9.1 Area Under A Curve P. 379 1-12

1. $f(x) = x^2$, $x=1$, $x=3$

$$\int_1^3 (x^2)\,dx$$

$$= \frac{1}{3}x^3 \Big|_1^3$$

$$= \frac{1}{3}(3)^3 - \frac{1}{3}(1)^3$$

$$= 9 - \frac{1}{3} = \frac{26}{3}\,u^2$$

2. $f(x) = 2x+4$, $x=-1$, $x=1$

$$\int_{-1}^1 (2x+4)\,dx$$

$$= x^2 + 4x \Big|_{-1}^1$$

$$= (1^2 + 4(1)) - ((-1)^2 + 4(-1))$$

$$= 5 - (-3) = 8\,u^2$$

3. $f(x) = \frac{1}{x}$, $x=2$, $x=6$

$$\int_2^6 \left(\frac{1}{x}\right)dx$$

$$= \ln x \Big|_2^6$$

$$= \ln 6 - \ln 2$$

$$= \ln 3\,u^2$$

4. $f(x) = e^x$, $x=-2$, $x=2$

$$\int_{-2}^2 (e^x)\,dx$$

$$= e^x \Big|_{-2}^2$$

$$= e^2 - e^{-2}\,u^2$$

5. $f(x) = \sin x$, $x=0$, $x=\frac{\pi}{2}$

$$\int_0^{\frac{\pi}{2}} (\sin x)\,dx$$

$$(-\cos x) \Big|_0^{\frac{\pi}{2}}$$

$$= \left(-\cos\tfrac{\pi}{2}\right) - (-\cos 0)$$

$$= (-0) - (-1)$$

$$= 1\,u^2$$

6. $f(x) = \sqrt{x}$, $x=1$, $x=4$

$$\int_1^4 \sqrt{x}\,dx$$

$$= \frac{2}{3}x^{3/2} \Big|_1^4$$

$$= \left(\frac{2}{3}(4)^{3/2}\right) - \left(\frac{2}{3}(1)^{3/2}\right)$$

$$= \frac{16}{3} - \frac{2}{3} = \frac{14}{3}\,u^2$$

7. $f(x) = x^{-3}$, $x=\frac{1}{2}$, $x=1$

$$\int_{1/2}^1 (x^{-3})\,dx$$

$$-\frac{1}{2}x^{-2} \Big|_{1/2}^1$$

$$= \left(-\frac{1}{2}(1)^{-2}\right) - \left(-\frac{1}{2}\left(\frac{1}{2}\right)^{-2}\right) = \left(-\frac{1}{2}\right) - (-2) = \frac{3}{2}\,u^2$$

9.1 - Continued

8. $f(x) = \cos 2x, \quad x=0, \ x=\frac{\pi}{4}$

$$\int_0^{\frac{\pi}{4}} \cos 2x \, dx \qquad \text{Let } u = 2x, \quad du = 2dx$$

$$= \frac{1}{2}\int_0^{\frac{\pi}{4}} \cos 2x \, 2dx$$

$$= \frac{1}{2}\left(\sin 2x\right)\Big|_0^{\frac{\pi}{4}}$$

$$= \left(\frac{1}{2}\sin 2\left(\frac{\pi}{4}\right)\right) - \left(\frac{1}{2}\sin 2(0)\right)$$

$$= \left(\frac{1}{2}\right) - (0) = \boxed{\frac{1}{2}u^2}$$

9. $f(x) = x\sqrt{25-x^2}, \quad x=0, \ x=$

$$\int_0^5 x\,(25-x^2)^{1/2} dx \qquad \text{Let } u = 25-x, \quad du = -2x\,d,$$

$$= -\frac{1}{2}\int_0^5 (25-x^2)^{1/2} -2x\,dx$$

$$= -\frac{1}{2}\left(\frac{2}{3}\right)\left(25-x^2\right)^{3/2}\Big|_0^5$$

$$= \left[-\frac{1}{3}\left(25-(5)^2\right)^{3/2}\right] - \left[-\frac{1}{3}\left(25-0\right)^{3/2}\right]$$

$$= [0] - \left[-\frac{1}{3}(125)\right] = \boxed{\frac{125}{3}u^2}$$

10. $f(x) = xe^{x^2}, \quad x=0, \ x=2$

$$\int_0^2 (xe^{x^2})dx \qquad \text{Let } u = x^2, \quad du = 2x\,dx$$

$$= \frac{1}{2}\int_0^2 e^{x^2} 2x\,dx$$

$$= \frac{1}{2}\left(e^{x^2}\right)\Big|_0^2$$

$$= \frac{1}{2}e^4 - \frac{1}{2}e^0$$

$$= \boxed{\frac{e^4-1}{2}\,u^2}$$

11. $f(x) = \dfrac{20x}{x^2+1}, \quad x=1, \ x=3$

$$= 10\int_1^3 (x^2+1)^{-1} 2x\,dx \qquad \text{Let } u = x^2+1, \quad du = 2x\,dx$$

$$= 10\left[\ln|x^2+1|\Big|_1^3\right]$$

$$= 10\left[\ln(3^2+1) - \ln(1^2+1)\right]$$

$$= 10\left[\ln 10 - \ln(2)\right]$$

$$= \boxed{10\left[\ln 5\right]u^2}$$

12. $f(x) = \cos^2 x \sin x + 1, \quad x=0, \ x=2\pi$

$$\int_0^{2\pi} (\cos^2 x \sin x + 1)\,dx \qquad \text{Let } u = \cos x \quad du = -\sin x\,dx$$

$$= -1\left(\frac{1}{3}\cos^3 x\right) + x\Big|_0^{2\pi}$$

$$= \left(-\frac{1}{3}(\cos 2\pi)^3 + 2\pi\right) - \left(-\frac{1}{3}(\cos 0)^3 + 0\right)$$

$$= \left(-\frac{1}{3} + 2\pi\right) - \left(-\frac{1}{3}\right)$$

$$= \boxed{2\pi\, u^2}$$