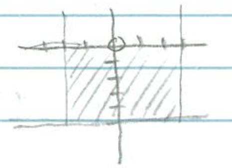
9.2 Area Above a Curve   P.381 1-21

1. $f(x) = -5, \quad x=-2, \; x=3$

$\int_{-2}^{3} -5 \, dx$



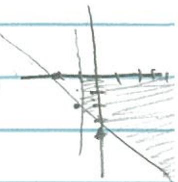
$= -5x \Big|_{-2}^{3}$

$= \left| -5(3) - (-5(-2)) \right|$

$= |-15 - 10| = |-25| = 25u^2$

2. $f(x) = -2x-4, \quad x=-1, \; x=$

$\int_{-1}^{6} (-2x-4)\, dx$



$= \left| -1x^2 - 4x \right|_{-1}^{6}$

$= \left[-1(36) - 4(6) \right] - \left[-1(-1)^2 - 4(-1) \right]$

$= [-36 - 24] - [-1 + 4]$

$= |-60 - 3| = 63u^2$

3. $f(x) = x^2 - 9, \quad x=1, \; x=3$

$\int_{1}^{3} (x^2 - 9)\, dx$

$= \frac{1}{3}x^3 - 9x \Big|_{1}^{3}$

$= \left(\frac{1}{3}(3)^3 - 9(3) \right) - \left(\frac{1}{3}(1)^3 - 9(1) \right)$

$= \left| (9 - 27) - (\frac{1}{3} - 9) \right|$

$= \left| (-18) - (\frac{-26}{3}) \right| = \frac{28}{3} u^2$

4. $f(x) = 2x - x^2. \quad x=2, x=3$

$\int_{2}^{3} (2x - x^2)\, dx$

$= x^2 - \frac{1}{3}x^3 \Big|_{2}^{3}$

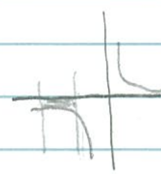
$= \left| \left[3^2 - \frac{1}{3}(3)^3 \right] - \left[2^2 - \frac{1}{3}(2)^3 \right] \right|$

$= \left| [9 - 9] - [4 - \frac{8}{3}] \right|$

$= \frac{4}{3} u^2$

5. $f(x) = \frac{1}{x}, \quad x=-4, \; x=-2$

$\int_{-2}^{-4} \frac{1}{x}\, dx$



$= \ln|x| \Big|_{-2}^{-4}$

$= \ln(4) - \ln(2)$

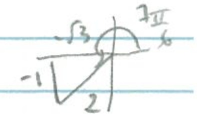
$\ln(2)$

6. $f(x) = \cos x \quad x = \frac{\pi}{2}, \; x = \frac{7\pi}{6}$

$\int_{\frac{\pi}{2}}^{\frac{7\pi}{6}} \cos x \, dx$

$= \sin x \Big|_{\frac{\pi}{2}}^{\frac{7\pi}{6}}$

$= \left| \sin \frac{7\pi}{6} - \sin \frac{\pi}{2} \right|$

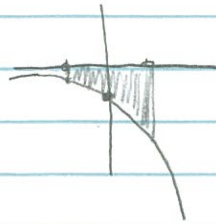


$= \left| -\frac{1}{2} - 1 \right| = \frac{3}{2} u^2$

9.2 - Continued

7. $f(x) = -e^{2x}$, $x=-1$, $x=1$

$\int_{-1}^{1} -e^{2x}\,dx$



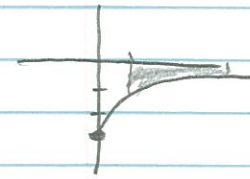
$= -\frac{1}{2} e^{2x} \Big|_{-1}^{1}$

$= \left[-\frac{1}{2}e^{2} - \left(-\frac{1}{2}e^{-2}\right)\right]$

$\frac{e^2 - e^{-2}}{2} \Rightarrow$ $= \frac{e^2 + e^{-2}}{2}$ ← -ve of this answer

8. $f(x) = \sqrt{x} - 3$, $x=1$, $x=4$

$\int_{1}^{4} \sqrt{x} - 3$



$= \frac{2}{3}x^{3/2} - 3x \Big|_{1}^{4}$

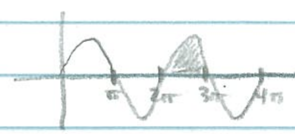
$= \left|\left(\frac{2}{3}(4)^{3/2} - 3(4)\right) - \left(\frac{2}{3}(1)^{3/2} - 3(1)\right)\right|$

$= \left|\left(\frac{16}{3} - 12\right) - \left(\frac{2}{3} - 3\right)\right|$

$= \left|\left(\frac{-20}{3}\right) - \left(\frac{-7}{3}\right)\right| = \frac{13}{3}u^2$

9. $f(x) = 3\sin\frac{1}{2}x$, $x = 2\pi$, $x = 3\pi$

$\int_{2\pi}^{3\pi} 3\sin\frac{1}{2}x\,dx$



let $u = \frac{1}{2}x$
$du = \frac{1}{2}dx$

$= -2(3)\cos\frac{1}{2}x \Big|_{2\pi}^{3\pi}$

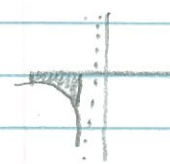
$= \left|\left(-6\cos\frac{1}{2}(3\pi)\right) - \left(-6\cos\frac{1}{2}(2\pi)\right)\right|$

$= \left|(-6(0)) - (-6(-1))\right|$

$= 6u^2$

10. $f(x) = \frac{4}{2x+1}$, $x=-4$, $x=-$

$\int_{-1}^{-4} 4(2x+1)^{-1}dx$



let $u = 2x+1$
$du = 2dx$

$= 2\ln|2x+1| \Big|_{-1}^{-4}$

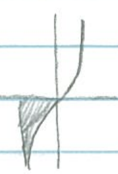
$= 2\left[\ln|(2(4)+1)|\right] - \left[\ln|(2(-1)+1)|\right]$

$= 2(\ln(7) - \ln(1))$

$= 2\ln 7$

11. $f(x) = x(x^2+1)^4$ $x=-2$, $x=0$

$\frac{1}{2}\int_{-2}^{0} (x^2+1)^4\, 2x\,dx$



$= \frac{1}{2}\left(\frac{1}{5}\right)(x^2+1)^5 \Big|_{-2}^{0}$

$= \left|\frac{1}{10}\left[((0)^2+1)^5 - ((-2)^2+1)^5\right]\right|$

$= \left|\frac{1}{10}[1 - 3125]\right|$

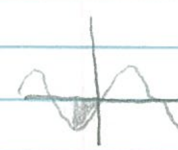
$= \frac{1562}{5}u^2$

12. $f(x) = e^{\cos x}\sin x$, $x = -\pi$, $x = 0$

$(-1)\int_{-\pi}^{0} e^{\cos x}(-1)\sin x\,dx$

let $u = \cos x$
$du = -\sin x\,dx$

$= (-1)e^{\cos x}\Big|_{-\pi}^{0}$



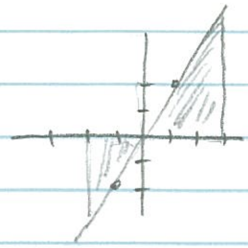
$= \left|-1\left[e^{\cos 0} - e^{\cos(-\pi)}\right]\right|$

$= \left|-1\left[e^{1} - e^{-1}\right]\right|$

$= e^{1} - e^{-1}\, u^2$

9.2- Continued

13. $f(x) = 2x, \quad x=-2, \ x=3$



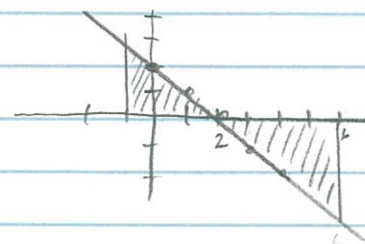
$$\int_{-2}^{3} 2x \, dx$$

$$= x^2 \Big|_0^3 + \left| x^2 \Big|_{-2}^0 \right|$$

$$= (3^2 - 0) + \left| 0^2 - (-2)^2 \right|$$

$$= 9 + 4 = 13 u^2$$

14. $f(x) = 2-x, \quad x=-1, \ x=6$



$$\int_{-1}^{6} (2-x) \, dx$$

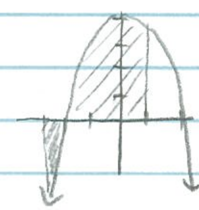
$$= \left(2x - \tfrac{1}{2}x^2\right)\Big|_{-1}^{2} + \left| \left(2x - \tfrac{1}{2}x^2\right)\Big|_{2}^{6} \right|$$

$$= \left[\left(2(2) - \tfrac{1}{2}(2)^2\right) - \left(2(-1) - \tfrac{1}{2}(-1)^2\right)\right] + \left[\left(2(6) - \tfrac{1}{2}(6)^2\right) - \left(2(2) - \tfrac{1}{2}(2)^2\right)\right]$$

$$= \left[(2) - \left(-\tfrac{5}{2}\right)\right] + \left|\left[(-6) - (2)\right]\right|$$

$$= \frac{9}{2} + 8 = \frac{25}{2} u^2$$

15. $f(x) = 4-x^2, \quad x=-3, \ x=1$



$$\int_{-3}^{1} (4-x^2) \, dx$$

$$= \left| \left(4x - \tfrac{1}{3}x^3\right)\Big|_{-3}^{-2} \right| + \left(4x - \tfrac{1}{3}x^3\right)\Big|_{-2}^{1}$$

$$= \left| \left[\left(4(-2) - \tfrac{1}{3}(-2)^3\right) - \left(4(-3) - \tfrac{1}{3}(-3)^3\right)\right] \right| + \left[\left(4(1) - \tfrac{1}{3}(1)^3\right) - \left(4(-2) - \tfrac{1}{3}(-2)^3\right)\right]$$

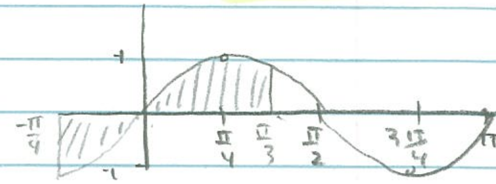
$$= \left| \left[\left(-8 + \tfrac{8}{3}\right) - (-12 + 9)\right] \right| + \left[\left(4 - \tfrac{1}{3}\right) - \left(-8 + \tfrac{8}{3}\right)\right]$$

$$= \left| \left[-\tfrac{7}{3}\right] \right| + [9]$$

$$= \frac{34}{3} u^2$$

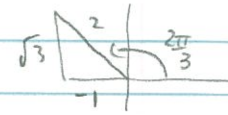
9.2 Continued

16. $f(x) = \sin 2x, \quad x = \frac{-\pi}{4}, \; x = \frac{\pi}{3}$



$$\int_{-\frac{\pi}{4}}^{\frac{\pi}{3}} \sin 2x \, dx \qquad \text{let } u = 2x \quad du = 2dx$$

$$= \left| -\tfrac{1}{2}(\cos 2x)\Big|_{-\frac{\pi}{4}}^{0} \right| + -\tfrac{1}{2}(\cos 2x)\Big|_{0}^{\frac{\pi}{3}}$$

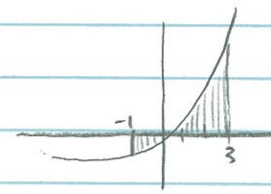


$$= \left| -\tfrac{1}{2}\left[(\cos 2(0)) - \cos 2\left(\tfrac{-\pi}{4}\right)\right] \right| + \left[-\tfrac{1}{2}\left(\cos 2\left(\tfrac{\pi}{3}\right) - \cos 2(0)\right)\right]$$

$$= \left| \tfrac{-1}{2}(1-0) \right| + -\tfrac{1}{2}\left(-\tfrac{1}{2} - 1\right)$$

$$= \tfrac{1}{2} + \tfrac{3}{4} = \frac{5}{4} u^2$$

17. $f(x) = e^x - 1, \quad x = -1, \; x = 3$



$$\int_{-1}^{3} (e^x - 1)\, dx$$

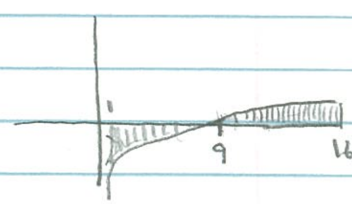
$$= \left| (e^x - x)\Big|_{-1}^{0} \right| + (e^x - x)\Big|_{0}^{3}$$

$$= \left| \left[(e^0 - 0) - (e^{-1} + 1)\right] \right| + \left[(e^3 - 3) - (e^0 - 0)\right]$$

$$= \left| \left[1 - 0 - e^{-1} - 1\right] \right| + \left[e^3 - 3 - 1\right]$$

$$= (e^{-1} + e^3 - 4)\, u^2$$

18. $f(x) = \sqrt{x} - 3, \quad x = 1, \; x = 16$



$$\int_{1}^{16} (\sqrt{x} - 3)\, dx$$

$$= \left| \tfrac{2}{3}x^{3/2} - 3x \Big|_{1}^{9} \right| + \tfrac{2}{3}x^{3/2} - 3x \Big|_{9}^{16}$$

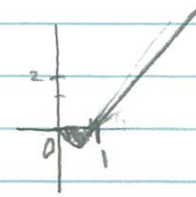
$$= \left| \left[\left(\tfrac{2}{3}(9)^{3/2} - 3(9)\right) - \left(\tfrac{2}{3}(1)^{3/2} - 3(1)\right)\right] \right| + \left[\left(\tfrac{2}{3}(16)^{3/2} - 3(16)\right) - \left(\tfrac{2}{3}(9)^{3/2} - 3(9)\right)\right]$$

$$\left| \left[(-9) - \left(\tfrac{-7}{3}\right)\right] \right| + \left[\left(-\tfrac{16}{3}\right) - (-9)\right]$$

$$20/3 + 11/3 = \boxed{31/3\, u^2}$$

9.2 - continued

19. $f(x) = x - \sqrt{x}$

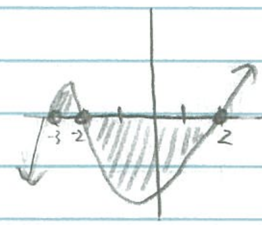


$$\left| \int_0^1 x - \sqrt{x} \; dx \right|$$

$$= \left| \frac{1}{2}(x)^2 - \left(\frac{2}{3}\right) x^{3/2} \Big|_0^1 \right|$$

$$= \left| \left(\frac{1}{2}(1)^2 - \frac{2}{3}(1)^{3/2} \right) - 0 \right|$$

$$= \left| -\left(\frac{1}{2} - \frac{2}{3} \right) \right| = \frac{1}{6} u^2$$



20. $f(x) = x^3 + 3x^2 - 4x - 12$

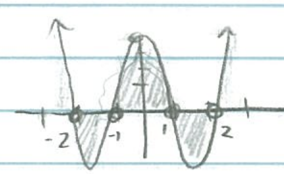
$$\int (x^3 + 3x^2 - 4x - 12) \, dx$$

$$= \left(\frac{1}{4}x^4 + x^3 - 2x^2 - 12x \right) \Big|_{-2}^{-3} + \left| \frac{1}{4}x^4 + x^3 - 2x^2 - 12x \Big|_{-2}^{2} \right|$$

$$= \left[(12) - \left(\frac{45}{4} \right) \right] + \left| (-20) - (12) \right|$$ (plug into calculator)

$$= \left(\frac{3}{4} \right) + (32) = \frac{131}{4} u^2$$

21. $f(x) = x^4 - 5x^2 + 4$



$$\int (x^4 - 5x^2 + 4) \, dx$$

$$= \left| \frac{1}{5}x^5 - \frac{5}{3}x^3 + 4x \Big|_{-2}^{-1} \right| + \left(\frac{1}{5}x^5 - \frac{5}{3}x^3 + 4x \right) \Big|_{-1}^{1} + \left| \left(\frac{1}{5}x^5 - \frac{5}{3}x^3 + 4x \right) \Big|_{1}^{2} \right|$$

$$\left| \left(\frac{-38}{15} \right) - \left(\frac{-16}{15} \right) \right| + \left(\frac{38}{15} - \left(\frac{-38}{15} \right) \right) + \left| \left(\frac{16}{15} \right) - \left(\frac{38}{15} \right) \right|$$

$$\frac{22}{15} + \frac{76}{15} + \frac{22}{15} = 8u^2$$