac{1}{4}x^2\right]_0^4$$

$$= \left(\frac{2}{3}(4)^{3/2} - \frac{1}{4}(4)^2\right) - (0)$$

$$= \frac{16}{3} - 4 = \frac{4}{3}u^2$$

14. $y = x^3$, $y = 2x^2 - x$

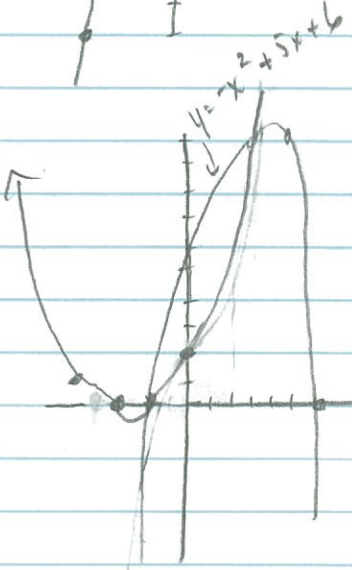


high low

$$\int_0^1 \left[x^3 - (2x^2 - x)\right]$$

$$\left[\frac{1}{4}x^4 - \frac{2}{3}x^3 + \frac{1}{2}x^2\right]_0^1$$

$$= \frac{1}{4}(1)^4 - \frac{2}{3}(1)^3 + \frac{1}{2}(1)^2 = \frac{1}{12}u^2$$



15. $y = x^2 + 3x + 2$, $y = -x^2 + 5x + 6$

$$\int_{-1}^{2} \left[(-x^2 + 5x + 6) - (x^2 + 3x + 2)\right] dx$$

$$\int_{-1}^{2} -2x^2 + 2x + 4 \; dx$$

$$= -\frac{2}{3}x^3 + x^2 + 4x \Big|_{-1}^{2}$$

$$= \left[-\frac{2}{3}(2)^3 + (2)^2 + 4(2)\right] - \left[-\frac{2}{3}(-1)^3 + (-1)^2 + 4(-1)\right]$$

$$\left(\frac{20}{3}\right) - \left(\frac{-7}{3}\right) = 9u^2$$